Long-Dated Agricultural Futures Price Estimates Using the Seasonal Nelson-Siegel Model

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Abstract

Over the counter (OTC) forward contracts are regularly traded by hedgers at maturities beyond the longest-dated futures contract. The presence of seasonality in agricultural commodities creates additional uncertainty for obtaining fair prices for OTC forward contract trades beyond the liquid futures strip. This paper employs an augmented Nelson-Siegel function to obtain seasonal agricultural commodity price estimates for OTC forward contracts beyond the longest available maturity of exchange traded futures contracts. A multifactor seasonal Nelson-Siegel model is chosen due to its internally consistent and parsimonious functional form. The Nelson-Siegel approach is used to model seasonally adjusted corn, cotton and sugar forward prices for OTC contracts out to five years maturity calibrated against shorter-dated futures contracts. Residual and contract liquidity testing indicates that the seasonal model provides efficient estimates of contract prices beyond the futures strip which allows agricultural commodity hedgers to obtain fair prices for OTC forward contracts.

Keywords: Commodity prices, Futures, Nelson-Siegel function, Seasonality, Liquidity, Cotton, Corn, Sugar

1. Introduction

Agricultural commodities are traded in both a spot market and also in the form of futures contracts through various exchanges. Healthy levels of liquidity in the shorter-term contracts allow the efficient trading of such securities but longer-dated contracts are generally much less liquid. Beyond the longest-dated futures contract buyers and sellers must engage in off-exchange or over-the-counter (OTC) forward contracts. For instance the longest-dated contract for trading sugar futures available on the NYBOT is two years beyond which any hedging or speculation activity must take place through bilateral forward contracts. This paper examines an approach for estimating OTC forward contract prices of seasonally-affected commodities at maturities beyond the longest quoted liquidly traded future contract. In particular, we utilize an augmented Nelson-Siegel function to model agricultural commodity futures prices in order to obtain theoretical OTC forward prices for contracts beyond the longest available maturity of the exchange traded contracts. A multifactor Nelson-Siegel model is chosen due to its internally consistent and parsimonious functional form. Much of the existing literature ignores contract tenors beyond the last traded futures strip while simultaneously acknowledging the large volume of longer-dated trades (Sørensen, 2002). Such trades are assumed to occur at a linearly interpolated price modeled using 18-month and 2-year contract prices but they ignore the inherent seasonality in the price behavior of the commodity. This study offers an efficient approach to model the price behavior, volatility and seasonality of agricultural commodity futures contracts to fill a significant gap in the literature.

Most agricultural commodity prices exhibit some form of mean reversion and seasonal behavior. To cater for these characteristics, the usual approach in financial econometrics has emphasized the use of models that utilize arbitrage relationships across collections of assets, see Schwartz (1997). During this process, the entire term structure is modeled in terms of a few underlying and possibly unobserved factors. The typical approach begins by assuming a functional form for a set of underlying state variables, namely the commodity spot price, and then assuming a stochastic convenience yield and a stochastic interest rate. A term structure of agricultural commodity futures contract prices can then be derived using observed contract prices calibrated to a constrained

set of market factors. The convenience yield is treated as a flow of benefits to the physical commodity owner that do not accrue to holders of futures positions, analogous to the flow of dividends or coupons to stock or bond holders respectively.

However this approach makes many assumptions concerning market factors and does not always provide consistent estimates of fair prices beyond the longest exchange-traded contract maturity. Furthermore, arbitrage models are usually unable to incorporate inherent seasonality into their structures. To obtain an efficient term structure of prices beyond the observed futures strip, we employ a multifactor seasonal Nelson-Siegel approach that assumes a general functional form to model observed futures prices constrained by the term structure of volatility prices. This function allows for OTC forward prices and associated volatilities to be derived for maturities beyond the final date for the quoted contract values. The accuracy of the model is tested using a comparison with exchange-traded contract prices that lie beyond typical maturities and is found to provide an efficient estimate for seasonally-affected commodity forward contracts. In addition, the efficiency of the model is tested using liquidity measure comparisons between exchange traded futures contracts and OTC forward contracts from commodity trader databases. A measure for the liquidity of the contracts confirms that the specified model delivers efficient price discovery estimates of contracts beyond exchange traded maturities.

Section 2 discusses commodity price behavior. Section 3 introduces the model and discusses the methodology. Section 4 proposes a testing procedure and presents the results. Section 5 offers some concluding remarks.

2. Commodity Futures Price Behavior and Modeling Approaches

The production cycle of agricultural commodities combined with the unavoidable role of climate and weather produce very distinctive behavior in commodity prices. Using observed futures prices, this paper demonstrates that the inclusion of systematic seasonal variation improves the ability of time-series models to predict OTC forward prices beyond the quoted futures strip, using a low-parameterized seasonal function calibrated via the application of constraints to the term structure of forward volatilities. The distinct difference between futures prices and forward prices is maintained through the model calibration process.

Trading agricultural futures contracts instead of spot contracts has many advantages. First, agricultural futures are exchange-traded standardized contracts devoid of counterparty risk. Second, since contracts can easily be closed out or rolled over, delivery can be avoided meaning that shipping, storage, and insurance need not be necessary. Third, long and short positions are equally straightforward to transact. A fourth potential benefit is that futures returns do not just stem from movements in the underlying spot prices, but may contain a futures specific component as well, resulting from deviations of the futures price from the expected future spot price, to which the futures price will eventually converge as the contract matures.

When futures prices are a fair reflection of expected future spot prices, the expected futures return will be zero. When hedging demand is particularly strong, however, a discrepancy may arise as hedgers may be willing to accept a less favorable futures price in return for being able to fix their future spot price. When hedgers on balance sell (buy) futures this puts downward (upward) pressure on the futures price. The result is a futures price, which is lower (higher) than the expected future spot price and which therefore offers buyers of the futures contract a positive (negative) premium.

Modeling approaches for commodities are typically borrowed from interest rate term structure models. Term structure estimation has been traditionally implemented with static models that only use current bond prices or yields without regard to past information. Some methods such as Nelson and Siegel (1987) and Svensson (1994), assume a parametric functional form for the forward rates. Other methods, for instance McCulloch (1971, 1975) and Fisher, Nychka and Zervos (1994) use non-parametric spline-based interpolation methods to calculate the term structure. Arbitrage-free methods as used in Schwartz (1997) and Ramos Ribeiro and Hodges (2004) also provide robust term structure dynamics however the calibration to the short rate and deterministic convenience yields, along with the use of stochastic short rates and convenience yields, greatly increases the complexity of the model. Empirical evidence shows that in well developed fixed-income markets where numerous bonds are traded every day for different maturities, these static methods generate yield curves that accurately fit current bond transactions, see Bliss (1997). While a high level of confidence in the goodness-of-fit levels of observed prices is desirable in a term-structure model, of equal importance is the stability of the term structure curves obtained from the model for deriving fair prices through the seasonal cycle of an agricultural commodity. The stability of a given model can be quantified by observing the sequence of daily term structure estimates implied by the model. It might well be the case that the model fits existing bond prices or yields extremely well but it may imply large daily movements of yields for long-dated maturities that are not traded.

To analyze the stability of the modeled term-structure curves we compare the term structure of volatilities from the model with observed volatility from a dataset. For agricultural commodity futures in contrast to interest rate modeling, a parametric approach is preferred due to the presence of a strong seasonality component in prices, which cannot be readily incorporated into a non-parametric or arbitrage-free term structure model. Therefore a parametric model is employed as the most appropriate to use for the estimation of fair OTC forward rates beyond quoted futures contract prices.

