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Some Difficulties in Understanding Negative Numbers Faced by Students: A Qualitative Study Applied at Secondary Schools in Indonesia

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Abstract

This study revealed how students' understanding of negative numbers and identified their difficulties related with the concept of integer and its counting operation as part of identifying epistemological obstacles about negative numbers. Even though teachers have explained counting operation procedure of integer, but there was concept misunderstanding among students. The concept difference between what was comprehended by respondents in their learning process and knowledge science has resulted in wrong perceptions about the negative numbers. In this article, the authors explained how these misunderstanding in concepts occurred among students and how these ideas were expressed by the students in solving the problem which was related with counting operation of negative integer. This study was part of Didactical Design Research using qualitative approach in negative number learning by involving 96 students of 7th grade as participants in three different schools. This study showed some difficulties by the student to understanding negative numbers in terms of prerequisite knowledge, understanding concepts, procedures, principles and problem solving.

Keywords: negative numbers, epistemological obstacles, mathematical understanding

1. Introduction

In daily practice, students naturally experience the situation which is called as learning obstacles. These learning obstacles can be experienced by students with various ability levels and groups. If we examine, learning obstacles are not only experienced by students who has 'low' ability but also can be experienced by students who has 'good' ability. Even mathematicians have entire domains of mathematics with which they do not feel comfortable (Einsenberg, 2002). Students start their learning process in an environment that is unbalanced and full of difficulties and obstacles just like human society so the new knowledge comes from the skill to adapt to the new circumstances and stimuli and a new reaction to the environment is the proof that a learning process has taken place (Manno, 2006). This obstacles need to be identified to find out the way to overcome it and determine the strategies to reduce or minimize this obstacle.

Balacheff (Brown, 2008) stated that mathematical error in students occurs from the way student to adapt as their knowledge response in a *milieu* or it's surrounding. Brousseau (2002) refers to obstacle from theory of Bachelard (1938) and Piaget (1975) about "error", that error and failure have a role which is not simple. This type error is unexpected and uncertain, which is called obstacles. This error is part of knowledge acquisition. Learning obstacle, according to Brown (2008) is a process to know (obtain the knowledge). In other word, this obstacle can be avoided because this obstacle is important part in acquiring new knowledge.

Many factors can cause learning obstacles among the students. One of them is learning pattern which is delivered by teachers as a pedagogical aspect (Widdiharto, 2008). In learning activities, context usage is usually needed to use by teachers and it is a synergy in imparting a concept to students. However, context usage which is not balanced to understanding its mathematics content results in the context which is functioned as example of application only. Suryadi (2013) suggested that if context only used as application, then student only imitated procedure which was showed by the teachers without understanding concept and when it was confronted to different situations, the students were most likely unable to answer and solve them, then it resulted in what is

called as learning obstacles.

In Indonesia, negative integer is encountered in mathematics curriculum at the beginning of secondary school, ie at the seventh grade (Kemdikbud, 2013). Some other countries, such as Singapore and Japan, also provide these materials in seventh grade. Researchers conducted a short interview to some 7th grade students and teachers of mathematics in secondary schools. According to them, even though it seems easier in grade seventh and teachers in secondary schools, this material is quite difficult to be understood, particularly in operation of number which involves negative number. Unlike positive number, negative number has not perception referential which is clear, and therefore, students should try harder to learn about negative number (Blair et al, 2012). Many of these are very good models of the positive and negative numbers but difficulties arise when we use them to illustrate the operations of addition, subtraction, multiplication and division (Galbraith, 1974).

When solving the problem, students should firstly understand the related concepts (Schmidt, 1997). Students who experience misunderstanding when receiving a learning concept for the first time, not only influence him/her when he/she learns that concept but will be result in next learning which is development of that concept. Basic concept which will be understood well by the students will give influence in understanding a context in higher level. Research about learning difficulties which is experienced by students will give underlying picture in developing effective intervention in teaching and learning activity, and therefore create hopes to improve mathematics teaching and learning in school and also for the development of knowledge itself (Li & Li, 2008).

Based on that view, this study was intended to find out students' understanding about the definition of negative numbers in daily life. This study is also more specifically to the students' difficulties in understanding the concepts of negative numbers and their counting operation.

2. Literature Review

2.1 The Concept of Negative Numbers

Negative numbers always become an interesting topic, particularly from mathematics didactic point of view (Thomaidis, 1993). The need for negative numbers arises both in the everyday world of measurement and in the mathematical world of number (Galbraith, 1974). The concept which is imparted in respondents thus far is that negative number has nature which is different to positive number which finally become obstacle in homogenization of (N, +) ∪(N, -). Two concepts which are different about positive and negative number will bring us to opposite pole and different domain. Thomaidis explained that the term 'positive label' and 'negative label' did not mean that positive and negative numbers but 'what is added' and 'what is subtracted'. Therefore, the natures of positive number operation actually are not prevailed in negative number.

