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Valuing Private Companies: A DEA Approach

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Abstract

Traditionally, company valuation methods are based on discounted cash flows, market prices, comparable sales and even liquidation values, but these are known to have a number of shortcomings. The application of data envelopment analysis (DEA) to finding companies comparable to the one under examination and thereby predicting the market values of such companies can be considered to be an extension of the market-based approaches. As a result of using DEA, companies are classified into either an "inefficient" or "efficient" group, for each of which we assume that a corresponding upper or lower bound of its market value exists, respectively. Furthermore, the market value of an inefficient company is expressed in a form of a range of values, which are calculated by utilizing firm's reference set defined by DEA. As the validation to the modelling part, inefficient companies correctly classified their market values into the evaluated variation range up to 36.93%, and 66.20% of the actual market values are below their upper bounds. On the other hand, 41.67% of the time DEA can find a lower bound of their market values for efficient companies based on inefficient peer comparisons. The results show that using DEA in valuing private companies is a relatively advanced method, and could prospectively play an active role in company valuation.

Keywords: company valuation, data envelopment analysis (DEA), market value, efficiency

1. Introduction

Business valuation in North America is a multi-billion-dollar business. In 1996, $650 billion (USD) was invested in mergers and acquisitions (M&A). The total amount was up to $861 billion in 2009 (Schmid et al., 2012) and dramatically increased to $1,114 billion by 2011 (Schmid et al., 2013). There are many reasons for evaluating a business, but in most cases it is required to obtain an impartial value of the business. Improper valuations can lead to large losses for either the buyer or seller.

The basic underlying methodology of DEA is a fractional linear programming technique based comparison of a group of DMUs (decision making units) under analysis, be the DMUs banks, hospitals, and many other entities or in this case companies. DEA allows multiple inputs and outputs for the DMUs and provides efficiency scores for them, which can be used to distinguish efficient DMUs from others. For an inefficient DMU, DEA indicates a distinct improvement scheme by projecting it to the efficient frontier where the projection is usually a combination of efficient units, and thus is regarded as the target of improvement for an inefficient unit. The motivation of the current research, namely incorporating DEA theory in this effort originates from a series of inherent virtues in DEA. For instance, the first and foremost one is that it is non-parametric; hence, it does not require the specification of a functional form. Secondly, DEA is a non-biased method which incorporates a cluster of relevant factors in a single model and none of them is given preference. In addition, it treats the DMUs with unrestricted weights which make a rather fair judgment on the importance of such variables.

Given this opportunity, DEA could be used as an ideal tool for finding a group of similar and comparable companies for the one under valuation. The primary objective of this research is to value private companies using DEA. A secondary objective is to find the factors that determine Market Capitalization (MC) value for companies and to find the firms that can be used for comparison purposes in valuation utilizing DEA. To develop an appropriate approach to this task, at first we value public companies because here, we have actual stock market based values that can be used for our purposes. The technique uses DEA to find comparable market traded firms, for which the appropriate market value ranges can be calculated. As the firm for which we had found a value, can be validated as it has a known market value readily available. Once the models are validated we could apply them to valuing private companies in the future.

The data set originally came from reports of *Value Line Inc.* in 2013 which included the financial statements of more than 6,000 American companies (listed on U.S. stock markets). Note that the financial statement data will lag by a longer period, since the most recent financial statements available in January 2013 for a company with December year-end; these financials will be the December 2011 statements. Neglecting incomplete data points, 500 companies were randomly selected from the original dataset as our research target. The data includes SIC (Standard Industrial Classification) codes from 1000 to 9975. The analysis was carried out in a variety of industries covering Metals & Mining, Petroleum, Food Processing, Clothing, Medicine, etc. in total, 86 different codes.

The rest of this paper is organized as follows. Section 2 offers a short literature survey on company evaluations. Section 3 contains a survey of the relevant DEA methodology. The methodology developed in the current research including the details of the DEA models and the analysis of the data is illuminated in Section 4. Section 5 explains the results. The paper ends up with Section 6 which sums up our contributions.

2. Literature Survey

Value is the highest price that a knowledgeable buyer, at arm's length, is willing to pay in an open and unrestricted market. The need for business valuation may arise from a variety of circumstances and the different circumstances will require different valuations. Such reasons may include IPOs (Initial Public Offering), mergers and acquisitions (M&A), estate planning, taxation, and many other similar needs.

Traditional company valuation methodologies include the asset-based approach, the discounted cash flow method, and the comparable firms approach. The advantages and disadvantages of these methods are stated briefly in Table 1 as shown below (Booth, 2000).

