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Mathematical Models and Methods for Infrastructure Investments: A Critical Analysis of the Models and Methodologies for Infrastructure Investments, Proposed by EDHEC Business School and Moody's

Cecilia Mackie[1]

[1] Mackie Developments, LLC, USA

Correspondence: Cecilia Mackie, Mackie Developments, LLC, USA. E-mail: info@mackiedevelopments.com

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Abstract

The work of seeking more appropriate models for infrastructure finance is critical due to the fact that there is a gap between the amount of infrastructure the world will need in order to support expected rates of economic growth ($3.3 trillion spent annually) and what they are currently investing ($2.5 trillion spent annually). The G20 hope the private sector can fill this gap. One of the problems with the mostly private market for infrastructure project finance is that there is currently no standard, or particularly appropriate financial model in use. The EDHEC Infrastructure Institute (EDHEC) has published over twenty reports on the subject of debt and equity infrastructure investments, which contain mathematical models that were created to develop benchmarks for these kinds of investments. Moody's have also published research that discusses the methodology they use for rating infrastructure project finance. This paper is a critical analysis of the research published by EDHEC and Moody's on the subject of valuing and rating infrastructure project finance. Through their work EDHEC have taken a big step towards standardising valuation methods in a hitherto opaque market. Their model is significantly more robust than Moody's static rating system, which does not take into account the dynamic and path-dependent nature of risk and returns in infrastructure projects.

Keywords: infrastructure finance, G20 initiatives, project bonds, infrastructure bonds, infrastructure equity, project finance, EDHEC Infrastructure Institute, Moody's

1. Introduction

For this study we will be applying a qualitative methodology called critical analysis. Critical analysis is an established research method. The Institution for Engineering and Technology (IET 2021) defines critical analysis as the detailed examination and evaluation of another person's ideas or work (Lima 2021). Dodgson (2021) states that the research process for conducting a critical analysis literature review has three phases (a) the deconstruction phase (b) the analysis phase and (c) the reconstruction phase. Critical analysis is a common methodology used in the field of quantitative finance (Daníelsson, 2000, Norreklit, 2000), in the finance field more generally (Mitsilegas, 2008, Shaoul, 2005, Goldstein, 1971, Szymanski, 2014, Broadbent, 1991) and in related fields (Lima, 2021).

EDHEC is a French business school that was established over one hundred years ago and was ranked as one of the top three business schools in the world by the Financial Times in 2018. Their *EDHEC Infrastructure Institute, Singapore* division (hereinafter shortened to EDHEC) have created over a dozen reports on the subject of debt and equity infrastructure investments, which contain mathematical models that were created to develop benchmarks for these types of investments. Until EDHEC's involvement benchmarks for this sector did not exist. Benchmarks tell asset owners and managers what returns they can expect from their money. Benchmarks also give owners and managers a means to judge performance after their investments have borne fruit. Without benchmarks it is unrealistic to expect mainstream participation among institutional investors in infrastructure. EDHEC claimed to have been working on the largest infrastructure investment data set in the world at the time of the publication of their model (Blanc-Brude 2016). Fourteen reports were carefully considered, which were picked out on the basis that they were considered to contain the essential parts of EDHEC's debt and equity models for infrastructure investments. Two reports on Moody's methodology for the same finance vehicle were also analyzed, so readers may compare different approaches that are being taken in the field. This paper contains the critical analysis of five of the reports produced by EDHEC. The full-length version of this paper, which contains a critical analysis of fourteen of the reports produced by EDHEC, can be found at www.mackiedevelopments.com.

EDHEC's reports are hard to understand without significant review and even then the model is not always spelled out. Several mathematical inconsistencies were found. With all of that said, moments of ingenuity abound. No conventional model fits the specific nature of infrastructure project finance as well as EDHEC's model, due to its treatment of project finance loan covenants like technical defaults, among other things. Infrastructure project finance data does not suit statistical methods due to data paucity. In additional, EDHEC have found that infrastructure investments result in non-linear cash flows based on three different tests and have thus concluded that standard cash flow models are inappropriate for modeling infrastructure project finance.

EDHEC's solution for coping with the idiosyncracies of infrastructure project finance is to utilise Bayesian Inference to model cash flows, for both their equity and their debt models. After that a structural credit risk model is used for the debt component (using the value of the asset, which consists only of cash flows, to calculate the default risk) and a State-Space model coupled with Kalman Filtering is used for the equity component. Overall it is worth contemplating this model and adapting it if necessary. It is also worth conducting empirical studies that use past data to demonstrate that this model is superior to models that are in standard-use, in terms of its ability to predict cash flow distributions and valuations. One of the problems with the (mostly private) market of infrastructure project finance is that there is currently no standard (or particularly appropriate) model in use. This is bound to cause problems with respect to transparency, investor confidence and ultimately the evolution of liquid markets. Understanding the model is important for any infrastructure finance professional and this report was written in part to explain the mathematics in a simpler fashion. This report was also written to pose substantive questions regarding how exactly the model is to be used in practice, so that the model may be utilised by outside parties. In addition, it presents questions regarding the validity of the quantitative and qualitative choices that were made by EDHEC when the model was constructed, so it can be adapted as outside analysts see fit. With respect to EDHEC's benchmarks (and the metrics calculated for them), which went live in the summer of 2019 and are available for use by subscription, the insights provided in this report reveal the extent to which EDHEC's models can be relied upon in their current state.

After writing this report thirty pages of inquiries were submitted to EDHEC. That submission, named "Requests for Clarification," can be found in the Appendix A of this report. The Requests for Clarification document was sent to the director of EDHECInfra and as yet it has remained unanswered.

The work of seeking more appropriate models for infrastructure finance is critical due to the fact that there is a gap between the amount of infrastructure the world will need to support expected rates of economic growth ($3.3 trillion spent annually) and what they are currently investing ($2.5 trillion spent annually) and the G20 hope the private sector can fill this gap. Emerging economies will account for 60% of this infrastructure gap. For the last several years the G20 have been developing resources to facilitate a greater involvement of private investors in infrastructure. Notwithstanding this impetus from the world's largest governments, the participation of the world's largest institutional investors is a negligible 0.7% (World Bank Group, 2018). In the EDHEC 2019 investor's survey that received responses from asset owners that manage over $10 trillion in infrastructure assets collectively, most asset owners and managers were still using inadequate absolute or relative benchmarks. This implies that institutional investors have not yet begun to use EDHEC's benchmarks or models.

The report is laid out as follows. After this brief introduction the reports are taken one by one, summarised and any issues that were found are discussed in depth. Following that a section titled Discussion, lists that main points that have arisen as a result of this critical analysis. A conclusion follows thereafter.

2. Reports

2.1 Report #1 - Construction Risk in Infrastructure Project Finance (February 2013)

In February of 2013 EDHEC released an analytical report of construction risk in infrastructure. This report begins by going over traditionally procured infrastructure construction risk studies that have been conducted in the past. Traditional government projects systematically overrun in terms of cost, even before accounting for the fact that the principal of each project is incentivized to reveal only data that puts him in a favorable light. On top of this cost overruns can lead to more cost overruns cost leading to a positively skewed distribution. At times they have been seen to exceed initial estimates by 200%.

Public Private Partnerships (PPPs), which use a financial structure called *Project Finance,* (Note 1) have been introduced to counteract construction risk because they explicitly transfer it from the public to the private sector.

In this report on the subject EDHEC presents compelling data showing that when this *Project Finance* structure is used (including but not limiting the analysis to PPPs) the construction risk is zero. This is probably because incentives are well aligned. In *project finance* the builder of the project is often a shareholder of the Special Purpose Entity (SPE), which is the controlling entity in *project finance* and shareholders would be contractually obliged to inject capital if costs overrun. It is specifically the SPE that is pointed to as being immune from non-diversifiable construction risk. This means that investors in the debt and equity of the project, which yield income and dividends from the SPE, are protected from construction risk.

For this report EDHEC looked at 75 projects with consistent financial data definitions due to the fact that the data all comes from a single bank, NATIXIS. For the purposes of this study construction risk is defined as the ratio of contracted value at financial close against the actual cost at contract completion. The projects come from diverse sectors in all five continents and range in size from $12 million to $24 billion. As expected the data was positively skewed which means the median result will be a better indicator than the calculated average, of how the projects in the middle of the data set perform. The most crucial takeaway here is that "*The median of construction risk is not statistically different from zero at the 1% confidence level"* (World Bank Group, 2018).

Interestingly the researchers go on to regress four contract features that are used to protect the SPE against construction risk (Note 2) found in the data set of these 75 projects that were funded by the NATIXIS bank. They find that the presence of these incentive mechanisms does not significantly correlate with reduced cost overruns suggesting that they could be removed.

Notwithstanding the fact that the *project finance* structure results in better outcomes it is still relatively uncommon.

"[In the] National Construction Contracts and Law Survey 2012 for the UK construction sector (NBS 2012) … on average 60% construction contracts are still traditionally delivered, 29% are design and build, and only 11% are referred to as 'other' contract types" (World Bank Group, 2018).

2.2 Report #2 - Benchmarking Long-Term Investment for Infrastructure (June 2014)

This enlightening report introduces EDHECs primary goal, which is to enable more private investment in infrastructure. The report launches into the existential quandary pertaining to investments in this sector by explaining that there are problems in defining what infrastructure investments even are. It is extraordinary to discover that the most commonly traded financial instruments thought to represent them, do not properly reflect investments in the underlying asset. The most common instruments are comprised of listed and unlisted infrastructure equity products that come in the form of listed infrastructure indices and unlisted infrastructure Private Equity (PE) funds (Blanc-Brude, 2014). Whereas investors assume infrastructure gives them exposure to long-term, inflation-protected assets with stable cash flows EDHEC aggregate research that shows that neither listed infrastructure nor PE infrastructure funds do these things effectively. The case is thus made that a clear solution must be found to properly define these assets before modelling work can begin.

Beyond that a caveat may serve those already invested in those forms of assets. Research has shown that returns that are reported by PE managers are not indicative of the returns that actually manifest after you invest. Regarding listed instruments, corporate bonds issued to utilities will not have the characteristics of loans extended to infrastructure projects. These common infrastructure investments do not deliver the unique risk-return profile that infrastructure is thought to be capable of bringing to an investment plan.

Project finance is the financial structure that *"represents the bulk of infrastructure projects today."* (Note 3) A key feature of the *project finance* structure is that there is no recourse for investors beyond claims on the Special Purpose Entity (SPE), which is especially created for each project. Investors therefore rely entirely on the cash flows and assets of the project for recouping their investment and the SPE is restricted in taking on more debt. Leverage is high notwithstanding this, which indicates that the expected probability of default is low. Contracted covenants give the lender control over the project's contractor and cash distributions if things do not go as planned. These standard features mean that a population of projects with this SPE structure can be expected to behave in a homogenous fashion, which can give rise to an asset class that can prove useful to money managers. Perhaps unsurprisingly the majority of project finance equity and debt is privately held. The project's equity is generally owned by either the construction company that is building the project, or by funds or private investors.

EDHEC claim that little or no work has been done in the area of creating a modelling framework for *project finance*. (Note 3) This mathematical layout is necessary in order to derive the financial indicators that are currently used in modern finance to construct portfolios. Without a way to calculate financial parameters like correlation and risk, assets cannot be integrated into an asset allocation plan and investment can only be done blindly. One of the logical starting points is to develop a way to benchmark *project finance* investments. This will allow investors to see what returns they can expect from such investments *a priori*. Also, it will allow investors to gauge whether their investment has performed well against the investment's asset class *a posteriori*. Asset managers are heavily reliant on both of these processes in all investments they make. However, when one begins to analyse *project finance*, many problems arise.

Perhaps the greatest obstacle is the fact that existing *project finance* data, which must be interpreted to model the asset class, is not particularly helpful. Firstly, there are practically no purchase or sale transactions through which to glean information on value. Secondly, it is likely that only ten years of cash flow data can be collected on infrastructure projects (as of the time of this report,) yet these projects have typical lifespans of 20 or 30 years. These shortcomings are coupled with the fact that the data is not in the public domain so the cooperation of a number of participants may be required. Also, only dozens of investable infrastructure projects are created every year in the most active markets so data paucity affects the statistical significance of mathematical calculations. (Note 4).

