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Government Expenditure Financing, Growth, and Factor Intensity

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Abstract

By shedding light on the factor intensity, this paper incorporates the Romer (1986)-type knowledge spillover technology into the Uzawa (1961, 1963) two-sector model of consumption and investment goods and studies the effect of the ratio of government expenditure to total output on the economic growth rate under three types of tax financing schemes: lump-sum tax financing, income tax financing, and consumption tax financing. We find that a rise in government expenditure with lump-sum tax financing has an ambiguous effect on the balanced growth rate depending on the factor intensity between the sectors. The balanced growth rate decreases (increases) with a rise in government spending if the consumption (investment) goods sector is capital-intensive. Moreover, the result of consumption tax financing is equivalent to lump-sum tax financing, while an increase in the government expenditure with income tax financing reduces the balanced growth rate. Our two-sector model with lump-sum tax or consumption tax financing seems to be able to provide a channel through which to explain the mixed empirical findings.

Keywords: government expenditure financing, factor intensity, two-sector model, endogenous growth

JEL Classification: E62, H61, O41, O38

1. Introduction

The relationship between government expenditure and macroeconomic performances has been extensively explored within an intertemporal optimizing framework over the last two decades. One strand of the studiesput their emphases on the consequences of government expenditure policies under alternative financing approaches [Turnovsky (1992), Van der Ploeg and Alogoskoufis (1994), Devereux and Love (1995), Palivos and Yip (1995), Turnovsky (1996, 2000), Gokan (2002), and Chang et al. (2004), among others]. Depending on model specifications, the multiform methods used to finance a given government expenditure lead to different real effects on relevant economic variables. To enrich the studies in this field, this paper incorporates knowledge spillover technology of theRomer (1986)-type in the Uzawa (1961, 1963) two-sector model of consumption and investment goods to examine the roles of factor intensity and relative price in evaluating the effects of alternative government expenditure financing scheme.The study of this issue may be justified for three reasons.

First, following the remarkable contributions of Romer (1986) and Lucas (1988), numbers of studies have paid attention to the effects of different government expenditure financing schemes on the economy's growth rate where the steady-state growth rate is endogenously determined in the economy. Most of these studies adopt the specification of the one-sector AK technology [Van der Ploeg and Alogoskoufis (1994), Palivos and Yip (1995), Turnovsky (1996, 2000), and Gokan (2002)]. (Note 1). With a one-sector growth model and consumption tax financing, Turnovsky (1996) finds that an increase in government consumptionexpenditure does not affect the steady-state growth rate (p.31), and an increase in government investment expenditure improves the steady-state growth rate (p.39). Moreover, Turnovsky (2000) introduces an endogenous labor-leisure decision into a simple AK growth model and shows that an increase in either government consumption expenditure or government investment expenditure with lump-sum tax financing improves the steady-state growth rate. While the one-sector model has the virtue of tractability, it can notproperly capture the fact that the economy consists of a multi-sector environment, such as consumption goods and investment goods sectors. (Note 2)

Second, plenty of studies use the Uzawa (1965)-Lucas (1988) two-sector model with joint accumulation of physical capital and human capital to discuss the effect of tax policies in an endogenous growth framework [King and Rebelo (1990), Rebelo (1991), Jones et al. (1993), Devereux and Love (1994), Stokey and Rebelo

(1995), Bond et al. (1996), and Mino (1996)]. Obviously, the effect of different government expenditure financing schemes is less addressed. To our knowledge, Devereux and Love (1995) make a first attempt to examine the impact of government spending financed by either lump-sum or income tax on economic growth rate under a two-sector endogenous growth model with endogenous labor-leisure decision. Their conclusions suggest that a permanent increase in government expenditure improves the steady-state growth rate under lump-sum tax financing, but worsens the steady-state growth rate under income tax financing. By contrast, Chang et al. (2004) introduce the motive of status-seeking into the two-sector endogenous growth model and show that a negative relationship exists between government expenditure and the long-run growth rate under lump-sum tax financing. Although the studies have employed a two-sector model, the propositions they established does not embody the well-known feature in a two-sector model such as factor intensity between the sectors. As the empirical evidence shown in Jones (2003), the industry-level capital shares are quite discrepant in the United States. In addition, the capital shares in investment goods sectors are not necessarily greater than those in consumption goods sector (we summarize the empirical data of 1996 in Jones, 2003 in Table 1). Motivated by the empirical evidence, following Uzawa (1961, 1963), the present paper construct a two-sector model of consumption and investment goods in which both sectors use physical capital and labor as inputs in a Romer (1986)-type technology to highlight the important features of factor intensity and relative price between the sectors in evaluating the growth rate effect of government expenditure policy under alternative financing schemes.

Third, the empirical studies which emphasize the relationship between government expenditure and the economic growth rate do not enable us to reach a definite conclusion. Kormendi and Mequire (1985) use data for forty-seven countries and find that there is no significant cross-sectional relationship between average government consumption expenditure and the growth rate of real GDP. Recently, Miller and Russek (1997) divided the effects of government expenditure based on alternative financing modes and found that the relationship between government expenditure and economic growth is empirically mixed. From this perspective, the existing theoretical works clearly can not provide a general explanation for the diverse outcomes in the empirical studies. Thus, it is a worthwhile task for us to find a potential vehicle to explain the empirical evidence.

Based on the two-input two-good model, our findings can be summarized as follows. First, the growth effect of a rise in the government expenditure with lump-sum tax financing depends on the factor intensity between sectors. If the consumption (investment) goods sector is capital-intensive, then the steady-state growth rate declines (increases). This ambiguous result for the steady-state growth rate sharply contrasts with the results of Devereux and Love (1995), Turnovsky (2000) and Chang et al. (2004). Second, an increase in government expenditure with income tax financing leads to a deterioration in the steady-state growth rate. The finding is consistent with Devereux and Love (1995). Third, in contrast to Turnovsky (1996), an expansion in government spending with consumption tax financing has an ambiguous effect on the steady-state growth rate, which is equivalent to that with lump-sum tax financing. Furthermore, the theoretical outcomes with lump-sum tax or consumption tax financing are capable of providing a channel to explain the diverse empirical results.