Table 1. Valuation approaches – advantages and disadvantages

Approach	Advantage	Disadvantage
Comparable Firms (CF)	Best when a highly comparable group is available Units are close in both size and business type	The whole sector may be over/under valued There are too few comparable examples Insufficient recent transactions
Discounted Cash Flow (DCF)	Has firm theoretical basis Easy to compare competing opportunities. Finance people readily understand it	Estimating future cash flows is difficult at best Estimating interest rates in the future is uncertain
Asset-Based	Looks at all the underlying values in the firm's assets Conservative, not likely to be criticised Traditional method, people are comfortable with it	More relevant if the assets can be liquidated readily Does not work for initial IPOs Small firms are disadvantaged Service firms are difficult to value this way Growth rates in high-tech firms are not included

Factors analyzed in asset valuation are the market value for fixed assets, leasehold improvements, owners' benefits, goodwill (intellectual property, patents, trademarks, copyrights, etc.) and inventory. The sum of these factors will give an estimate of the market value for the business. Booth (Booth, 2000) listed the results of a survey of large companies as to which method they used for evaluation. Table 2 shows the respondents' preferences which are rated by corresponding scores in a range from 1 to 7.

Table 2. Desirability of different valuation approaches

Method	Score (1 – 7)
DCF	6.1
CF	5.7
Projected Earnings	4.4
Revenue Multiples	4.1
Earnings Multiples	4.0
Book Value	3.6
Liquidation Approach	2.3

There is a large body of literature on corporate evaluation techniques but it does not serve any useful purpose to go into great details of all these issues in this paper. Hence, we will explain only the basic principles of a few of the most often used methods.

2.1 Discounted Cash Flow

The DCF method calculates the expected future income stream of a company discounted back to present value, at a risk-adjusted rate. It is based on the Gordon Price Model (Gordon & Shapiro, 1956) and is mathematically represented as follows.

$$P = \sum \frac{D_n}{(1+k)^n} \tag{1}$$

Where P is the price, D_n is the dividend in year n, and k is the cost of equity or discount rate. Another way of representing DCF is to show that the DCF formulation values only the assets that give rise to the free cash flow.

$$V_o = \sum_{t=1}^{T} \left(\frac{C_t}{(1+K)^t} + \frac{V_t}{(1+K)^t} \right) \tag{2}$$

Where t is the time period, C represents the free cash flow, K is the discount rate, and V_t is the terminal value.

For most small and high-tech companies, future earning potential is more important than past performance. That is why calculating DCF is such an important issue when evaluating companies. The DCF method is used predominantly for companies that are not asset intensive, like the services sector. In the DCF method, certain variables and unknowns must be calculated which include: the amount and duration of future cash flow, growth potential of the cash flow, risks associated with them, and the discount rate to apply to arrive at the net present value (Mullen, 1990). Other factors to be incorporated into this approach include risk within the industry, quality and size of the customer base, length of time the company has been in business, and their levels of technology.

One of the problematic aspects of this method is the selection of an appropriate discount rate. For private companies, the discount rates are chosen based on the perceived riskiness of the business. In order to determine an appropriate discount rate, several following methods can be employed: CAPM (Capital Asset Pricing Model), WACC (Waited Average Cost of Capital) and analyst estimates.

Two major problems in the DCF procedure are the discount rate used and the estimation of future cash flows. Earnings are affected by fluctuations within a certain industry and the state of economy. Past earnings have no guarantee about future earnings. The risk free rate (RFR) is usually acquired from business news, but it is usually correct for only a short time period, and has to be adjusted to imitate the growth of the firm for longer time horizons.

Researchers who reported on their use of DCF models include, Kaplan et al. (1995), who prefers DCF rather than Cash Flow (CF) approach; they concluded that CAPM and DCF yielded about the same accuracy as the CF approach; Mullen (Mullen, 1990) asserts that future cash flows are more important than the past but the risk of the future must be considered, so DCF is his preferred method; Grubbström (Grubbström & Kingsman, 2004) used DCF to determine ordering quantities of a product while its price is changing; Related application can also be found in Jennergren's study (2008), where DCF is employed in firm valuation.

2.2 Comparable Firms–The Market Based Approach

The comparable firms (CF) approach or market approach as it is sometimes known, typically involves comparison with publicly traded companies. This is done by taking the earnings per share (EPS) of the firm under consideration, using the average or median P/E ratio of the complementary publicly traded firms and then multiplying the P/E ratio with the EPS to get the projected share price. Earnings forecast, market to book, and price to sales are other factors that can be used instead of P/E ratios.

Many studies examine the use of the P/E ratio for comparable firms. Boatsman and Baskin (1981) compared the accuracy of two different types of P/E models. The first model finds a random company in the same industry. The second model uses a company from the same industry but with similar ten-year average growth rate of earnings. They found the accuracy of the second model to be better.