The Nelson-Siegel (NS) model, in its simplest form, can be used as a parametric model to fit a term structure of futures prices, see Nelson and Siegel (1987). The NS model can be extended and used for modeling agricultural commodity futures with strong levels of seasonality. For this analysis we improve on the fundamental NS model in a number of ways. Firstly, we include a term to explicitly model the seasonality of agricultural futures prices. Seasonality is evident when examining the residuals after applying the basic NS model to agricultural commodity futures prices. The presence of seasonality in agricultural commodity futures prices can be examined in contrast to the absence of seasonality observed in the time series of the shortest-dated futures contracts. We also explicitly model the daily volatility term structure of agricultural commodity futures contracts. From the resulting term structure model we can estimate the expected value of longer-term volatilities for OTC forward contracts. These OTC forward volatility estimates are used to dynamically constraining the term structure constructed using the NS model. The estimation process ensures close agreement between forecast volatilities, and those observed from OTC forward contract prices. Historical comparisons of this approach with observed historical data demonstrate a good fit to futures contracts, exhibit less volatile forward price estimates and results in improved stability of price behavior.

3. Data and Research Method

3.1 Data

The adjusted NS model, hereafter referred to as the NS seasonal model, has been refined to accommodate seasonal pricing effects and the volatility term structure of agricultural commodity prices. The model is built and calibrated using historical prices of agricultural commodity futures contracts. The seasonally-affected commodities used for this analysis include corn, cotton and sugar. Futures commodity price history for cotton and sugar was obtained from the New York Board of Trade historical price database and the futures price history for corn was obtained from the Chicago Board of Trade historical price database for the period July 2001-July 2006. Details of each contract type are given in the Appendix. The first seven contracts for each commodity are used in the calibration of the model. The first contract is referred to as C1, the seventh contract as C7 and the longest-dated forward contract estimate at 5-year maturity is referred to as C20, indicating that we will obtain 13 forward contracts, C8-C20, using the seasonal NS model, beyond the seven traded futures contracts, C1-C7.

3.2 The Model

The basic NS model is generally used for parsimoniously fitting a term structure of traded securities. The basic NS can be discretised from the family of NS models as

$$f_k(t) = \alpha_{0,k} + \alpha_{1,k} \frac{1 - e^{-t/\tau}}{t/\tau} + \alpha_{2,k} e^{-t/\tau}, \tag{1}$$

where $\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}$ and τ are the parameters to be estimated at a rate of t, see Nelson and Siegel (1987).

However, applying a basic NS model to seasonally-dependent commodity futures prices results in a strong seasonal pattern observed in the residuals. Figure 1 shows the estimated model residual from 2002-2006 between the observed fourth NYBOT sugar #11 futures contracts and the fourth contract modeled using the basic NS model. The fourth futures contract has approximately one year to maturity and seasonal effects are generally easier to detect at this maturity. We observe a slight seasonal pattern with negative residuals each year around March and peaks in the residuals in September.

The seasonality of agricultural commodity futures prices is more apparent than the spot prices of the same commodity due to short-term supply and demand differences and intraday market-specific factors. A common approach to modeling seasonality in a time series is to use a sinusoidal term with a calibrated amplitude, frequency and phase. After de-trending via the NS function, the residuals plot indicates that the frequency and

phase adjustment parameter should be a constant or a deterministic function of time. In particular a constant frequency of one year is used for calibrating the seasonal phase parameter. The next section will show that we can estimate the phase simply from historical data and establish a deterministic function of time that approximates the phase. The phase is estimated equal to zero and that the seasonal cycle can be inferred to begin in at the same point each year.

3.3 Calibration

3.3.1 Parametric model for volatility

To estimate the OTC forward price curve beyond maturities of exchange traded futures contracts, we require constraints to ensure long-dated volatilities are within reasonable limits which ensure that the modeled values conform to reasonable volatilities. Our approach is to estimate a parametric model based on the historically estimated volatilities of liquid futures contracts each day. The realized volatilities for each contract are determined from the last 20 daily log-returns. The choice of $d=20$ returns for realized volatilities was made to be as consistent as possible with the observed implied volatility of at-the-money options on each futures contract. A scaling factor of $\sqrt{252}$ is used to annualize the results, consistent with market practice.

The general parametric model used to estimate the volatility term structure from realized volatilities is

$$\ln(v_{k,t}) = \lambda_{0,k} + \lambda_{1,k} \frac{\left(1 - e^{-\gamma_k t}\right)}{\gamma_k t} + \varepsilon_{k,t}, \qquad (2)$$

where t denotes the tenor in years, k is an index denoting the day, $v_{k,t}$ is the realized volatility, and the error term $\varepsilon_{k,t}$ has zero mean and constant variance. The parameters of the model to be estimated are $\lambda_{0,k}$, $\lambda_{1,k}$ and γ_k. The approach used to estimate the term structure from the realized volatilities is a nonlinear-weighted least-squares scheme. The weighting scheme used gives a weight of 0.5 to the shortest-dated and most liquid futures contract, with all other contracts receiving an equal weight of 0.083. These weights were chosen as they have been shown to represent the general relationship between volatilities within a term structure, see Bliss (1997). The model described in (2) is linear in the parameters $\lambda_{0,k}$ and $\lambda_{1,k}$ if we have an estimate of γ_k, so we estimate $\lambda_{0,k}$ and $\lambda_{1,k}$ via weighted least squares conditional on our current estimate of γ_k. To arrive at an estimate of γ_k we must employ a one-dimensional search over a region with a specified initial value. The lower bound is assumed to be 1.96, which is obtained via the solution to the equation

$$\frac{\left(1 - e^{-z\phi}\right)}{z\phi} = 0.25 \ ,$$

using a simple Newton-Raphson scheme, where $z = 2$. The upper bound of 10 is similarly chosen, solving the same equation but with a right-hand side constraint of 0.05. The choices of 1.96 and 10 implicitly assume that less than 25 percent, but more than 5 percent of the decay in the curvature function occurs after two years, hence $z = 2$. Two years is approximately the tenor of the longest dated futures contract with sufficient liquidity traded on each exchange.

To estimate γ_k a coarse grid search over the region of 1.96< γ_k <10 is performed, using the three grid points that bound the maximum of the likelihood function as inputs to the golden search procedure, which iteratively converges towards the estimate of γ_k. This calibration is performed at a daily frequency.

3.3.2 Parametric forward price model and estimation

The basic Nelson Siegel function described in (1) can also be specified as

$$f_k(t) = \alpha_{0,k} + \alpha_{1,k} e^{-\mu_k t} + \alpha_{2,k} t e^{-\mu_k t} + \varepsilon_k, \qquad (3)$$

where $f_k(t)$ is the forward price at tenor t, in years, for the k-th day. The model is parameterized by long, short and medium-term components via $\alpha_{0,k}$, $\alpha_{1,k}$ and $\alpha_{2,k}$ respectively. The rate of decay of the forward prices is primarily governed by the nonlinear parameter μ_k. To model the seasonal pattern evident in the residuals after fitting model (3) we include an exponentially-decaying sinusoidal term with constant frequency of one year, amplitude $\alpha_{3,k}$ and phase $\alpha_{4,k}$. That is

$$f_k(t) = \alpha_{0,k} + e^{-\mu_k t}\left\{\alpha_{1,k} + \alpha_{2,k}t + \alpha_{3,k}\sin(2\pi t + \alpha_{4,k})\right\} + \varepsilon_k, \tag{4}$$

where $f_k(t)$ is the forward price at tenor t for the k-th day. The parameters $\alpha_{0,k},...,\alpha_{4,k}$ are estimated using ordinary least squares after an estimate for μ_k is obtained and an implicit constraint on the long-term forward price is applied.

