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A Scorecard to Detect Financial Leverage Profitability

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Abstract

This study investigates how balancing internal and external financing sources can create economic value. We set a financial scorecard, consisting of the Cost of Debt (COD), Return on Investment (ROI), and the Cost of Equity (COE). We show that COE should be a cap for COD and a floor for ROI in order to increase the Net Present Value at Weighted Average Cost of Capital and the Adjusted Present Value of the levered investment. However, leverage should be carefully monitored if COD and ROI go off the grid. Situations where leverage has the opposite effect on value creation and the Equity Internal Rate of Return are also discussed. Illustrative examples are given. The proposed model aims to help corporate management in financial decisions.

Keywords: capital structure, financial leverage, profitability

1. Introduction

A long-standing question in corporate management is how to balance internal and external financing sources in levered industrial investments in order to increase creation of added value (see Brealey et al., 2016 among others).

By setting up a Key Perfomance Index (KPI) dashboard consisting of the Cost of Debt (COD), the unlevered Cost of Equity (COE) and Return on Investment (ROI), we formalize the intuitive condition that debt cost should be sufficiently cheap and investment should be sufficiently profitable. We show that COE shoud be a cap for COD and a floor for ROI. If KPIs go off this grid, external funding sources require careful monitoring. In fact, leverage may destroy added value, which is measured by the Net Present Value at Weighted Average Cost of Capital (NPV at WACC) and the Adjusted Present Value (APV).

Secondly, we discuss the impact of leverage on Equity Internal Rate of Return (Equity IRR). We show that situations may exist where leverage pushes Equity IRR up and destroys economic value at the same time. These findings are illustrated by didactic examples.

In light of this evidence, this study proposes a test to signal project financial leverage profitability, able to support the Chief Financial Officers in making fast decisions.

This paper is organized as follows. Section 2 describes the framework and collects basic notations. In Section 3 sufficient and necessary conditions for leverage to generate positive and increasing value are examined. Linkages with Equity IRR are also discussed. In Section 4 the findings are illustrated using didactic examples. Section 5 concludes.

2. The Framework

Financing is an essential part of operating any business, in fact a firm's potential for growth is limited without adequate access to financing (Rahaman, 2011).

By using the terminology (Note 1) introduced by Lutz and Lutz (1951), we restrict our analysis to PICO projects characterized by a unique cash outflow (point input) at $t_0 = 0$ and providing distributed cash in-flows (continuous outputs) at future dates. The investment will be supported with a POCI loan, providing a single cash inflow (point output) at $t_0 = 0$ requiring a number of future repayments for principal and interests (continuous output).

Here is the notation that will be used throughout the document. Let's consider an economic agent (i.e. a firm) facing the opportunity of investing in an industrial project A that promises at time t_s, with $s = 0,...,n$, free operating cash-flow a_s, with the usual convention that $a_s < 0$ means that at time t_s there is a money outflow, while $a_s > 0$ a money inflow and $a_s = 0$ no cash movement.

For simplicity but without loss of generality, we can assume that:

- a single project generates *free operating cash-flow* a_s at times t_s where $s = 0,...,n$;
- initial project outlay $a_0 = -1$;
- initial time $t_0 = 0$;
- initial unitary borrowing $f_0 = 1$ at time $t_0 = 0$ asks for payments $f_s \le 0$ at subsequent epochs t_s where $s = 1,..,n$.

At the beginning, the project is $\alpha \cdot 100\%$ debt financed and $(1-\alpha)\cdot 100\%$ equity financed with $0 \le \alpha \le 1$. If $\alpha = 0$ the project is all-equity financed; whereas if $\alpha = 1$ it is all-debt financed.

The *debt financing stream* reads $[\alpha, \alpha f_1, ..., \alpha f_n]$. The *equity financing* at initial time $t_0 = 0$ is $e_0 = a_0 + \alpha f_0 = -1 + \alpha = -(1-\alpha)$.