Kim and Ritter (1999) use sales, earnings, operating cash flow and book value to determine company value, as well as comparable firm multiples. They tested the accuracy of the comparable firms method by using two sets of comparable firms. The first set consisted of recent IPOs in the same industry and the second set was chosen by a research boutique (Note 1). The results showed that the second set worked better than the first, and applying forecasted earnings worked better than using historical earnings. They also tested the hypothesis that young firms are harder to value by splitting the data into 'age' groups and applying the second set of comparable firms. They found that the valuation errors of the comparable firm multiples are noticeably smaller for the older firms than for the younger firms, especially when using earnings.

Kim and Ritter used the geometric mean of the research boutique's comparable firm P/E multiples as the explanatory variable in regressions using the three P/E ratios of the IPOs as the dependent variables (Kim & Ritter, 1999). The three P/E ratios, using the midpoint of the POP (preliminary offer price) for the IPO, are the last 12 months, current fiscal year's forecast, and next year's forecast of the EPS. Comparing the regressions, the average prediction error falls from 55% to 43.7% to 28.5% respectively, and the percentage of firms that are valued within 15% of the actual multiple increases. Younger firm valuation using forecasted earnings works better than using historical earnings, as there is a potential growth opportunity that needs to be addressed.

Three main issues with this approach are the P/E ratios employed, the use of the right multiplier for the industry and the availability of a highly comparable group of companies. P/E ratios are dynamic and vary widely over time and among companies. The problems with P/E ratios and the right multiplier can be remedied by looking at industry trends and the broader market to forecast trends and adjust historical P/E ratios accordingly.

Recently, there are many different methods used to value firms (Wang et al., 2013), but each person who values companies is quick to point out that a combination of these methods works best. Therefore, no study can be done in isolation, different methods should be applied and a range of values reached, as supported by the work of Mullen (1990), Fitchett (1988) and Ovens (1959).

3. Data Envelopment Analysis

DEA is a linear programming technique for calculating the relative performance of a population of operating entities, commonly referred to as DMUs. This technique considers multiple inputs and outputs simultaneously. Essentially, DEA "compares" the information available from a group of DMUs with all others in the group optimizing each individual DMU with the objective of calculating a discrete, piece-wise linear, "best practice" frontier.

In DEA, the efficiency of a DMU is obtained by evaluating a ratio of weighted outputs to weighted inputs, subject to the constraint that no DMU can acquire weights that would make another DMU have a ratio greater than 1.0. Charnes et al. (1978) extended Farrell's single output/input technical efficiency measure (Farrell, 1957) to multiple inputs and outputs and they defined this concept mathematically as follows (this is known as the ratio form):

$$\begin{aligned} \rho_o &= \max_{u,v} \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ \text{s.t.} \quad & \frac{\sum_{r=1}^{s} u_r \; y_{rj}}{\sum_{i=1}^{m} v_i \; x_{ij}} \le 1, \; (j = 1, \ldots, n) \\ & u_r, v_i \ge 0, \; (r = 1, \ldots, s; i = 1, \ldots. m) \end{aligned} \tag{3}$$

where ρ_o is the relative efficiency of the DMU_o under evaluation, j is the DMU index, r and i are the indices for output and input, respectively. We assume there are n DMUs, and for each of which s outputs and m inputs are assigned. The weights are represented by u_r for output and v_i for input variables. If $\rho_o < 1.0$, the DMU is considered to be inefficient and if $\rho_o = 1.0$, then that DMU is efficient and thus will be on the efficient frontier.

The above ratio form was then transformed into a linear programming model to avoid the inherent non-linearity. That formulation is a constant returns to scale form (CRS) and that was later followed by a number of other models, the most important of which was Banker's (1984) variable returns to scale (VRS) model. For a good overview of the developments since these first two major models were introduced, see the book by Cooper, Seiford and Tone (2007).

DEA is a form of frontier analysis that recognizes that some DMUs perform below optimum levels. In this method, each DMU is optimized against all other DMUs. That is, the linear program assigns weights to the variables such that each DMU looks the "best" it can be. DEA constructs the efficiency frontier and calculates the efficiency score for each DMU based on its distance from the frontier. For radial DEA models the score indicates the proportion by which the unit can increase its outputs without consuming any more inputs, or conversely, the proportion by which it can decrease its inputs and maintain its current output level.