We estimate the model described in (4) after trigonometric simplification, when assuming that the decay parameter μ_k is known, fixed or determined. To estimate the parameters $\alpha_{0,k},...,\alpha_{4,k}$ in (4), we use ordinary least squares after an estimate of μ_k is obtained with an implicit constraint on the long-term forward price. We consider each step of this process in turn.

3.3.3 Imposing a long-term forward price constraint

We assume that the z-year forward price volatility determined from the last 20 days of forward price curves in (4) is equivalent to the z-year volatility term structure of (2). The justification for a 20-day history was provided in Section 3.2. Some algebra leads to the relationship

$$\ln\left(\frac{f_k(z)}{f_{k-1}(z)}\right)^2 = \frac{d}{T_d}\left(\exp\left\{\lambda_{0,k} + \lambda_{1,k}\frac{1-e^{-z\gamma_k}}{z\gamma_k}\right\}\right)^2 - \sum_{i=2}^{d}\ln\left(\frac{f_{k-i+1}(z)}{f_{k-i}(z)}\right)^2, \tag{5}$$

for the i-th day, where $d = 20$ and $T_d = 252$. This relation specifies the squared log-return for the most recent period in terms of the estimated volatility and the volatility realized from historical z-year forward prices. The value of z is chosen to be 5 years, which extends the futures curve over three years beyond the seventh futures contract, C7.

If the right-hand-side of (5) is less than zero, which implies that we cannot get an agreement between the model and the realized volatilities, the best approximation is given by $p_k(z) \equiv p_{k-1}(z)$. If the right-hand-side of (4) is positive, then we can see that

$$f_k(z) = f_{k-1}(z)\exp\left[\pm\sqrt{\left(\frac{d}{T_d}\left\{\exp\left(\lambda_{0,k} + \lambda_{1,k}\frac{1-e^{-z\gamma_k}}{z\gamma_k}\right)\right\}^2 - \sum_{i=2}^{d}\ln\left(\frac{f_{k-i+1}(z)}{f_{k-i}(z)}\right)^2\right)}\right] \tag{6}$$

where the sign of the exponent is chosen to minimize the estimated residual sum of squares. The $f_k(z)$ is then added to the quoted futures prices and tenors for day k and then used in subsequent estimation.

3.3.4 Calibrating linear parameters $\alpha_{0,k},...,\alpha_{4,k}$

Assuming an estimate for μ_k we can manipulate (4) to obtain the relation

$$f_k(t) = \theta_{0,k} + \theta_{1,k}e^{-\mu_k t} + \theta_{2,k}te^{-\mu_k t} + \theta_{3,k}\sin(2\pi t)e^{-\mu_k t} + \theta_{4,k}\cos(2\pi t)e^{-\mu_k t} + \varepsilon_k. \tag{7}$$

The form of (7) is completely linear in the parameters $\theta_{0,k},...,\theta_{4,k}$ so with an assumed estimate of μ_k we can use ordinary least squares for estimation. To arrive at the parameters $\alpha_{0,k},...,\alpha_{4,k}$, we must transform the estimates of $\theta_{0,k},...,\theta_{4,k}$. Applying the transformations $\alpha_{0,k} = \theta_{0,k}$, $\alpha_{1,k} = \theta_{1,k}$, $\alpha_{2,k} = \theta_{2,k}$, $\alpha_{3,k} = \sqrt{\theta_{3,k}^2 + \theta_{4,k}^2}$ and $\alpha_{4,k} = \arctan(\theta_{3,k}/\theta_{4,k})$ results in the linear structure of (7), which greatly simplifies the calibration process.

3.3.5 Calibrating the nonlinear parameter μ_k

The value of μ_k is constrained to lay within the range 0.5 and 4.0 and initialize μ_k by the previous day's value. These values are chosen to represent the range in which 95 percent of empirically observed estimates are expected to occur. A coarse grid search is performed over this range, where at each step the function to be minimized is the least squares cost function. If an interior point of the region is found, the grid point and its two

nearest neighbors are used as input to the golden search routine. At each step all other parameters are estimated by least squares, conditional on the given μ_k .

4. Results

The model was calibrated to cotton, corn and sugar futures prices. In the interests of brevity and for ease of illustration, only the parameter estimates of the NYBOT sugar #11 futures contracts are provided here, however the seasonal phase parameters for corn and cotton are provided. The parameter results for corn and cotton may be obtained from the author upon request. In Figures 2 to 5 we show the parameter estimates of $\alpha_{0,k},...,\alpha_{3,k}$ for NYBOT sugar #11 futures prices plotted over the period 2002-2006. The constant parameter $\alpha_{0,k}$ illustrated in Figure 2 is a proxy for the level of sugar prices, re-estimated using the seasonal NS model each day. The daily variability is very stable over time, and in close agreement with the estimated volatility term structure at very long lags. Figures 3, 4 and 5, in contrast to Figure 2, represent the strength of short-, long- and medium-term trends and seasonality. The parameter trends for corn and cotton are similar to the observed results for sugar.

Figure 6 illustrates the strong seasonality component of the phase parameter $\alpha_{4,k}$ modeled for the NYBOT sugar #11 contracts. The trend is more or less truncated between $-\pi$ and π as specified in the construction of the seasonal NS model. The beginning of the seasonal cycle evident in sugar futures prices occurs in March each year, while for corn and cotton it is in November, as evident in Figures 7 and 8. Each commodity shows strong seasonality profiles, however the profile of sugar is more stable over the final two years than corn and cotton. The consistency in the phase estimates emphasizes that our assumption of seasonality in futures prices is an empirically reasonable assumption. There are some instances of instability for all three commodities during 2005-6 however forward contract prices and volatilities from the estimated model around this period are quite stable. The parameters exhibit high levels of correlation between each other on days of observed instability, and any volatile effects subsequently cancel each other out. In general though, such linear dependence is not apparent and the parameters are relatively stable and significant through time.

Finally the parameter trace of μ_k for NYBOT sugar #11 futures contract prices over the period is shown in Figure 9. We observe that the imposed upper and lower limits of 4.0 and 0.5 are seldom reached and although the parameter fluctuates through time, the fluctuations are consistent with the expected rate of decay of price movements due to seasonal variation.

For confirmation that the models in equations (2) and (4) produce reasonable estimates Figures 10 and 11 illustrate historical values and the corresponding 5-year estimates for NYBOT sugar #11 prices and volatilities. In particular, Figure 10 shows the daily realized volatilities for the nearest-term futures contract C1, the longest-dated futures contract C7, and the estimated 5-year volatility for the forward contract C20 from (2). The volatilities of the observed futures and the 5-year forward estimates are uniformly lower as we increase the tenor, as is observed in agricultural futures markets. Figure 11 illustrates that the 5-year forward price, considered separately or compared to C1 or C7, also provides reasonable results for the range considered.