The rationale for these results is as follows. An increase in the government expenditure through lump-sum tax or consumption tax financing gives rise to a resources-withdrawal effect which leads to a change in the relative price between goods and then induces a movement in production factors from one sector to another. With the Stolper-Samuelson theorem, an increase in the price of one sector's output leads to a more than proportional increase in the price of the factor used intensively in that sector. As a result, depending on the factor intensities of different sectors, a change in the relative price leads to an alteration in the capital share-labor ratio. The alteration in the capital share-labor ratio further affects the marginal productivity of capital which may be beneficial or harmful to economic growth. Therefore, the steady-state growth effect of government expenditure financed by a lump-sum or consumption tax hinges on the factor intensity between the sectors. With regard to the effects under the income tax financing scheme, there are two channels for the government expenditure to affect the economic growth. The first relates to the induced increase in the income tax rate and the second is due to the alteration in the relative price. Although the second channel has an ambiguous effect on the economic growth, the growth-retarding effect that arises from the first one always dominates the second one. Therefore, a rise in government spending with income tax financing retards the economic growth rate.

The reminder of the paper is organized as follows. In Section 2, we construct the perfect-foresight equilibrium in the Uzawa-Romer endogenous growth framework under a balanced government budget rule in which taxes adjust endogenously to finance a given government expenditure. Section 3 examines the effects of government expenditure on the steady-state relative price and the economy's balanced growth rate under alternative financing schemes. Finally, Section 4presents the main findings of our analysis and concludes.

2. The Analytical Framework

We establish a two-sector economy in which one sector produces investment goods and the other produces consumption goods. The economy consists of a continuum of infinitely-lived identical agents and a government. Agents have common preferences, optimize decisions on the basis of perfect foresight, and employ a production technology of Romer (1986)-type. The government collects taxes (a lump-sum tax, income tax, and consumption tax) to support its spending which is a fraction of total output. The details of this economy are described as follows.

2.1 The Optimization of Representative Agent

The objective of a representative agent is to maximize the discounted sum of future instantaneous utilities:

$$\int_0^\infty \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt \tag{1}$$

where c is consumption, $\sigma \geq 1$ is the inverse of the intertemporal elasticity of substitution, (Note 3) and $\rho \in (0,1)$ is a constant rate of time preference.

At each instant of time, the representative agent is bounded by two types of constraints. First, we assume that the time endowment of the representative agent is normalized to unity and l_I (l_C) is the labor time allocated to the investment (consumption) goods sector. Thus a time constraint can be expressed as:

$$l_I + l_C = 1 \tag{2}$$

Second, a flow constraint links capital accumulation to any difference between its gross income (the total output) and its gross expenditure (consumption, taxes, and capital depreciation). It can be described by:

$$\dot{k} = (1-\tau)(y_I + p y_C) - (1+\theta) pc - \delta k - T \tag{3}$$

where an overdot denotes the time derivative, k is the capital stock, y_I is the output of investment goods sector, y_C is the output of consumption goods sector, p is the relative price of consumption goods in terms of investment goods, τ is the income tax rate, θ is the rate of consumption tax, T is the lump-sum tax, and δ is the depreciation rate of capital. The production function of each sector in the economy is the Romer (1986)-type specification of the knowledge spillover effect which is specified as follows:

$$y_I = A_I (sk)^\alpha l_I^{1-\alpha} \bar{k}^{1-\alpha}, \tag{4a}$$

$$y_C = A_C [(1-s)k]^\beta l_C^{1-\beta} \bar{k}^{1-\beta}, \tag{4b}$$

where $A_I > 0$, $A_C > 0$, $0 < \alpha < 1$, $0 < \beta < 1$, $\bar{k}$ is the average economy-wide level of capital stock, s is the fraction of capital stock allocated to the investment goods sector. By using equations (2), (4a) and (4b), equation (3) can be rewritten as:

$$\dot{k} = (1-\tau)\{A_I (sk)^\alpha l_I^{1-\alpha} \bar{k}^{1-\alpha} + pA_C [(1-s)k]^\beta (1-l_I)^{1-\beta} \bar{k}^{1-\beta}\} - (1+\theta) pc - \delta k - T \tag{5}$$

The optimizing problem for the representative agent is to maximize equation (1) subject to equation (5) and the initial capital holdings $k_0 > 0$. Letting λ be the co-state variable associated with equation (5), the current-value Hamiltonian can be expressed as:

$$H = \frac{c^{1-\sigma}-1}{1-\sigma} + \lambda\{(1-\tau)[A_I s^\alpha k^\alpha l_I^{1-\alpha} \bar{k}^{1-\alpha} + pA_C (1-s)^\beta k^\beta (1-l_I)^{1-\beta} \bar{k}^{1-\beta}] - (1+\theta) pc - \delta k - T\}$$

The first-order conditions necessary for this optimizing problem are:

$$c^{-\sigma} = \lambda (1+\theta) p \tag{6a}$$

$$\alpha A_I s^{\alpha-1} l_I^{1-\alpha} k^{\alpha} \bar{k}^{1-\alpha} = p\beta A_C (1-s)^{\beta-1} (1-l_I)^{1-\beta} k^{\beta} \bar{k}^{1-\beta} \tag{6b}$$

$$(1-\alpha) A_I s^{\alpha} l_I^{-\alpha} k^{\alpha} \bar{k}^{1-\alpha} = p(1-\beta) A_C (1-s)^{\beta} (1-l_I)^{-\beta} k^{\beta} \bar{k}^{1-\beta} \tag{6c}$$

$$-\dot{\lambda} + \lambda\rho = \lambda\{(1-\tau)[\alpha A_I s^{\alpha} l_I^{1-\alpha} k^{\alpha-1} \bar{k}^{1-\alpha} + p\beta A_C (1-s)^{\beta} (1-l_I)^{1-\beta} k^{\beta-1} \bar{k}^{1-\beta}] - \delta\} \tag{6d}$$

together with equation (5) and the transversality condition of k :

$$\lim_{t\to\infty} \lambda k e^{-\rho t} = 0 \tag{6e}$$

Equation (6a) implies the equality between the marginal utility of consumption and the tax-adjusted shadow value of capital stock. Equations (6b) and (6c) respectively determine the optimal allocation of capital and labor between two sectors which requires that the marginal productivity of capital and labor in terms of investment goods should be equalized across sectors at each point of time. Equation (6d) is the Keynes-Ramsey rule governing the optimal choice between consumption and capital accumulation by equating the marginal return on consumption and the after-tax net rate of returns on capital.