EDHEC believe the answer to all of these issues is to use Bayesian Inference techniques. Bayesian Inference is used by EDHEC for its infrastructure models in two different respects. They use it to calculate the probability of default. They also use it to calculate the cash flows for equity and debt. A binomial distribution is selected to represent the probability of default because a binomial only has two outcomes. Calculating a cash flow distribution is more complex than calculating a binomial distribution because there are many possible outcomes so a different distribution is used. The Bayesian Inference treatment used for the probability of default and for cash flow distributions is covered in EDHEC's report on Cash Flow Dynamics (Report #5) and I will cover the mathematics when I summarize that report.

On a separate point, this report relates to two important values, Probability of Default (PD) and Distance to Default (DD) but the description is unfathomable and it is not accompanied by any equations in this report or in others. More detail on this is given.

The rest of this report describes the methods that will be used for the debt and equity models. I will be covering that information in the debt and equity reports that follow.

2.3 Report #3 - Unlisted Infrastructure Debt Performance Measurement (Aug 2014)

EDHEC's debt model for infrastructure investments is contained within a report by Blanc-Brude & Hasan called *Unlisted Infrastructure Debt Performance Measurement (Aug 2014).* When I refer to pages in the explanation below I am referring to pages of this report.

In this report EDHEC start out by criticizing common debt valuation models. A top choice for analysts is to use *capital budgeting* models. These models compare the initial investment with future cash flows in different ways. Examples of *capital budgeting* models include calculating the time it will take to pay the investment back with future cash flows *(Payback Period,)* or calculating whether the present value of future cash flows exceeds the investment *(Net Present Value.)* The problem with making your investment decision using *capital budgeting* models is that they all use a constant rate of return on your money and a constant loss given default, but EDHEC believe that both of these values vary throughout the life of the project.

EDHEC's carefully considered models ostensibly resolve a lot of the problems that exist in current valuation methods. The SPE consists of claims on future cash flows. The SPE also owns the infrastructure asset but this often has no value *"...outside of the contractual framework…"* For this reason, EDHEC argue that a *structural credit risk* model should be used. A *structural credit risk* model simply means that the default risk is calculated using the value of the assets, which in this case are the future cash flows. A commonly used *structural credit risk* model is the Merton Model.

Another crucial feature of *project finance* debt is the fact that because lenders are typically highly leveraged, they have debt covenants specified in the contract for protection, which ultimately impact recovery rates (the amount of money recovered after a default.) Examples of these options/covenants include *"lockups"* which allow lenders to block the equity dividend and *technical default,* which gives lenders the right to restructure the debt. These contract features tend to use different levels of the Debt Service Coverage Ratio (DSCR) as a trigger. Thus lenders keep track of the DSCR, although EDHEC has struggled to obtain that data.

$$DSCR = CFADS/DS$$

Where CFADS is Cash Flow Available for Debt Service and DS is Debt Service.

Once we learn this it is logical to assume that "*The model should incorporate path dependent cash flows driven by* *embedded options."* Accordingly, many metrics in the EDHEC debt model depend on the DSCR process.

The report then divides *project finance* vehicles into two types. *Contracted Infrastructure* (for which payments depend on a fixed sum from a government entity) and *Merchant Infrastructure* (for which payments depend on market demand.) *Project finance* has a tail, which allows investors to receive payments beyond the maturity of the loan, in cases where payments fall behind. *Contracted Infrastructure* has leverage of about 90%, a flat DSCR and a shorter tail. This can be modelled as a normal distribution.

$$DSCR_t = E[DSCR] + \sigma(DSCR)dW_t$$

Is E(DSCR) the expected value for DSCR or is it a function of DSCR? If it is the later what is the function? I have asked EDHEC to clarify (Appendix A, part 3.1).

Merchant Infrastructure has leverage of between 70 and 80%, a rising and increasingly volatile DSCR and a longer tail. EDHEC model it as the following lognormal distribution:

$$d(DSCR_t)/DSCR_t = \mu dt + \sigma dW_t$$

Equation 5 above is taken from a report on the subject of *risk pricing* written by Shaun S. Wang. (Note 5) Within Wang's report *the random movement of the price of an asset* is described by Equation 5, which is written above however in EDHEC's report the price of the asset has been substituted by the DSCR. EDHEC do not define dW_t, μ and σ in their report.

Wang's report explains that *"dW_T is a random variable drawn from a normal distribution with mean equal to zero and variance equal to dt."* In that report μ is defined as the *rate of return of the asset price* and σ is *the volatility of the asset return*. Remember that EDHEC have substituted the asset price for the DSCR. Note that the left hand side of the equation is also equal to the rate of return of the DSCR, or μ. I have a number of issues in relation to these equations. I have asked EDHEC to clarify them (Appendix A, part 3.1).

After using Equations 5 and 6 to define the cash flow distributions these distributions will be undergoing the Wang transform in order to value the underlying asset. The Wang transform gives a risk-free value for the distributions, which is equivalent to discounting the cash flows. This is a practical solution for valuing cash flow distributions when the excess returns cannot be calculated. The following two equations from EDHEC's report are Wang transforms:

Risk Neutral Distribution of $DSCR_T$:

$$F^*(DSCR_T) = N\,(N^{-1}[F(DSCR_T)] + \lambda_T)$$

Where:

$F(DSCR_T)$ and $F^*(DSCR_T)$ are the physical and risk neutral distributions of $DSCR_T$.

The value EDHEC gives for λ_T is questioned (Appendix A, part 3.3).

I have asked EDHEC what the function for N is (Appendix A, part 3.2).

Risk Neutral Probability of Default q(t,T):

$$q(t, T) = N\,(\,N^{-1}[p(t, T)] + \lambda_T\,)$$

Where:

p(t,T) is the physical probability of default,

q(t,T) is the risk-neutral probability of default,

EDHEC follow Equation 15 with the following statement:

"[F*(DSCR)] *follows the same distribution (normal or lognormal) with a shifted mean μ - λ σ. Hence, the risk* [free] *distribution of the DSCR would* [be] *the same as the physical distribution of the DSCR but with a shifted mean."*

Let me try to make sense of that. As you know Wang constructs a logarithmic distribution where μ is defined as the *rate of return of the asset price* (Equation (4) on p. 7 in Wang's report and Equation 5 on p. 12 in EDHEC's report, which has been written out above). He then states that:

$$\lambda = \frac{(\mu_i - r)}{\sigma_i}\sqrt{T}$$

The above equation should look familiar because the Sharpe ratio is constructed in the same fashion:

$$\lambda_i = \frac{(E(R_i\ ) - R_f)}{\sigma_i}$$

Rearranging Equation (6) on p. 7 of Wang's report gives:

$$\mathbf{r}\ (= \mathbf{R_f}) = \ \mu_i - \frac{\boldsymbol{\lambda}\ \boldsymbol{\sigma_i}}{\sqrt{\boldsymbol{T}}}$$

The risk free rate of return is equal to the rate of return of the asset with a shifted mean. Ergo the risk free (a.k.a. risk neutral) rate of return has a *"shifted mean μ - λ σ."*

EDHEC give the following equation for λ, which is clearly copied from Wang's report:

$$\lambda_T = \frac{(\boldsymbol{\mu}-\mathbf{r})}{\boldsymbol{\sigma}}\sqrt{\boldsymbol{T}-\boldsymbol{t}}$$

As you can see EDHEC's value for λ contains the value μ however Equation 6 on p. 12 (written out above), which describes infrastructure projects with a contracted business model, does not contain μ. Only Equation 5 on p. 12 (written out above) contains the value μ and that equation applies to cash flows of merchant projects. That means that the λ given by EDHEC does not necessarily apply to contracted projects. I have asked EDHEC to clarify what λ equates to for Equation 6 and for Equations 14 and 15. I have also asked EDHEC to clarify what σ_i and μ_i equate to since they are parameters involved in calculating the value of λ for these three equations (Appendix A, part 3.3).

The reason for using these transforms in explained in an earlier report on Benchmarks (Report #2 in Appendix A) (Blanc-Brude, 2014):

"…valuing project debt implies taking into account future cash flows in different states of the world, including post renegotiation, implying actual discount rates from initial lending decisions and a measure of risk is not possible. Instead, [we model] *the subjective probabilities that investors assign to future risky cash flows.*

These probabilities are known as the risk neutral probabilities, and the valuation method is called risk neutral valuation, … (i.e. [discounting] *the future cash flows at the risk free rate)…….*

Risk-neutral valuation adjusts for risk aversion by assigning a lower probability to riskier cash flows, and hence decreasing their expected value under the risk neutral distribution, instead of discounting the expected values under the physical distribution at a higher discount rate — both approaches are indeed equivalent."

In other words you use this method to conduct valuations for debt instruments. Typically you discount cash flows to calculate the price/value of an asset. Because you cannot calculate the discount rate for cash flows of debt instruments you calculate the probability of each cash flow so that riskier cash flows are given lower probabilities and thus lower values, which is equivalent to discounting them with higher discount rates.

In the body of their reports EDHEC never mention Wang transforms. I believe they should mention them because without studying Wang's report the above four equations from EDHEC's report would have been unfathomable.

Moving on, a fairly intuitive approximation of the Merton Model published by Crosbie and Bohn is cited by EDHEC to calculate the distance to default. In Crosbie and Bohn's equation a default asset value is deducted from the current value of assets to determine the distance to default. EDHEC substituted assets for Cash Flow Available for Debt Service (CFADS) and the asset value "default point" for the Debt Service (DS) since that is the trigger point for a default in project finance.

$$DD_t = (CFADS_t - DS_t^{BC}) / (\sigma_{CFADSt} * CFADS_t)$$

The Value of a Loan in Project Finance:

a) Payments made until restructuring, refinancing or maturity (including final payment at maturity)

b) Value of restructured SPE after technical default

c) Value of restructured SPE after hard default

d) Value of refinanced SPE

On p. 16 and 17 EDHEC define four payout functions but these do not match the calculations that follow in the report. Because of this I have adjusted my version of the payout functions in a) through d) above so that they match the mathematical guidance given by EDHEC in their report. Let's take them one by one.

a) Payments made until restructuring, refinancing or maturity (including final payment at maturity)

This does not need a calculation because it is simply the existing debt schedule.

b) Payments for a restructured SPE after technical default

Equation 15 on p. 14 (written out above) will be used to calculate the outcome of the restructuring after a technical default. This is how the calculation is implemented. The calculation pivots around the debt holder's original IRR, which must remain fixed. The original value of the debt, which is calculated using the original debt schedule and the original IRR, also remains fixed. A risk neutral DSCR distribution is calculated using the original debt schedule. Then risk neutral DSCR distributions are calculated with a variety of new potential debt schedules to find one where the risk neutral value of the new debt schedule plus the cost of restructuring exceeds the risk neutral value of the original debt schedule.

c) Payments for a restructured SPE after hard default

In the event of a hard default the debt owners control the SPE and have four choices:

1. Liquidation of the SPE;
2. Switch of equity stakeholders (owners);
3. Sale of loan;
4. Restructuring of debt and equity.

EDHEC explain that default in *project finance* projects is rare and when it does happen the most common result is a restructuring (Blanc-Brude, 2014) (however this appears to contrast with data from a later report (Note 27.) Thus, EDHEC spends some time explaining how to calculate equity and debt values after a restructuring. The model cleverly links all resulting values to the liquidation value of the project since the debt holders will weigh their options against the possibility of liquidating the SPE. EDHEC have drawn an ingenious chart on p. 20 (figure 3) showing the equity and debt values on one axis and the liquidation value on the other (the NPV axis in figure 3 is *"the liquidation value of the SPE at time of default")* so you may visualize the resulting values of the debt and the equity as the liquidation value falls.

$\hat{V}$ is the value debt holders get for the SPE in the event if is liquidated.

V is the value of the restructured SPE.

$\breve{V}$ is the value of the SPE under the existing debt schedule.