The *equity cash flow* (*ECF*) generated by the project at time t_s is given by the difference between free operating inflow and the debt repayment outflow

$$e_s = a_s + \alpha f_s \text{ , for } s = 1,...,n.$$

And we can assume that $e_s \ge 0$, so that no further capital is required during the project's life-time (Note 2). For all-equity financed projects, the most conceptually best analysis tool from the stockholders' perspective is the *Net Present Value* (*NPV*) method (see Brealey et al., 2016 among others).

The created economic value of an all-equity financed project *A* corresponding to $\alpha = 0$, is calculated by discounting free operating project cash-flow *A*, given by

$$NPV_A(i) = \sum_{s=0}^{n} a_s (1+i)^{-t_s}$$

where *i* is the *discount rate*. This definition highlights the crucial role played by the discount rate in profitability valuation. The proper discount rate is *COE*, meant as shareholders' required rate of return on an equity investment for the period from 0 to t_n.

3. Levered Investment Valuation

NPV has to be corrected for debt reimbursement costs if the project is partially supported by external funding. The prevailing methods are based on calculating *NPV* at *WACC*, labelled *WACC* method and calculating *NPV* of net equity cash-flow, labelled *Adjusted Present Value* (*APV*) method. Here we demonstrate that the profitability tracking rule is just the same for both the methods. Let's consider the following KPIs:

- *ROI* defined as Internal Rate of Return (*IRR*) of the free operating project cash-flow A; and
- *COD* defined as *IRR* of the debt cash-flow *D*.

ROI and *COD* exist and are unique for PICO projects and POCI financing.

3.1 WACC and APV Methods

3.1.1 WACC method

The most common method used by executives is *NPV* at *WACC* (see for example Copeland et. al. 1996). $WACC_\alpha$ rate for a $\alpha \cdot 100\%$ debt financed project is defined as

$$WACC_\alpha = (1-\alpha)\cdot COE + \alpha \cdot COD \quad \text{where } 0 \le \alpha \le 1.$$

That can be rewritten as

$$WACC_\alpha = COE + \alpha \cdot (COD - COE)$$

So, added value is just given by

$$NPV_A(WACC_\alpha) = \sum_{s=0}^{n} a_s \cdot \left(1 + COE + \alpha \cdot (COD - COE)\right)^{-t_s}. \quad (1)$$

The following is true for PICO projects:

- *For cheap loan condition*, i.e. $COD < COE$, discounting factors in (1) increase in leverage α. It means that $NPV_A(WACC_\alpha)$ value increases in leverage α, so debt should be taken at the *maximum value* permitted;

- *For expensive loan condition,* i.e. $COD > COE$, discounting factors in (1) decrease in leverage α . It follows that $NPV_A(WACC_\alpha)$ value decreases in leverage α , so debt should be limited to the *minimum value* necessary.

If $COD = COE$, leverage α has no impact on $NPV_A(WACC_\alpha)$ value. $NPV_A(WACC_\alpha)$ is equal to the $NPV_A(COE)$ generated by all-equity financed project.

Discounting free operating cash-flow a_s with $s = 1,..,n$, at $WACC_\alpha$ implicitly assumes that the levered investment maintains the debt percentage α invariant over time. However, severe distortions in valuation of the present value may occur if capital structure changes.

3.1.2 APV method

Following the seminal ideas of Myers (1974) the *APV* approach has been independently formalized and extended by Grubbstrom et al. (1991) and Peccati (1989) (see Myers, 2015). *APV* is defined as *NPV* of *net equity cash-flow (ECF)* at the discount rate *i*:

$$APV_{A+\alpha D}(i) = \sum_{s=0}(a_s + \alpha f_s)(1+i)^{-t_s} = \sum_{s=0} a_s(1+i)^{-t_s} + \alpha \sum_{s=0} f_s(1+i)^{-t_s}$$

with $0 \le \alpha \le 1$. By re-writing the above formula we get

$$APV_{A+\alpha D}(i) = NPV_A(i) + \alpha NPV_D(i) \tag{2}$$

where $NPV_A(i)$ is the net present value of free operating project cash-flow *A* at the discount rate *i* if the project is all-equity financed; and $NPV_D(i)$ stands for the net present value of a unitary debt cash stream *D* at the discount rate *i*.