Since all agents are assumed to be identical, in a symmetric equilibrium all agents own the same amount of capital, and hence $k = \bar{k}$ is true in equilibrium. Equations (4a), (4b), (5) and (6b)-(6d) are accordingly rewritten as:

$$y_I = A_I s^{\alpha} l_I^{1-\alpha} k, \tag{7a}$$

$$y_C = A_C (1-s)^{\beta} l_C^{1-\beta} k, \tag{7b}$$

$$\dot{k} = (1-\tau)\{A_I s^{\alpha} l_I^{1-\alpha} + pA_C (1-s)^{\beta} (1-l_I)^{1-\beta}\} k - (1+\theta) pc - \delta k - T \tag{7c}$$

$$\alpha A_I \left(\frac{s}{l_I}\right)^{\alpha-1} = p\beta A_C \left(\frac{1-s}{1-l_I}\right)^{\beta-1} \tag{7d}$$

$$(1-\alpha) A_I \left(\frac{s}{l_I}\right)^{\alpha} = p(1-\beta) A_C \left(\frac{1-s}{1-l_I}\right)^{\beta} \tag{7e}$$

$$-\dot{\lambda} + \lambda\rho = \lambda\{(1-\tau)[\alpha A_I s^{\alpha} l_I^{1-\alpha} + p\beta A_C (1-s)^{\beta} (1-l_T)^{1-\beta}] - \delta\} \tag{7f}$$

2.2 The Government

The government is assumed to impose taxes (a lump-sum tax, income tax, or consumption tax) to finance its expenditure and maintains a continually balanced budget at each point in time. Hence, the government's budget constraint can be expressed as:

$$G = \tau(y_I + py_C) + \theta pc + T \tag{8}$$

where G is government expenditure. In order to ensure sustained steady-state growth, following Devereux and Love (1995) and Palivos and Yip (1995), we specify government expenditure to be a fixed ratio of total output, i.e., $G = g(y_I + py_C)$, $0 < g < 1$. Equation (8) can thus be rewritten as:

$$(g-\tau)(y_I + py_C) = \theta pc + T \tag{9}$$

Furthermore, government purchases are assumed to be composed of investment goods and consumption goods, i.e., $G = G_I + pG_C$, where G_I (G_C) denotes government expenditure of investment (consumption) goods. Without loss of generality, both types of government expenditures are set as fixed ratios of the respective output and both ratios are set equally, that is: $G_I = gy_I$ and $G_C = gy_C$.

We then depict our policy experience as follows: throughout the paper, the government controls the expenditure ratio g as a policy instrument and lets the corresponding tax endogenously adjust to maintain the balanced government budget. To be more specific, following Palivos and Yip (1995), if the government implements a lump-sum tax financing scheme, the lump-sum tax T will adjust endogenously to finance the increase in government expenditure and maintain the balanced government budget while the income tax rate τ and consumption tax rate θ are set to be zero. As for the income tax financing scheme or the consumption tax financing scheme, the method of manipulation is similar to that in the lump-sum tax financing scheme.

2.3 The Equilibrium

We turn to derive the equilibrium dynamics of the economy. By combining equations (3) and (9) with equations (7a) and (7b), the consumption goods and investment goods markets are respectively in their equilibrium through the flexible adjustment of relative price. That is:

$$c = (1-g)A_C(1-s)^{\beta}(1-l_I)^{1-\beta}k \tag{10a}$$

$$\frac{\dot{k}}{k} = (1-g)A_I s^{\alpha} l_I^{1-\alpha} - \delta \tag{10b}$$

Next, from equations (7d) and (7e), we have:

$$\frac{s}{l_I} = v(p)\,,\ \ v_p = \frac{dv}{dp} = \frac{1}{(\alpha-\beta)p}v(p) \gtrless 0\,, \quad \text{if } \alpha \gtrless \beta \tag{11a}$$

$$s = s(p)\,,\ \ s_p = \frac{ds}{dp} = -\beta(1-\alpha)\frac{1}{(\alpha-\beta)^2 p}v(p) < 0 \tag{11b}$$

$$l_I = l_I(p)\,,\ \ l_{I p} = \frac{dl_I}{dp} = -\alpha(1-\beta)\frac{1}{v(p)(\alpha-\beta)^2 p} < 0 \tag{11c}$$

The rationale of equations (11a)-(11c) is as follows. When the relative price of consumption goods increases, the consumption (investment) goods sector will increase (decrease) its output and the production factors will move from the investment goods sector to the consumption goods sector. Hence both s and l_I will decrease. However, the effect of the relative price on the capital share-labor ratio ($\frac{s}{l}$)is ambiguous, which in turn is determined by factor intensity. (Note 4) As documented by the Stolper-Samuelson theorem, if the consumption goods sector is capital-intensive ($\alpha < \beta$), then the higher p makes capital to be more expensive relative to labor, thereby leading both sectors to use less capital and more labor. Thus the capital share-labor ratio in both sectors declines. By contrast, if the investment goods sector is capital-intensive ($\alpha > \beta$), then the higher p makes labor to be more expensive relative to capital and hence the capital share-labor ratio in both sectors rises.

Furthermore, by taking the log derivatives of equation (6a), the optimal change in consumption is expressed as:

$$\frac{\dot{c}}{c} = -\frac{1}{\sigma}\left[\frac{\dot{\lambda}}{\lambda} + \frac{\theta}{1+\theta}\frac{\dot{\theta}}{\theta} + \frac{\dot{p}}{p}\right] \tag{12}$$

Then, taking log derivatives of equation (10a) and substituting the resulting equation into equation (12), the growth rate of the relative price is obtained as:

$$\frac{\dot{p}}{p} = \phi\left[(-\frac{1}{\sigma})(\frac{\dot{\lambda}}{\lambda} + \frac{\theta}{1+\theta}\frac{\dot{\theta}}{\theta}) - \frac{\dot{k}}{k}\right] \tag{13}$$

where $\phi = \dfrac{\sigma}{1+\Omega\sigma} > 0$ and $\Omega = \dfrac{py_{Cp}}{y_C} = -[\beta \dfrac{s_p}{1-s(p)} + (1-\beta)\dfrac{l_{Ip}}{1-l_I(p)}]p > 0$. Equation (13) is a generalized equation which governs the movement of the relative price under alternative government expenditure financingschemes. Moreover, we will show that equation (13) is a one-dimensional differential equation of p under the corresponding financing scheme.