L and R are the liquidation and restructuring costs respectively.

The liquidation value is therefore $\hat{V} - L$. The relationship between the liquidation value and the equity and debt values is divided into five sections. We will start with the highest liquidation value (moving along the NPV axis from right to left.) In all cases the equity holders will have to give the debt holders at least the value of liquidation since the debt holders always have the choice of liquidating after a hard default. Note that there appears to be an assumption throughout that that $\breve{V} = V$. This is confirmed on the graph. However, in an earlier part of the report EDHEC say "*If the market value of the new debt schedule* [V]*, net of rescheduling costs, exceeds the market value of the original debt schedule,* [$\breve{V}$]*, the new debt schedule is preferred;"* which conflicts with this assumption. For more detail on this and other problems with this section of EDHEC's report on debt see Appendix A, part 3.4.

1. If the liquidation value is greater than the value of the restructured SPE the debt holders will liquidate and the equity owners will receive nothing.

$\hat{V} - L > V$ (region 1 of figure 3 on p. 20)

$V^D = \hat{V} - L$

$V^E = 0$

2. If the liquidation value is less than the value of the restructured SPE then restructuring will be considered. Depending on the liquidation value, the equity and debt will either restructure and receive a new debt schedule or keep the existing debt schedule. As the liquidation value drops equity holders will receive a greater proportion of the SPE.

$\hat{V} - L < V$

The four options that fit into this category are as follows:

(a) If the liquidation value is less than the value of the restructured SPE but is greater than the value of debt under the existing schedule it is worth restructuring to release this value. The debt holders keep the liquidation value and the equity can keep any remaining value.

$\hat{V} - L > \breve{V}^D$ (region 2 of figure 3 on p. 20)

$V^D = \hat{V} - L$

$V^E = V - (\hat{V} - L)$

(b) Neither (a) nor (c) below, hold (region 3 of figure 3 on p. 20)

$$V^{D} = \breve{\boldsymbol{V}}^{D}$$

$$V^{E} = \breve{\boldsymbol{V}}^{E}$$

(c) If the liquidation value is so small that the value of the restructured SPE minus the liquidation value (which must be given to debt holders) and minus the costs of restructuring is greater than the value of the equity stake under the existing debt schedule then the restructuring will release value. However, this appears to be incorrect since equity holders would do a deal with the debt holders to redistribute the existing debt schedule as they did in part (a) to avoid paying the restructuring costs and share any savings. If they did so, both equity and debt lines in region 4 would move up half a notch, which is a win win for both.

$$V - (\widehat{\boldsymbol{V}} - L) - R > \breve{\boldsymbol{V}}^{E} \qquad \text{(region 4 of figure 3 on p. 20)}$$

(d) Here the liquidation value is so low it is less than half the value of the restructured SPE. Presumably the equity owners and debt holders must both approve the restructuring and are evenly matched in their ability to make demands over the value of the SPE, which is consequently split down the middle.

$$0.5\, V > \widehat{\boldsymbol{V}} - L \qquad \text{(region 5 of figure 3 on p. 20)}$$

$$V^{D} = \mathbf{0.5}\ V$$

$$V^{E} = \mathbf{0.5}\ V$$

EDHEC did not explain these equations in plain language. See Appendix A, parts 3.8 to 3.13.

d) Value of refinanced SPE

$$\mathbf{P}\ (\boldsymbol{\tau}) = \ (\mathbf{1}\ +\ \mathbf{c})\sum_{\boldsymbol{i}=\boldsymbol{\tau}}^{\boldsymbol{T_D}} \mathbf{e}^{-\mathbf{IRR}(\ \mathbf{i}\ -\ \boldsymbol{\tau}\ )}\mathbf{DS}_{\boldsymbol{i}}$$

Where c = refinancing costs and IRR is the original IRR of the loan.

Here EDHEC is simply saying that in the event of a refinancing the value received is simply the value of the debt at that time. Therefore, the financial benefits achieved from any future refinancings and/or defaults that result beyond this refinancing horizon do not appear to be accounted for within this valuation model. This is an acceptable approximation.

Obviously this model would be laborious to implement by hand so a computer algorithm is suggested and illustrated in Figure 4 of p. 26. You start with the base case debt schedule (DS) and you pick the right DSCR model (merchant or contracted.) The DS and DSCR give the CFADS. (Note 6) Next you look at the debt covenants of the loan, which include the technical default threshold and other features such as reserve accounts, cash sweeps and claw back provisions. Figure 4 shows you how the CFADS is used to make equity payments, which are affected by payments to the reserve account and cash sweeps to the senior debt holders. According to the notes the algorithm is also affected by claw back provisions but details are not specified. Based on the Figure 4 it looks like none of the CFADS remains at the end of any given time period because it has either been paid out to the shareholders or the senior debt holders. I have asked EDHEC if this is so.

The algorithm is intuitive. You begin at t = 0 and you cycle through the algorithm in each time period. The CFADS distribution acquired above is used to "*project CFADS paths for future periods.*" For each path, the CFADS values in each time period are compared with the technical and default thresholds so that parts b) and c) of the model can be applied where necessary. Refinancing is triggered "*if the SPV has transitioned into a sufficiently low risk environment,*" "*for each projected CFADS path.*"

The whole point of the report is to be able to value the loan. This is subtly addressed in the following statement:

"*The total value of a PF loan is the expected present value of the weighted sum of these 4 payout functions under the risk-neutral measure, discounted at the risk-free rate.*" (Note 7)

As explained earlier the weightings are calculated by using Equation 15, which assigns a lower value to cash flows that have higher risk but many parts of Equation 15 and associated Equations 5, 6 and 14 have not been explained. I have asked EDHEC about this.

"The objectives of this paper are: ... 3. To derive the most relevant return and risk measures for long-term debt investors and regulators: per period loss, value-at-risk (VaR), expected shortfall or Conditional VaR (CVaR), expected recovery rates, duration, yield, and z-spread." (Note 8)

Equations for Duration, Expected Loss, Recovery Rate, yield and z-spread are all laid out in the Appendix but "*per period loss, value-at-risk (VaR), expected shortfall or Conditional VaR (CVaR)"* are not given in this report by EDHEC. I have asked EDHEC to explain how they plan to calculate these missing metrics.

2.3.1 Discussion

EDHEC's debt model values a project by calculating the value of a project based on different outcomes and summing the weighted values of each outcome using a weighting that has accounted for the probability of each outcome. This probability is calculated using a method called *"risk neutralization"* which is equivalent to discounting a valuation to give its value at an earlier point in time. That is because *"risk neutralization"* causes the probability to be lower for outcomes that have higher risk. Ergo it assigns a lower valuation those higher risk outcomes. It is equivalent to discounting because higher risk means a higher discount rate, which results in a lower value after discounting.

Having read my summary on this report (Report #3) alongside the referenced parts of Appendix A you will know that some of the details of this process are unclear, therefore I have asked EDHEC to confirm that the summary of the debt model given in the above paragraph is not inaccurate.

Parts of this report on EDHEC's debt model need to be fleshed out but overall the debt model is well constructed. It is plain to see that a model, which accounts for both the explicit functionality of debt covenants as well as accounting for the path taken by homogenous and exhaustive DSCR profiles (like those of the merchant and contracted business models) is likely to be more accurate than models that do neither, like many infrastructure models that are commonly used today.

2.4 Report #4 - The Valuation of Privately-Held Infrastructure Equity Investments (Jan 2015)

EDHEC's equity model for infrastructure investments is contained within a report by Blanc-Brude and Hasan called *The Valuation of Privately-Held Infrastructure Equity Investments (Jan 2015).* When I refer to pages in the explanation below I am referring to pages of this report.

Equity investments in infrastructure have a dynamic risk profile due to several factors. Deleveraging happens as time goes by and this is coupled with decreasing duration. There is also a resolution of uncertainty (e.g. construction completion). EDHEC argue that there are three main valuation methods used in academia and they are all inadequate. These include: observing infrastructure transactions (which are too infrequent); using public market assets as proxies (which defeats the object of considering infrastructure investments as instruments that are unlike stocks and can thus provide diversification benefits) and cash flow approaches, which use past cash flow data. A standard cash flow approach is not possible in infrastructure due to the fact that most cash flows are unknown because they lie in the future. (Note 9)

As a solution to this EDHEC suggests that we model future cash flows as best we can and then calculate rates of return using initial transaction prices. The data we have at our disposal for infrastructure projects consists of the initial transaction price; partial cash flows going back 10 years or so (Note 10) and some investment characteristics needed to separate these investments into sub-groups.

The Institute proposes a metric that will be used widely throughout this report. This is the Equity Service Coverage Ratio (ESCR), which is the ratio of the expected (or realized) cash flow divided to the original (base case) cash flow forecast.

$$ESCR^i_t = C^i_t / C^0_t$$

The cash flows for the project, C^i_t are derived using the above equation. (Note 11) This is because the base case distribution, C^0t is known and the ESCR distribution is modelled using Bayesian Inference. EDHEC explain how they use Bayesian Inference to model the ESCR distribution within this report but they also cover the same method when they model debt cash flows (Debt Service Coverage Ratio - DSCR) in their report on the Cash Flow Dynamics of Project Debt (Report #5). Since the mathematics is identical I will cover it when I summarize Report #5.

An important note on nomenclature here, EDHEC use five different terms that are equivalent and they use them interchangeably throughout their reports. Those five terms are: *discount factors*, *risk-premia*, *excess returns*, *the State Vector* and *the term structure.* I will always refer to them in this report as *excess returns* to avoid confusion. *Excess returns* are an investment's rate of return minus the risk free rate. A main impetus for developing the equity model is to enable the calculation of *excess returns.* Three different symbols are used to represent *excess returns* throughout this EDHEC report:

$$\lambda, \Theta, \Lambda$$

λ is an *excess return* value for a single point in time. When there is a time series of *excess returns* I call that a *time series of excess returns*, which can come in matrix form. θ and Λ are both matrices (specifically, a time series) of *excess returns*, which EDHEC may refer to as *the State Vector* or *the term structure.* EDHEC use the words *dividends* and *cash flows* interchangeably too.

Getting back to *excess returns*, studies have shown that *"the use of a constant discount rate compared to the use of a term structure of time-varying discount rates… can lead to mis-pricings well over 50%"* (Blanc-Brude 2015). It is therefore advisable that a term structure (a time-series) *of excess returns* is calculated.

EDHEC suggest the use of a *state-space model* to deliver a *time-series of excess returns.* This is a model that contains two equations, which are called the State Equation and the Observation Equation. The State Equation equates the *time-series of excess returns* from a previous state to the *time-series of excess returns* in a new state. What exactly is a state? Each subsequent state contains the data from a new transaction, k. By referring to the *time-series of excess returns* as θ_k, throughout this report, EDHEC make things very confusing because analysts could reasonably assume this means the *time-series of excess returns* for transaction k, but this is not so. θ_k is the *time-series of excess returns* of a group of infrastructure investments. To complicate things further, although EDHEC refer to k as *a transaction* (Blanc-Brude 2015), each subsequent state θ_k appears to contain the data from a *group* of new transactions (e.g. 20 transactions.)

"*...all the model's parameters are estimated on a rolling basis as new observations are made, e.g. every 20 [transactions]... capturing the evolution of the market at regular intervals..."* (Blanc-Brude 2015).

However this conflicts with the following statement on p. 50:

"*[the] state or transition equation ... describes the dynamics of this state, from one observation (transaction) to the next."*

It highlights just a few of the numerous instances in the report where EDHEC has stated that each new state contains information from a single new observation/transaction and other instances where they have stated that each new state integrates the data from a group of observations/transactions. I have asked EDHEC to clarify.

