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Transition Process of Procedural to Conceptual Understanding in Solving Mathematical Problems

Fatqurhohman[1,2]

[1] Department of Mathematics Education, PGRI Banyuwangi University, Indonesia

[2] School of Postgraduate, The Learning of Malang, Malang, Indonesia

Correspondence: Fatqurhohman, Department of Mathematics Education, PGRI Banyuwangi University, Jl. Ikan Tongkol 22 Banyuwangi, Indonesia. E-mail: frohman86@gmail.com

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Abstract

This article aims to describe the transition process from procedural understanding to conceptual understanding in solving mathematical problems. Subjects in this study were three students from 20 fifth grade students of SDN 01 Sumberberas Banyuwangi selected based on the results of the students' answers. The transition process from procedural to conceptually based on three aspects: (1) identify problems in the use of an algorithm, (2) the process algorithm, (3) connect multiple concepts to transform into another shape through the symbolic/picture representations. The results showed that the majority of students (18 students out of 20 students) only meets two (2) aspects of 10 students (50%) can identify the algorithms and the use of algorithms, 8 students (40%) able to use algorithms and connect with other forms. While other students (two students from 20 students) of 10% that meet only three aspects of the transitions. Thus, understanding the procedural has an important role in developing a conceptual understanding. Because the component/aspect of procedural understanding exist on components/aspects of conceptual understanding. Thus, the association acquired several components/aspects that support the process of transition from procedural understanding to a conceptual understanding.

Keyworsd: procedural understanding, conceptual understanding, mathematical problems

1. Introduction

1.1 Background

Mathematical is also a form of science that are arranged in a systematic, logical and structured. Each student must learn mathematics with understanding (Marchionda, 2006; Isleyen & Isik, 2003). Variety of mathematical concepts can be understood only after the students have acquired skills in using a procedural concept which then led to a better understanding (Rittle-Johnson et al., 2001). The skills to use the procedure is necessary so that students do not encounter obstacles and learning objectives in class right on target, especially in learning mathematics (Rahaimah & Noraini, 2013; Mousley, 2004).

Some studies show that success in solving mathematical problems is supported by ideas that allow a deep understanding (Stylianou, 2002; Parkinson & Redmond, 2002). In solving the problem, students need the support of a strategy that will govern the interpretation and manipulation of information in analyzing and interpreting for the selection procedures and making the right decisions with the ability to think (Johnson, 2010; Lager, 2006; Geary, 2004). At a deep level of understanding of the students began to be able to make the connection between mathematical ideas and make a generalization of the concept armed with the basic capabilities already understands (Potter & Kustra, 2012).

There was a significant relationship between procedural and conceptual understanding of mathematics (Hutkemri, 2010; Hiebert & Lefevre, 1986). procedural and conceptual understanding is also important in school mathematics, especially in problem solving (Krulik & Rudnick, 1996; NWREL, 2000). Thus, procedural understanding can assist conceptual understanding in mathematics (Lim, 2002).

Procedural understanding is the ability to solve problems by using a procedure that is efficient, accurate, true to add, subtract, multiply and divide (Askew, 2012, p. 55; Kilpatrick et al., 2001, p. 18). While, conceptual understanding is the ability to classify objects that make up the concept, applying the concept in the algorithm, presents the concept in various forms of mathematical representation, and linking the various concepts (Effandi

& Norliza, 2009; Romero & Mari, 2006; Chappell & Killpatrick, 2003). In addition, a conceptual understanding of knowledge that involves a thorough understanding of the basic concepts in mathematics an algorithm (Marchionda, 2006; Angel, 2007).

Some aspects of your procedural understanding: ability to reason through a situation, ability to use the algorithm in a problem situation, ability to perform noncomputational skills such as rounding and ordering, ability to verify the truth of the procedure, and select and implement the appropriate procedures (National Assesment of Educational Progress, 2003). Kilpatrick et al. (2001, p. 18) suggests three aspects of the conceptual understanding that comprehension of mathematical concepts, operations, and relations. Comprehension of mathematical concepts can be done by identifying and using appropriate concepts to solve problems. Operations can be known through the use of shape and determine the results of operations that fit correctly. Relations can be done through a representation (picture or symbol numeric). Thus, the process of transition from procedural understanding to conceptual understanding of using the three aspects, which can identify problems in the use of an algorithm, the process an algorithm, connecting some of the concepts to transform into another shape through the representation of a symbol or picture. These three aspects are believed to represent the students 'understanding of the ability of a procedural and heading ability of the students' understanding conceptually.