Negative integer is part of integer, Z. In set N = {0, 1, 2, 3,…} from non-negative integer there is N = {1, 2, 3, 4….} which is positive number in which and for all a, b, c, έ Z, the operation nature in positive number are prevailed. For each a ε Z where -a ε Z it is prevailed that $a + (-a) = 0$ (invers of addition), then it can be written to become $a - b$ for $a + (-b)$ (Hungerford, 2000). The concepts of negative number like this can pose problem in respondent to understand it. Statement of $a - b = a + (-b)$ is difficult to be understood by students if they not introduced first by addition invers. Through this invers concept, it can be determined that $a - (-b) = a + b$. For some secondary school mathematics textbooks, the operation nature of integer addition and subtraction is written directly in the form of formula.

In particular, Sfard (1991) analyzed mathematical concepts in two underlying aspects namely structural and operational which was refer to two things separated: "object" and "process". Sfard showed that there are two stages of children's learning development in order to understand the numbers. When children learn number concept, they start from counting first. This is "process" stage, which is natural and relatively easy for child. Anyway, a child should change counting process toward abstract number concept. This is called "object" stage. Sfard argued that child should "transfer" from process to object in order to understand the concept entirely. In case of negative number, according to Sfard, child ability to think about a process entirely can be assessed through respondent ability in joining basic process with counting operation namely its ability to do arithmetic manipulation of negative and positive number.

The negative sign can take on at least three meanings in mathematics: unary, binary, and symmetric functions (Bofferding, 2014; Vlassis, 2008). Bofferding (2014) uses the term "minus" generically to refer to the "-" symbol, "subtraction sign" to refer to the binary meaning of the minus sign, and "negative sign" to refer to the unary meaning of the minus sign.The three concept dimensions of negative number which was explained by Altiparmark and Ozdogan (2010), namely: 1) the negative case which is focused on meaning of number, direction and quantity of number (numerical system), 2) meaning of counting and number line which focused on

counting operation which connected between what have been understood and the real world (we limited it only on counting operation of addition and subtraction) and the use of number-line as concrete illustration which described context of a situation, and 3) interpretation and explanation, that was how student understand the connection of negative number in a context, interpret, make generalization, determine calculation operation, and connect a context to another context.

2.2 Understanding and Epistemological Obstacles in Mathematics

In mathematics learning, understanding was a fairy common word spoken and often became the object of study in mathematics education. Understanding revealed to the meaning that the materials taught to the students not only as are citation, but also the students could understand the concept of the subject matter. Sierpinska (1990) regards that understanding as an act that involved in a process of interpretation, this interpretation being a developing dialectic between more elaborate guesses and validations of these guesses. The process of understanding starts with a guess which we further try to justify and validate. In the course of validation the guess may be improved, changed, or rejected. The new guess is then subjected to justification and validation. It continues as a spiral until it should be understood considered to have been appropriate.

In many cases the understanding and epistemological obstacles are two ways concerning about the same thing (Sierpinska, 1990), epistemological obstacles will appear if they are paired with understanding. Epistemological obstacles look backwards, attention focused on what is wrong in the ways of knowing. Understanding is the way to find and to know in the future perspective. Children will pass through both, if we look from the perspective of his past knowledge then we will see how these children overcome these obstacles. But if we look from the perspective of the future knowledge then we will look their understanding. Overcoming epistemological obstacles and understanding are two images that complement each other through epistemological analysis of mathematical concepts.

Concept of epistemological obstacles occurs in knowledge philosophy which is pioneered by Bachelard (Schneider, 2014). Brousseau then developed epistemological obstacles notion into didactic mathematics as one of learning obstacles. According to Brousseau (2002), based on its cause, learning obstacles are categorized into three types, namely: (1) onto genic obstacle; (2) didactical obstacle; and (3) epistemological obstacle. Ontogenic obstacle is obstacle which is related to child mental development stage in accord with its age and biological development. Some children sometimes lack of ability which is needed for cognitive aim related with age. If its obstacle derived from slow mental development (and not because pathology situation), then it will disappear together with its growth. Didactical obstacle is the obstacles which occur as result of learning choice which related with education system. This obstacle can be avoided through development of alternative learning approach (which is called by Brousseau as didactical engineering). The Introduction of integers by using a horizontal number line can cause conceptual onto genic and didactical obstacle (Suryadi, 2016). In understanding of number, onto genic obstacle, partly due to the inability of the children experienced the process of sense making, especially interpreted the 0 and negative integers. Forming process of the concept image of 0 and negative integers are difficult to represent in a concrete that make didactical obstacles, so it requires more thinking allows children. The last, epistemological obstacle is obstacle which can be avoided because of its important role in constructing its knowledge concept itself. According to Duroux (Brouseau, 2002, p. 99), epistemological obstacle is one's knowledge is limited to a certain context only. If the person is faced with a different context, the knowledge becomes unusable or she/he had difficulty to use it (Suryadi, 2016). Brown (2008) stated that "Epistemological obstacles can be construed as faulty ways of thinking but such a perspective ignores their importance, their developmental necessity, and their productivity in specific settings."