The State Equation:

$$\theta_k = H_k\theta_{k-1} + B_k u_k + w_k$$

- H_k "*captures the expected return autocorrelation between individual transactions (market sentiment)."*
- $B_k u_k$ "*captures the effect of change in discount rates due to a change in perceived risk profile (the conditional volatility of dividends) between transactions, with u_k denoting the change in risk profile, and B_k denoting the associated repricing of risk."*
- w_k "*is the state "estimation error" which can be interpreted as capturing the range of investor valuations of comparable assets in individual transactions."*

The Observation Equation relates new observable data from a new transaction to the *time-series of excess returns.*

The Observation Equation:

$$y_k = F_k\theta_k + v_k$$

- y_k captures the observations in state/transaction(s) k
- F_k are the *expected cash flows*, discounted using state vector θ_k of discount-factors
- v_k is the *observation error*, which "*captures the extent to which expected cash flows are not well documented at the time of valuation."*

These equations are pivotal to the equity model and it is thus critical they are explained well. Since EDHEC have failed to articulate and present them in clear language there is value in doing so. Explaining the concept of the state space model can be done easily because this whole model revolves around a basic tenet that any analyst can understand. The combination of current price and future cash flow data can be used to arrive at rates of return. Therefore, resting our attention on *the observation equation* first ($y_k = F_k\theta_k + v_k$) one can posit that if F_k is a future cash flow and θ_k is a discount rate (which is the inverse of a rate of return) then that obviously makes y_k a value/price (if v_k is an error term).

There is a missing piece to the puzzle. I was correct in saying that F_k is a time series of future cash flows. However, as we know, the state vector θ_k is not a time series of returns, it is a *time series of excess returns*. This means that y_k is not exactly a price, but it's close. The equation for y_k does not look intuitive at first sight but once you follow the derivation in the Technical in the left hand column of p. 91 (which I have proof read) you will see that it is fine. I have simplified it here:

$$y_k = D'.\ (1 - A.\ R^f_t) - P_{tc} \qquad \text{(Equation A)}$$

Adapted from the equation in the top of the right hand column on p. 55

Where:

- 1 and A are matrixes of different shapes that only contain the number 1, so for simplicity they can be ignored.
- D' is a time-series of expected cash flows laid out in a matrix that has 1 row and t columns (a.k.a. F_k)
- R^f_t is a time-series of risk free rates.
- P_{tc}, is the price of the investment at the end of the years of construction, which I will call the *post-construction price*. tc is the time period when the first dividend is paid thus there are no cash flows to consider during the construction years. EDHEC posit that you have to discount P_{tc} at the risk free rate to arrive at the initial transaction price, P_k (a.k.a P_{t0}.)

You will see that the unnumbered equation at the top of the second column of p. 55 is equivalent to Equation A above, except that there is an error in EDHEC's equation in the EDHEC report. P_k is defined as the price at the transaction close on p. 55, therefore it must grow in value during the forthcoming construction years by multiplying it by $1 + R_f$, not $1 - R_f$ as EDHEC has stated in this equation. You can see that the preceding equation, which is called equation 3.7 and is found on the same page, is written correctly.

EDHEC have made these reports very difficult to digest as you can see in the notes I have made in the footnote on this page (Note 12). These notes may prove useful if you are attempting to read EDHEC's reports.

On p. 59 EDHEC start to layout what seems at first to be an entirely new model. In one sentence let's recap what we have encountered so far. We calculate y_k using Equation A (into which you input a post-construction price P_{tc} and a cash flow distribution) and then we input that into the observation equation of the state-space model, $y_k = F_k\theta_k + v_k$ using a cash flow distribution F_k to arrive at excess returns θ_k. For this "new model," introduced on p. 59 the institute is instead "*deriving a time-series of prices ... per transaction*" using the cash flow distribution and cash flow volatilities. Then they derive the excess returns θ_k (a time-series of $\boldsymbol{\lambda_t}$) from those prices. In other words, up until now we have started with known prices but this time we *calculate* the prices. The end goal is to arrive at excess returns θ_k but the route is different this time. Let's take a look at the formulas.

$$\boldsymbol{P_t} = \sum_{\tau=1}^{T} \frac{C_{t+\tau} - \frac{1}{\phi^{\tau}\left(\sum_{i=1}^{\tau} [\![\gamma_{t+i}) \sigma_{C_{t+\tau}}]\!]\right.}}{\prod_{j=0}^{\tau-1}\left(1 + r^f_{t+j}\right)}$$

t is a *"a given point in time during the lifecycle of an investment project"*

$\boldsymbol{\tau}$ is a measure of "*the remaining periods in the investment*" and takes values from $\boldsymbol{\tau} = 1,\ldots T$

$$\boldsymbol{\lambda_t} = \frac{P_{t+1} + C_{t+1}}{P_t} - r^f_t - 1$$

$$\boldsymbol{\sigma_{\lambda_{t-1}}} = \sigma_{\lambda_t} - \frac{\lambda_t - \phi\lambda_{t-1}}{\gamma_t}$$

You will note that equation 3.19 contains the parameters φ and γ. These parameters are first introduced on p. 57 in a proof that leads to equation 3.12 on p. 58. In equation 3.12. φ is the coefficient of the previous time-period's excess return value. It can also be called the return autocorrelation and you've come across it before. φ is H_k from the state equation. In equation 3.12 γ is the coefficient of $\dot{\boldsymbol{\sigma}}^i_t$ and since $\dot{\boldsymbol{\sigma}}^i_t$ is the change in the volatility of returns, and the volatility of returns is the chosen measure of risk, γ can be called the repricing of risk. You've come across that before too. γ is B_k from the state equation.

Assuming I understand EDHEC correctly here, if you are an analyst and you need to find the rate of return of a new (merchant or contracted) infrastructure investment you go about it as follows. First of all you acquire the merchant or contracted φ and γ parameters from EDHEC, who have been refining the φ and γ parameters by applying equations 3.19, 3.20 and 3.21 to their dataset of infrastructure investments. (Note 13) I have asked EDHEC to confirm that I am correct here.

Looking at Equation 3.19 you will also note that you need the estimated future cash flows for the project *and* the volatility of each cash flow, which is odd. (Note 14) EDHEC do not explain how you must calculate the volatility of each expected cash flow anywhere in this report and I have asked for an explanation as to how it is done.

After using equation 3.19 to calculate a time-series of prices, you use this time-series of prices to calculate a time-series of excess returns, λ_t, using equation 3.20 et voila, you have θ^- (a time-series of λ_t), the *estimate* for the *time series of excess returns,* which is needed for the Kalman Gain. You then use equation 3.21 to calculate $\boldsymbol{\sigma_{\lambda_t}}$. This gives you the u_k required for the state equation (a.k.a $\dot{\sigma}^i_t$ which is written on p. 92 as $\sigma_{\lambda_t} - \sigma_{\lambda_{t-1}}$). This will be used to calculate θ^- using Equation B below, when we come on to the Kalman Filtering process next.

At first glance the above equations definitely look complex but their derivations on p. 92 and 93 start with basic discounted cash flows. Unfortunately, I have found two mistakes in the proof, which could invalidate Equation 3.19. Equation 3.19 is an equation that is critical to EDHEC's equity model. I have explained these mistakes in the footnote of this page. (Note 15) More Errata found in Equations 3.19, 3.20 and 3.21 can be found.

To recap, take note that we now have two methods by which to calculate excess returns, θ^-. The first method involves using Equation A (written above) in combination with *the observation equation*. The discounted cash flow (DCF) model of Equations 3.19, 3.20 and 3.21 is the second. Both methods will ultimately be used in the model.

The final piece of the puzzle involves the use of a method called Kalman Filtering. This is a much used method which helps you estimate the value of a variable by predicting it before you measure it and then calculating it using both the estimate and the measurement, weighting each based on their expected accuracy. It is used when measurements are so inaccurate that using them in combination with an estimate will produce better results. For this reason the Kalman Filter is sometimes considered to be an *estimator.* In tracking the trajectory of an aircraft, sensor readings can be quite inaccurate (or noisy) so Kalman Filtering is often used.

In our case instead of needing to find out the spatial coordinates of an aircraft's trajectory we need to find a *time-series of excess returns*, θ_k. Let me demystify this final part of the model, which incorporates everything you have read so far in this report.

Here are the Kalman Gain equations. They look complicated at first but don't worry, I'm going to walk you through them. You may notice that I had to make some adjustments. (Note 16)

$\hat{\theta}_k^+ = H_k\ \hat{\theta}_k^- + B_k u_k$ *(prior state estimate)* ($\lambda^i_t = \varphi\lambda^i_{t-1} + \gamma_t\ \dot{\sigma}^i_t$) (Equation B)

$v_k = y_k - F_k\ \hat{\theta}_k^-$ *(observation error)* ($v_k = y_k - \hat{y}_k$ (Equation C)

$K_k = P^-_k F_k^T S_k^{-1}$ *(Kalman Gain)* (Equation D)

$\hat{\theta}_k^+ = \hat{\theta}_k^- + v_k K_k$ *(posterior state estimate)* (Equation E)

Covariances:

$P_k^- = (H_k P_{k-1}^- H_k^T + Q_k)$ (prior state covariance) (Equation F)

$S_k = F_k P_k^- F_k^T + R_k$ (observation error covariance) (Equation G)

$P_k^+ = (1 - K_k F_k P_k^-)$ (posterior state covariance) (Equation H)

For the purposes of interpreting the Kalman Gain Equations (Equations B, C, D and E) I imagined that a greater covariance means a variable is more prone to error. This is because the larger the covariance of an element the greater the chance that element receives a lesser weighting. This is explained more fully in the footnote (Note 17).

An important problem lies in the fact that EDHEC do not spell out how you calculate the variable R_k or the initial value of P_k^-. They also confuse the reader by making mistakes when they define these variables. (Note 18)

EDHEC lay out the steps involved in one cycle of the Kalman Filtering process on p 64 and 65. These are supplemented with a flow chart called Figure 12 on p. 64. Once again, EDHEC's steps are ambiguous and confusing and I have asked the institute questions on the subject. However, there are a couple of things I can say for sure. EDHEC wants you to do the following:

1. $\hat{y}_k$ is *estimated.* Unfortunately, EDHEC do not properly explain how this is done and there are several routes that can be taken.

2. y_k is calculated by putting data from *observations* into Equation A.

The most important errors and ambiguities can be summed up as follows:

1. It is unclear which steps and how many steps (and what steps) EDHEC use to update excess returns, $\hat{\theta}_k^-$ in each iteration. There is a conflict between what is indicated in the written steps and what is illustrated in Figure 12. In the written steps it appears that $\hat{\theta}_k^-$ is updated in three steps in every iteration. In Figure 12, $\hat{\theta}_k^-$ only appears to be updated in two steps. I have asked EDHEC to clarify exactly how many steps are taken to update $\hat{\theta}_k^-$ in every iteration (Appendix A, part 4.18.3).

2. It is unclear how EDHEC are estimating $\hat{y}_k$. There are several routes that could be taken. The written steps on p. 64 and p. 65 indicate that EDHEC are using Equation 3.19 to estimate a transaction price. This could be used to calculate $\hat{y}_k$ using Equation A. Figure 12 conflicts with this by suggesting $\hat{\theta}_k^-$ from the prior iteration is used to estimate $\hat{y}_k$ using the observation equation $\hat{y}_k = F_k\ \hat{\theta}_{k,}$ which is a second alternative. A third alternative could be used to estimate $\hat{y}_k$. You could use Equations 3.19 and 3.20 to estimate $\hat{\theta}_k^-$ as indicated in Step 3 of the written steps and then use this to estimate $\hat{y}_k$ using the observation equation $\hat{y}_k = F_k\ \hat{\theta}_k$. I have asked EDHEC to specify the exact route taken. This and more issues are raised regarding the estimation of $\hat{y}_k$ in Appendix A, part 4.17, Step 3.

3. EDHEC talk about calculating *"the average transaction price"* and their *"variance"* or *"range"* using the Kalman Gain. It is unclear what route they take to calculate prices and they never show you how to calculate the variance/range of prices. See Appendix A, part 4.18.1 for a fuller explanation.

4. It is unclear what cash flows are used for the calculation. This is explained in more detail in Appendix A, part 4.18.2.