In Figures 12 and 13 a sample of the realized and estimated volatility term structure and the corresponding futures and estimated OTC forward curves is provided for NYBOT sugar #11. Corn and cotton volatility and futures term structures are of a similar appearance. A forward volatility and futures price curve can be produced for every day in the period. The model fit for the volatility term structure is generally reasonable over the period and the forward price curves fit the observed futures contract prices reasonably well. Table 1 provides the residuals obtained by computing the difference between modeled forward contract price volatility and the volatility estimated at each tenor using the general parametric model (2) for corn, cotton and sugar contracts. In relation to the typical volatility of commodity futures contracts the residuals are small and significant, particularly for corn and cotton futures contracts. The significance of the sugar contract residuals decreases at longer tenors, due to higher volatility and seasonality inherent in long-dated sugar OTC forward prices. This indicates that the forward volatility curve obtained using the seasonal NS model closely resembles the volatilities expected of forward rates, assuming the general parametric model in (2) using observed futures contract volatilities is a good representation of the volatility term structure. Given that the seasonal NS model relies heavily on the forward volatility curve for the calibration process the results in Table 1 confirm that a stable set of volatilities is being used over the estimation period. Importantly, the fitted curves are also very stable throughout the period. This provides a robust structure for deriving fair prices of OTC forward contracts three years beyond the longest-dated exchange-traded future contract observed in the market. Table 2 provides the OLS estimates for $\alpha_{0,k}, \alpha_{1,k}, \alpha_{2,k}, \alpha_{3,k}$ and $\alpha_{4,k}$ using the transformation from (4) to (7) for the NYBOT sugar #11 futures curve over the period 2001-2006 with the relevant t-statistics. The results show generally

statistically significant values for the parameters over each tenor indicating stability in the generated forward curve.

4.1 Backtesting

Since no exchange-traded contracts are available for maturities beyond two years it is difficult to quantify the likely differences in values for long-dated deals. For sugar futures, quotes are sometimes available for the eighth contract and on rare occasions the ninth contract. Corn and cotton futures have more regular quotes for both the eighth and ninth contracts. These contracts are relatively illiquid and may not reflect the similar underlying true price discovery that earlier-dated contracts exhibit. However, we can observe differences between the model's estimation of a contract price at the eighth contract tenor, C8 and the actual value of the eighth contract expiring at the same maturity. At various times, quotes for the eighth contract are available and a simple backtest can be performed. We use the seasonal model with re-calibrated parameters to estimate daily contract values for C8 and compare this with the quoted market price for C8. The difference between the estimated price and the actual price is referred to here as the residual.

Figure 14 shows a histogram of the residuals for NYBOT sugar #11 futures. The residuals are approximately normally distributed which demonstrates that the seasonal NS model generates accurate estimates for C8. To formally test the normality of the residuals using the seasonal model for each commodity, we use the D'Agostino-Pearson (DAP) omnibus test for normality. This test is superior to the Jarqua-Bera, Kolmogorov-Smirnov and the Shapiro-Wilk tests for normality, see D'Agostino and Stephens (1986). The DAP test is specifically designed to detect departures from normality, without requiring that the mean or variance of the hypothesized normal distribution be specified in advance. The DAP test first analyzes data to determine skewness and kurtosis of the distribution. It then calculates how far each of these values differs from the value expected with a normal distribution, and computes a single p-value from the sum of the squares of these differences, see D'Agostino and Stephens (1986). The full test is a combination of the D'Agostino skewness test and Anscombe-Glynn kurtosis test. The DAP test statistic is $K^2 = Z_{g1}^2 + Z_{g2}^2$, where Z_{g1}^2 is a test of the symmetry of the distribution and Z_{g1}^2 is a test of the kurtosis of the distribution. The K^2 statistic has approximately a chi-squared distribution, χ^2 with 2 degrees of freedom when the population is normally distributed. A significance level of 5 percent is used to determine the significance of the distribution. The residual normality test results for each commodity are given in Table 3. The results indicate that the residual between the estimated and traded price at C8 for sugar, and both C8 and C9 for corn and cotton is normally distributed and is within reasonable bounds. The model therefore provides an adequate representation of likely contract values at C8 for sugar, and C8 and C9 for corn and cotton. It is inferred from this that contract prices estimated using the seasonal model beyond C8 are efficient estimates of these prices.

The same process is performed for obtaining residuals between the quoted market price for C8 and an estimate for C8 obtained by simply linearly extrapolating (flat-line) from C7 for sugar which is a common practice in the risk management arm of investment banks and commodity funds. The histogram and normal distribution are illustrated in Figure 15, and the results of the DAP test are given in Table 3. The residuals observed in Figure 15 and Table 3 show that the simple flat-line extrapolation more often than not results in a residual value of zero, which implies that the quoted market value of C8 is simply equal to the market value of C7. The residuals are heavily skewed and leptokurtic indicating that the simple extrapolation method from C7 significantly departs from expected market prices and C8 and C9. Thus setting the value for C8 to be equal to the value for C7 is a comparatively poor fit than the seasonal model. There is a strong possibility that the illiquidity of C8 provides no incentive for market participants to trade this contract at a different price to C7. However the exchange traded illiquidity is compensated by the availability of OTC forward contracts beyond the furthest traded contract, C7. Nevertheless, a simple extrapolation results in non-normal residuals and is therefore a poor fit to the traded contract data.

4.2 Market Efficiency and Liquidity

A measure of the efficiency of the seasonal model beyond the furthest traded contract can be performed using a simple order-based measure for liquidity, see Amihud and Mendelson (1986) and Aitken and Comerton-Forde (2003). The bid–ask spread represents the cost that an investor must incur in order to trade immediately. That is, to purchase (sell) an asset investors must cross the spread and hit the existing ask (bid) orders in the schedule. This is an effective and accurate method of calculating the liquidity of an asset. By calculating this cost as a percentage of the asset price (relative spread), liquidity may be compared across assets with different prices. The relative bid-ask spread statistics for each commodity are provided in Table 4. The bid-ask spreads for the exchange-traded futures contracts, C1-C7 were obtained from the respective exchange historical databases. The

bid-ask spreads for forward contract prices, C8-C20 were obtained from a commodity broker employing the seasonal NS model for trading and hedging OTC CBOT corn and NYBOT cotton and sugar futures contracts beyond C7. The relative spreads are obtained by dividing the bid-ask spread at each tenor by the published closing price at that tenor using the seasonal NS model. As shown in Table 4, the relative spreads widen as the maturity increases. This is to be expected, particularly for OTC contracts. However the relative spreads of the forward contracts are reasonably small, particularly when compared to the exchange-traded futures contracts. This indicates that both the spread and market-traded prices are reasonably efficient estimates of forward contracts beyond the longest-maturity futures contract.

5. Concluding Remarks

This study has augmented the basic Nelson-Siegel term structure model to better estimate seasonal agricultural commodity forward prices beyond the longest-maturity futures strip in two important ways. Firstly we explicitly modeled the seasonality present within futures contracts and then developed a simple term structure model for futures contract volatilities, using this parametric model as a constraint within the forward curve estimation. The results of historical simulation, and an analysis of model calibration, indicate that the model is stable and provides an excellent fit to existing futures contracts and anticipated forward contracts. The efficiency of the forward contract prices has been confirmed via a test between exchange-traded futures prices and model-derived forward prices, along with a simple measure for the liquidity of the derived forward contracts. The seasonal NS model allows market participants to obtain efficient estimates of agricultural commodity contract prices beyond the maturity of exchange-traded futures contracts, which will improve the ability of the market to trade and hedge longer-dated contracts. The ability to achieve price discovery in agricultural commodities with limited liquidity levels provides hedgers with the confidence to obtain fair prices for OTC forward contracts beyond the futures strip.