Cornu (2002) argued that epistemological obstacles are not related with learning approach which is used by teacher, but as result of mathematical concept nature itself. The argument proposed by Cornu refers to Bachelard (1938) who explained that epistemological obstacles occured both in history of scientific thinking development or in education activity. According to him, epistemological obstacle is important thing to obtain knowledge and it is found in history of concept development. Therefore, educators (in this case, teachers) should learn deeply epistemological obstacle in history of mathematical development in order to understand student obstacle in learning the concept. Those two things will complement each other as revealed by Brousseau. According to him, the most important things is to learn deeply epistemological obstacle by identifying concept error which is experienced by students which then compared to obstacle in its mathematical history to determined its epistemology character (Brousseau, 2002).

In terms of epistemological obstacles and understanding of the concept, Schneider (2014) explains that epistemological obstacles have some relations with conception or concept error in the form of mental object,

concept image, and student's point of view or thinking. It often happens that mathematical concept possessed by students is based on definition they know from their point of view. This may become one of student's obstacles in thinking mathematically to solve the problem (Vinner, 2014). This opinion is also revealed by Schmidt (1997) that difficulty often occured when students' ideas are different from the definitions accepted by experts. Initial concept which is understood by students sometimes not suited with concepts of scientist. Concept differences between what is understood by students in their learning process and knowledge science which cause concept misunderstanding or misconception (X. Li & Y. Li, 2008). From a child's point of view, this is normal and reasonable concept based on their experience in different context or in their activity in daily life thus it is not error from child's point of view (Fujii, 2014).

Related with concept of a material, there are some possibilities that cause learning obstacles, namely: (1) prerequisite knowledge; (2) concept understanding; (3) procedure; (4) principle; and (5) problem solving (Depdiknas, 2007). Procedure, principle and problem solving more related with structured knowledge which is student ability to apply concept (Widdiharto, 2008), included here the ability to translate the problem, identify problem solving scheme, determine strategy and solve the problem with appropriate algorithm. In relation with number, Bruno and Martinon (1999) use term 'numerical knowledge' which refer to concept, procedure, representation and algorithm.

Studies conducted on negative numbers showed the result that negative numbers have posed the problems for many high school students (Almeida & Bruno, 2014; Bofferding, 2014; Vlassis, 2008). The problems of negative numbers were caused by structure, position which was unknown, type of number or its context. Altiparmark and Ozdogan (2010) can reveal the difficulty in understanding concept of negative number that is understand numerical system, its direction and its amount, understand operation of arithmetic, and difficulty which is related with negative sign. Those three things are tightly related with abstract concept. This abstract concept of negative number is known as result in epistemological obstacle in development of mathematical knowledge (Larsen, 2012).

3. Objectives of Study

The underlying concept in mathematical material need to get more attention, because basic concept which can be understood well by students give influence in understanding learning material in higher level. In accord with what had been explained by Brousseau, researcher considered that it is important to identify student difficulty in understanding negative number. Through this identification, it is hoped that it can force teacher to create learning situation which is effective. The aims of this study are:

1) To study students' understanding about the definition of negative numbers in daily life.
2) To identify students' difficulties in understanding the concepts of negative numbers and their counting operation.

4. Method

4.1 Participants

Totally 96 students (11-13 years old) in 7th grade in three different schools in Palembang, Indonesia, were involved as participants. Researchers choose this grade because negative number material as part of integer subject matter had been given in 7th grade. Respondents consisted of 58 students of two different Public Junior High Schools (32 in Group A and 26 in Group C) and 38 students of Private Junior High School (Group B).

4.2 Data Collection

4.2.1 Instrument

Researchers observed students' epistemological obstacles by investigated their difficulties in understanding negative numbers by administering test to 96 students of seventh grade which had learned about integer. 25 problems (fill in blank and essay) were given to respondents related with integer and operation of integer. The questions were designed to identify the difficulties of respondents in understanding negative numbers which comprise aspects of prerequisite knowledge, concept of integer, counting operation procedure of integer, principle understanding, and problem solving which is adapted from diagnosis instrument of mathematical learning difficulty according to Depdiknas (2007) and Widdiharto (2008). Researchers also adapted three dimensions of negative number concept revealed by Altiparmark and Ozdogan (2010) as guidance in determining indicator of each difficulty aspect.

Instrument pass through validation qualitatively in material domain, construction and language by involving two mathematics education experts and three mathematics teachers. After revision, instrument was defined suitable to

be used in accord with the aim of education.