3. Long-Run Growth Effects of Alternative Financing Approaches

By using the information mentioned in the former section, the effect of government expenditure under alternative financing approaches on the steady-state relative price and growth rate is investigated as follows.

3.1 Lump-Sum Tax Financing

Under a lump-sum tax financing scheme ($\tau = \theta = 0$), the endogenous adjustment of T clearly has no direct effect on the optimal behavior of agents and the dynamics of the relative price. From this perspective, T can be recursively determined by equation (9). Consequently, the system can be expressed by equation (13) with $\dot{\theta} = 0$ as follows:

$$\frac{\dot{p}}{p} = \phi\ [(-\frac{1}{\sigma}\frac{\dot{\lambda}}{\lambda}) - \frac{\dot{k}}{k}] \tag{14}$$

Employing equations (7f) and (10b) with (7d) and (11a)-(11c), we have:

$$-\frac{1}{\sigma}\frac{\dot{\lambda}}{\lambda} = \frac{1}{\sigma}\{\alpha A_I[v(p)]^{\alpha-1} - \rho - \delta\} \equiv \Pi(p) \tag{15a}$$

$$\frac{\dot{k}}{k} = (1-g)A_I[s(p)]^{\alpha}[l_I(p)]^{1-\alpha} - \delta \equiv \Lambda(p,g) \tag{15b}$$

where $\Pi_p = \dfrac{\partial \Pi}{\partial p} = -\dfrac{1}{\sigma}\alpha(1-\alpha)A_I[v(p)]^{\alpha-2}v_p \gtrless 0$, if $\alpha \lessgtr \beta$, $\Lambda_p = \dfrac{\partial \Lambda}{\partial p} = (1-g)A_I[v(p)]^{\alpha-1}[\alpha s_p + (1-\alpha)v(p)l_{Ip}] < 0$, and $\Lambda_g = \dfrac{\partial \Lambda}{\partial g} = -A_I[s(p)]^{\alpha}[l_I(p)]^{1-\alpha} < 0$. Notice that, in equation (15a), $\alpha A_I[v(p)]^{\alpha-1}$ is the marginal productivity of capital, thus $-\dfrac{1}{\sigma}\dfrac{\dot{\lambda}}{\lambda}$ can be regarded as the demand for investment goods. Based on equation (11a), when the relative price of consumption goods increases, the capital share-labor ratio declines (increases) and thus the after-tax marginal productivity of capital improves (deteriorates) if $\alpha < \beta$ ($\alpha > \beta$). Accordingly, the demand for investment goods is an increasing (decreasing) function of p and the upward-sloping (downward-sloping) locus $-\dfrac{1}{\sigma}\dfrac{\dot{\lambda}}{\lambda}$ when $\alpha < \beta$ ($\alpha > \beta$) can be illustrated in Figure 1 (Figure 2). In addition, $\dfrac{\dot{k}}{k}$ in equation (15b) refers to the capital stock which can be used by agents and hence is regarded as the supply of investment goods. According to equations (11b) and (11c), an increase in p makes the production factors move from the investment goods sector to the consumption goods sector and thus decreases the output of investment goods. As a consequence, the supply of investment goods is an increasing function of p and the downward-sloping locus $\frac{\dot{k}}{k}$ can be illustrated in Figures 1 and 2. (Note 5).

Substituting equations (15a) and (15b) into equation (14), we have a one dimensional differential equation of p as follows:

$$\frac{\dot{p}}{p} = \phi\ [(\Pi(p) - \Lambda(p, g)] \tag{16}$$

Equation (16) implies that the adjustment of p depends on the relative strength between the demand for and the supply of investment goods. Given that p is a jump variable, the dynamic stability of the system claims that the characteristic root of equation (16) should be positive to ensure a unique perfect-foresight equilibrium path, i.e.

$$\Pi_p - \Lambda_p > 0 \tag{17}$$

where $\Lambda_p < 0$. When the consumption goods sector is capital-intensive ($\alpha < \beta$), $\Pi_p > 0$ is true and equation (17) is obviously satisfied (Figure 1). However, when the investment goods sector is capital-intensive ($\alpha > \beta$), Π_p and Λ_p are both also negative, and equation (17) claims that $\frac{\dot{k}}{k}$ must be steeper than $-\frac{1}{\sigma}\frac{\dot{\lambda}}{\lambda}$ (Figure 2).

In the balanced-growth equilibrium, the economy is characterized by $\dot{p} = 0$. Hence p is at the steady-state value, namely, $\overline{p}$, and c and k exhibit a common growth rate, $\overline{\gamma}$. By substituting $\dot{p} = 0$ into equation (16), $\overline{p}$ can be obtained as follows:

$$\Pi(\overline{p}) = \Lambda(\overline{p}, g) \tag{18}$$

To ensure the existence and uniqueness of the steady-state equilibrium, by referring to Figures 1 and 2, the following condition is claimed: (Note 6)

$$\frac{1}{\sigma}(\alpha A_I - \rho - \delta) < (1 - g)A_I - \delta \tag{19}$$

Given the conditions in equations (17) and (19), we derive that an increase in the government expenditure ratio will lower $\overline{p}$ by using equation (18) as follows:

$$\frac{d\overline{p}}{dg}|_T = \frac{\Lambda_g}{\Pi_p - \Lambda_p} < 0 \tag{20}$$

In Figures 1 and 2, an increase in government spending from g_0 to g_1 gives rise to a decrease in the supply of investment goods [$\frac{\dot{k}}{k}(g_0) \rightarrow \frac{\dot{k}}{k}(g_1)$], and thereby p decreases accordingly.