5. EDHEC say "*The first phase of the Kalman filter consists of predicting the state at the next iteration … before observing the data … but taking into account any change in the state control variables … A new state and a new covariance error matrix are thus computed.*" The process for calculating this *"covariance error matrix"* is never specified. The closest thing I can find to it is Ψ, which is a function of parameters R_k, Q_k, φ and γ_t according to Equation 6.25 on p.93. On p. 93 Ψ is only ever referred to as *"a vector of unknown parameters"* but it does contain the covariances of the error terms of the state and observation equation, R_k and Q_k. Several issues relating to the parameters R_k, Q_k, φ and γ_t and to the function Ψ are found in Appendix A, part 4.18.4.

6. As we already know in each Kalman Filtering iteration, y_k is calculated from *observations* and $\hat{\theta}_k$ is estimated. In each iteration you will use Equation A to calculate y_k from *observations* from n transactions. Since you are observing n transactions every time you cycle through the filter you might be producing n values of y_k, which will give you a distribution of y_k values. A second problem arises in understanding whether $\hat{\theta}_k$ is a single value or a distribution of values, which has a trickledown effect on the other steps in the filtering process. See Step 3 of the Kalman Gain Filtering Steps in Appendix A, part 4.17 for more details.

7. Problems relating to the Kalman Gain Steps are covered in Appendix A, part 4.17. More requests for clarification and errata relating to EDHEC's use of Kalman Gain Filtering can be found in Appendix A, part 4.18.

EDHEC never explicitly state or imply that their equity model incorporates the all-important debt covenants present in project finance. Also, the Equity Service Coverage Ratio (ESCR), the cornerstone of the equity model is never explicitly expressed in terms of the Debt Service Coverage Ratio (DSCR), around which the debt model revolves (although it could be assumed that the expected cash flow (C^i_t) is equal to the cash flow available for debt service (CFADS) and equate these two variables using basic algebra to give ESCR in terms of DSCR.) DSCR levels trigger hard and technical defaults. I have asked EDHEC to justify these important omissions to the report on the equity model (Appendix A, part 4.21).

2.4.1 Discussion

In the pages of this report EDHEC walk you through a multi-step process with the primary goal of estimating excess returns, θ_k. Firstly the Equity Service Coverage Ratio (ESCR – which is multiplied by the known base case cash flows to give you a project's expected cash flows) is introduced in order to model cash flows using Bayesian Inference. A logarithmic distribution of the ESCR is assumed. Cash flows will be discussed in more detail in the next report. Excess returns, θ_k are estimated using a discounted cash flow model (Equations 3.19, 3.20 and 3.21) requiring projected cash flows and cash flow volatilities as inputs. Then θ_k is updated iteratively as follows. The State equation ($\theta_k = H_k\theta_{k-1} + B_k u_k + w_k$) is used to produce an estimate for θ_k, which accounts for autocorrelation (H_k) as well as changes in perception and pricing of risk ($B_k u_k$). Then using Kalman Filtering an estimate for θ_k is created which is partially composed of the estimate derived from the state equation and partially composed of current observations. The weighting of each of these two components (estimate or observation) is determined based on the variance of each component, which indicates the reliability of the component. Kalman Filtering is a method commonly used to estimate variables when measurements are noisy and cannot be fully relied upon.

If you have read my report on this model you will know that some of the details of this process are not explained clearly, therefore I have asked EDHEC to confirm that the summary of the equity model given in the paragraph above is accurate (Appendix A, part 4.22).

After all of this work EDHEC never explicitly define a process for valuing an investment although one can assume that with the term structure of excess returns and the cash flow distribution one can simply sum discounted cash flows. I have asked EDHEC whether this is the way they intended for investments to be valued (Appendix A, part 4.23).

EDHEC have gone to great pains to create this elaborate model and there are moments where their mathematics are impressive. However the sheer complexity of the model leaves more room for error than there would be if EDHEC stuck to a simpler, more familiar method and there are numerous substantive issues that put the model in jeopardy.

- Should the ESCR have been modelled as a logarithmic? See the footnote for more detail. (Note 19)
- Is the omission of the "volatility of the volatility of the cash flow" in the Discounted Cash Flow model substantive enough to discredit the model? See Appendix 4.12.
- Does the use of a linear relationship between excess returns and market returns, found within the derivation of the State Equation, discredit the model due to the well-known "*non-linear*" nature of the data? See Appendix A, part 4.24.
- Does the model overly rely on estimated data and can the model be discredited for this reason? See Appendix A, parts 10.2 and 13.1.

One of the principal aims of this model is to arrive at estimates for excess returns. A basic multi-factor model (which EDHEC end up using in a future report) can do that. As you read on you will find that EDHEC present a total of three different models to derive excess returns. Excess returns are first modelled in this Equity report (Report #4). In their Broad Market Equity Indices report (Report #8) EDHEC derive excess returns using a new version of the Capital Asset Pricing Model (CAPM). In their Asset-Pricing Methodology report (Report #10) EDHEC posit yet a third way to derive excess returns in a straightforward multifactor-model in which they estimate factor returns and factor exposures to arrive at an overall return. At no point have they elaborated on why they have developed different models to do the same thing nor do they explain which model they use to calculate excess returns for their custom benchmarks. I have asked EDHEC about this (Appendix A, part 10.17).

2.5 Report #5 - Cash Flow Dynamics of Private Infrastructure Project Debt (March 2016)

In this report EDHEC focus only on cash flow distributions for their debt model so we are working backwards here having already digested the debt model report that was published by EDHEC two years prior to this report. Recall that the following parameters are used in EDHEC's debt model:

$$CFADS = DSCR \times DS$$

Where:

CFADS = Cash Flow Available for Debt Service

DSCR = Debt Service Coverage Ratio

DS = Debt Service (a constant)

EDHEC analyse data from a broad range of infrastructure projects across a fifteen-year time span (1999 – 2014) and claim theirs is the largest dataset available for research, as of the date of this report. There were 207 projects contained in their data set at the time of this report. They argue that the data does not have the properties necessary to make it appropriate for standard linear models.

"...our DSCR data exhibits non-constant variance (the Breush-Pagan test rejects the null hypothesis of homoskedasticity), significant auto-correlation (the Durbin-Watson test rejects the null hypothesis of no autocorrelation in the regression residuals) and non-normal residuals (The Shapiro test rejects the null of Gaussian residuals)" (Blanc-Brude 2016).

On top of this the data is sparse which makes it less appropriate for statistical modelling. As I said before while summarizing the report on Benchmarks (Report #2) EDHEC believe the answer to all of these problems is to use Bayesian Inference. Bayesian Inference will allow you to improve your predictive accuracy as you update your results over time and this is important when the data set you are starting out with is small and unreliable as is the case with infrastructure investments. Bayesian Inference has two added advantages. It is not a linear model and it does not resolve problems using statistics.

Here is a short introduction to Bayesian Inference. Bayesian Inference allows you to update a probability with new data that has come to light. In a nutshell a *posterior* probability (an updated probability) equals a *prior* probability (the original probability) multiplied by a factor that accounts for the impact of the new data on that original probability.

posterior = *prior* x "data impact factor"

The mathematical way of saying this is:

$$p(A|B) = p(A)L(B|A)$$

Where:

p(A|B) is the posterior (the updated distribution)

p(A) is the prior or conjugate prior (the original distribution)

L(B|A) is the likelihood

The nomenclature (A|B) simply means "the probability of hypothesis A given B." In this case the new data corresponds to the letter B. If the *prior* distribution comes from the same family as the *posterior* distribution (e.g. both are normal distributions) the *prior* is referred to as the *conjugate prior*.

If you remember I said that EDHEC use Bayesian Inference for two main reasons. (Note 20) They use it to calculate ratios relating to the cash flow distributions for equity and debt (which are the DSCR for debt and the ESCR for equity). As I explained in the equity report (Report #4) both the ESCR and the DSCR are calculated using the same exact treatment. EDHEC also use Bayesian Inference to calculate the probability of default. A binomial distribution is selected to represent the probability of default because a binomial only has two outcomes. The probability of default only has two outcomes because the project is either in default or it isn't. Academic research has shown that a binomial *likelihood* calls for the use of a beta distribution as the *conjugate prior* (and the *posterior*.) Prior research has also given us the parameters of the beta distribution. Once we know these parameters we can calculate the average value for the probability of default and the variance of the probability of default. The average value and the variance will more commonly be referred to as the mean (m) and precision (p), where the precision is the inverse of the variance.

Moving on to cash flows, calculating a cash flow distribution is more complex than calculating a binomial distribution because there are many possible outcomes. Strictly speaking the debt *and* equity cash flow ratios (the DSCR and the ESCR) are not cash flows but cash flows can easily be derived from them as I have shown previously so I will call their distributions, "cash flow distributions" for simplicity as EDHEC have done. EDHEC claim that both the DSCR and the ESCR for *project finance* have a lognormal distribution. (Note 21) Previous academic research has shown that when the *likelihood* is a lognormal distribution, the *conjugate prior* distribution is Gamma-Normal (as is the *posterior*.) Gamma-Normal distributions come with several parameters (also derived in prior academic research), which are used to calculate values for the *mean* and *precision* of the cash flow distributions.

A two-step process is proposed. In the first step the project is determined to be in either a safe, risky or default state. The second step involves using the Bayesian Inference model for cash flows described above to determine the parameters of the cash flow distribution when the project is in the lognormal state, which is the risky state. Intuitively the project is in its default state when DSCR < 1 (which only happened five times across the 207 projects) and is in its safe state when the cash flow level rises beyond a certain point and the project is considered risk-free. Projects move into the safe state 15% and 10% of the time for contracted and merchant project types respectively. Again, since the data ostensibly reveals that the cash flow data has a logarithmic distribution in the risky state, it is the cash flow distribution for the lognormal/risky state, which will be calculated using Bayesian Inference.

So we have established that a Gamma-Normal distribution will be the *conjugate prior* when using Bayesian Inference to calculate the mean and precision of the cash flow distribution (either the DSCR or the ESCR). (Note 22)

Next EDHEC reference a report called "*A Compendium of Conjugate Priors*," published in 1997 in which the author Daniel Fink presents the Gamma-Normal *conjugate prior* (which has the same form as the *posterior*) in terms of several parameters, α, β, μ and δ. Fink then presents those same parameters in a set of equations in which a given parameter in the *updated* Gamma-Normal distribution (the *posterior*) is expressed in terms of that same parameter in the *original* Gamma-Normal distribution (the *conjugate prior*). The equations are written out below so you will see what I mean (Equation 3.8 on p. 43). The mean (m) and precision (p) of the cash flows will eventually be expressed in terms of the updated parameters of the conjugate prior, α, β, μ and δ. EDHEC have copied the Gamma-Normal *conjugate prior* and the equations for the parameters of that *conjugate prior* from p. 35 of Fink's report and here they are.

The *Conjugate Prior* and the *Posterior* are described by the Gamma-Normal distribution, which is:

$$\Pr(m, p \mid \alpha, \beta, \mu, \delta) = \frac{p^{\alpha-1}\exp\left(-\frac{p}{\beta}\right)}{\Gamma(\alpha)\beta^{\alpha}} \left(\frac{p\delta}{2\pi}\right)^{\frac{1}{2}} \exp\left(-\frac{p\delta}{2}(m - \mu)^2\right)$$

To update the four parameters α, β, μ, δ of the *prior* so they become the four parameters of the *posterior*, the following equations are used. As you will see, the new data (in the form of variables N, $\bar{Y}$and SS which will be defined below) will be used to update the parameters α, β, μ, δ.

$$\alpha^{+} = \alpha + \frac{N}{2}$$

$$\beta^{+} = \left(\frac{1}{\beta} + \frac{SS}{2} + \frac{\delta N(\bar{Y}-\mu)^2}{2(\delta + N)}\right)^{-1}$$

$$\mu^{+} = \frac{\delta\mu + N\bar{Y}}{\delta + N}$$

$$\delta^{+} = \delta + N$$

Fink's report does not contain the derivations for these equations.

As I said above, these equations show you how the new data (in the form of variables N, $\bar{Y}$and SS) is used to update the DSCR distribution. The nature of that data is explained in this quote:

"Following Fink (1997), the sufficient statistics (required data) to update a prior distribution are the number of observations N, mean of Y [= $\bar{Y}$]= $[\sum_{n=1}^{N} \ln(Y)_n]$ */N and SS the sum of squared deviation of the log data about m;"*

This sentence may contain two errors (Appendix A, part 5.1).