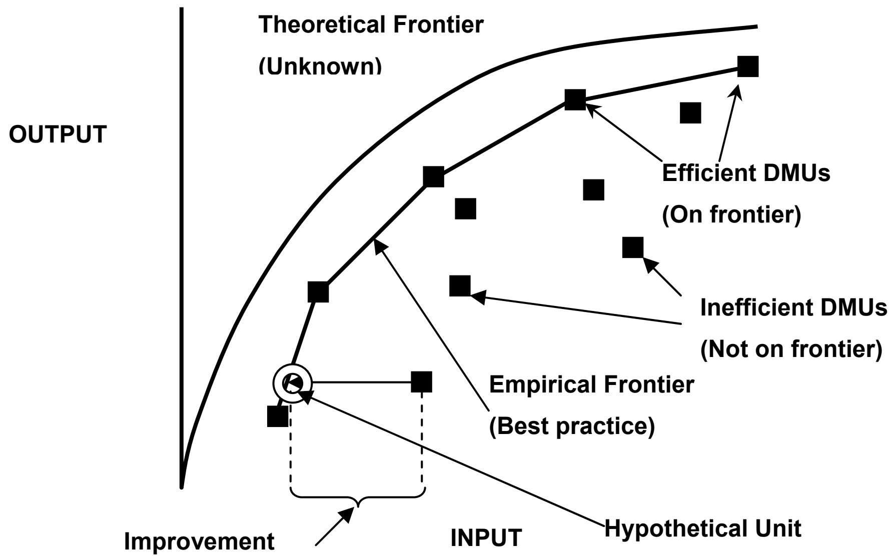

Figure 1. The DEA envelopment frontier

3.1 Variable Returns to Scale

The graphical representation of a one input and one output VRS model, see Figure 1, identifies the efficient as well as the inefficient DMUs in the sample. The efficient frontier, or envelopment surface, consists of the best practice DMUs. The hyperplanes connecting the efficient units form the frontier and the inefficient units are enveloped within. DEA links the estimation of the technical efficiency and the production frontier. Somewhere above the efficient frontier is the theoretical frontier (shown as a curve in Figure 1). This frontier is beyond the reach of human performance, as there is always a chance for performance improvement when human output is involved. The DMUs that are enveloped by the best practice frontier are considered to be inefficient. For each inefficient DMU, the DEA analysis identifies the sources and the levels of inefficiency for both the inputs and outputs by comparing the inefficient unit with a single, hypothetical, reference unit formed from the linear combination of efficient units that operate under similar conditions. This comparison determines the potential improvement obtainable by the inefficient DMU.

The VRS model (Banker, 1984) distinguishes between technical and scale inefficiencies by estimating pure technical efficiencies at the given scale of operation and by identifying whether increasing, decreasing, or constant returns to scale possibilities are present for further exploitation. This model interprets the efficiency of the DMUs with the underlying assumption that a VRS mechanism exists among the population of observations. It incorporates the notion that an increase in the inputs does not necessary translate to the same proportional increase in outputs throughout all scales of operation.

3.2 VRS SBM (Slacks-Based Measure)

Another set of DEA models are not- radial, and sometimes these are referred to as additive DEA, of which SBM is considered to be the representative because of its wide application in many domains. The VRS SBM is distinguished from CRS SBM by adding an extra constraint "$\sum\lambda = 1$" in order to realize variable returns to scale, and it is similar to the relationship between CCR and BCC. The VRS SBM model rather than CRS SBM or other radial DEA models is selected in this research for the reasons that we will explain in Section 4 after we introduce the company valuation method.

At first, we give a brief introduction to the original SBM model. Assume there are n DMUs in the current system to be analyzed, of which input and output vectors are represented by an $m \times n$ matrix $\boldsymbol{X}$ and an $s \times n$ output matrix $\boldsymbol{Y}$. The number of inputs and outputs are denoted by m and s respectively. Thus the efficiency score of DMU_o under evaluation takes the following form:

$$\rho = \min_{\lambda, s^-, s^+} \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{y_{ro}}}$$
$$\begin{aligned} s.t. \quad & \boldsymbol{x}_o - \boldsymbol{s}^- = \boldsymbol{X\lambda} \\ & \boldsymbol{y}_o + \boldsymbol{s}^+ = \boldsymbol{Y\lambda} \\ & \boldsymbol{e\lambda} = 1 \\ & \boldsymbol{\lambda} \geq 0, \boldsymbol{s}^- \geq 0, \boldsymbol{s}^+ \geq 0 \end{aligned} \tag{4}$$

where $\boldsymbol{s}^+$ and $\boldsymbol{s}^-$ are the vectors for output and input slacks respectively, and $\boldsymbol{e\lambda} = 1$ imposes a convexity constraint to allow different returns to scales on the efficient frontier. This model differs from radial DEA models which only allow proportional changes to all inputs or outputs, estimate DMU_o from all attributes without proportional assumption. Furthermore, Tone (2001) shows that the above SBM model satisfies the following four properties: (P1) *Units Invariance*: The optimal value of the objective function is independent of the units in which the inputs and outputs are measured. (P2) *Monotony*: The efficiency of a DMU is monotonically decreasing along with the increase in each slack to either input or output. (P3) *Reference Set Dependence*: The efficiency of a DMU should be measured only by consulting its corresponding reference set. (P4) *Charnes-Cooper Transformation*: The above original nonlinear SBM model in (4) can be transformed into a linear one using Charnes-Cooper transformation.