Furthermore, with $\dot{p} = 0$ in equation (14), the balanced growth rate $\overline{\gamma}$ is expressed as follows:

$$\overline{\gamma} = \frac{\dot{k}}{k} = (-\frac{1}{\sigma}\frac{\dot{\lambda}}{\lambda}) \tag{21}$$

Thus, by employing equations (21) and (15a) in the steady state and equation (20), theeffect of an increase in the government expenditure ratio on $\overline{\gamma}$ is derived as follows:

$$\frac{d\bar{\gamma}}{dg}|_T = \Pi_p \frac{d\bar{p}}{dg}|_T \gtrless 0 \text{ , if } \alpha \gtrless \beta \tag{22}$$

From equation (20), p decreases, thus the investment (consumption) goods sector tends to increase (decrease) its output and the production factors move from the consumption goods sector to the investment goods sector (both s and l_I increase). As mentioned before in the Stolper-Samuelson theorem, the decrease in p induces an adjustment in the capital share-labor ratio depending on the factor intensity. If the consumption goods sector is capital-intensive ($\alpha < \beta$), a decrease in p causes labor to be more expensive relative to capital, and hence both production sectors enhance the capital share-labor ratios, which in turn deteriorates the marginal productivity of capital and thus the steady-state growth rate decreases. By contrast, if the investment goods sector is capital-intensive ($\alpha > \beta$), a decrease in p makes capital to be more expensive relative to labor and hence the capital share-labor ratios are reduced in both production sectors, which in turn improves the marginal productivity of capital and leads to an increase in the steady-state growth rate. This ambiguous result of the steady-state growth rate runs in sharp contrast to Devereux and Love (1995), Turnovsky (2000) and Chang et al. (2004). In addition, it can be viewed as a potential means of explaining the mixed empirical results [Miller and Russek (1997)]. From the above analysis, we obtain the following proposition:

Proposition 1. (*The Effect of Lump-Sum Tax Financing*) *When the government implements a lump-sum tax financing scheme in a two-sector economy with consumption goods and investment goods, an increase in the government expenditure ratio lowers the steady-state relative price of the consumption good, and has an ambiguous effect on the steady-state growth rate depending on the factor intensity.*

3.2 Income Tax Financing

Under income tax financing ($T = \theta = 0$), $\tau = g$ can be obtained from equation (9). Here, the supply of investment goods ($\frac{\dot{k}}{k}$) is still represented by equation (15b), but the demand for investment goods ($-\frac{1}{\sigma}\frac{\dot{\lambda}}{\lambda}$) with $\tau = g$ becomes:

$$-\frac{1}{\sigma}\frac{\dot{\lambda}}{\lambda} = \frac{1}{\sigma}\{(1-g)\alpha A_I[v(p)]^{\alpha-1} - \rho - \delta\} \equiv \Gamma(p,g) \tag{23}$$

where $\Gamma_p = \frac{\partial \Gamma}{\partial p} = -\frac{1}{\sigma}(1-g)\alpha(1-\alpha)A_I[v(p)]^{\alpha-2}v_p \gtrless 0$, if $\alpha \lessgtr \beta$, $\Gamma_g = \frac{\partial \Gamma}{\partial g} = -\frac{\alpha}{\sigma}A_I[v(p)]^{\alpha-1} < 0$.

Hence, the dynamic system can also be expressed by substituting equations (23) and (15b) into equation (14) as follows:

$$\frac{\dot{p}}{p} = \phi\ [(\Gamma(p,g) - \Lambda(p,g)] \tag{24}$$

Similar to the analysis for the lump-sum tax financing scheme, the dynamic stability of the system and the uniqueness and existence of the steady-state equilibrium claim that $\Gamma_p - \Lambda_p > 0$ and $\frac{1}{\sigma}[(1-g)\alpha A_I - \rho - \delta]$ $< (1-g)A_I - \delta$, respectively.

Under the income tax financing scheme, the effect of an increase in the government expenditure ratio on $\bar{p}$ is derived from equation (24) with $\dot{p} = 0$:

$$\frac{d\bar{p}}{dg}|_\tau = -\frac{\Gamma_g - \Lambda_g}{\Gamma_p - \Lambda_p} = -\frac{1}{\Gamma_p - \Lambda_p}[s(\bar{p}) - \frac{\alpha}{\sigma}]A_I[v(\bar{p})]^{\alpha-1} \gtrless 0 \text{ , if } s(\bar{p}) - \frac{\alpha}{\sigma} \lessgtr 0 \tag{25}$$

Now, we go straight to the analysis of the steady-state effect using Figures (3a)-(3b) and (4a)-(4b).Obviously, under the income tax financing scheme, a rise in government expenditure not only gives rise to a resource-withdrawal effect which decreases the supply of investment goods [$\frac{\dot{k}}{k}(g_0) \to \frac{\dot{k}}{k}(g_1)$], but also leads to a deterioration in the after-tax marginal productivity of capital via an increase in the income tax rate which decreases the demand for investment goods [$-\frac{1}{\sigma}\frac{\dot{\lambda}}{\lambda}(g_0) \to -\frac{1}{\sigma}\frac{\dot{\lambda}}{\lambda}(g_1)$],as shown in Figures 3a-3b and 4a-4b. As a consequence, the effect of an increase in the government expenditure ratio on the steady-state relative price of consumption goods depends on the relative strength between the decreases in the supply of and the demand for the investment goods, i.e., $s(\overline{p}) \lesseqgtr \frac{\alpha}{\sigma}$. When the extent of the decrease in supply is greater (less) than that of the decrease in demand [$s(\overline{p}) > (<) \frac{\alpha}{\sigma}$], the relative price of consumption goods goes down (up) [Figures 3a and 4a (3b and 4b)].