1.2 Objectives of the Study

Procedural understanding has important aspects of the conceptual understanding of learning mathematics, the purpose of this study is to describe the process of transition from procedural understanding to conceptual understanding of of solving mathematical problems.

2. Method

2.1 The Participants

Subjects in this study were three students from 20 fifth grade students of SDN 01 Sumberberas Banyuwangi selected based on the results of the students' answers. The transition process from procedural to conceptually based on three aspects: (1) identify problems in the use of an algorithm, (2) the process algorithm, (3) connect multiple concepts to transform into another shape through the symbolic/picture representations. Based on these criteria, selected 1 (one) student each of the criteria be used. It is considered be representative of aspects the transition procedural to conceptual understanding.

2.2 Instruments

The instrument used in this study is all about tests and structured interviews. Test questions about the description given in the form that is tailored to a predefined aspect. While structured interviews necessary to dig or search for more in-depth understanding of the transition process prosesural to students' conceptual understanding. So in this way will be able to help and obtain complete information in analyzing the results.

Instrument test items as follows.

> Adi berada 13 meter di kiri tiang bendera. Ati berada 6 meter di kanan Adi. Sedangkan Popy berada 9 meter di kiri Ati. Jika posisi tiang bendera dianggap titik nol, berapa meterkah jarak Popy dari tiang bendera? Berikan penjelasan dari jawaban anda!

Translate in English:

> Adi is located 13 meters to the left of the flagpole. Ati is 6 meters on the right Adi. While Popy is 9 meters to the left Ati. If the flag pole position is considered the zero point, how many meters distance Popy from the flagpole? Please provide an explanation of your answers!

Table 1. Components of trancitions the procedural-conceptual understanding

Komponen Transisi	Indikator/Sub-Component
Identify problems in the use of an algorithm	Finding a logical basis for using an algorithm or specific operations.
	Finding a logical basis for applying an algorithm to solve the problems
The process algorithm	Verify the use of algorithms/computing
	Selecting the use of algorithms/computational precise and correct
Connect multiple concepts to transform into another shape through the symbolic/picture representations	Shows the relationship/linkages with some of the concepts in the process an algorithm to solve the problem
	Shows interpretation through other forms of representation to solve the problem correctly

2.3 Procedure

Problems test was given to all participants (20 students) who voluntarily agreed to participate in solving a given problem. Before conducting the test, students were given instructions/rules read by researchers, are: (1) students are given 30 minutes to solve the problem; (2) students are not allowed to collect the answer before the appointed time finish; and (3) students are not allowed to ask or see the results of his friend work. After completing the test, researchers examined the results and conduct a structured interview to students who meet the specified three aspects. The interviews were conducted one day after the test is performed.

2.4 The Data Analysis

Data is obtained from answers to students who meet the three aspects of the transition, are identify problems in the use of an algorithm, the process algorithm, and connect multiple concepts to transform into another shape through the symbolic/picture representations. Data were analyzed quantitatively and qualitatively. Quantitative data to show the percentage of each aspect of the transition. While, qualitative data to describe the process of transition based on the specified aspect. The instructions coding used in qualitative data are shown in Table 2 below.

Table 2. Coding of aspects transition the procedural-conceptual understanding

Coding (student)	Description
S1	Students 1 is identify the problem
	Students 1 is using the algorithms/calculations
S2	Students 2 is using the algorithms/calculations
	Students 2 is connect multiple concepts through the representations
S3	Students 3 is identify the problem
	Students 3 is using the algorithms/calculations
	Students 3 is connect multiple concepts through the representations

3. Results and Discussion

3.1 Results Analysis Data

The results of the data obtained were analyzed quantitatively and qualitatively. Analysis of quantitative data obtained to show the percentage of each aspect of the transition. While the analysis of qualitative data to describe the process of transition based on the specified aspect.