Since formula (2) involves the net equity stream, the appropriate discount rate is *COE*. Discounting at *WACC* rate would be conceptually incorrect because the debt cost is already incorporated in free operating project cash-flow (see Krüger et al., 2015). From here on, we will use the short notation $NPV_D = NPV_D(COE)$ and $APV_{A+\alpha D} = APV_{A+\alpha D}(COE)$. We can prove (see Appendix A) that this is valid for PICO projects:

- *For cheap loan condition*, i.e. $COD < COE$, NPV_D is positive. Debt creates positive value and leverage should be taken at the *maximum value* permitted;
- *For expensive loan condition,* i.e. $COD > COE$, NPV_D is negative. Debt destroys value and leverage should be limited to the *minimum value* necessary.

If $COD = COE$, NPV_D is null, so debt has no influence on value creation.

Now we are ready to set a cap to debt cost and a floor to investment return to guarantee that leverage produces positive and increasing economic value, according to both *WACC* and *APV* methods.

Result: *Let a PICO project with free operating cash-flow* a_s *,* $s = 0,1..,n$ *and POCI financing with debt cash-flow* f_s *at time* t_s*, with* $s = 0,1..,n$ *. Let the initial capital invested* $a_0 = 1$ *is* $\alpha \cdot 100\%$ *debt financed and* $(1-\alpha)\cdot 100\%$ *equity financed, with* $0 \le \alpha \le 1$*.* NPV_A *at* $WACC_\alpha$ *and* $APV_{A+\alpha D}$ *are both positive and increasing in leverage at any* α *, with* $0 \le \alpha \le 1$*, if and only if*

$$COD \le COE \quad and \quad ROI \ge COE \tag{3}$$

See the Appendix B for proof.

The double condition (3) simply formalizes the intuitive guideline that leverage creates positive and increasing value if *loan is cheap* and *project is profitable*. It is worthwhile noting that profitability is detected by the same balanced scoreboard track (3) even though the *WACC* and *APV* methods are grounded on different assumptions (see Cigola and Peccati, 2005).

External financing has to be handled with care if the double condition (3) is weakened. Specifically:

- *For cheap loan and unprofitable project*, i.e. $COD < COE$ and $ROI < COE$: leverage creates positive or null value and the project destroys value. These two opposite effects may partially compensate each other and final value may be either positive or negative;

- *For expensive loan and profitable project*, i.e. $COD > COE$ and $ROI \geq COE$: leverage destroys value and the project creates a positive or null value. Again, final value may turn positive or negative according to the leverage level.

3.2 Equity IRR Criterium

A popular KPI for gauging equity profitability is the return on equity defined as the interest rate earned by equity in one period. Recently, this KPI has been extended to multi-period investments (see Beal, 2000). $Equity\ IRR_\alpha$ is defined as the *IRR* of net equity cash flow of a $\alpha \cdot 100\%$ debt financed project. The double condition (3) implies $ROI > COD$, and that implies that leverage increases $Equity\ IRR_\alpha$ (see Farinelli et al., 2017). Then the double condition (3) implies that leverage increases NPV_A at *WACC, APV* and $Equity\ IRR_\alpha$. However, the simple condition $ROI > COD$ that guarantees that leverage increases $Equity\ IRR_\alpha$, is not sufficient to ensure the double condition (3). For expensive loans (i.e. $COD > COE$) leverage has an opposite effect on value creation and $Equity\ IRR_\alpha$. In fact, leverage increases $Equity\ IRR_\alpha$ thanks to the project profitability (i.e. $ROI > COD$), but at the same time, it decreases NPV_A at *WACC* and $APV_{A+\alpha D}$ due to expensive loans.