By using equations (21), (23) and (25) in the steady state, we next obtain: (Note 7)

$$\frac{d\overline{\gamma}}{dg}|_{\tau} = \Gamma_p \frac{d\overline{p}}{dg}|_{\tau} + \Gamma_g = \frac{1}{\Gamma_p - \Lambda_p}\frac{1}{\sigma}(1-g)\alpha A_I{}^2[v(\overline{p})]^{2(\alpha-1)} s_p < 0 \tag{26}$$

Equation (26) indicates that an increase in government expenditure based on income tax financing definitely causes the steady-state growth rate to deteriorate. The rationale is that an increase in government expenditure based on income tax financing affects the after-tax marginal productivity of capital through two channels. First, due to the increase in the income tax rate, the after-tax marginal productivity of capital deteriorates. Second, the alteration in p further affects the marginal productivity of capital. If the consumption goods sector is capital-intensive ($\alpha < \beta$), the decrease (increase) in p leads both sectors to raise (reduce) the capital share-labor ratio and in turn decreases (increases) the marginal productivity of capital. Obviously, if the marginal productivity of capital decreases, the second effect reinforces the first growth-retarding effect (Figure 3a). However, if the marginal productivity of capital increases, the two effects are in conflict with each other and the second one is dominated by the first (Figure 3b). As a result, an increase in the government expenditure ratio using income tax financing always reduces the steady-state growth rate.

Nevertheless, if the investment goods sector is capital-intensive ($\alpha > \beta$), the economic explanation for the steady-state effects on the relative price and the growth rate is similar to the case where $\alpha < \beta$. To save space, we only present the graphical illustrations which are shown in Figures 4a and 4b. This finding is consistent with Devereux and Love (1995). Accordingly, we have the following proposition:

Proposition 2. (*The Effect of Income Tax Financing*) *When there is a two-sector economy composed of consumption goods and investment goods, an increase in the government expenditure ratio based on income tax financing has an ambiguous effect on the steady-state relative price of consumption goods which depends on the relative extent of the difference between* $s(\overline{p})$ *and* $\frac{\alpha}{\sigma}$. *Furthermore, it results in a deterioration in the steady-state growth rate.*

3.3 Consumption Tax Financing

Under consumption tax financing scheme ($T = \tau = 0$), the endogenous consumption tax is derived from equation (9) as follows: (Note 8)

$$\theta = \theta(p, g) \tag{27}$$

where $\theta_p = -\frac{\theta}{p}[\Omega + \frac{y_I}{y_I + py_C}] < 0$ and $\theta_g = \frac{y_I + (1+\theta)py_C}{(1-g)py_C} > 0$. Based on equation (27), the rate of change in the consumption tax rate can be derived as:

$$\frac{\dot{\theta}}{\theta} = -[\Omega + \frac{y_I}{y_I + py_C}]\frac{\dot{p}}{p} \tag{28}$$

Furthermore, by substituting equation (28) into equation (13), we have:

$$\frac{\dot{p}}{p} = \frac{\phi}{\Psi}[(-\frac{1}{\sigma}\frac{\dot{\lambda}}{\lambda}) - \frac{\dot{k}}{k}] \tag{29}$$

where $\Psi = \frac{1}{(1+\Omega\sigma)(1+\theta)}[\theta\frac{py_C}{y_I + py_C} + (1+\Omega\sigma) + \Omega\theta(\sigma - 1)] > 0$, moreover, $(-\frac{1}{\sigma}\frac{\dot{\lambda}}{\lambda})$ and $\frac{\dot{k}}{k}$ are still represented as equations (15a) and (15b). By comparing equation (29) with equation (14), it is clear that the conditions for the dynamic stability and the uniqueness and existence of the steady-state equilibrium under consumption tax financing are similar to those under lump-sum tax financing, and thus we omit them to save space. Furthermore, by substituting $\dot{p} = 0$ into equation (29), we can sequentially derive that the steady-state effects of an increase in the government expenditure ratio under consumption tax financing are equivalent to those under lump-sum tax financing. To be more specific, in referring to Proposition 1, a consumption-tax-financed increase in the government expenditure ratio gives rise to an ambiguous effect on the steady-state growth rate that depends on the factor intensity. This result contrasts with Turnovsky (1996).

Proposition 3. (*The Effect of Consumption Tax Financing*) *In a two-sector economy with consumption goods and investment goods, a consumption-tax-financed increase in the government expenditure ratio gives rise to an ambiguous effect on the steady-state growth rate that hinges on factor intensity.*

3.4 Discussion

Based on sections 3.1-3.3, we can summarize that, regardless of the financing scheme, $-\frac{1}{\sigma}\frac{\dot{\lambda}}{\lambda} = \frac{\dot{k}}{k}$ has to be satisfied in the steady state. Generally, the above relationship can be expressed as:

$$\frac{1}{\sigma}\{(1-\tau)\alpha A_I[v(\overline{p})]^{\alpha-1} - \rho - \delta\} = (1-g)A_I[s(\overline{p})]^{\alpha}[1 - l_I(\overline{p})]^{1-\alpha} - \delta \tag{30}$$

furthermore, the steady-state relative price can be derived with equation (30) as:

$$\overline{p} = \overline{p}(g, \tau) \tag{31}$$

Obviously, under the lump-sum tax (consumption tax) financing scheme, the endogenous T (θ) does not affect $\overline{p}$. Thus, the steady-state effects of an increase in the government expenditure ratio under consumption tax financing are equivalent to those under lump-sum tax financing, i.e.

$$\frac{d\overline{p}}{dg}|_T = \frac{d\overline{p}}{dg}|_\theta \text{ and } \frac{d\overline{\gamma}}{dg}|_T = \frac{d\overline{\gamma}}{dg}|_\theta \tag{32}$$

However, when the income tax financing is implemented, an increase in the government expenditure ratio raises the income tax rate endogenously ($\frac{d\tau}{dg}|_\tau > 0$). By employing the concept of the *Le Chatelier Principle* and equation (31), we obtain the following equation: (Note 9)

$$\frac{d\overline{p}}{dg}|_\tau = \frac{d\overline{p}}{dg}|_T + \frac{d\overline{p}}{d\tau}|_T \cdot \frac{d\tau}{dg}|_\tau \tag{33}$$

furthermore, the effect on the steady-state growth rate is expressed as: (Note 10)

$$\frac{d\bar{\gamma}}{dg}\Big|_{\tau} = \frac{d\bar{\gamma}}{dg}\Big|_{T} + \frac{d\bar{\gamma}}{d\tau}\Big|_{T} \cdot \frac{d\tau}{dg}\Big|_{\tau} \tag{34}$$

where $\frac{d\bar{\gamma}}{d\tau}\Big|_{T} < 0$. (Note 11) As a result, the following relationship is obtained:

$$\frac{d\bar{\gamma}}{dg}\Big|_{\tau} < \frac{d\bar{\gamma}}{dg}\Big|_{T} \tag{35}$$