Unfortunately, neither Fink nor EDHEC give you the equations for the most important parameters, which are the mean, m and the precision, p (which again, is the inverse of the variance) of the cash flow distributions. This is important because the conjugate prior above, Pr(m, p | α, β, μ, δ) is written in terms of the four parameters α, β, μ, δ *and* m and p but the most important metrics which you must calculate are the mean, m and the variance (which is the inverse of the precision, p) of the updated cash flow distribution. EDHEC show that m and p are a function of their parameters (Note 23) but the actual equations are not given. Evidently EDHEC have calculated m and p as you can see from Figure 27 and Table 10 on p. 52 so I have asked them to express m and p in terms of α, β, μ, δ (Appendix A, part 5.2).

In the Equity Report EDHEC justify giving the same mathematical treatment to the ESCR and the DSCR distributions:

"In Blanc-Brude (2014b), we show empirically that the debt service cover ratio (DSCR) in non-recourse infrastructure project finance is lognormally distributed, and since the free cash flow of the firm is simply the product of the DSCR with the debt service (a constant), we can reasonably argue that the free cash flow of the firm and its dividends have a similar function form." (Blanc-Brude, 2015).

The only problem with this is that EDHEC use the treatment for the ESCR distribution but ESCR is not "*the free cash flow of the firm"* nor is it "*its dividends."* It is the ratio of base case to realized or expected dividends ($ESCR^i = C^i / C^0$), which cannot be expected to have the same mathematical form as a project's dividends alone. I have asked EDHEC to reconcile this apparent conflict (Appendix A, part 4.20).

Let's now use Bayesian Inference to calculate the probability of default. EDHEC determine that it is also necessary to use Bayesian Inference to calculate the probabilities that the project is in each state (safe, risky and default). To simplify the situation EDHEC ask us to pretend there are not three but two states (risky and default) which would mean that the probability is a binomial distribution. On p. 48 EDHEC then proceed to copy another equation from David Fink's report *"A Compendium of Conjugate priors."* This time they copy the beta distribution (a.k.a. the *beta prior density*) from pages 10 and 11 of Fink's report. The derivations for these equations are not found in EDHEC's or Fink's report.

To use Bayesian Inference to calculate the probability that the project is in each state EDHEC layout an intuitive equation that explains how to calculate $p_{t,r,}$, which is the probability that a project will be in a risky state at time t.

$$p_{t,r} = p_{t-1,r}\,\pi_{rr} + (1 - p_{t-1,r})\,\pi_{dr}$$

Equation 3.9 (Note 24)

Then you take the parameters of the beta distribution, (which is being used as the conjugate prior):

$\pi_{ii} = a_{ii} + n_{ii}$

$\pi_{ii} = a_{ii} + N_i - n_{ii}$

$\pi_{ii} = \alpha_{ii}/(\alpha_{ii} + \beta_{ii})$

Equations 5.5, 5.6 and 5.7 (Note 25) (which are the value of the parameters given in Fink's report.(Note 26))

Where N_i denotes the total number of transitions *starting from* state i and

n_{ii} denotes the total number of transitions from state i to state i and

i can either be the r (risky) or the d (default) state.

Thus to complete one iteration you first plug n_{ii} and N_i into equations 5.5, 5.6 and 5.7 to arrive at a value for each version of π_{ii} after which you would plug the values of π_{ii} into equation 3.9 on p. 46 (written out above) to update your prior values, $p_{t-1,i}$ to create posterior values, $p_{t,i}$. You will need to estimate the initial value for $p_{t-1,}$ which is the prior for the first iteration… et voila! As you can see this will allow you to calculate the probability of being in the risky state, $p_{t,r}$ and the probability of being in the default state, $p_{t,d}$.

EDHEC have decided they don't want to use the above method to calculate the probability of transitions out of default because there are too few defaults (there were only 5 defaults in their entire dataset), which means the choice of prior will dominate. This is because when using Equations 5.5, 5.6 and 5.7 values for N_d and n_{di} will be zero, which will mean that for much of the time π_{ii} will be equal to its own prior. The problem with this is that the first prior is an estimate therefore your choice of prior will dominate results.

In a few sentences let me summarize the alternative method EDHEC use to calculate transitions from default. Then I will explain the mathematics. For this calculation they incorporate all three states (risky, safe and default.)

1. First EDHEC devise an indirect way to calculate the number of projects in default at a given point in time, n_d.

2. Then using data from Moody's, they estimate the number of projects that remain in default, n_{dd} as well as the number of projects that transition from a default state to a risky or safe state, n_{drs} (where $n_{drs} = n_{dr} + n_{ds}$).

3. Once you have N_d and n_{di} you can use Equations 5.5, 5.6 and 5.7 to find values of π_{dd} and π_{drs}, which are the probabilities of transitioning from a default to a default state (π_{dd}) and from a default state into a risky or safe state (π_{drs}).

Ok, let's begin.

1. To calculate the number of defaults using recent observations EDHEC manipulate this intuitive line of algebra:

$$N = n_d + n_r + n_s = p_d N + n_r + n_s$$

and end up with this:

$$n_d = p_d\,(n_r + n_s)/(1 - p_d)$$

The proof appears to be incorrect and I have asked EDHEC to explain their lines of algebra (Appendix A, part 5.4).

Where N is the total number of observations

n_d, $n_r + n_s$ are the number of projects in the default, risky and safe states respectively

p_d is the probability of default

Assuming Equation 5.2 is correct you can see that it allows you to indirectly calculate the number of defaults, as long as you have a value for the probability of default, p_d. On p. 55 of the report you can see that applying Bayesian Inference to the data to calculate the DSCR distribution has allowed EDHEC to plot the probability of hard default based on the probability that the DSCR drops below the 1.0 level. This gives you the probability of default, which is required for Equation 5.2. Equation 5.2 then gives you the number of projects in default at a given point in time, n_d.

2. Next, to estimate the number of transitions from the default state, EDHEC suggest this should be derived from Moody's assertion in 2015 that the time to emergence from default in projects is 2.3 years.

"..we assume that each project has a 10% probability of emergence from default within the first year, a 50% probability of emergence in the second year conditional on not having emerged in the first year, and a 40% probability of emergence from default in the third year conditional on not having emerged in the first two years. This leads to an average time of emergence of 2.3 years."

We use this data and the data that was derived in the first section (a value for the number of projects that are in default) to calculate n_{dd} and $n_{d,rs}$. From this you can calculate the number of projects that transition *from* the default state, n_{di} (= n_{dd} and $n_{d,rs}$).

3. Once you have n_{dd} and $n_{d,rs}$ (= $n_{dr} + n_{ds}$) this will allow you to use Equations 5.5, 5.6 and 5.7 on p. 59 (written out earlier) to find probabilities for transitions π_{dd} and $\pi_{d,rs}$. These give you the probability that a project will exit the default state.

EDHEC do not explain what happens when you adapt the model to deal with three states not two, which will mean that the binomial distribution is no longer appropriate. This might be resolved by always condensing the three states into two states by treating two of the states as if they are one. That means that you would join either rs, sd or dr together. This was done above when EDHEC estimated transitions into and out of default. In this case they grouped the risky and safe state together and called it rs. This may allow you to use a binomial distribution for three states. I have asked EDHEC how they are dealing with calculations involving three states (Appendix A, part 5.7).

EDHEC never explicitly explain how the probability of transitions into/out of default (which we calculated above) are integrated into the model. Are the cash flows multiplied by this probability? I have asked EDHEC (Appendix A, part 5.8). On another point if the default probabilities are year dependent (e.g. the *prior* is the probability of transition into default in the first year of a loan's life) is this process repeated for all transitions in each operational year?

Because of the autoregressive characteristics of the cash flows of infrastructure debt EDHEC posit that the mean DSCR in a new period is equal to the realized DSCR from the prior period. However, when a project enters into default it is no longer easy to predict DSCR dynamics. EDHEC opt to resolve this issue by calculating a weighted sum of the *project* DSCR and the (contracted or merchant) *family* DSCR to arrive at a projected DSCR distribution. The weightings are calculated by taking the (unreliable) *realized* DSCR upon emergence from default and calculating the sum of the squared difference between that and the *project* DSCR prior. This is labelled SS_p. Then EDHEC calculate the sum of the squared difference between the *realized* DSCR and the *family* DSCR prior, which is labelled SS_f. As you will see from the following equation, if the *realized* DSCR is dramatically different from the *project* DSCR prior, the *project* DSCR prior will contribute less to the projected mean. The operation is conducted on the mean, μ and the precision, p of the projected DSCR distribution:

$$\mu^{+} = \frac{SS_f}{(SS_p + SS_f)} \cdot \mu_p^{+} + \frac{SS_p}{(SS_p + SS_f)} \cdot \mu_f^{+}$$

$$p^{+} = \frac{SS_f}{(SS_p + SS_f)} \cdot p_p^{+} + \frac{SS_p}{(SS_p + SS_f)} \cdot p_f^{+}$$

I have corrected these equations because they are written incorrectly in the report (Appendix A, part 5.10).

After covering the final parts of the model from Chapter 5 of this report I'm going to jump back to Chapter 4 where EDHEC begin to implement the model using data from real projects. As EDHEC crunch their data in this section there is some confusion. Written descriptions don't appear to match the Tables and Figures and it is unclear what they mean when their charts refer to *"years"* which could mean calendar or operational years. The issues are laid out in Appendix A, part 5.11 and I asked EDHEC to address them.

When this methodology is applied to the data Merchant projects have a lower probability of being in the risky state and therefore each state is less stable than it is for contracted projects. Another aspect that is worthy of note is path dependency, which is an important feature in infrastructure projects. Perhaps unsurprisingly merchant projects can diverge strongly from the mean of the population making their trajectory more path-dependent while contracted projects are more homogeneous,

Using such a model means that technical defaults can now be predicted where otherwise they could not be. In one example EDHEC assume that DSCR levels of 1.15, 1.10, and 1.0 correspond to dividend lockup, soft default, and hard default thresholds. The ability to halt dividends and to restructure debt before default is an important advantage for debt holders and should not be ignored.

"Having derived the entire distribution of DSCRs at each point time, we can now predict technical as well as hard defaults, an important improvement on the reduced form approach employed by rating agencies. …technical defaults are the most common in project finance and they trigger valuable options to step-in for creditors, which largely explains the high reported level of recovery in infrastructure project debt."

Additional Requests for Clarification and Errata can be found in Appendix A, part 5.

2.5.1 Discussion

This report explains the method EDHEC use to calculate projected values of the Debt Service Coverage Ratio (DSCR), a cash flow distribution for project finance debt. Among other things it allows us to model the probability of transitions into and out of default. According to EDHEC, Moody's and Standard and Poor's use default numbers across batches of projects to calculate credit risk, with no attention given to a specific project's characteristics and performance. There is no question that a model that adjusts the on going cash flows of a project in real time so it may identify important default triggers (as EDHEC's model does) has the potential to be a superior model.

2.6 Report A - Moody's Generic Project Finance Methodology, Moody's (2010)

At the time of this report this method was only being used with about 40 issuers.

In this report Moody's revealed that the main factors they use to determine their ratings for project finance listings are:

1. Long-Term Viability
2. Stability and Predictability of Cash Flows
3. Operations & Event Risk.
4. Key Financial Metrics (e.g. the Average Annual Debt Service Coverage Ratio (AADSCR); and resilience to declining revenues or increasing costs)

After a score is generated using these factors (for which the exact calculations used, if any, are not given) this score can be notched upwards or downwards based on "structural considerations" such as liquidity, refinancing risk and "project finance features," for which specifics, are again not given. The explanations regarding the individual notching factors do give the sense that except for the odd quantitative threshold, decisions are mostly reliant on the judgment and interpretation of the person assigned with the task.

Interestingly there is the possibility of further downward notching if the project does not include a list of standard project finance features including:

1. Entity created to exclusively engage in the project
2. Security on contracts, assets and revenues
3. Standard covenants restricting further indebtedness and so on.