Table 1. The possibility aspect of students' difficulty problem source

The possibility of problem source	Indicator	Test items number
Prerequisite knowledge	Unable to distinguish type of number sets (set of integer and set of whole number)	1
	Unable to distinguish positive integer and negative integer	2
	Unable to do simple counting operation in integer	3
Understanding the integer concept	Unable to distinguish activity which describe integer and non-integer	4
	Unable to determine which number bigger than two integers	5
	Unable to order the number and position of number in number-line	6, 8
	Unable to write connection sign of two numbers correctly	7
	Unable to represent integer in a context	9
	Unable to give context example which represent integer	10
Operation procedure of integer addition and subtraction	Unable to do counting operation of integer addition	11, 13, 15
	Unable to understand invers concept in addition	12
	Unable to determine the value which is unknown in counting operation	16, 18
	Unable to do counting operation of integer subtraction	14, 17
Understanding the principle	Unable to state disparity of two integer	19
	Unable to determine position of object as result of counting operation	20
	Unable to use properties of counting operation in integer	21
Problem solving	Unable to represent a problem in daily life in the form of integer counting operation which involve negative number and strategy which is appropriate to solve it.	22, 23, 24,25

To obtain the accurate data of respondent understanding about negative number, researcher give problem which is related with procedure, principle, problem solving strategy in addition and subtraction operation which involve negative number.

4.2.2 Procedure

Test was given to all respondents as participants who agreed to take the test. Each respondent obtained item sheet and answer sheet and were not allowed to use calculator to ensure the objectivity and authenticity. Respondents are free to choose the number of item which will be solved first. Test implementation was also supervised by each math teacher in that class. Time provided is 90 minutes.

4.3 Data Analysis

Researchers gave score with 0 until 4 scales for each item. Therefore, the whole score was 100. This score was given based on the quality of respondent's answer to each item. This score was converted become 0 or 1 only with criteria 0 if score before are 0, 1, or 2 and 1 if score before are 3 or 4. This conversion considered that respondent who get score 0 – 2 not fully understand the intention of item, false in concept, principle or procedure. As for respondents who get score ≥ 3 are respondents who generally had answered correctly, even though there is miscalculation, but they had understand the intention of item, used concept, procedure and principle correctly.

5. Results

Analysis is done based on component of difficulty aspect to describe how many respondents who experience difficulty based on average score in each number of item. This analysis then continued by determining percentage of respondents who experience difficulty in each group. In the whole, researchers see the difficulty which almost distributed in all respondents in aspects of concept, procedure, principle and problem solving. This description showing information about difficulty experienced by respondents based on its source of problems in

negative number is presented below.

Table 2. Performance in every difficulties aspect of negative number

N =96	Prerequisite knowledge	Concept of integer	Procedure of counting operation	Principle related with concept and procedure	Problem solving
The numbers of item	3	7	8	3	4
Mean	54.78%	51.24%	74.92%	89.11%	86.67%
Median	58	55	87	93	93.5
Standard deviation	39.80	26.40	25.75	11.63	17.97

In Table 2, it was seen that difficulty which most frequently occur was in aspect of principle related with concept and procedure (89.11%), and problem solving which were the applications of concept, procedure, and principle understood by respondents (86.67%).

5.1 Prerequisite Knowledge

In average, there is 54.78% respondents who experienced this first problem with deviation standard is 39.80. This was because the difference which was very significant among its indicator (see Table 3).

Table 3. Percentage of respondents who comply indicator in first difficulty aspect

Indicator	Item number	The numbers of respondent who comply the indicator	Percentage
Unable to distinguish type of number sets (integers and whole number)	1	57	59.38
Unable to distinguished positive integer and negative integer	2	94	97.92
Unable to do simple counting operation in integer	3	7	7.29

From Table 3, it can be said that almost all students have understood concept of simple arithmetic counting operation. The highest error occured in item to asked respondent to identify negative integer and positive integer in a group of number (see Figure 1).

Number 2. Put a mark "O" to positive integer and "□" for negative integer in group of numbers as follow: 1, -2, -1, 7, ¾, 14,5, 13, -26, 8 1/2, 29, -4 ¼ , 32

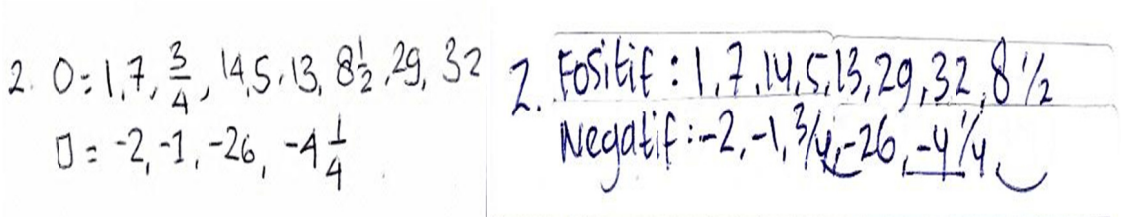


Figure 1. Gilang and Bella's answers

As a prerequisite knowledge, which must be mastered, the students should be able differentiate between integers and fractions, and can differentiate positive and negative numbers too. Gilang even included fractions into

integer. Whereas Bella categorizing positive fractions into negative integer. Their answers show that actually they can distinguish positive number and negative number but they do not know the difference between integer and fraction.