4. Numerical Illustrations

To ascertain the impact of leverage on NPV_A at *WACC*, $APV_{A+\alpha D}$ and $Equity\ IRR_\alpha$ we discuss a didactic case. Let the project *A* be structured as in Table 1. At the start, project *A* requires an outflow of €1000; and promises €600 a year later and €700 two years later.

Table 1. Project *A* cash flow

Time	0	Year 1	Year 2
Project *A*	-1000	+600	+700

The results $ROI = 18.88\%$. Let *A* be $\alpha \cdot 100\%$ debt financed and $(1-\alpha)\cdot 100\%$ equity financed. Debt is reimbursed in one year. Let compute *NPV* at *WACC* and *APV* as defined in (1) and (2), respectively. For $\alpha = 0$, the project is all-equity financed. It results $WACC_{\alpha=0} = COE$ and NPV_A at $WACC_{\alpha=0}$ is equal to $NPV_A(COE) = 112.70$.

Case I: Cheap loan and profitable project. Let $COD = 8\%$, $COE = 10\%$ and $ROI = 18.88\%$.

Double condition (3) applies, so both debt and project create positive value. It follows that the higher the leverage α, the higher positive NPV_A at $WACC_\alpha$ and positive $APV_{A+\alpha \cdot D}$ values, for any α. Since $COD < ROI$, the higher α, the higher $Equity\ IRR_\alpha$, as well. We can conclude that optimal leverage strategy is to debt financing *at maximum level*. That is illustrated in Table 1, where as α increases, positive NPV_A at $WACC_\alpha$, positive $APV_{A+\alpha D}$ and $Equity\ IRR_\alpha$ increase, as well.

Table 2. NPV_A at $WACC_\alpha$, $APV_{A+\alpha D}$ and $Equity\ IRR_\alpha$ increase in α, for any α

	$\alpha=10\%$	$\alpha=20\%$	$\alpha=30\%$	$\alpha=40\%$	$\alpha=50\%$
NPV_A at $WACC_\alpha$	115.73	118.78	121.87	124.97	128.10
$APV_{A+\alpha D}$	114.35	116.00	117.66	119.31	120.96
$Equity\ IRR_\alpha$	19.66%	20.57%	21.64%	22.92%	24.47%
	$\alpha=60\%$	$\alpha=70\%$	$\alpha=80\%$	$\alpha=90\%$	$\alpha=100\%$
NPV_A at $WACC_\alpha$	131.26	134.45	137.66	140.90	144.16
$APV_{A+\alpha D}$	122.61	124.27	125.92	127.57	129.23
$Equity\ IRR_\alpha$	26.42%	28.95%	32.387%	37.41%	45.83%

The model informs that optimal leverage is for $\alpha=100\%$. That means that *at initial time* equity should not be invested in the project *A*. Equity should be invested only a year latter when debt repayment asks for €1080. Since the project revenue is of only €600, the difference should be covered by an equity outflow of €480. Equity is rewarded by €700 a year after. That financing strategy makes $NPV_A\left(WACC_{\alpha=1}\right)=NPV_D\left(COD\right)=144.16$, $APV_{A+\alpha D}=129.23$ and $Equity\ IRR_{\alpha=1}=45.83\%$.

Table 3. NPV_A at $WACC_\alpha$, $APV_{A+\alpha D}$ and $Equity\ IRR_\alpha$ achieve maximum value if at $t=0$ project *A* is all-debt financed

Time	0	Year 1	Year 2
Project *A*	-1000	+600	+700
Debt $\alpha=100\%$	+1000	-1080	0
Equity	0	-480	+700

Case II: Cheap loan and unprofitable project. Let $COD=15\%$, $COE=20\%$ and $ROI=18.88\%$.

