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An Empirical Analysis of the Property Catastrophe Reinsurance

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Abstract

Following a series of costly catastrophes, including Hurricane Harvey and Hurricane Irma in 2017 and the Sichuan Earthquake in 2017, the purchase of property catastrophe reinsurance has become a major topic of debate. Many techniques for selecting an optimal retention and upper limit level have been proposed, but no entirely satisfactory method has been devised. Therefore, in practice, the setting of retentions and upper limits is still more a matter of judgment than science. In this study, we examine the determinants of property catastrophe excess-of-loss reinsurance retentions and limits for property-liability insurance companies in the U.S. insurance industry. A cross-sectional model is estimated using two-stage least squares regression. The regression analysis shows that most coefficients have the hypothesized signs and are significant. This study is the first research that provides clear evidence to support the relationship among retentions, upper limits, and co-reinsurance rates.

Keywords: catastrophe reinsurance, co-insurance, retention, upper limit

1. Introduction

Over the past 10 years, the importance of property reinsurance in the property-liability insurance industry has growth because the exposures to natural catastrophes such as hurricanes and earthquakes have increased dramatically in the United States.[1] Following a series of costly catastrophes, including Hurricane Harvey and Hurricane Irma in 2017 and the Sichuan Earthquake in 2017, primary insurers have experienced a difficult time in obtaining catastrophe coverage due to the capacity limits of the U.S. and worldwide reinsurance market. Furthermore, the insurers' exposure to natural catastrophes is constantly increasing because demographic trends and rising property values escalating the concentration risk in catastrophe-prone areas. Many experts expect that the insured losses from catastrophic disasters in the future are beyond any figures previously imagined. As a consequence, the purchase of property treaty reinsurance, especially catastrophe excess-of-loss reinsurance, has become a major topic of debate.

The purchase of excess-of-loss reinsurance treaty is a high-profile annual event involving a significant amount of investigation, analysis, and negotiation. Many techniques for the optimal retention and upper limit level have been developed, but no entirely satisfactory method has been devised. (See Table 1.) Therefore, in practice, the setting of retentions and upper limits is still more a matter of judgment than science.

The purpose of this study is to investigate how the primary insurers make their excess-of-loss reinsurance decision. More specifically, this study is designed to identify the factors affecting excess-of-loss reinsurance retentions and upper limits in the U.S. insurance industry. Reinsurance studies can give us insights into how other financial intermediaries may compete with reinsurers in providing property catastrophe coverage.

The paper is organized in the following manner. Section 2 shows the literature review. Section 3 provides theory and alternative hypotheses for catastrophe excess-of-loss reinsurance retentions and limits. In Section 4, the data to be used are explained, and then models presented. Estimation results are shown in Section 5. The main findings and limitations of the study are summarized in Section 6.

[1]The loss ratio for property insurance for 1989, 1992, 1994, 2017 are very high reflecting catastrophic losses caused by Hurricane Hugo, Hurricane Andrew, the Northridge Earthquake and Hurricane Harvey, respectively.

Table 1. List of Retention Studies

Author	Year	Article (Book) Title	Journal	Types of Re	Approach
Borch, K.	1960	An Attempt to Determine the Optimum Amount of Stop Loss Re.	TICA	Stop Loss	Minimum Variance
Vajda, S.	1962	Minimum Variance Reinsurance	AB	Proportional	Minimum Variance
Verbeek, H.	1966	On Optimal Reinsurance	AB	Non-proportional	Minimum Variance
Borch, K.	1961	The Utility Concept Applied to the Theory of Insurance	AB	Proportional	Expected utility
Pesonen, M	1984	Optimal Reinsurance	SAJ	Proportional	Expected utility
Gerber, H	1984	Chains of Reinsurance	IME	Proportional	Expected utility
Zecchin, M	1987	Calculation of the Maximum Retentions in XL Reinsurance	IME	Pro/Non-pro	Expected utility
Paulsen et al.	1994	Properties of Functions of the XL Retention Limit with Application	IME	Non-proportional	Expected utility
Borch, K.	1961	Reciprocal Reinsurance Treaties	AB	Pro/Non-pro	Expected Utility/Game Theory
Lemaire et al.	1981	The Core of a Reinsurance Market	AB	Proportional	Expected Utility/Game Theory
Finetti, B.	1940	II Problema dei pieni	Attuari	Non-proportional	Ruin Probability
Waters, H.	1983	Some Mathematical Aspects of Reinsurance	IME	Pro/Non-pro	Ruin Probability
Waters, H.	1996	Reinsurance and Ruin	IME	Pro/Non-pro	Ruin Probability
Centeno, L.	1991	An Insight into the XL Retention Limit	SAJ	Non-proportional	Ruin Probability
Centeno, L.	1997	XL Re. and the Probability of Ruin in Finite	AB	Non-proportional	Ruin Probability
Ramlau-Hansen	1988	A Solvency Study in Non-life Insurance	SAJ	Non-proportional	Ruin probability
Dickson&Waters	1997	Relative Reinsurance Retention Levels	AB	Pro/Non-pro	Ruin probability
Bowers et al.	1986	Actuarial Mathematics		Non-proportional	Ruin Probability
Daykin et al.	1994	Practical Risk Theory for Actuaries		Pro/Non-pro	Ruin Probability
Beard et al.	1969	Risk Theory		Proportional	Ruin Probability/Utility
Buhlmann, H.	1970	Mathematical Methods in Risk Theory		Pro/Non-pro	Ruin Probability/Utility
Gerber, H	1979	An Introduction to Mathmatical Risk Theory		Combination	Ruin Probability/Utility
Heerwaarden	1989	Optimal Reinsurance in Relation to Ordering of Risks	IME	Stop Loss	Stop-loss Order
Denuit et al.	1998	Optimal Reinsurance and Stop-loss Order	IME	Stop Loss	Stop-loss Order
Rantala, J.	1989	On Experience Rating and Optimal Reinsurance	AB	Pro/Non-pro	Stochastic Control Theory
Hojgaard et al.	1997	Optimal Pro. Re. Policies for Diffusion Models	SAJ	Proportional	Stochastic Control Theory
Cummins & Roy	1985	A Stochastic Simulation Model for Reinsurance Decision		Non-proportional	Stochastic Simulation
Jang & Powers	2001	Catastrophe Reinsurance Retentions and Limits		Pro/Non-pro	Ruin Probability/Utility