In detail, in group B all respondents give wrong answer, group A 97% and group C 96%, which means that there are only two students of 96 respondents who answer correctly. Respondent can distinguish which one is negative number and which one is positive number, but almost all respondents cannot distinguish integer and fraction number even though all respondents answer this item.

5.2 Understanding of the Integer Concept

The possibility of problem source in this aspect related to basic concept of integer, that is respondent understanding about the meaning of integer, order and position of number in number-line as general context which is often used, and representation of integer. In average, respondents who experience obstacle in this aspect is 51.24% with highest obstacle is that they are unable to distinguish activity which describe integer and non-integer (see Table 4).

Table 4. Percentage of respondents who were comply the indicators of second difficulty aspect

Indicator	Item number	The numbers of respondents who comply the indicator	Percentage
Unable to distinguish activity which describe integer and which describe non integer	4	82	85.42
Unable to determine the number which is bigger than two integers	5	32	33.33
Unable to order number and position of number in number-line	6	57	59.38
	8	29	30.21
Unable to write connection sign of two numbers correctly	7	23	23.96
Unable to represent integer in a context	9	50	52.08
Unable to give context example which represent integer	10	58	60.42

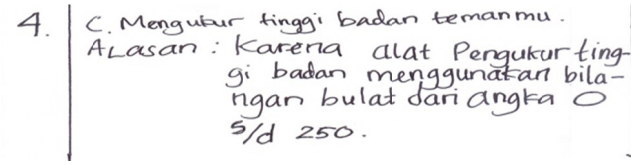
The problem can be seen in respondent understanding towardinteger when confronted to activity which describe integer (see Figure 2).

Number 4. What do you think, which one which describe integer in following activity. Give the reason of your answer.

a. Count the amount of books in cupboard.

b. Count the amount of children present in your class.

c. Measure the height of your friend.



Translate:

c. Measure the height of your friend.

Reason: Because the height is measured using integers from 0 to 250

Figure 2. Nugrah's answer

Nugrah choose (c) as activity which describe integer with reason that height measurement tool is integer from 0 until 250. Nugrah do not think that in measuring height it is possible that we will obtain decimal number. It is

seen here that Nugrah do not understand the meaning of integer.

To find out whether respondent can represent round umber into a context, then researcher ask respondent to determine the position of place by describing the drawing sketch of situation. This question is continued by giving some numbers to respondent, then respondent is asked to write the example of event in daily life which represent the meaning of that number (see Figure 3).

Number 9. Determine the position of a place which is determined as follow by making the drawing sketch of situation:

a. 150 meter above sea surface.

b. Three floors under basement.

c. Five steps backward.

Number 10. Give example in daily life the events which represent: (a) 15, (b) -10, (c) 0

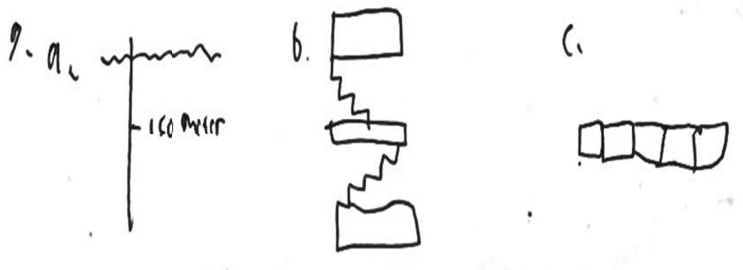


Figure 3. Andi's answer

Andi tried to represent what he think about 150 meter above sea surface, three floors under basement, and five steps backward. Generally, he can represent those three contexts with correct intention. But Andi does not explain which one surface, basement and initial point before moving backward. On the other word, the sketch he draws has not determined the direction of context appropriately. Andi then empty the answer of number 10 that asked him to represented 15, -10, and 0 in everyday life. There are more than 50% respondents have difficulty to represented these numbers in daily life.

5.3 Counting Operation Procedure of Addition and Subtraction

In this component, there are four items to diagnosis whether respondent understand counting operation procedure of addition and subtraction which involve negative integer with four indicators to find out whether respondent has obstacle in this aspect. The result show that in average, respondents who experience obstacle in this aspect are 74.92% with 95.83% respondents unable to do counting operation of integer subtraction, then followed in indicator which determine the value of number which is unknown as much as 93.75% (see Table 5).

Table 5. Percentage of respondents who were complied the indicator of third difficulty aspect.