Through the years, DEA has been used to study the efficiency of over 50 industries including, for example, schools, banking, fast food retail, healthcare, insurance, credit unions, highway road maintenance, and capital budgeting projects. Over 6,000 papers have been written by academics on a wide variety of theoretical and practical advances in DEA. However, in this paper we will only show one such work as this is directly relevant to our efforts.

Zhu (2000) conducted a thorough study on DEA and company efficiency. In this work he examines the financial performance of the Fortune 500 companies using DEA. The data is from the April 1996 issue of Fortune Magazine. The investigation found that the top-ranked companies by revenue do not necessarily perform efficiently, and that only 3% of these companies were on the best-practice frontier.

Eight factors were used in evaluating companies: revenues, profits, assets, number of employees, stockholders' equity (SE), market value, earnings per share and total return to investors. Zhu (2000) tested three models in order to obtain an overall performance index. The first model, called profitability, had three inputs (employees, assets, and stockholders' equity) and two outputs (revenues and profits). The second model, named marketability, had two inputs and three outputs, (namely the outputs from the first model were considered as the inputs into this one), and the new outputs were market value, EPS and total return to investors. The third model combined the first two models, it used the inputs from the first model and the outputs from the second model, and thus it had three inputs (employees, assets, and stockholders' equity) and three outputs (market value, earnings per share and total return to investors). This third model was meant to represent the overall performance/efficiency.

Upon further investigation into scale efficiency and returns-to-scale, Zhu (2000) found that the top 20 companies were not only scale inefficient, but they also operated in a DRS region, while most relatively small companies operated in an IRS region. Sensitivity analysis showed which production factor would not affect an efficient company's financial performance while all other production factors are kept at their current levels.

4. DEA & Company Valuation

The main motivation for this research comes from a Ph.D. dissertation by Simak (2000) on this subject and the follow up work by Anadol (2000). In this work, Simak tested the hypothesis that DEA could be used for private company valuation and proposed a DEA model to identify similar public companies to the one under scrutiny. The model used data on 51 public manufacturing companies. The resulting peer groups from the DEA analysis are the used to identify the similar companies. The peer group for each inefficient company is identified and other companies that have the same peer groups are collected. A difference indicator variable δ_{ij} between company *i* and *j* is introduced as follows.

$$\delta_{ij} = \sum_k (\lambda_{ik} - \lambda_{jk})^2 \tag{5}$$

where *k* is the coefficient of all the efficient companies. The lower the δ_{ij} value, the more similar the companies are. He also examined the closeness of the efficiency scores. By looking at the MC of these public companies and testing if the DEA analysis found peer companies with similar MCs, the factors can be combined to reveal the best possible peer(s) for the subject-company. The results were encouraging, provided that reasonably similar public companies were available for reference.

In Simak's research, input-oriented VRS BCC model is used which focuses on maximizing the DMUs performance by proportionately reducing its inputs, or in other words, produces the observed outputs with the optimum resource level. We still use the VRS model, because it is apparent that a VRS efficient frontier is constructed by more DMUs than the CRS resulting in more reference sets to classify DMUs. The difference is we use the SBM model instead of BCC because, (1) Both inputs and outputs play important roles in deciding the efficient status for a DMU, and it is difficult to say which part is more critical. (2) Oriented DEA models (either input-oriented or output-oriented) put only inputs or outputs in the objective function as controllable factors, on the contrary, SBM puts all inputs and outputs in the objective function and perceive the efficiency levels from all attributes. (3) Proportional change for inputs or outputs sometimes may not be realistic for some companies.

DEA based valuation will allow multiple inputs and outputs to be used in the same model. The DEA comparable firms model is an extension of the market approach to find comparable companies. DEA is often used to find the most efficient units but it can also be used to find the appropriate peer groups. This paper extends the approach introduced by Simak (2000) and Anadol (2000), but it differs in several respects. First, factors used for valuation are different. Second, two new techniques are introduced, one to value efficient companies and the other to determine upper bounds for the inefficient companies. No estimation of the value of efficient companies has ever been attempted before to our knowledge.

4.1 The Valuation Model

Before the formulation of the DEA model, a number of assumptions regarding inputs and outputs had to be incorporated into the valuation model.

The Inputs

Variable selection is a central issue in formulating a DEA model. As Zhu (2000) has shown, there is substantial flexibility in what to include as inputs and/or outputs. Frequently, a particular variable may be included as an input in one model while it is an output in another. The basic issue is that DEA minimizes inputs and maximizes outputs, so the analyst should keep in mind what is a beneficial formulation for a DMU. Here, we chose total assets and total liabilities as inputs.