There are two limitations to the approach used in this analysis. Firstly the parametric model requires constraints to ensure long-dated volatilities are within reasonable limits to ensure the model conforms to reasonable volatilities. During highly volatile market activity or when expected inventories are low resulting in large differences in volatility between near-term and far-dated contracts, the volatility term structure may become unstable and calibrating the model may result in large standard errors of estimates. Secondly the model assumes limited liquidity premiums applied to long-dated contracts which may not hold during periods of significant market illiquidity for contracts beyond the futures strip.

Areas for future research include the application of the model to other non-agricultural commodity futures (or forward) contracts that exhibit seasonality, particularly energy commodities. The model could also be extended to cater for other features that impact on commodity markets such as business cycles and inventory levels.

References

Aitken, M., & Comerton-Forde, C. (2003). How should liquidity be measured? *Pacific Basin Finance Journal,* 11. 45-59. http://dx.doi.org/10.1016/S0927-538X(02)00093-8

Amihud, Y., & Mendelson, H. (1986). Asset pricing and the bid ask spread. *Journal of Financial Economics*, 17. 223-249. [Online] Available: http://www.sciencedirect.com/science/article/B6VBX-45N500Y-35/2/b50b2546c0a63268bbcfd8d62d8a3301

Bliss, R.R. (1997). Testing term structure estimation methods, *Advances in Futures and Options Research,* (9). 197-231. [Online] Available: http://www.frbatlanta.org/filelegacydocs/wp9612a.pdf

D'Agostino, R.B., & Stephens, M.A. eds. (1986). *Goodness-of-fit Techniques*. New York, Dekker.

Duffie, D., & Singleton. K. (1999). Modeling term structures of defaultable bonds. *Review of Financial Studies*, 12. 687-720. http://dx.doi.org/10.1093/rfs/12.4.687

Fisher, M.E., Nychka, D., and Zervos, D. (1994). Fitting the term structure of interest rates with smoothing splines, Working paper, Federal Reserve Board. [Online] Available: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=6260

McCulloch, J. H. (1971). Measuring the term structure of interest rates. *Journal of Business,* 44. 19-31. http://dx.doi.org/10.1086/295329

McCulloch, J.H. (1975). An estimate of the liquidity premium. *Journal of Political Economy,* 83. 95-119. http://dx.doi.org/10.1086/260308

Nelson, C.R. Siegel, A.F. (1987). Parsimonious modeling of yield curves. *Journal of Business,* 60(4). 473-489. http://dx.doi.org/10.1086/296409

Ramos Ribeiro, D., & Hodges, S.D. (2004). A two-factor model for commodity prices and futures valuation, January, EFMA Basel Meetings Paper.

Schwartz, E.S. (1997). The stochastic behavior of commodity prices: implications for valuation and hedging. *Journal of Finance*, 52 (3). 923-973. http://dx.doi.org/10.2307/2329512

Sørensen, C. (2002). Modeling seasonality in agricultural commodity futures. *Journal of Futures Markets*, 22. 393-426. http://dx.doi.org/10.1002/fut.10017

Svensson, L. (1994). Estimating and interpreting forward interest rates: Sweden 1992-4. Discussion paper. Centre for Economic Policy Research (1051). [Online] Available: http://www.cepr.org/pubs/dps/DP1051.asp

Table 1. Forward volatility curve residuals, 2001-2006

						Forward Volatilities							
	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20
CBOT Corn #2	0.52%	0.65%	0.56%	0.37%	0.18%	1.74%	1.41%	1.13%	0.87%	0.65%	0.41%	0.20%	0.03%
	(4.89)	(3.72)	(2.89)	(2.56)	(2.21)	(2.45)	(3.42)	(2.67)	(3.69)	(3.41)	(4.12)	(3.66)	(2.91)
NYBOT Cotton #2	1.10%	0.53%	0.21%	0.58%	0.71%	0.83%	0.94%	0.41%	0.11%	0.15%	0.12%	0.23%	0.72%
	(2.67)	(2.87)	(3.18)	(3.04)	(3.51)	(3.07)	(2.91)	(2.90)	(2.51)	(2.63)	(2.89)	(2.49)	(2.08)
NYBOT Sugar #11	1.40%	2.06%	2.03%	2.61%	2.01%	2.23%	2.58%	2.58%	2.49%	3.33%	3.03%	3.69%	3.35%
	(2.07)	(1.97)	(2.41)	(2.09)	(2.11)	(1.91)	(1.88)	(2.31)	(2.10)	(1.95)	(1.77)	(1.60)	(1.69)

Mean residuals between modeled forward contract price volatility and the volatility estimated at each tenor using the general parametric model (2) for corn, cotton and sugar, over the period 2001-2006. The general parametric model is used to estimate the volatility term structure from realized futures volatilities. The respective t-statistics are in parentheses.

Table 2. OLS estimates of $\alpha_{0,k}$, $\alpha_{1,k}$, $\alpha_{2,k}$ $\alpha_{3,k}$ and $\alpha_{4,k}$, 2001-2006

Contract	$\alpha_{0,k}$	$\alpha_{1,k}$	$\alpha_{2,k}$	$\alpha_{3,k}$	$\alpha_{4,k}$
C1	11.70	4.07	5.15	0.15	-2.00
	(3.454)	(3.990)	(2.334)	(3.010)	(-1.870)
C2	11.62	4.29	5.03	0.17	-1.97
	(3.561)	(3.748)	(4.090)	(3.904)	(-1.919)
C3	11.89	3.64	4.46	0.13	-2.07
	(3.980)	(3.770)	(3.390)	(4.098)	(-1.960)
C4	12.04	3.83	4.46	0.17	-2.09
	(2.991)	(4.018)	(4.904)	(4.127)	(-2.806)
C5	12.47	3.42	4.26	0.20	-2.10
	(3.213)	(3.784)	(4.778)	(3.673)	(-2.667)
C6	13.25	2.11	3.55	0.21	-2.24
	(3.391)	(4.551)	(2.811)	(3.496)	(-2.390)
C7	13.44	1.46	3.16	0.13	-2.43
	(2.671)	(2.869)	(2.901)	(2.741)	(-1.896)
C8	13.26	1.77	3.15	0.10	-2.53
	(2.778)	(2.673)	(2.349)	(1.980)	(-1.994)
C9	13.40	1.60	2.96	0.09	-2.57
	(3.118)	(3.756)	(1.912)	(1.982)	(-2.010)
C10	13.83	0.87	2.62	0.09	-2.59
	(2.417)	(2.090)	(3.213)	(1.993)	(-2.991)
C11	14.06	0.62	2.83	0.09	-2.70
	(2.907)	(3.317)	(3.067)	(2.029)	(-2.258)
C12	14.20	0.65	2.94	0.10	-2.97
	(2.545)	(2.793)	(2.997)	(1.988)	(-1.999)
C13	14.32	0.64	3.02	0.09	-3.01
	(2.767)	(3.011)	(3.043)	(1.957)	(-2.677)
C14	14.58	0.34	3.21	0.11	-3.13
	(1.989)	(1.946)	(2.887)	(2.319)	(-2.341)
C15	14.73	0.32	3.42	0.10	2.78
	(2.316))	(2.442)	(2.980)	(2.121)	(2.102)
C16	14.90	-0.02	3.08	0.10	2.52
	(2.567)	(-1.871)	(3.709)	(2.097)	(1.787)
C17	14.93	0.29	2.89	0.06	2.66
	(2.139)	(1.671)	(3.656)	(1.896)	(1.382)
C18	15.15	-0.18	2.99	0.12	2.47
	(1.760)	(-0.991)	(3.609)	(1.290)	(1.430)
C19	15.29	-4.15	16.81	2.71	1.33
	(1.871)	(-2.996)	(2.223)	(1.452)	(1.558)
C20	15.59	-3.53	14.63	2.27	1.38
	(2.190)	(-3.568)	(4.551)	(2.019)	(1.998)

Estimates of $\alpha_{0,k}$, $\alpha_{1,k}$, $\alpha_{2,k}$ $\alpha_{3,k}$ and $\alpha_{4,k}$ from equation (4) using the transformation of equation (7) for each future contract (C1-C8) and modeled forward contract (C9-C20) using the general parametric model (2) for sugar, over the period 2001-2006. The respective t-statistics are in parentheses.