Based on Equations (32) and (35), we have:

Proposition 4. (*The Policy Recommendation*) *In the perspective of growth, regardless of the factor intensity, when government attempt to finance a government purchase, a lump-sum tax financing scheme (consumption tax financing scheme) is always better than the income tax financing scheme.*

4. Concluding Remarks

By using the Uzawa (1961; 1963) two-sector model of consumption and investment goods with a Romer (1986)-type knowledge spillover technology, we examine the growth rate effect of an increase in the government expenditure ratio through three types of tax financing: lump-sum tax financing, income tax financing, and consumption tax financing. The results show that, a rise in the government expenditure ratio based on lump-sum tax financing has an ambiguous effect on the balanced growth rate depending on the factor intensity between the sectors. The balanced growth rate decreases (increases) with a rise in government spending if the consumption (investment) goods sector is capital-intensive. The outcome for consumption tax financing is equivalent to that for lump-sum tax financing while an income-tax-financed increase in the government expenditure ratio definitely decreases the economy's long-run growth rate. Since the empirical results reported by Kormendi and Mequire (1985) and Miller and Russek (1997) vary quite considerably, our two-sector model that is based on lump-sum tax or consumption tax financing appears to be able to provide a channel through which to explain the mixed empirical findings.

Table 1. Capital shares for 2-digit U.S. industries [summarized form Jones (2003)]

Industry	Construction	Furniture and Fixture	Motor Vehicle	Trade	Transportation	Textile Mill Products
Capital Share	9.4	21.5	21.7	22.7	25.1	25.3
Industry	Agriculture	Electrical Machinery	Food and Kindred Products	Communication	Finance and Insurance	Petroleum and Coal Products
Capital Share	41.5	42.6	46.1	51.7	58.1	63.1

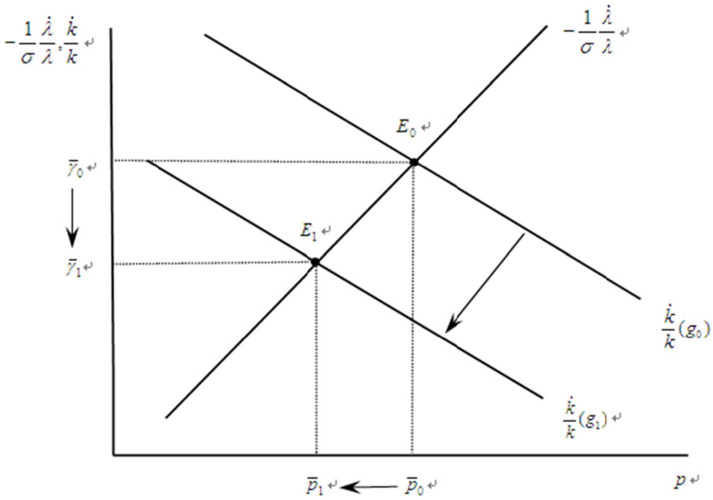

Figure 1. The effect of an increase in the government expenditure ratio with a lump-sum tax financing: $\alpha < \beta$

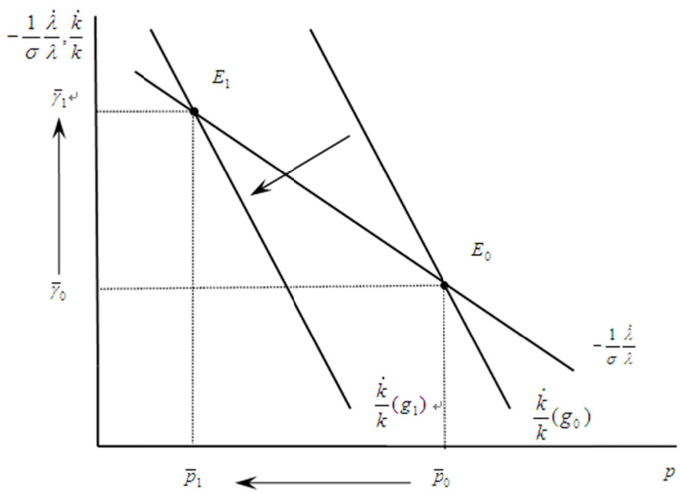

Figure 2. The effect of an increase in the government expenditure ratio with a lump-sum tax financing: $\alpha > \beta$

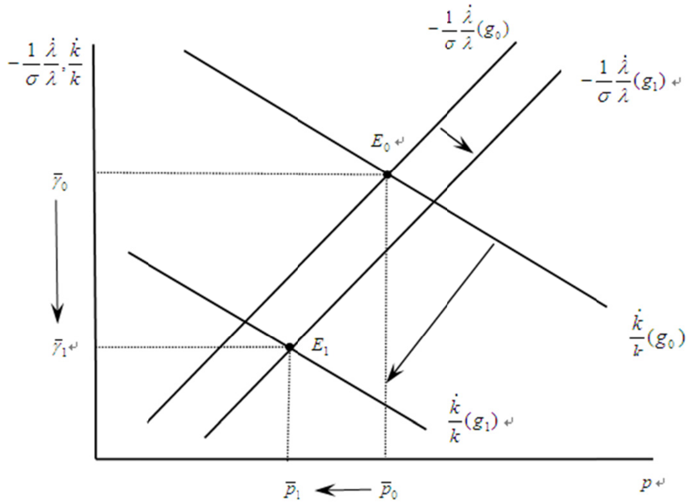

Figure 3a. The effect of an increase in the government expenditure ratio with an income tax financing: $\alpha < \beta$ and $s(\bar{p}) > \alpha/\sigma$