For the outputs, the choices were: cash flow from operations, net income and net sales. The latter two terms are commonly used in all manner of work studying the earnings potential of a company. And, of course, in the formulation of the DEA model, the larger these two factors are the better the performance of the business. The inclusion of such measures also allows the analyst to study the operating conditions of these corporations and to determine the firms' viability.

The model constructed here aims to represent company operations. Inputs and outputs selected for the model are represented in Figure 2. This operational representation is the basis for the formulation of the DEA model.

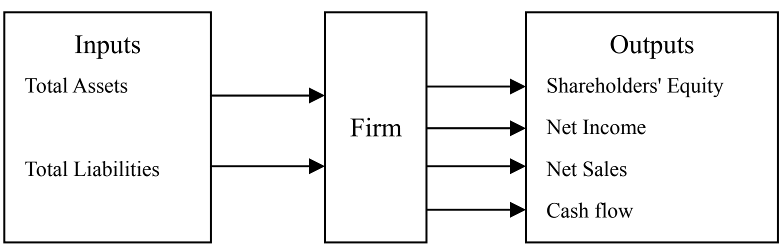

Figure 2. DEA-Valuation model

As to the negative values encountered in this research, as many low performance companies may have negative values in SE (Shareholders' Equity), NI (Net Income) and CF (Cash Flow), since SBM is not capable of dealing with negative values, and is not translation invariant (which means we can't translate such negative values into positive ones by adding an amount of positive number to all attributes), we split such output into positive and negative parts. For example, SE was split into SE^+ and SE^-, where SE^+ was defined as output as its usual meaning, but SE^- was defined as input. This method is essentially saying that SE^- could be regarded as an *inflow*, and therefore should be minimized to improve the company's operating efficiency. However SE+ is viewed as a usual *outflow* which should be maximized. Therefore the inputs/outputs of the model after revision are shown in Figure 3.

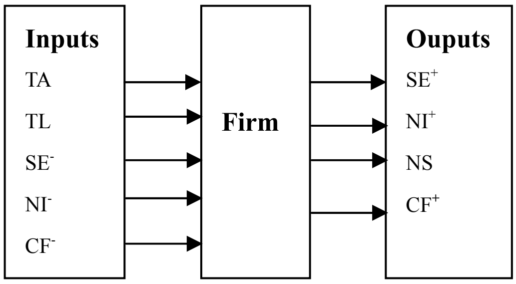

Figure 3. DEA inputs/outputs after modification

5. Analysis

The analysis is carried out in two steps. First, the SBM model is run to classify the companies as efficient or inefficient. Then the two classifications are examined separately.

For the inefficient companies, peers are found based on the other inefficient companies having the same efficient companies as their peers. The difference indicator is applied to all peers in turn to establish the δ values. The hypothetical market value for the subject company is established by multiplying the δ values with its firm's MC and then summing these values. Moreover, an upper bound can also be found for the inefficient companies and this upper bound is also based on lambda relationships, but in this case between the inefficient company and its efficient company peers. Theoretically, a comparable efficient company should have a greater MC than an inefficient firm and this is what is computed here.

For the efficient companies, a lower bound can be found by finding all the companies using it as a peer, again relying on the assumption that an inefficient firm will have a lower MC than an efficient firm. However, the sum of λ values will most likely be > 1.0; therefore, these must be normalized so that they do sum to 1.0. Now computation of the lower bound can be achieved by simply multiplying these normalized λ values by the market values of their respective firms and then summing them.

Based on the random 500 data points we selected, for each company, we now had their associated efficiency, lambda, slack and projections. Totally, 32 companies get an efficiency score of 1.0, while 468 companies are in the inefficient group. The MC is brought back into the model only to validate the findings. Validation was a simple comparison with the listed firm's MC.

5.1 Inefficient Company Analysis

Hence, the 468 inefficient DMUs were chosen for the analysis. To illustrate the method, the peer group for the company, Cheniere Energy Inc., was found and these consisted of all the efficient companies it was compared to, as well as all the inefficient companies that were compared to the same group of efficient companies. In the case for Cheniere, the peer companies were AFC Enterprises, Alaska Communic., Fifth & Pacific Co., Lin TV Corp., Moody's Corp., Taubman Centers Inc. and Team Health Hldgs. The lambda values going across the table for each DMU should sum to unity in the VRS model; this is due to the convexity constraint. Table 3 illustrates the peer group along with efficiency scores and lambda values.