Table 3. D'Agostino-Pearson omnibus test for normality of eighth futures commodity contract residuals, 2001-2006

Model Residuals					
	CBOT Corn #2		NYBOT Cotton #2		CBOT Sugar #11
	C8	C9	C8	C9	C8
Skewness	-0.1552	0.2770	0.1243	0.2706	-0.0981
Excess Kurtosis	-0.2568	-0.7128	-0.3767	-0.2484	0.4240
DAP	1.3521*	5.0928*	1.6980*	2.9555*	4.8940*
Flat-Line Extrapolation Residuals					
	C8	C9	C8	C9	C8
Skewness	-1.5923	-1.5648	-1.2348	2.5375	-0.0567
Excess Kurtosis	9.0950	4.2560	4.9820	6.0325	1.3565
DAP	4643.01	1395.38	1545.94	3107.34	407.30

D'Agostino-Pearson (DAP) omnibus test for normality for the residuals of the traded eighth and ninth contract and the eighth and ninth contracts estimated using a seasonal Nelson-Siegel model for corn and cotton futures prices over the period 2001-2006. Sugar futures prices are compared with only the eighth contract. The eighth contract approximately represents the two-year futures contract maturity, and the ninth contract approximately represents the 2¼-year maturity. The critical value is obtained from a chi-squared distribution, χ^2 with 2 degrees of freedom. The * indicates significance at the 5 percent level. Skewness and excess kurtosis statistics are also provided. The residuals from a simple flat-line extrapolation of futures contract prices from C7 are also provided.

Table 4. Relative spreads of futures and forward contracts, 2001-2006

Futures Prices													
	C1	C2	C3	C4	C5	C6	C7						
CBOT Corn #2	0.48%	0.46%	0.45%	0.89%	1.09%	1.07%	1.05%						
NYBOT Cotton #2	0.58%	0.57%	0.55%	0.91%	.99%	1.09%	1.02%						
NYBOT Sugar #11	0.41%	0.39%	0.54%	0.54%	0.76%	1.14%	1.53%						
Forward Prices													
	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20
CBOT Corn #2	1.05%	1.15%	1.12%	1.18%	1.94%	2.02%	2.85%	2.00%	3.35%	3.98%	3.41%	3.85%	4.54%
NYBOT Cotton #2	1.15%	1.26%	1.42%	1.28%	1.84%	1.18%	2.65%	2.10%	3.37%	2.97%	3.41%	3.76%	3.51%
NYBOT Sugar #11	1.92%	1.29%	1.66%	1.66%	1.03%	1.42%	2.80%	2.19%	2.57%	2.94%	2.95%	2.73%	3.10%

The relative spreads, calculated for the bid-ask spread as a percentage of the asset price, for the exchange traded contracts (C1-C7) and the seasonal Nelson-Siegel model forward contracts (C8-C20) obtained from a commodity broker employing the seasonal NS model, over the period 2001-2006.
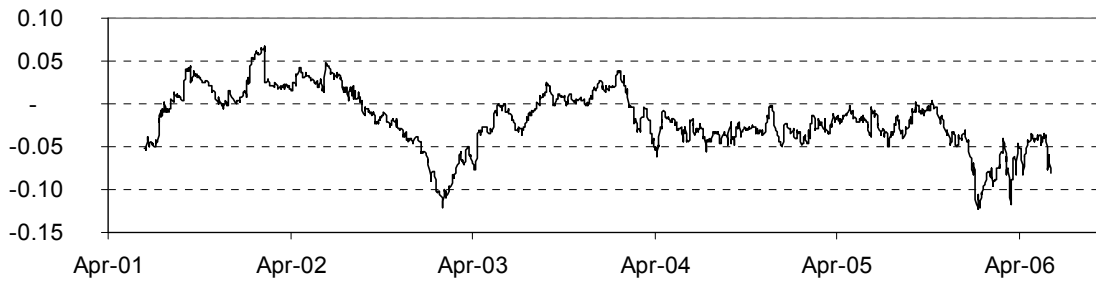
Figure 1. Residuals between the fourth traded contract and the basic Nelson-Siegel function estimated for the fourth NYBOT sugar #11 futures price, 2001-2006. The fourth contract approximately represents one-year to maturity. A slight seasonal pattern in the residuals can be detected
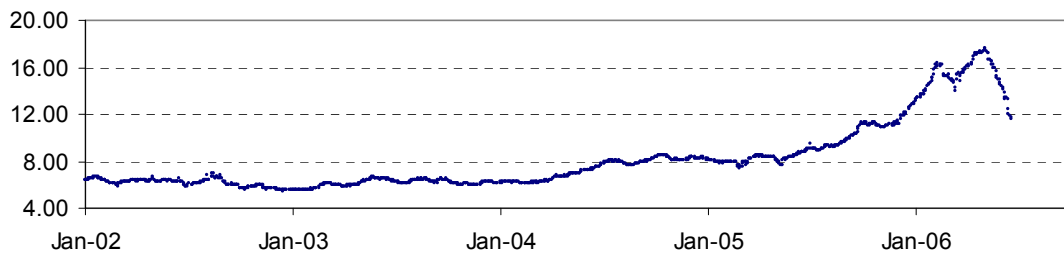