3.1.1 Quantitative Data

Percentage results of the students' answers are categorized based on each transmission components prosesural-conceptual understanding can be identified in Table 3 below.

Table 3. Percentage on the answers in every aspect of transition procedural-conceptual understanding

Coding every aspect of transition	Students (n)	Average (%)
(1), (2)	10	50
(2), (3)	8	40
(1), (2), (3)	2	10

(1) Identify problems in the use of an algorithm

(2) The process algorithm

(3) Connect multiple concepts through the representations

Based on Table 3. Obtained illustrates that 10 students (50%) can identify the using the algorithm and the use of an algorithm, followed 8 students (40%) able to use algorithms and connect it to another form. Moreover, very few students can use a third aspect of the transition that is a number of 2 students (10%). By identifying the problem, students will be able to plan the use of the procedure (an algorithm) to solve the problems, then the results of an algorithm, students can check back through the relationships among concepts in the identification and algorithms used. Checking back through the relationships between concepts and an algorithm by changing to another form, such as using images to clarify the results of an algorithms.

3.1.2 Qualitative Data

(1) The results of the students' answers (S1)

Based on the analysis of quantitative data S1 obtained that 10 students (50%) can identify the using the algorithm and the use of algorithms. After the results are corrected in order to determine the procedural understanding that refers to the indicators set which is then amplified by the transcript of the interview, the obtained results can be seen in Figure 1, as follows.

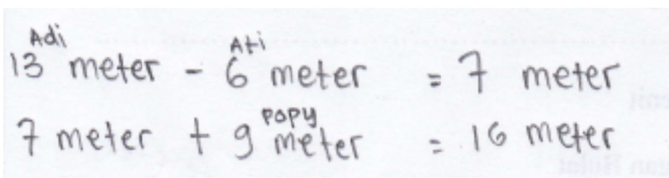

Figure 1. S1 (identifying problems and the use of algorithms)

Translate in English.

Adi distance of 13 meters and the distance Ati 6 meters (on the right/left Adi), so the distance Ati is 7 meters from Adi. Then, a distance of 9 meters Popy towards Adi, the obtained result is 16 meters.

Based on Figure 1. S1 has not shown any aspect of the transition from procedural to conceptual understanding good. S1 has not provided the identification and use of an algorithm in correctly. In addition, an explanation of the use of the procedure have not been able to give the right answer. Is the distance Popy located on the left or right of the flag pole ?. After conducting further interviews, S1 disclose that Popy is located to the left of the flag pole or a negative value are -16 meters (the symbol number).

In other words, the use of algorithms to heading selesaian given in Figure 1 is a systematic or students using the usual procedure used before. While, the transition to a conceptual understanding shown is the problem identification and use of the operations of addition/subtraction is diasumsikannya still not showing anything specific. Such as, Adi and Ati position on the flag pole as the initial reference usage operations. S1 does not explain the final result (the position Popy) from selesaiannya that position Popy is 16 m door (left or right) flagpole. Thus, S1 is said to not meet the specified aspect properly, although some aspects have been fulfilled.

(2) The results of the students' answers (S2)

Based on the analysis of quantitative data S2 obtained that 8 students (40%) able to use an algorithm and connect with other forms (representation). The results of the students' answers S2 can be seen in Figure 2, as follows.

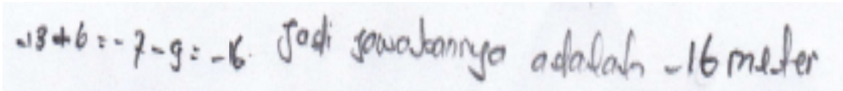

Figure 2. S2 (using of algorithms/calculations)

Translate in English.

$$-13+6=-7-9=-16$$

So, the answer is -16 meters.

Based on Figure 2. The results of the answers S2 is believed that having a good conceptual understanding. This gives the results of the use of algorithms that worked S2 by using a negative sign (-) and symbols (16). Based on the interview given S2 that the sign is written gives meaning Popy is located to the left of the flagpole. While, a symbol that is written gives meaning the distance between the flagpole with the Popy position as far as 16 meters. Then, the S2 has other explanations using representation through the a number line to connect multiple concepts (knowledge) of understanding. The result can be seen in Figure 3 as follows.