Indicator	Item number	The numbers of respondents who comply the indicator	Percentage
Unable to do counting operation of integer addition	11	41	42.71
	13	82	85.42
	15	64	66.67
Unable to understand the concept of invers in addition	12	30	31.25
Unable to determine unknown value in counting operation	16	87	90.63
	18	90	93.75
Unable to do counting operation of integer subtraction	14	92	95.83
	17	85	88.54

In many answers, respondent apply commutative properties in addition to subtraction. The procedure of counting operation has not fully mastered well by respondents (see Figure 4).

Number 14. Fill in the blank in order to become correct sentence.

a. 36 – 17 = ...– 36 = ...

b. -24 – 8 = -8 – ... = ...

c. -10 – 12 – 5 = ...– ... = ...

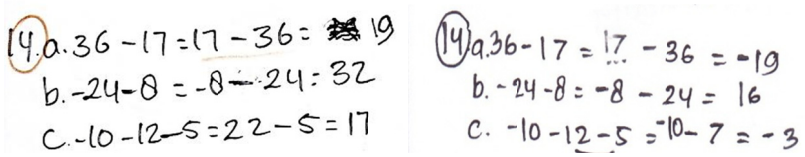


Figure 4. Ika and Endah's answers

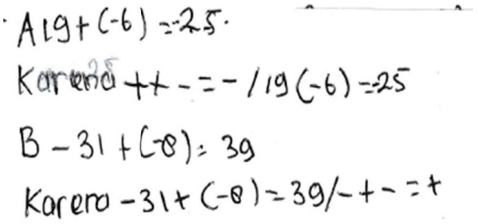
Ika and Endah gave 36 −17 = 17 – 36 as their answers, as commutative properties in addition (it should be 55 – 36). It is similar with -10 – 12 – 5, it should be (-10 – 2) – 5 not – 10 – (12 – 5). From the result of those two questions, in general respondents only know commutative properties in positive number addition only, they have not understand the meaning of commutative and associative properties in counting operation of integer.

The mistaken perception about concept of counting operation is experienced by many respondents. In average, 85.42% has not understood this concept even though some of them have understood the procedure of counting operation (see Figure 5).

Number 15. Finish the counting operation below:

a. 19 + (-6) = ...

b. -31 + (-18) = ...



A. 19 + (-6) = -25. Becauce (+) + (-) = - , 19 (-6) = -25
B. -31 + (-8) = 39. Becauce -31 + (-8) = 39, (-)(+) = +

Figure 5. Nanda's answer

Nanda's answer shows that she added all numbers without considering the value of positive and negative number. Nanda explained the reason as follow:

Researcher : Nanda, I want to ask you about your answer in item 15. The question of 15(a) what is that?
Nanda : 19 + (-6) equal with.....
Researcher : What the result do you get?
Nanda : Min twenty five.
Researcher : Why is it so? Please explain how do you get it?
Nanda : (Write 19 + (-16) on a piece of paper). Here it is, 19 is positive whereas 16 is negative.
Researcher : And then?
Nanda : Positive if it meet negative then the result is positive. Thus, it get minus 25.
Researcher : Mmmm…what about (b) ?
Nanda : negative number minus 31 meets negative number minus 8 equal with 39.

Researcher : Why is it so?
Nanda : If negative meet negative then the result is positive, isn't it?

The same case occured in item of addition counting operation as follow (see Figure 6).

Number 13. Count the addition of number as follow:

a. 12 + (19) + 15

b. -23 + 18 + (-35)

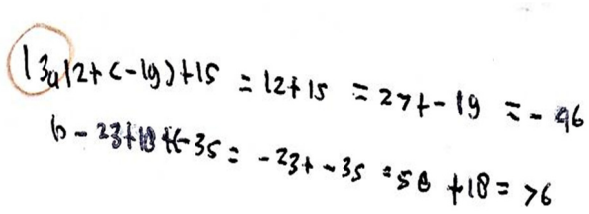


Figure 6. Joni's answer

Joni is among of clever student in his class. Joni's answer showed that he had understood associative properties in addition with method $a + b + c = (a + c) + b$, a, and c are numbers which have the same sign but falsein getting the result of his counting operation. According to him, if a + (-b) then it will result in positive number because negative which is side by side with negative will result in positive number. This is a concept which made Joni's answer and some other children become false. From the correctness of final answer they get, in detail 75% respondents of group A still false in making value of counting operation, group B is 53% and group C is 77%.

5.4 Understanding the Principle

Understanding the principle means respondent's principle understanding toward concept and procedure of counting operation in integer which is related with certain context. On average, there are 89.11% of respondents who have difficulties in this aspect. The highest obstacle occur in third indicator, all respondents unable to use the properties of counting operation in integer (see Table 6).

Table 6. Percentage of respondents who comply the indicator of forth difficulty aspect

Indicator	Item number	The numbers of respondents who comply the indicator	Percentage
Unable to state disparity of two integer	19	79	82.29
Unable to determine the position of an object as result of counting operation	20	80	83.33
Unable to use properties of counting operation in integer	21	96	100

To identify whether respondents are able to apply concept and procedure of counting operation, researcher pose the items as follow:

Number 21. The figure below is magical rectangle with addition operation. The amount of numbers in each row and column always the same. Determine the value of a and b.