Due to cheap debt conditions (i.e. $COD<COE$) external financing creates value. However equity should not be invested in the project *A*, because project return is lower than the equity return (i.e. $ROI<COE$). The values NPV_A at $WACC_\alpha$ and $APV_{A+\alpha D}$ may be negative. Due to cheap debt conditions, value creation switches from negative to positive for sufficient high levels of external financing. NPV_A at $WACC_\alpha$ *is positive for* $\alpha\geq 30\%$; and $APV_{A+\alpha D}$ *is positive for* $\alpha\geq 40\%$, see Table 4. Leverage increases *Equity IRR* because $COD<ROI$. In conclusion, *Equity IRR* criterium goes hands in hands with *NPV* at *WACC* and *APV* criteria.

Table 4. NPV_A at $WACC_\alpha$, $APV_{A+\alpha D}$ and $Equity\ IRR_\alpha$ increase in α.

	$\alpha=10\%$	$\alpha=20\%$	$\alpha=30\%$	$\alpha=40\%$	$\alpha=50\%$
NPV_A at $WACC_\alpha$	-6.46	-1.25	4.07	9.49	15.03
$APV_{A+\alpha D}$	-8.10	-4.63	-1.16	2.31	5.79
$Equity\ IRR_\alpha$	19.16%	19.48%	19.86%	20.31%	20.85%

	$\alpha=60\%$	$\alpha=70\%$	$\alpha=80\%$	$\alpha=90\%$	$\alpha=100\%$
NPV_A at $WACC_\alpha$	20.67	26.42	32.29	38.28	44.38
$APV_{A+\alpha D}$	9.26	12.73	16.20	19.68	23.15
$Equity\ IRR_\alpha$	21.52%	22.36%	23.47%	25%	27.27%

Optimal leverage strategy consists in maximizing external financing. Maxiumum value creation is achieved if $\alpha=1$, i.e. the project *A* is entirely debt financed at $t=0$. A year after, the debt reimbursement of €1150 is paid back by the project revenue of €600 and by equity of €550, see Table 5. For $\alpha=1$, $Equity\ IRR_{\alpha=1}=27.27\%$ and $APV_{A+\alpha D}=23.1481$ reach their maximum values.

Table 5. NPV_A at $WACC_\alpha$, $APV_{A+\alpha D}$ and $Equity\ IRR_\alpha$ achieve their maximum values if at $t=0$ project is all-debt financed $\alpha = 100\%$

Time	0	Year 1	Year 2
Project A	-1000	+600	+700
Debt $\alpha = 100\%$	+1000	-1150	0
Equity	0	-550	+700

Case III: Expensive loan and profitable project. Let $COD = 15\%$, $COE = 10\%$ and $ROI = 18.88\%$.

Loan is expensive (i.e. $COD > COE$) consequently leverage destroys value. Project is profitable (i.e. $ROI > COE$) then creates value and all-equity financed project is the most profitable strategy. Because of $ROI > COD$, then $Equity\ IRR_\alpha$ increases with α (see Farinelli et al., 2017). In conclusion, an increase in leverage α moves NPV_A at $WACC_\alpha$ and $APV_{A+\alpha\cdot D}$ down and, at the mean time, $Equity\ IRR_\alpha$ up. So *WACC* and *APV* methods identify leverage strategies which are in conflict with *Equity IRR* criterium. In conclusion, debt should be: (1) limited to *the minimum necessary*, if you follow *WACC* and *APV* methods, but (2) it should be augumented at *the maximum level permitted*, if you follow *Equity IRR* criterium.

Table 6. NPV_A at $WACC_\alpha$ and $APV_{A+\alpha D}$ decrease in α, whereas $Equity\ IRR_\alpha$ increases in α, for any α

	$\alpha = 10\%$	$\alpha = 20\%$	$\alpha = 30\%$	$\alpha = 40\%$	$\alpha = 50\%$
NPV_A at $WACC_\alpha$	105.23	97.91	90.73	83.71	76.82
$APV_{A+\alpha D}$	108.56	104.43	100.30	96.171	92.04
$Equity\ IRR_\alpha$	19.16%	19.48%	19.867%	20.31%	20.85%