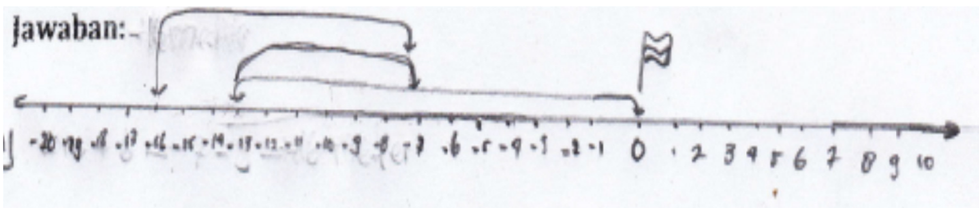

Figure 3. S2 (connect multiple concepts through the representation)

Based on Figure 3. S2 does not give a full description. After further interviews. S2 makes clear that a flagpole as a starting point, the position of Adi is 13 meters to the left of the flagpole. Then position Ati 6 meters to right of Adi position. Furthermore, the position Popy 9 meters to the left of the position Ati (3 meters to the left of the position of Adi). Representation of the results using a number line that S2 is assumed to already be able to use his understanding conceptually, although it has not been able to give a full explanation. It is believed to be a transition process that occurs is the use of representations by connecting multiple concepts (knowledge) possessed can improve student understanding.

(3) The results of the students' answers (S3)

Based on the analysis of quantitative data S3 obtained that very few students can use a third aspect of the transition is to identify problems in using an algorithm, the use of an algorithm, and connect it to any other form (representation) for 2 students (10%). S3 answer results indicate that the position of each Adi, Ati, and Popy from the flagpole by connecting some of the concepts to other forms through the are picture representations using a number line to make it easier to position Adi, Ati, and Popy. The result can be seen in Figure 4 below.

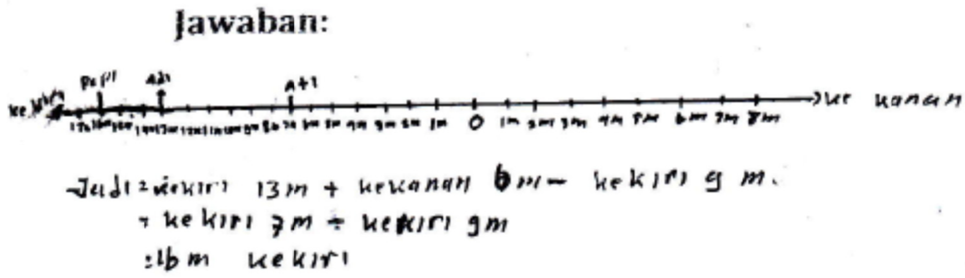

Figure 4. S3 (connecting several concepts through the representation and process algorithm)

Translate in English.

Positively defined direction to the right and left directions to the defined negative

The position of the flagpole; Ati 9 meters, Adi position 13 meters, Popy 16 meters.

So; = left to right 13 meters + 6 meters - left 9 meters.

= Left 7 meters + left 9 meters.

= 16 meters to the left.

Based on Figure 4. S3 provides an answer using the number line operation. S3 easily determine the starting position to the end position according to the information problems are understands. S3 determine the final outcome of the initial position (flagpole) to position Ati 9 meters, Adi position 13 meters, 16 meters final position Popy Popy (left of the flag pole) though no explanation are the displacement/shift position. But information or explanations given S3 can be explained from the results of an algorithm. So that others can easily understand that thought S3. In addition, S3 provides another explanation by giving examples to do with the answers he got. S3 explanation results can be seen in Figure 5 below.

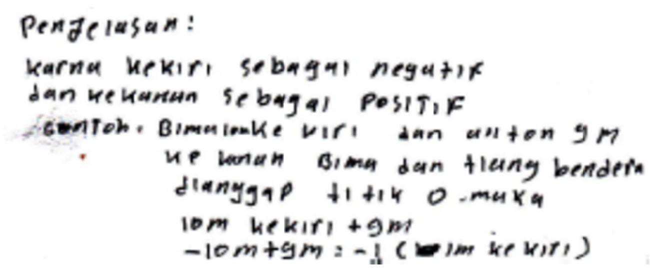

Figure 5. S3 (identifying problems and process an algorithm)

Translate in English.