-7	4	*b*
-4	*a*	-3
10		0

Beside respondents must determine the counting operation which is appropriate, they also firstmust make sure that all column and row is totaling -1. This is which is missed from respondents' answer (see Figure 7).

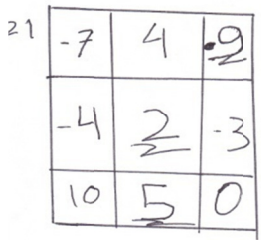


Figure 7. Zaky's answer

Zaky fill in the box which is blank without passing through counting operation procedural which is appropriate. He should firstly add -7 + (-4) + 10 and got (-1). In this case all respondents unable to identify that -7 + (-4) + 10 = -4 + *a* + (-4) + 10 = -7 + 4 + *b*. Zaky found difficulty in abstracting the properties of that mathematical object. This is parallel with obstacle before in determining unknown number in counting operation in which almost all respondents cannot do it. This fact showed that he had difficulty in applying the nature of integer counting operation in solving counting operations.

In another problem, respondents used number line to represent a context. Actually this way is effective to determine position of certain object. The obstacle occur when respondent define the direction of number mistakenly (see figure 8).

Number 20. Before moving, a lift in the building is on fourth floor. Now that lift is on sixth floor under its original position. If basement is considered as 0, on what floor the position of lift right now?

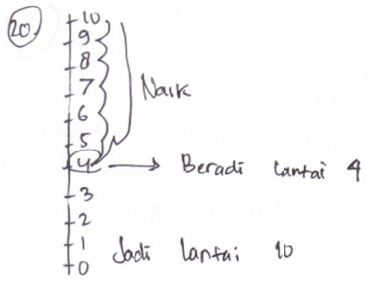


Figure 8. Santi's answer

Santi used number line and start in number 4 to represent the original position of lift. According to her answer, at first itlift was on the 4th floor then moved up as much as six floors and now is on 10th floor. Here, it seems that Santi did not know the intention of item that the position of lift now is under original position as negative number. Other respondents know the intention of item and are able to write the operation which is appropriate but false in determining the result of its operation. Most of respondents unable to applied the counting operation in a context, 84% respondents of group A, 87% of group B, and 78% of group C.

5.5 Problem Solving

Researchers gave four forms of daily problem to see respondent difficulty in representing a problem context in counting operation of integer and procedure and strategy which was appropriate to solve it. In a whole, 86.67% respondents found obstacle in this aspect (see Table 7).

Table 7. Percentage of respondent which comply the indicator of fifth difficulty aspect

Indicator	Item number	The numbers of respondent who comply the indicator	Percentage
Unable to represent a problem in daily life in the form of integer counting operation which involve negative and positive numbers which is appropriate to solve it.	22	96	100
	23	58	60.42
	24	85	88.54
	25	94	97.92

This problem posed is contextual problem which is often encountered by students. The problem which close with child habitual is believed can develop the stage of child's formal operational thinking. The problem posed require respondent to be able to understand the intention of task first, then identify the problem, construct mathematical object, and then determine strategy to get solution. The results show that in average almost all respondents in each group find obstacles in solving the problem mathematically (see Figure 9).

Number 22. A motor boat move at 35 km/hour speed on calm water in a river. In flowing water, this motor boat can move at 40 km/hour speed against the stream. If stream of river water at that time has 5 km/hour speed, how fast this motor boat move by passing through a tree in the edge of river stream when a) against the stream and b) in the same direction with stream?

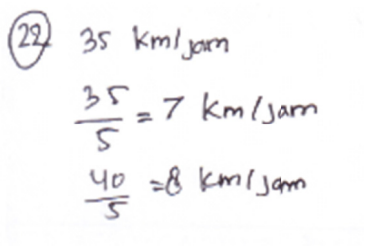


Figure 9. Erika's answer

Erika used division operation to answer this problem. Erika and all other respondents did not understand the meaning of 'against the stream' and 'in the same direction with stream' as representation from direction which symbolize addition and subtraction. The finding obtained was 26 respondents (68%) in group B did not give answer at all. Some respondents tried to answer this problem by adding 40 and 35, multiply 40 with 5, or another form of operation.

In another problem, there was also the same problem in which respondent unable to understand the problem, determine the scheme and strategy in solving, and choosing appropriate algorithm.

6. Conclutions and Discussions

Introducing negative number to students require them to change deeply rooted conception about number which is built in all elementary schools (Almeida and Bruno, 2014). When negative number subject matter is given, conceptual and procedural change occurred which involve knowledge namely there is number which is less than zero. Most children who not familiar with number as something real for them will experience disequilibrium or imbalance in their cognitive thinking. The main change was the transition from a concrete viewpoint to a formal viewpoint (Vlassis, 2008).