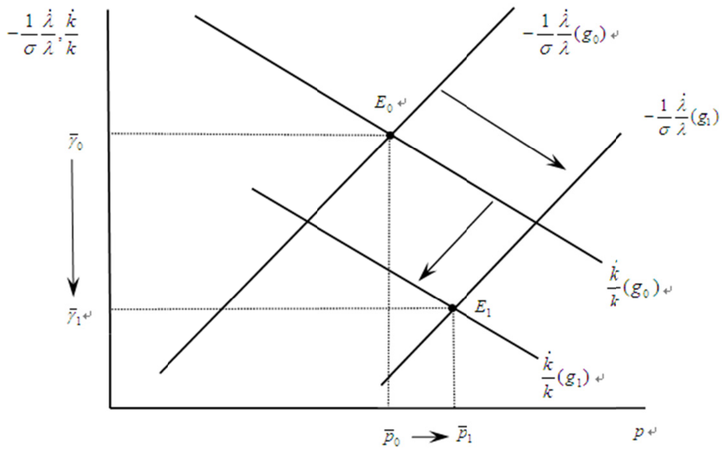

Figure 3b. The effect of an increase in the government expenditure ratio with an income tax financing: $\alpha < \beta$ and $s(\bar{p}) < \alpha/\sigma$

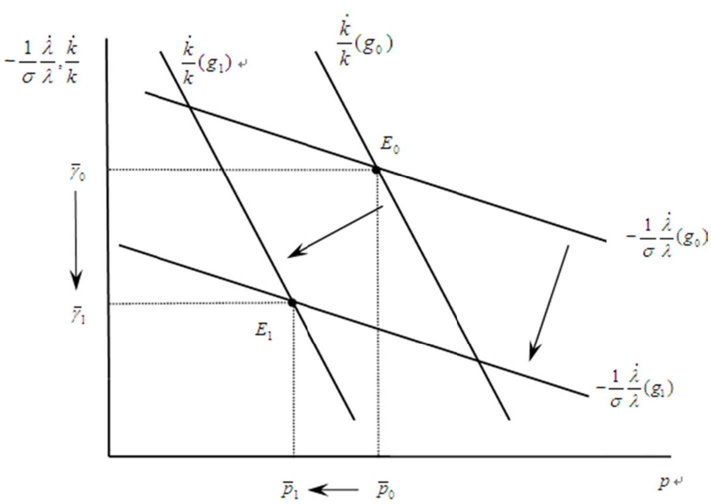

Figure 4a. The effect of an increase in the government expenditure ratio with an income tax financing: $\alpha > \beta$ and $s(\bar{p}) > \alpha/\sigma$

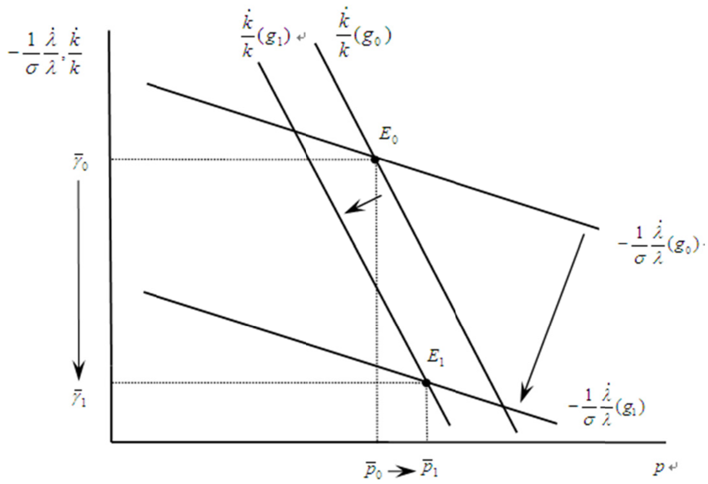

Figure 4b. The effect of an increase in the government expenditure ratio with an income tax financing: $\alpha > \beta$ and $s(\bar{p}) < \alpha/\sigma$

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Notes

Note 1. Van der Ploegand Alogoskoufis (1994), Palivos and Yip (1995), and Gokan (2002) analyze the relative merits of alternative forms of government expenditure financing under a monetary endogenous growth model.

Note 2. The definition of consumption goods we use here is: the economic goods or services purchased by households to satisfy their wants and desires, such as food, clothing, motor vehicles, communications and finance and insurance. Moreover, the definition of investment goods is: the economic goods or service used by firms as inputs to produce their product, such as construction (plant), electrical machinery, and equipments.

Note 3. The setting of $\sigma \geq 1$ implies that the elasticity of intertemporal substitution is located in the range between 0 and 1, which is supported in the empirical studies [Vissing-Jorgensen (2002), and Attanasio et al. (2002)], for a comprehensive survey, see Hasanov (2005).

Note 4. From equations (7d) and (7e), $\frac{1-\alpha}{\alpha}\frac{s}{l_I}=\frac{1-\beta}{\beta}\frac{1-s}{1-l_I}$ is true. This implies that the qualitative effect of p on the capital share-labor ratio $\frac{1-s}{1-l_I}$ in the consumption goods sector is similar to that in the investment goods sector.

Note 5. The concept of the graphical analysis we use here is in the spirit of and in accordance with that in the Solow-Swan growth model. Please refer to Barro and Sala-i-Martin (2004) or Romer (2005) for the detail.

Note 6. Since $\lim_{p\to 0} v = 1$, $\lim_{p\to 0} s = 1$, and $\lim_{p\to 0} l_I = 1$, from equations (15a) and (15b), $\lim_{p\to 0} \Pi(p) = \frac{1}{\sigma}(\alpha A_I - \rho - \delta)$ and $\lim_{p\to 0} \Lambda(p,g) = (1-g)A_I - \delta$ is true.

Note 7. The relationship $v_p = \frac{1}{l_I}[s_p - v(p)l_{Ip}]$ is used in deriving equation (26).

Note 8. By substituting equation (10a) into equation (9) with $T = \tau = 0$ and using equation (10b), we have: $g(y_I + py_C) = \theta p(1-g)y_C$ Taking the total differentiation of the above equation and using $y_{Ip} + py_{Cp} = 0$ and $\Omega = p\frac{y_{Cp}}{y_C} > 0$, we derive equation (27).

Note 9. The *Le Chatelier Principle*is introduced into economics analysis by Samuelson (1947), for a detail specification, please refer to Silberberg and Suen (2001).

Note 10. This equation is similar to equation (16a) in Turnovsky (2000), p. 200.

Note 11. From equations (31) and (15b), $\frac{d\overline{p}}{d\tau}|_T > 0$ and $\frac{d\overline{\gamma}}{d\tau}|_T < 0$ can be easily derived.