	$\alpha = 60\%$	$\alpha = 70\%$	$\alpha = 80\%$	$\alpha = 90\%$	$\alpha = 100\%$
NPV_A at $WACC_\alpha$	70.07	63.45	56.97	50.61	44.38
$APV_{A+\alpha D}$	87.90	83.77	79.64	75.51	71.37
$Equity\ IRR_\alpha$	21.52%	22.36%	23.47%	25%	27.27%

5. Conclusion

In this paper we have created a scorecard to fast track leverage effects on profitability. The profitability requirement that "debt should be fairly cheap and the project fairly profitable" is formalized by the double condition that *COE* should be a cap for *COD* and a floor for *ROI*.

Under these circumstances, leverage increases *NPV* at *WACC*, *APV* and *Equity IRR*. However, *Equity IRR* criterium may conflict with *WACC* and *APV* methods if this double condition is relaxed and the loan is expensive. Leverage may increase *Equity IRR* and bring down *NPV* at *WACC* and *APV* at the same time. Brief didactic examples illustrate the results.

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Appendix A

NPV_D has the same sign as *COD* and *COE* spread. For POCI loans with $f_0 = +1$ and $f_s \le 0$ for all $s = 1,..,n$. It follows that $NPV_D(i)$ is an increasing function in the discounting rate *i.* Since $NPV_D(COD) = 0$:

- Cheap loan condition $COD < COE$ makes $NPV_D = NPV_D(COE) > 0$;
- Expensive loan condition $COD > COE$ makes $NPV_D = NPV_D(COE) < 0$;

and for $COD = COE$, then $NPV_D(COE) = NPV_D(COD) = 0$. Q.E.D.

Appendix B

Assume cheap loan condition, i.e. $COD < COE$.

1) First consider the *WACC* method.

In Appendix A it has been shown that under cheap loan condition $COD < COE$, $NPV(WACC_\alpha)$ is increasing in leverage α. We should now set conditions to guarantee that $NPV(WACC_\alpha)$ is positive for any α, with $0 \le \alpha \le 1$. Let note that by definition of *ROI*, it results.

$$DCF_A(ROI) = -1 + \sum_s a_s (1 + ROI)^{-t_s} = 0$$

For PICO investments $a_s \ge 0$ for $s = 1,..,n$, $NPV(WACC_\alpha)$ is positive for any α, with $0 \le \alpha \le 1$ if and only if:

$$(1 + WACC_\alpha)^{-t_s} - (1 + ROI)^{-t_s} > 0 \quad \text{for } s = 1,..,n.$$

Then if

$$(1 + WACC_\alpha)^{-1} - (1 + ROI)^{-1} > 0,$$

it results $ROI > WACC_\alpha$ i.e.

$$ROI > (1-\alpha)\cdot COE + \alpha \cdot COD \quad \text{for all } 0 \le \alpha \le 1.$$

Since above holds for all $0 \le \alpha \le 1$ and $COD < COE$, then $ROI > COE$.

Due to the initial assumption $COD < COE$, the double condition (3) comes out.

2) Now consider the *APV* approach.

By definition (2), $APV_{A+\alpha D} = APV_{A+\alpha D}(COE)$ is a linear function in leverage α. Then, $APV_{A+\alpha D}$ is positive and increasing in leverage α, if and only if NPV_A and NPV_D are both positive. For PICO projects, $NPV_A(i)$ is a decreasing function of the discount rate *i* such that $NPV_A(ROI) = 0$. Then $NPV_A(COE) > 0$ if and only if

$ROI > COE$, so *COE* must be a floor for *ROI*. Analogously, $NPV_D(COE) > 0$ if and only if $COD < COE$, so *COE* must be a cap for *COD*. Q.E.D.

Notes

Note 1. Such terminology is constructed "from the viewpoint of the project". Consequently, when the project asks for money, we say that there is an input (in the project). When the project pays some positive cash-flow, we say that it produces an output.

Note 2. This condition characterizes the so called normal equity cash flow.