From result of study about student difficulties in understanding negative number, there is obstacle experienced by respondents in each aspect. The main obstacle which most frequently occur is in concept of integer particularly which is related with negative number. Most respondents recognize the number only as formal symbol in mathematics but they do not realize that daily activity is embodiment of the number itself. Learning obstacle which possibly occurs is respondents not get used to represent integer context and make a meaning of integer in daily life. At the difficulties on the concept of integer in Table 4, more than 59% of respondents cannot sort of negative numbers correctly. Most of them put a negative number with the wrong order. They argued that-11 is smaller than -19 and -27, so the sequence of numbers became -11, -19, -27. Meanwhile they were right in to sort of positive numbers. When students learn that negative integers exist, they might accept that negative

integers are less than zero but argue that -5 is greater than -3 because -5 is further away from zero than -3 is, just as 5 is further away from zero than 3 is (Bofferding, 2014). For them, negative number is something abstract so when represent that number into concrete form they experience obstacles in translating it.

Furthermore, based on Table 4 also found that more than 50% of respondents unable to represent negative number in all contexts. The abstract nature of negative number become main epistemological obstacles which occur in students (Almeida & Bruno, 2014; Altiparmak & Ozdogan, 2010; Blair , 2012; Larsen, 2012). This can occur because thus far student is introduced to number concept as a real thing. Negative number which has different sign with positive number cannot be imagined in reality by students. The problem aroused in understanding the sign of number play a role in determining strategy which is used to solve the problem because the sign which opposite is more complex compared to the sign which is similar (Almeida & Bruno, 2014).

The procedure of counting operation has highest figure as one problem source of student's difficulties. Based on Table 5, showed that 95% respondents who are unable to finish calculation the operation of addition with negative number. The concept of counting operation occured because there was false perception held by students. Perhaps this false concept is result of perception they received from explanation which is given by teachers in the form operation of $a + (-b)$ to become $a - b$. Respondents consider that positive number which side by side with negative will result in negative number. As for the form operation of $a - (-b)$. The false perceptions become the main problem of arithmetic counting operation in students. Bofferding (2014) stated that when students interpreting movements as "more negative" or "less negative," they might ignore the direction of "positive" or "negative" and move *more* or *less* based on the positive number line, or they might pay attention to moving in the positive or negative direction without paying attention to whether they should move "more" or "less" in that direction.

On the average based on Table 7, it found that more than 85% respondents having obstacle on problem solving which one of difficulties related to negative numbers. The general problem encountered is that respondents are unable translate information which contain in this problem so respondents do not understand counting operation procedure which is appropriate to describe problem condition. When they are able to understand the intention of problem, the main difficulty is determining strategy which is appropriate to solve the problem. The problem is that most students are unable to interpret a context into negative number by counting operation of subtraction. Glaeser (Heeffer, 2011) state that some epistemological obstacles experienced by students in learning negative number namely inability to manipulate the numbers of negative and difficulty to give the meaning on negative number. Students find it difficult in making meaning of negative number because the numbers of objects surround them is symbolized as positive number. It makes the students difficulty when being confronted with integer operation involving negative number. This is strengthened by result of this study that 60.4% of respondents experienced difficulty when they give context example which represent integer which involve negative number.

The strategy to solve the problem become not suitable which result in respondent solve the problem by false procedure. From a study delivered by Hughes (1986), some students who are able to translate the problem into mathematical model can be false in determining the solution of that problem. Student's knowledge which is very limited about counting operation, skill to apply basic operation process into new problem become the causing factor why students are unable to solve the problem (Hughes, 1986, p.8). Some studies which had been conducted in Indonesia showed that student understanding toward number still low particularly in doing counting process (Purnomo et al., 2014).

Children in aged 11 years old experience transition from concrete thinking into abstract. As revealed by Piaget, children begin to be able to do symbolic manipulation and enhancement of abstract thinking, and have intellectual potential to do formal reasoning (Hill, 2011). From this study, it can be see that students in grade 7th are transition from elementary school into secondary school, when initially they think informal with concrete operation switch to formal operation which is more abstract. It is not surprising that understanding the concept of abstract negative number become obstacle for student. A process bridging those two matters will be very helpful for students to minimize the difficulties occurred.

Negative numbers are an abstract concept for which students need phenomenological guidance in order to avoid epistemological obstacles (Larsen, 2012). Heeffer (2011) states that it is important for mathematics teachers to understand the history of the epistemological obstacle of negative numbers in the history of development of mathematical concepts to explore their students' difficulties when learning concepts of negative numbers. Identifying students' difficulties in understanding the negative numbers allows teachers to design a studying of the negative numbers as a follow-up. In planning to teach a mathematical concept it is of the utmost importance

to determine the possible obstacles, particularly the endemic epistemological obstacles (Cornu, 2002). Therefore, an instructional design activity through the right didactical situation expected to minimize the students' difficulties on negative integers.

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