Explanation:

because the left and to the right as negative as positive.

Example: if Bima 10 meters to the left and to the right Antoh 9 meters Bima. Flagpole considered 0 (the starting point), then the 10-meter left + 9 meters to the right.

-10+9=-1 (1 meters to the left).

Based on Figure 5. The S3 provides an explanation or description which reflected the assumption that the value of 0 is a reference to a shift in direction or position toward the flagpole. Direction (position) left of the flag pole is defined as negative values and to the right is defined as a positive value. Thus, the assumptions made S3 facilitate the operation to obtain the final result. After making these assumptions, S3 to resolve the problem and the result is 16 meters (to the left of the flagpole).

3.2 Discussion

The results showed that there were 10 students (50%) can identify problems and use of algorithms with indicators of the transition is to find a logical basis for the use of an algorithm (certain operations) and apply the an algorithm in solving the problem, select the use of algorithms/computational precise and correct; 8 students (40%) able to use algorithms and connect it to any other form (representation) with indicators of the transition is to choose the use of an algorithm/computation is accurate and correct, shows the relationship/linkages several concepts to the algorithm in the process of solving problems, and show the interpretation through the representations to other forms in solving the problem in correctly. Moreover, very few students can identify the problem, the use of an algorithm, and connect multiple concepts into another form (representation) is as much as 2 students (10%) with indicators of the transition is to find a logical basis for using the algorithm (specific operation), select use an algorithm/computation is accurate and correct, and shows the interpretation through the other forms of representation to solve the problem in correctly.

The findings of this study showed that most of the students understanding of the procedural and little understanding conceptually students. The learning activities are the most important thing is to build understanding (procedural and conceptual) in the classroom (Mousley, 2004). This is because the understanding of mathematical concepts is an important part in the process of learning mathematics (Hasnida & Zakaria, 2011). According to Potter and Kustra (2012), at the level of deep understanding of the students began to be able to make the connection between mathematical ideas. The linkage between mathematical ideas is one component in the conceptual understanding.

According to Wisconsin (2007), with a deep conceptual understanding of students who can give meaning to mathematics through the ability to apply their knowledge. Developing procedural knowledge have a positive effect on conceptual understanding (Rittle-Johnson et al., 2001; Johann et al., 2005). Furthermore Mary and Heather (2006) argue for successfully resolve the problem, students must develop a better understanding of the procedural and conceptual. With a conceptual understanding, students see the relationship between concepts and procedures that can provide arguments to explain some facts. In addition, the conceptual understanding help students avoid many critical errors in problem solving. More specifically stated NCTM (2000, 20) that it when students gain conceptual understanding the specific mathematical topics, the students can see connections between concepts and procedures that can give reasons to explain some facts that are a consequence of other facts.

Other findings from the discussion explained that if students learn a procedure without understanding will require extensive training so that steps procedure can be performed easily and correctly. Thus, the use of the procedure can strengthen and develop conceptual understanding. Components transition from procedural understanding to conceptual understanding able to help the teacher comprehend the thinking of students in depth and know the relationship is strong enough to complement each other. So with some procedural transmission components such conceptual to identify problems in the use of an algorithm, the use of an algorithm, and connect with other forms (of representation) able to facilitate teachers in the learning process that refers to a conceptual understanding, although the level of students' understanding of different.

4. Conclusions

From the discussion described, it could be concluded that it procedural understanding has an important role in developing a conceptual understanding. This is due to the component/aspect of procedural understanding exist on components/aspects of conceptual understanding. Thus, from this association gained some components/aspects that support the process of transition from procedural understanding to conceptual understanding. Components/aspects of the transition in question are (1) identify problems in the use of algorithms with the indicator are find a logical basis for the use of an algorithm (certain operations) and apply an algorithm to solve problems; (2) process an algorithm with the indicator are verifies the use of algorithms/computational and choose the use of an algorithm/computation is accurate and correct; (3) connecting some of the concepts to another (representation) with the indicator are shows the relationship/linkages several concepts to the algorithm in the process of problem solving and interpretation through the representation to show another form in correctly.

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