Figure 2. Estimate of $\alpha_{0,k}$ using the seasonal Nelson-Siegel function calibrated for NYBOT sugar #11 futures prices, 2002-2006
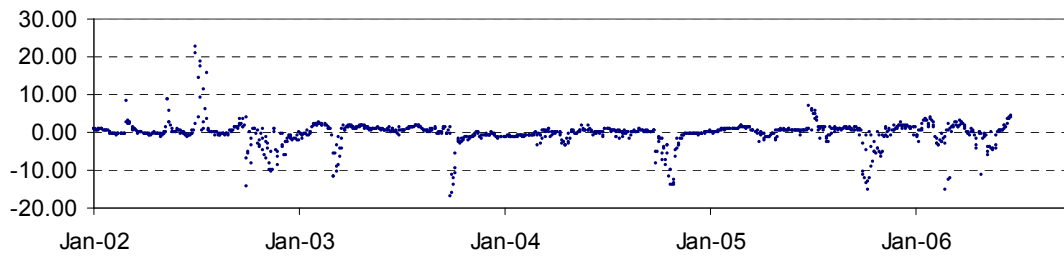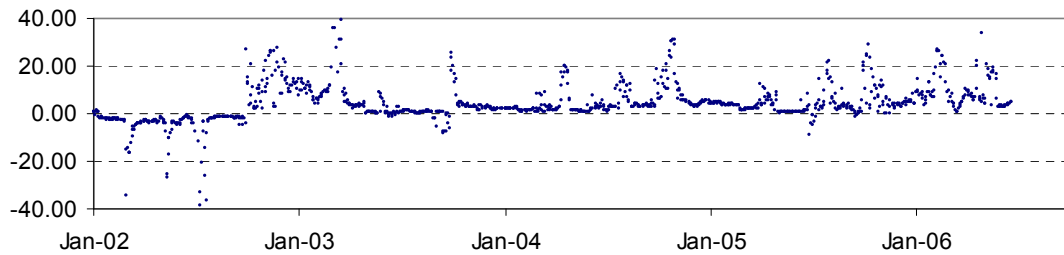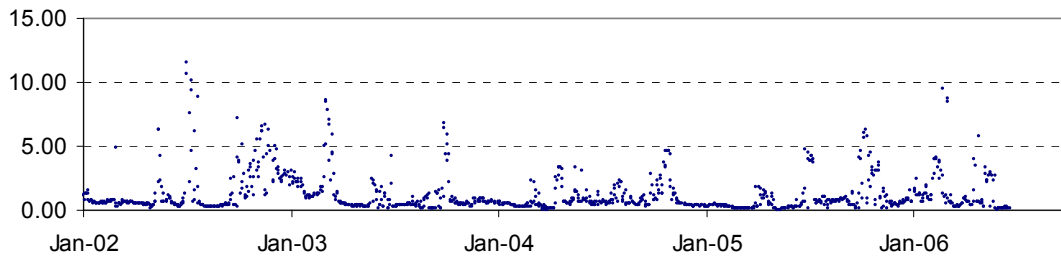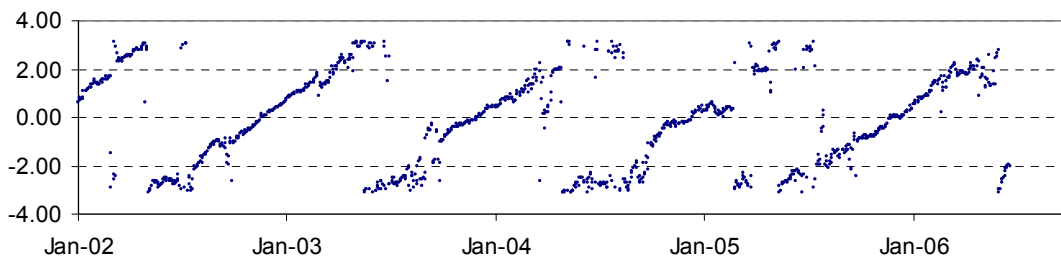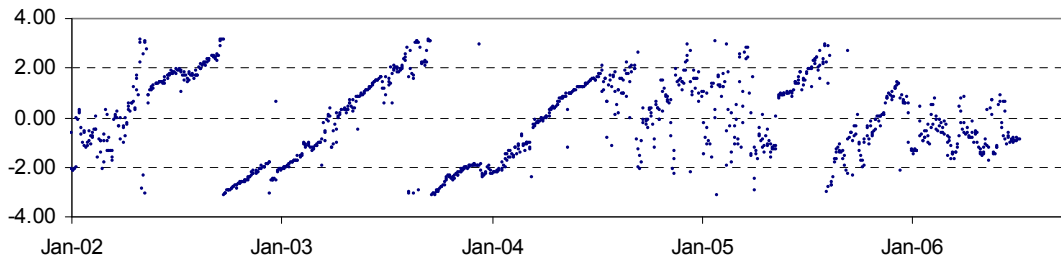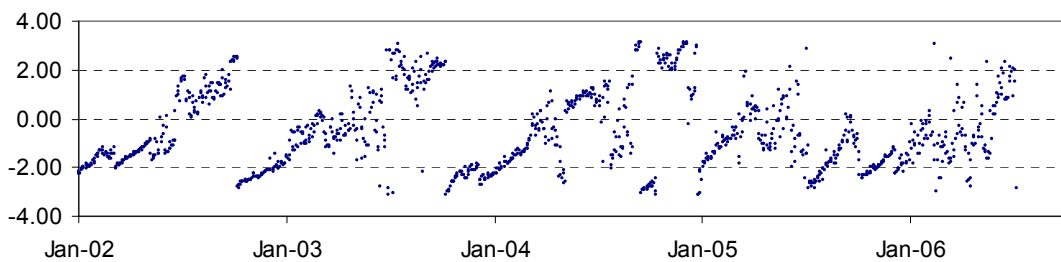

Figure 3. Estimate of $\alpha_{1,k}$ using the seasonal Nelson-Siegel function calibrated for NYBOT sugar #11 futures prices, 2002-2006
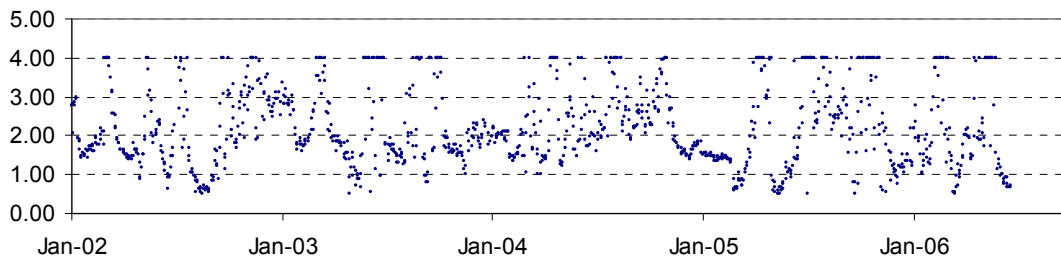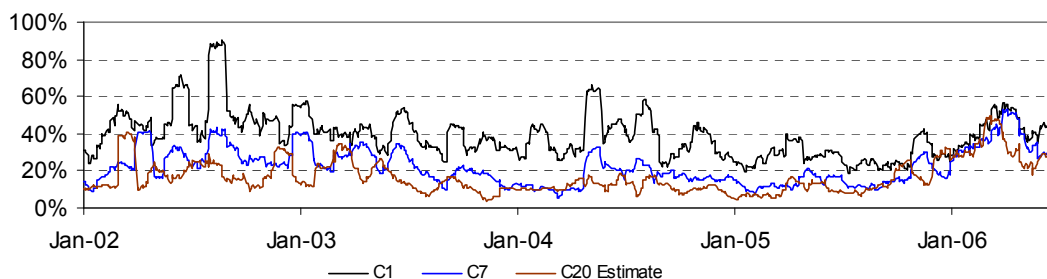

Figure 4. Estimate of $\alpha_{2,k}$ using the seasonal Nelson-Siegel function calibrated for NYBOT sugar #11 futures prices, 2002-2006

Figure 5. Estimate of $\alpha_{3,k}$ using the seasonal Nelson-Siegel function calibrated for NYBOT sugar #11 futures prices, 2002-2006

Figure 6. Estimate of $\alpha_{4,k}$ using the seasonal Nelson-Siegel function calibrated for NYBOT sugar #11 futures prices, 2002-2006

Figure 7. Estimate of $\alpha_{4,k}$ using the seasonal Nelson-Siegel function calibrated for CBOT corn futures prices, 2002-2006

Figure 8. Estimate of $\alpha_{4,k}$ using the seasonal Nelson-Siegel function calibrated for NYBOT cotton futures prices, 2002-2006

Figure 9. Estimate of μ_k using the seasonal Nelson-Siegel function calibrated for NYBOT sugar #11 futures prices, 2002-2006