Finally *Expected Loss* is calculated and the score is appropriately adjusted.

2.7 Report B - Operational Privately Financed Public Infrastructure (PFI/PPP/P3) Projects, Moody's (March 2015)

This report covers the rating methodology of Privately Financed Public Infrastructure projects that are in their operational phase. At the time of this report over half of the projects for which this methodology was applied were healthcare related, demonstrating the narrow application of this category of investment at the time of this report.

Four primary factors are aggregated to produce a result first. These are as follows:

1. Complexity of Project Operations and Performance Regime (i.e. standards that must be met to perform)
2. Strength of Contracts and Operations
3. Quality of Sub-contractor
4. Leverage and Debt Coverage Ratio

As before upward or downward notching is conducted with the next four factors:

1. Project Track Record (includes the quality of the working relationships and the project's operating performance)
2. Refinancing Risk
3. Structural Features (Reserves and Creditor Controls)

4. Off taker Considerations (produces only downward notching based on the credit quality of the government entity, often the sole source of the PPPs project revenues.)

There are several other possible notching factors beyond these.

3. Discussion

There is great value in EDHEC's model, however there are a number of problems that have arisen as their reports have been contemplated and these problems are highlighted below. Listing these issues is not intended to discredit the model but rather to suggest areas that may require attention. In a highly complex body of work such as this, errors of some kind are to be expected. The report numbers referred to below and the corresponding titles of the reports can be found in Appendix A.

1. There appears to be a serious problem regarding missing data, which would affect any attempt to conduct these studies and to construct EDHEC's benchmarks. Even the face value of debt, a fundamental starting point, may not be known for all projects. Report #10 on EDHEC's Asset-Pricing Methodology states "some of this information, especially 1/ the amortisation profile, 2/ debt maturity, 3/ the effective interest rate, or 4/ the outstanding face value, can be missing" (Blanc-Brude, 2018). The fact that a substantive portion of the data may need to be estimated is highly disconcerting. EDHEC's model discounts cash flows in order to value infrastructure assets. In doing so EDHEC's model already involves a lot of guesswork. Estimating a portion of the data makes things worse. I have asked EDHEC to specify what proportion of the data that EDHEC relies upon is missing (Appendix A, parts 10.2 and 13.1). The problem of missing data cannot be overstated. By having to estimate a substantive portion of the body of data, errors may be created that are large enough to potentially invalidate the entire model. It is critical that EDHEC address this issue. Although infrastructure investments are privately owned for the most part, the investment vehicles often revolve around government contracts. The World Bank, the OECD and the G20 have stated that "data gaps" are responsible for impeding private investment in infrastructure. Governments should mandate the submission of accurate data so this can be gathered by a neutral party like EDHEC, with the accompaniment of a confidentiality agreement. Benchmarks are critical to private participation and thus to the ability governments will have to meet the infrastructure needs of future populations.

2. In the Asset-Pricing Methodology report (Report #10 in Appendix A) the future Debt Service (DS) is said to be "most frequently missing." If the interest payments on the debt (the DS) and the principal value of the debt (the outstanding face value - see above) are missing it is hard to conceive how any work can be done with respect to valuing the debt.

3. In their Asset-Pricing Methodology report (Report #10 in Appendix A) EDHEC depart from their fundamental hypothesis that standard linear methods are inadequate for the modelling of infrastructure data due to data paucity and due to the nature of the data. The following question arises: why can a multi-factor model now be used when the data was deemed inappropriate for it in previous reports?

4. EDHEC's benchmarks are not investable. Project finance investments are large and heterogeneous. Diversification benefits only begin to happen after about fifty investments so matching the risk-adjusted returns of the benchmarks EDHEC has created is virtually impossible.

5. In their debt model, EDHEC does not calculate the rate of return/discount rate because valuations are done by summing probability weightings at refinancing and restructuring horizons beyond which visibility is limited, which rules-out the possibility of calculating an IRR.

6. In their report covering Revenues and Dividend Payouts (Report #6 in Appendix A) EDHEC try to compare the revenue and profit trends of infrastructure companies to those of non-infrastructure companies. For the report they choose to normalize revenues and profits by dividing by assets, however assets for infrastructure and non-infrastructure companies are fundamentally different. Infrastructure company's assets comprise of "outstanding contracted income to receive," which decreases to a zero value as the contract reaches completion. I call this the *decreasing average asset factor*. This factor causes normalized revenues and profits in infrastructure companies to appear to trend upward more than they should. In addition to this, infrastructure companies are unusually capital intensive. The disproportionately large asset base of an infrastructure company causes EDHEC's normalized revenues and profits to look smaller than they should, especially at the beginning. Together these two factors create an artificial upward trend in EDHEC's calculation of normalized revenues and profits of infrastructure companies. Many conclusions in this report are potentially flawed because the *decreasing average asset factor* has not been accounted for. The assertion is made that revenues and profits of infrastructure companies are more dynamic than those of non-infrastructure companies. This does not appear to be true when you look at the tables of results. One positive aspect of this report that is worthy of note is that revenues and profits of

infrastructure companies are less volatile from one calendar year to the next and thus are found to be less sensitive to the business cycle than revenues and profits of non-infrastructure companies. That conclusion probably holds after the suggested adjustments are made.

7. In the same report (Report # 6 in Appendix A) EDHEC use Bayesian Inference to model the dividend payout ratio (the ratio of the equity payout to the firm's revenues). They use mathematics from a referenced report by Daniel Fink but it has not been possible to confirm that the equations have been used correctly. They also make some assertions about the variance that do not make mathematical sense (Appendix A, part 6.9). After this they apply the model to the data and declare that infrastructure companies have higher payout ratios than a wide selection of registered companies. An elaborate model was not required to come to this conclusion because infrastructure companies consist of contracts, with finite lives at the end of which there is generally no remaining asset or capital value. Non-infrastructure companies will not pay out as large a portion of their revenue because whatever they reinvest will be duly reflected in the company's valuation, which can in turn be redeemed by stakeholders when the asset is sold. In contrast an infrastructure project has limited (if any) opportunities in which to invest any retained earnings. For this reason the proportion of an infrastructure project's revenue, which is paid out to equity stakeholders every year is higher. If one only measures the payouts of infrastructure and non-infrastructure companies, it is not a fair comparison.

8. According to Report #6 it seems that, from what we have been able to witness so far (we only have about 15 years of visibility), there are at least the same number of zero equity payouts than there are positive equity payouts in merchant projects. This arguably makes them poor investments especially since the value of the investment is fully tied up in equity payments (Appendix A, part 6.11).

9. In their Broad Market Equity Indices report (Report #8 in Appendix A) EDHEC calculate very high risk-adjusted returns for infrastructure compared with the risk-adjusted returns of a public equity index but in order to do this calculation they use a calculation for index variance that is questionable and must be justified. It is common knowledge that asset covariances must be known in order to calculate index (or portfolio) variance. EDHEC uses a calculation for asset covariances, which is an approximation, in order to then calculate the variance for infrastructure indexes. If this index variance is then compared to the variance of a "public equity reference index" that has been calculated using a more standard calculation for asset covariances, the comparison may be flawed. I have enquired as to why EDHEC think the comparison of variances that have been calculated using two fundamentally different forms of asset covariances is fair (Appendix A, part 8.8). The index variances that result from this calculation lead to very high risk-adjusted returns for infrastructure investments. Sharpe ratios of as much as 4 are reported by EDHEC for infrastructure investments compared with Sharpe ratios of less than 1 for the public equity reference index. The calculation for variance must be well justified in order to make the assertion that infrastructure investments have such high risk adjusted returns.

10. Throughout their reports EDHEC separate infrastructure investments into two groups called contracted and merchant projects based on whether a project receives fixed or demand-based payments respectively. They assume contracted projects have normal cash flow distributions and merchant projects have logarithmic cash flow distributions and they create separate mathematical models for each. In their Broad Market Equity Indices report (Report #8 in Appendix A) EDHEC demonstrate in Figure 14 of the report that the contracted and merchant business models do not relate to unique ranges of return volatility (or risk). This implies that the merchant and contracted categorisations may not be useful. It also implies that EDHEC's assumptions regarding attributing normal and logarithmic distributions to the contracted and merchant categories respectively, may need to be revisited.

11. Excess-returns and excess-return volatilities (which are EDHEC's main risk and return metrics) are calculated using three different methods throughout EDHEC's reports. Elaborate derivations justify a state-space model that models excess returns and excess return volatilities in the Equity report (Report #4 in Appendix A.) In their Broad Market Equity Indices report (Report #8 in Appendix A) EDHEC model excess returns and excess return volatility using a new version of the Capital Asset Pricing Model (CAPM.) In their Asset-Pricing Methodology report (Report #10 in Appendix A) EDHEC posits yet another way to derive excess returns in a straightforward multifactor-model in which they estimate factor returns and factor exposures to arrive at an overall return/discount rate. I have asked EDHEC which of these models they are using for their benchmarks (Appendix A, part 10.17).

12. In Report #4 in Appendix A, which contains the equity model, and in Report #5 in Appendix A EDHEC only model the Equity Service Coverage Ratio (ESCR) and Debt Service Coverage Ratio (DSCR) as lognormal distributions. However, in Report #3 in Appendix A, which covers the debt model, EDHEC state that only the merchant projects have a logarithmic distribution, whereas contracted projects have a normal distribution.

Therefore, the DSCR and ESCR for contracted projects have not been modelled by EDHEC.

Table 1. Summarizes the difference between EDHEC and Moody's approaches

Question	EDHEC	Moody's
Are all the assumptions valid?	No - The issues highlighted in the discussion section of this report regarding both risk and return metrics should to be addressed. With that said many sections of their model are well argued and soundly supported by reference material.	Yes - But largely because the model is so simplistic.
Does the model adequately measure firm specific risk and is path dependency considered?	Yes - It is specifically designed with this in mind.	No - The model is based on static factors which assumes a reversion to the population mean.
Does the absence of data limit the model?	Yes - the authors specifically aim to provide valuation metrics for traded and non-traded infrastructure projects, which leads to data paucity issues.	No - Their rating system is based on an abundance of publicly traded projects
Is it clearly presented and explained?	No - The reports are disorganised and difficult to understand without significant review.	Yes - But again this is largely because the model is so simplistic.
Is it biased in any way?	No - No conflicts are immediately apparent. The work of EDHEC is funded by investors.	Yes - Bond issuers pay for Moody's ratings and issuers won't want to pay for a bad rating so there is a clear conflict of interest.

4. Conclusion

Through their work EDHEC have made a significant contribution to creating standard valuation methods in a mostly private market for infrastructure projects. Data paucity makes this a particularly daunting challenge. Their model is significantly more robust than Moody's static rating system, which does not take into account the changing and idiosyncratic nature of risk and return in infrastructure projects.

Although EDHEC's work in valuing is ingenious and sound in many places, the robustness of the models and metrics suggested by EDHEC have been put into question in this report. In particular, the mathematical underpinnings of the equity model need to be reviewed. Key questions such as those highlighted in the Discussion section of this report should be addressed. Also EDHEC's debt and equity models need to be empirically tested against more traditional methods for valuing infrastructure project finance.

In comparison to EDHEC's models, Moody's rating methods are simplistic. No quantitative metrics beyond basic rankings can be derived from their work. Empirical work could also be undertaken to test the robustness of Moody's rating system, especially considering how much finance professionals rely on these ratings when making investments decisions.

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Notes

Note 1. "Project finance is defined by the International Project Finance Association (IPFA) as "the financing of long-term infrastructure, industrial projects and public services based upon a non-recourse or limited recourse financial structure where project debt and equity used to finance the project are paid back from the cash flow generated by the project" (IPFA 2013). This definition is consistent with the one used in the Basel-2 Accord (BIS 2005), which regulates project financing by banks." Taken from Construction Risk in Infrastructure Project Finance, February 2013, Frederic Blanc-Brude, Dejan Makovsek, p. 4.