Table 3. Subset of Cheniere energy inc. and distances

DMU	Score	MC (millions)	Efficient Peers (λ)		Dist (δ_{ij})
			Career Education	Kyocera Corp. ADR	
Cheniere Energy Inc	2.17E-05	3875.8	0.602575	0.397425	0
AFC Enterprises	2.76E-02	621.9	0.971965	2.80E-02	0.27
Alaska Communic.	1.92E-04	91.4	0.973477	2.65E-02	0.28
Fifth & Pacific Co.	6.82E-05	1390.1	0.902954	0.097046	0.18
Lin TV Corp.	1.29E-04	398.5	0.937096	6.29E-02	0.22
Moody's Corp.	1.97E-05	11113.8	0.271743	0.728257	0.22
Taubman Centers Inc.	2.79E-05	4868.6	0.532322	0.467678	0.02
Team Health Hldgs.	1.83E-02	1940.5	0.871511	0.128489	0.14

This peer set was then analyzed with the difference indicator. The distance from Cheniere to AFC Enterprises is computed as follows:

$$\delta_{ij} = \sum (0.602575\text{-}0.971965)^2 + (0.397425\text{-}2.80E\text{-}02)^2 = 0.27$$

In a similar manner, the remaining distances are computed for Cheniere's other six peers. All the peer companies were then compared in terms of their distance. Then two lowest ones were selected and their corresponding MC values were considered to be the interval within which Cheniere's MC will fluctuate. In this case, the analysis determined a range for Cheniere's market value based on its lowest δ peers of $4,868.6 million (Taubman Centers Inc. - δ = 0.02) to $1,940.5 million (Team Health Hldgs - δ = 0.14). Cheniere's real MC of $3,875.8 million is in this range. Such a range prediction was carried out for the other 467 inefficient companies, and the overall accuracy was 36.93%.

5.2 Upper Bound

A new method of determining market values based on the reference units for the inefficient companies is introduced, and it is suggested that this is an upper bound for the inefficient company under examination. To calculate an upper bound for Cheniere., we use only the market values of the *efficient* companies that it was being compared to, namely, Career Education and Kyocera Corp. ADR. The upper bound of Chenieres MC is computed by multiplying the efficient companies' MC with their λ values for Cheniere. The lambda values can be found in Table 3.

$$U_o = \sum_k \lambda_{ok} \cdot MC_k \tag{6}$$

where U_o is the upper bound for the inefficient company and λ_{ok} is the λ of the k-th efficient peer and MC_k is the MC of the k-th efficient peer.

This formula is exemplified by:

$$U_o = (0.22*217.8) + (0.45*1,6451.3) = \$6,669.4 \text{ million}$$

The $6,669.4 million value thus found is considered as an upper bound on the MC of Cheniere, since it is based on the real market values of the efficient companies. The base assumption made here is that an efficient company will always have a MC greater than an inefficient firm. Clearly, in practice, there will be exceptions to this assumption because the market is not perfect, but it is a reasonable approach. Similar analysis is carried out on the remainder of the inefficient companies in this subset. The results are summarized in Table 4:

Table 4. Cheniere energy inc. and the predicted market values

DMU	Real MC	Min	Max.	Upper Bound
Cheniere Energy Inc	3875.8	1940.5	4868.6	6669.399
AFC Enterprises	621.9	91.4	398.5	672.3304
Alaska Communic.	91.4	398.5	621.9	647.9827
Fifth & Pacific Co.	1390.1	398.5	1940.5	1793.196
Lin TV Corp.	398.5	621.9	1390.1	1238.886
Moody's Corp.	11113.8	3875.8	4868.6	12039.96
Taubman Centers Inc.	4868.6	3875.8	11113.8	7809.851
Team Health Hldgs.	1940.5	398.5	1390.1	2303.626

As can be seen from Table 4, the real MC of Cheniere is between the two peer values and below the predicted upper bound. AFC Enterprises (MC = \$621.9 million, peer range = \$91.4 – \$398.5 million, upper bound = \$672.3 million) is an example of a company not in the predicted peer range but is still below its predicted upper bound. This upper bound evaluation process was also executed for all inefficient DMUs, and 66.20% of DMUs' actual MC values are under the predicted upper bound.

5.3 Efficient Company Analysis

We now introduce the second new methodology to estimate the values of DEA efficient companies. This is a fundamentally different approach from what was used for the inefficient companies. Since efficient companies are not compared to other companies in DEA, one has to calculate the market value from the inefficient companies being compared to them. One can hypothesize that this way a lower bound on the market values can be found, again assuming that an efficient firm will always have a higher MC than an inefficient one. The lower bound was calculated based on the lambda values of all the inefficient companies being compared to the efficient company. Adjustments must be made for the fact that the lambda values on the column of the matrix do not sum to unity. The lambda values of the inefficient companies in relation to the one under examination are summed and then this sum was used to divide each lambda value to get a new normalized lambda

$$\tilde{\lambda} = \lambda / \sum \lambda .$$

$$L_i = \sum_j \tilde{\lambda}_{ij} \cdot MC_j \qquad (7)$$

where L_i is the lower bound of the efficient unit under scrutiny; $\tilde{\lambda}$ is the normalized λ; and MC_j is the j-th inefficient MC.