Figure 10. Historical NYBOT sugar #11 futures price volatilities for the nearest and furthest traded contracts (C1 and C7) plotted against the 5-year price estimated using the seasonal Nelson-Siegel function for 2002-2006
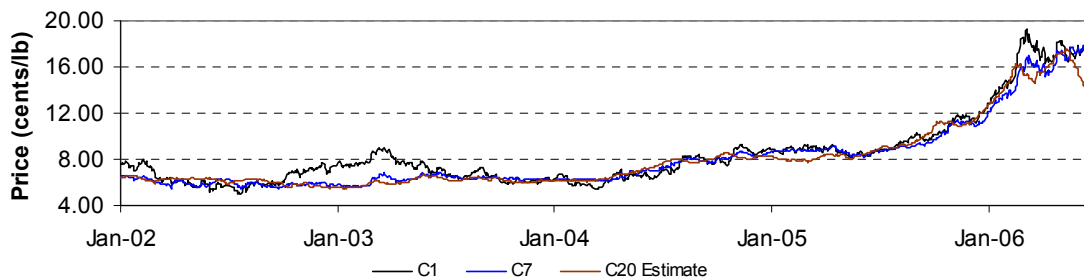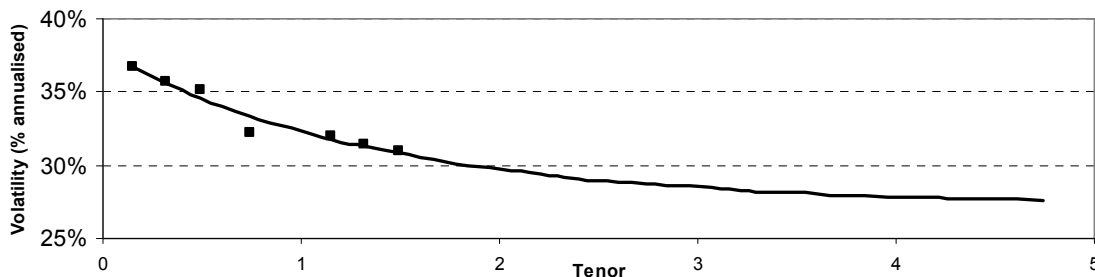

Figure 11. Historical NYBOT sugar #11 futures prices for the nearest and furthest traded contracts (C1 and C7) plotted against the 5-year price estimated using the seasonal Nelson-Siegel function for 2002-2006

Figure 12. Model volatility term structure of NYBOT sugar #11 futures prices (solid line) plotted against 20-day traded contract volatilities (C1-C7) using the seasonal model calibration procedure for 3 Jan 2006
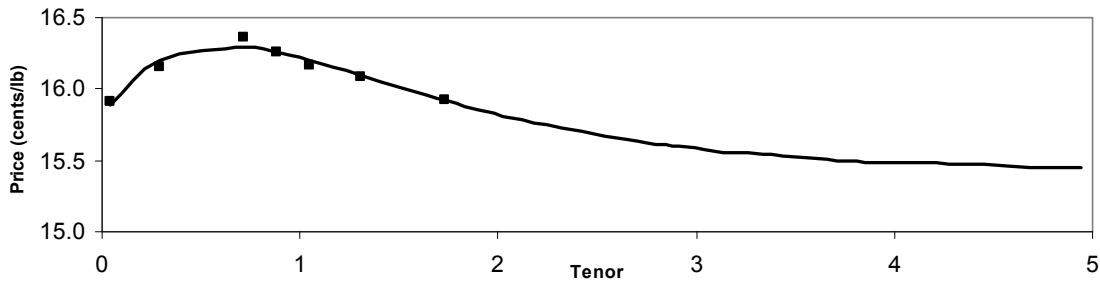
Figure 13. Modeled term structure for NYBOT sugar #11 futures prices (solid line) plotted against traded contracts (C1-C7) using the seasonal model calibration procedure for 3 Jan 2006
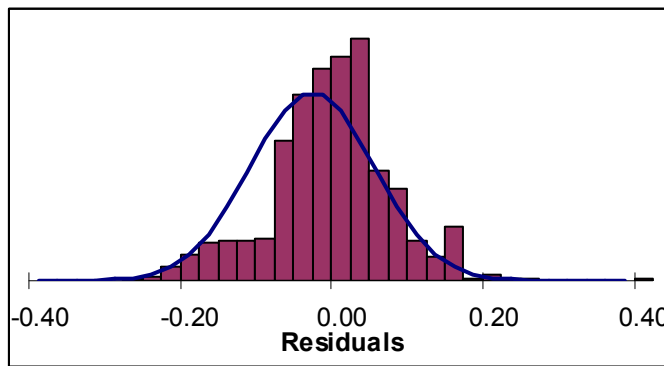

Figure 14. Histogram of residuals between modeled C8 NYBOT sugar #11 futures price and traded C8 futures price for 2002-2006, plotted against a normal distribution
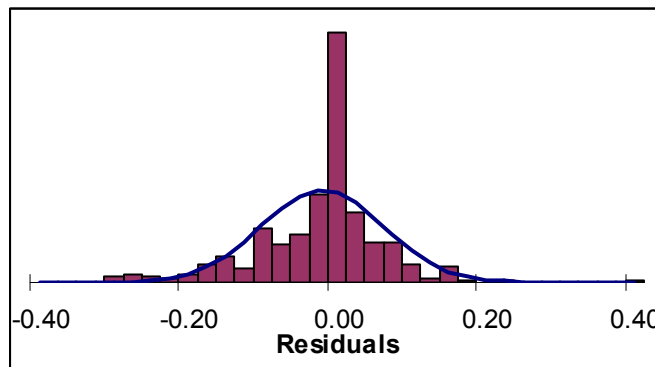

Figure 15. Histogram of residuals between extrapolated C8 NYBOT sugar #11 futures price from the C7 price and traded C8 futures price for 2002-2006, plotted against a normal distribution

Appendix A - Contract Specifications

CBOT Corn #2 Futures: This contract calls for the delivery of corn grade No. 2 Yellow at par, grade No. 1 yellow at 1 1/2 cents per bushel over contract price and grade No. 3 yellow at 1 1/2 cents per bushel under contract price. Price is quoted in cents per bushel and the contract dates are March, May, July, September and December. The daily price limit is twenty cents ($0.20) per bushel ($1,000/contract) above or below the previous day's settlement price. No limit is applicable in the spot month. Contract size is 5,000 bushels.

NYBOT Cotton #2 Futures: This contract calls for the delivery of cotton of certain minimum standards of basis grade and staple length (quality - strict low middling, staple length - 1 2/32nd inch). The minimum price movement is 1/100 of a cent (one "point") per pound below 95 cents per pound and 5/100 of a cent (or five "points") per pound at prices of 95 cents per pound or higher. Spreads may always trade and be quoted in one point increments (point value of $5/contract). The daily price limit is 3 cents above or below previous day's settlement price. However, if any contract months settles at or above $1.10 per pound, all contract months will trade with 4 cent price limits. Should no month settle at or above $1.10 per pound, price limits stay (or revert) to 3 cents/lb. In the spot month there is no limit on or after first notice day. Contract dates are March, May, July, October and December. Contract size is 50,000 pounds net weight.

NYBOT Sugar #11 Futures: This contract calls for the delivery of cane sugar, stowed in bulk, FOB from a number of participating countries. There is no daily price limit and contract dates are March, May, July and October. Contract size is 112,000 pounds (50 long tons). Prices are quoted in cents per pound and the minimum tick size is 1/100 cent/lb.