Note 2. The name of these four contract features are: Full Completion Guarantee; Construction Cap & Responsibility Liquidated Damages; Construction Delays Liquidated Damages (in Months); and Construction Performance Bond Liquidated Damages. From Construction Risk in Infrastructure Project Finance, February 2013, Frederic Blanc-Brude, Dejan Makovsek, p. 14

Note 3. Ibid., p. 21.

Note 4. "Realistically, we doubt that a bit more than a decade of observed cash flow data can be collected today (e.g. the project finance default studies undertaken by ratings agencies do not manage much longer time series). Likewise, given the private nature of such information, even with the active cooperation of a number of market participants, sample size in the cross-section will remain limited." p. 27 "Moreover, the number of observations remains low for the purpose of measuring cash flow volatility. Stochastic models can require large amounts of data points to populate probability distributions, but only dozens of investable infrastructure projects are created every year in the most active markets." p. 27 "an investor cannot instantaneously buy a basket of assets that is representative of investable infrastructure projects in existence at that point in time." p. 19. Taken from Frederic Blanc-Brude, Benchmarking Long Term Investments in Infrastructure.

Note 5. Wang, A Universal Framework for Pricing Financial and Insurance Risks, p. 7.

Note 6. There is a important conflict between the stages denoted in the body of text and the stages denoted in Figure 4. In the stages denoted in the body of the text the CFADS is risk neutralized at the beginning of the stages. I have asked EDHEC to resolve this conflict (Appendix A, part 3.5).

Note 7. This is taken from p. 17 of EDHEC's report.

Note 8. EDHEC make this statement on p. 4.

Note 9. Data extends backwards in time a maximum of 15 years and most projects have a lifetime of several decades.

Note 10. "infrastructure dividend cash flows can only be partially observed," p.22 of Blanc-Brude and Hasan, The Valuation of Privately-Held Infrastructure Equity Investments

Note 11."..the product of [the] expected value Et (ESCRt+τ) with a given investment's base case cash flows at time t+τ in a given project is the expected value of dividends at time t + τ in that project, given the information available at time t." p.27 of Blanc-Brude and Hasan, The Valuation of Privately-Held Infrastructure Equity Investments.

Note 12. In this report I have done the heavy lifting because the above equation for y_k has been referred to immediately, but I have to cross-reference many parts of the report in order to solve the riddle of how to implement the state space model. It took a long time to figure out that one would have to use the specific equation above to calculate y_k so that one would then be able to use the observation equation. To start off with, just to throw analyst's minds into a real twist, the first time the state and observation equations are introduced on p. 51 they are written incorrectly. Then there is so much peripheral mathematics to trawl through in the report that it is easy to miss the fundamental importance of the above equation for y_k when one comes to it. After that, the names of variables are changed unnecessarily and with frequency. To cite one example, cash flows are called C_t, F_k, D' and on p. 62 they are even called H_k by mistake. The report begins by calling states i at the start of the report (p. 27, 29, 81). Later in the report states are referred to using the letter k whilst calling time periods, i (p. 29, 51 and 59). After this k becomes a measure of time (p. 62). Go to Appendix A, part 4.5 to read the direct quotes.

EDHEC persistently refer to y_k as prices, which is misleading. EDHEC also like to refer to y_k as observables/observation(s), which is also misleading. Take another look at the equation for y_k above (Equation A.) The transaction price P_k that grows at the risk free rate to give P_{tc}, (which you need for Equation A) is observable but the expected cash flows D' are estimated future cash flows and are therefore not observable. It would have been helpful if the report referred back to Equation A (or the report's version of Equation A on p. 55) every time y_k was referred to in order to give this critical and complex variable proper context. See Appendix A, parts 4.2, 4.3 and 4.4 to read about problems surrounding references to the variable y_k.

Sections 3.4.1, 3.4.2 and 3.4.3 (p's 56 to 59) are an exercise in taking the state equation ($\theta_k = H_k\theta_{k-1} + B_k u_k + w_k$) and rewriting it twice with two completely new sets of greek letters. The state equation is rewritten as $\lambda^i_t = \varphi\lambda^i_{t-1} + \gamma_t \ \dot{\sigma}^i_t$ (equation 3.12 on p. 58). Then the state equation is again rewritten as $\Lambda_k = \Phi^{\delta t}\Lambda_{k-1} + \Gamma_k \ \dot{\Sigma}_k$ (equation 3.14 on. P. 58). The reason given for rewriting the second time (when they use Λ for excess returns) is that this is the *matrix formulation* (see section 3.4.2.) However, the report refers to the terms in the original *State Equation* as matrixes when it first introduces them on p. 51. The proofs in these pages could have been placed in the Technical Appendix and there was no apparent reason for adding a dozen superfluous greek letters.

Note 13. On p. 64 EDHEC say "recompute the desired parameters [me: this must mean φ and γ] on a continuous basis, thus capturing the evolution of the market at regular intervals." A few paragraphs later on p. 65 EDHEC say "we can thus measure the implied valuation (term structure of discount factors) and range valuations for a given type of infrastructure equity investment during a given reporting period." When they use the word types this must mean the two types they have defined, contracted or merchant projects.

Note 14. If you check the derivation on p. 92 and specifically look at the line in which the volatility of each cash flow is introduced in the left hand column of the page, you will confirm the volatility of each cash flow this is indeed what is being asked for here.

Note 15. The nominator of the second element of the right hand side of equation 6.20 in the right hand column of p. 92 should be:

$$\sigma_{C_T} - \sigma_{\sigma_{C_T}} \qquad \text{not} \quad \sigma_{C_T}$$

I agree with EDHEC that the elements C_{T-1} and C_T have a volatility, but σ_{C_T} also has a volatility (which has been ignored). This effect may not be small, which could potentially invalidate Equation 3.19. EDHEC have explicitly stated that there is "*non-constant variance*" (p. 36 Report #3 on Debt) in debt investments, which consist of debt service payments and that "*free cash flow of the firm is simply the product of the DSCR with the debt service (a constant),*" (p. 36 Report #4 on Equity). This implies that the dividends paid to equity investments have non-constant variance and the value of $\sigma_{\sigma_{C_T}}$ is not zero (Appendix, part 4.12). Once this correction is made how would EDHEC calculate σ_{C_T} and $\sigma_{\sigma_{C_T}}$ since σ_{C_T} cannot be calculated like a normal standard deviation? When standard deviation is constant it can be found by calculating the deviation of cash flows from their mean, but it isn't constant here.

Also, there is a mistake in the equation above Equation 3.19 on p. 59, which is the same as the equation above equation 6.22 on p. 93. The sum for γ in the nominator should be from $i = 0$ to $\tau - 1$ because in Equation 6.21 on p. 92 the time signatures are identical to the time signatures for the risk free rates. In fact you could substitute the letter j for the letter i so that both sums are $i = 0$ to $\tau - 1$ (Appendix, part 4.13). This is a simple change, however the fear is that in doing their calculations on their data EDHEC have used the wrong formula, which have caused them to produce erroneous results. The preceding two errata can be found in Appendix A, part 4.12 and 4.13. More errors in equations 3.19, 3.20 and 3.21 can be found in Appendix, part 4.16.

Note 16. The variable named b in these equations, is switched to v_k in figure 12 on p. 64 so I have substituted b for v_k below to reduce confusion. You may notice that I also had to change the labels for some of these equations to make them more intelligible, see Appendix A, part 4.8.

Note 17. Equation D for the Kalman Gain applies this weighting mechanism, which determines whether more weight will be given to $\hat{y}_k$, the *estimate* or to y_k, which was derived from *observations*. As you can see, it is affected by two covariances, which are given in Equations F and G. If you take a close look at the equation for the Kalman Gain, K_k, the observation error covariance, S_k, is inversely proportional to the Kalman Gain, so that when S_k is large the Kalman Gain is small. Since a small Kalman Gain will mean the information from the observations contained within y_k will have less impact, this is intuitive. Conversely the *prior state covariance*, P_k^-, is directly proportional to the Kalman Gain, K_k, so that when the *prior state estimate,* $\hat{\theta}_k^-$ has a higher covariance, P_k^- is large and the Kalman Gain is large. This causes the information from new observations to have a larger impact on the new (*posterior) state* and the *prior state estimate* to have a lower impact. This also makes sense. | Now turn your attention to Equation C for the *observation error,* v_k. As you can see, v_k is large if the observations have differed greatly from their estimated values. Looking now at the Equation E for the *posterior state estimate,* $\hat{\theta}_k^+$, you will see that $\hat{\theta}_k^+$ will vary greatly from the *prior state estimate,* $\hat{\theta}_k^-$, in the following circumstances:

- in the event that *observations* are dramatically different from their expected values/estimates (so when v_k is large)
- when the *observation error covariance* is small (so when K_k is large because S_k is small) and/or
- when the *prior state covariance* P_k^- is large (so when K_k is large because P_k^- is large).

As you can see, this is all very intuitive.

Note 18. On p. 62 they say "*v_k* [is] *the observation noise ... with covariance R_k.*" Remember that EDHEC have previously called v_k the observation error on p. 52 so they are switching its name and according to the Kalman Gain Equations on p. 63, v_k (which EDHEC intermittently call b) has covariance S_k (Equation G) not R_k.

Note 19. The following references are from Blanc-Brude and Hasan, The Valuation of Privately-Held Infrastructure Equity Investments. EDHEC posit that the ESCR has the same functional form as the DSCR (logarithmic) due to the fact that the free cash flow is the product of the DSCR and the Debt Service (DS – a constant) (see p. 36). However, this justification is disputable because the ESCR is not equal to the free cash flow. ESCR is a ratio of the base-case to expected or realized cash flow (see p. 27), which cannot necessarily be expected to have the same mathematical form as a project's free cash flow (Appendix A, part 4.20). Intuitively the ESCR which is a ratio of two cash flows (expected and base case) cannot be compared to the DSCR which is a ratio of a cash flow (CFADS) and the Debt Service (DS) which is fixed.

Note 20. Strictly speaking Bayesian Inference is used in more ways than this throughout EDHEC's reports but these are the most important elements of the model that use Bayesian Inference.

Note 21. I have disputed this in Appendix A, part 4.20.

Note 22. The mathematics for the debt and equity cash flows (the DSCR and the ESCR) are identical. The Equity Report (Report #4) shows all of these equations on pages 36 and 37. You will see that the ESCR distribution is denoted using the letter X. On p. 43 of the report on Cash Flow Dynamics for Debt (Report #3) only the likelihood function and the equations for all the parameters involved are shown. In Report #3 the DSCR distribution is denoted using the letter Y. When I summarized the equity report I explained that I would not go over the mathematics for the ESCR distributions because it would be repeated again in the report on Cash Flow Dynamics for Debt, which is being covered now.

Note 23. On p. 42 the nomenclature in equations 3.5 and 3.6 EDHEC expresses m as a function of μ, δ and p as a function of α, β without writing out the full equations.

Note 24. If you take a look at the Equation in the report you will note that I have changed the equation to make it clearer. I have added subscript r to the p values so it is clear that this p_r is the probability that the project will be in the risky state. I have simplified the time signatures since $p_{t+i} = p_{t+\tau-1}$. I have also simplified the second element on the right hand side. If the project starts in default there are only two possibilities. It either stays in a default state or it goes to a risky state. Therefore

$\pi_{dr} + \pi_{dd} = 1$. It is also curious to note that since:

$\pi_{dr} + \pi_{rr} = 1$ and

$\pi_{rd} + \pi_{dd} = 1$ and

$\pi_{rd} + \pi_{rr} = 1$ then

$\pi_{rr} = \pi_{dd}$ (= 1 - π_{dr} = 1 - π_{rd}) and

$\pi_{dr} = \pi_{rd}$

Note 25. I had to correct Equation 5.7 in which EDHEC used p_{ii} erroneously instead of π_{ii}.

Note 26. Daniel Fink, “A Compendium of Conjugate Priors,” Equation 29, p. 11.

Note 27. EDHEC’s report called Revenues and Dividend Payouts in Privately Held Infrastructure Investments (Report #6 in Appendix A) appears to show that there are at least the same number of zero equity payouts as there are positive equity payouts in merchant projects.

Appendix A

Found at: http://bit.ly/3HrCcBj