Costco Wholesale was analyzed first which had 5 companies being compared to it. For example, Costco has a lambda relation of 0.05857 to Microchip Technology, the sum of all the lambda relations equals to 0.510109. Using the $\tilde{\lambda}$, Costco's new lambda score is (0.05857 /0.510109) 0.11482. This same procedure was repeated for all the other companies' lambda values, thus the new sum of the lambda values equals one. The new normalized lambdas of the firms were then multiplied by their market values and summed. The results are shown in Table 5:

Table 5. Analysis of efficient costco wholesale

DMU	MC	λ	New $\tilde{\lambda}$	Lower bound contribution
Microchip Technology	6286.1	0.05857	0.11482	721.77
priceline.com	30419.1	1.69E-03	0.003313	100.7785
Silver Wheaton	12208.3	5.34E-02	0.104684	1278.014
Southern Copper	31995.6	0.390618	0.765754	24500.76
VMware Inc.	40524.9	5.83E-03	0.011429	463.1591
SUM:		0.510109	1.00	27064.48

For Costco, the predicted lower bound of MC is \$27,064 million, and its real market value is \$42,578 million which clearly above this limit. Of the 32 DMUs with 1.0 efficiency score, 12 only take themselves as reference set. Such efficient DMUs do not make any sense for lower bound evaluation. Similar analyses were carried out on the remaining 19 efficient companies.

In our simple example Costco only has 5 inefficient peers which is the least number compared to other efficient companies. For other efficient DMUs, dozens of inefficient peers is quite normal. For some efficient DMUs, like Career Education, China Agri-Business Inc., LUKOIL and Spectrum Group Intl Inc., they even have more than 100 inefficient peers because of the large number of entries we are using.

6. Conclusions

With a global increase in mergers and acquisitions and the number of companies going public, the need for valuation tools and methodologies increases. Determining value is a significant part of any business, be it for selling a business, owner dispute settlements or for mergers. Value estimates can vary widely and to insure a fair value for the business a number of issues need to be addressed.

Valuation is often a subjective task performed by professional valuators, business analysts, accountants, lawyers and investment bankers. The existing methods for determining this value consist of forecasting growth and future cash flows, or optimizing the value of owned assets by liquidation or finding similar companies and adjusting their forecasts and growth expectations to arrive at a value. The goal of this paper was to develop a methodology that evaluates the market value of corporations utilizing DEA to find comparable firms and to apply this methodology to assess the worth of private companies.

A DEA analysis of 500 companies randomly abstracted from more than 6000 American companies covering a comprehensive spectrum of industries in 2013 was studied. Valuation models for predicting market ranges were developed. Different approaches were constructed based on the performance of the company in the DEA model. Inefficient companies had their most similar peers identified which lead to a prediction of their market value range and an upper bound for their market value was also computed. For the efficient companies, a lower bound market value prediction approach was introduced.

The results were encouraging when a highly comparable peer group existed. Results showed that overall 34% of the inefficient companies were within the range predicted by the method. Since the MC values of companies were originally collected from the company reports where a part of the data may not actually reflect the realistic status of the companies. Thus this percentage could be even higher depending on more accurate records of MC values. Overall, the calculated upper bounds were greater than the real market value for the inefficient companies 66% of the time (Similarly this number could also be higher). For the efficient companies the lower bound was less than the real market value 42% of the time, however there were instances where it was not and this could mean that the companies are undervalued.

The data we are using have huge differences in each attribute, for example, the lowest value in "total liability" is \$0.1 million for American Energy Production In., IEH Corp., HemaCare Corp. et al. and the highest value is up to \$101,828 million for Ford Motor. For total asset, the lowest is \$0.2 million for Financial Content Inc. and the highest is \$224,044.1 million for Pfizer Inc. Such huge differences also exist in other attributes. Thus the results should be typical and trustable. On the other hand, the accuracy could be increased a lot if the ranges of attributes are narrowed down. We also did analysis by groups of big industries, like drug, finance and petroleum which have more than 100 companies. The results are almost the same, which means this method is not sensitive to industry sectors.

In summary, the DEA-Valuation model gave us considerable comfort when predicting appropriate market ranges. As with all valuations, this is another tool that businesses have at their disposal, and a combination of the different valuation methods will give better ranges of value. However, a complete study using different methodologies such as DCF, CF, and asset valuation, with all the appropriate parameters applied would be the best way to compare all the methods against one another. Furthermore, since the MC of a company is always changing, using dynamic DEA models to predict the trend of MC could be a sequential research topic in the near future.

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Note

Note 1. Company that specializes in valuing IPOs.