2. Literature Review

A number of alternative approaches to studying the reinsurance retentions and limits have been previously suggested in the literature. Traditionally, many risk theorists try to develop the optimal retention model with mathematical methods. However, all of the theoretical methods have not been generally accepted since they are too simplistic for practical use. Other empirical studies have analyzed the retention from more of the demand for reinsurance perspective. Although these studies try to cover all kinds of reinsurance, they fail to find proper methods for reinsurance retentions and limits.

In recent years, to explain the demand for catastrophe reinsurance, various hypotheses have been proposed. Gron (1999) examines the determinants of insurer demand for catastrophe reinsurance. She finds that there is relationship between the price and the reinsurance provisions: as prices increase, insurers increase retentions, decrease upper limits, and increase co-reinsurance rates. Kleindorfer and Kunreuther (1999) develop a hypothetical model for insurers (K-K model), which was originally introduced by Stone (1973). They indicate that an insurer usually varies its portfolio size and the required retention and limit level under a probable maximum rule (PML). That is, the K-K model suggests that the insurance underwriter operationalizes the survival constraint by choosing the portfolio size and the reinsurance amount so that the estimated ruin probability is less than some predetermined value, p. Reinarz, et al. (1990) and Webb, Harrison and Markham (1997) also show that the upper limits for property catastrophe reinsurance contracts are usually set based on the catastrophe PML in practice.

Risk theorists traditionally suggest that safety loading, premiums, and surplus are closely related to the retention levels. For example, Daykin, et al. (1994) show that the retention is an increasing function of the safety loading, the premiums, and the surplus, all else being equal. Many theorists introduce rules of thumb to estimate appropriate retention level. Some of them suggest that the retention should be a small portion of the premium ($M = \gamma P$); others that it should be proportional to the surplus ($M = \delta U$) or proportional to the safety loading and surplus ($M = \zeta\lambda U$). Also, risk management theory and financial distress theory suggests that firm characteristics, such as catastrophe exposures, operating results, leverage, default risk, and liquidity are substantially related to the amount of reinsurance.

The major losses from Hurricanes Andrew and Iniki in 1992 generated several immediate responses. Powers, et al. (2007) reviews the private sector and public sector reaction. In the private sector, unsatisfied demand for catastrophe coverage prompted the capitalization of new reinsurance companies in Bermuda. Additionally, a shortage of property excess-of-loss reinsurance capacity has led to the development of alternative insurance-based securities such as catastrophe bonds and catastrophe derivatives.

3. Theory and Hypotheses

In this section, we investigate catastrophe reinsurance in more detail to develop alternative hypotheses for excess-of-loss reinsurance retentions and limits. Catastrophe reinsurance is well-suited for the study of excess-of-loss reinsurance retentions and limits because it is relatively well-defined and a common form of reinsurance in the current markets.[2] Furthermore, one primary insurer's catastrophe reinsurance program is fairly similar to another primary insurer's.

Traditionally, primary insurers reduce their exposure to catastrophic risk by purchasing catastrophe reinsurance. However, many experts, as well as the U.S. federal government, believe that this traditional property reinsurance market is no longer able to handle the larger mega-catastrophes. Prior to 1989, there had never been a catastrophe in the United States costing over $1 billion. Since 1989, there have been more than 10 including Hurricane Harvey($20 billion), Hurricane Andrew ($15 billion) and the Northridge Earthquake ($12 billion). This also compares with cumulative insured losses from natural catastrophes in the decade prior to those events of only about $25 billion. In recent years, several studies indicate that these catastrophe losses will grow and increasingly threaten insurers with insolvency, since rising property values and demographic trends are escalating the concentration of risk in catastrophe-prone regions. With the limitations on the reinsurance coverage that primary insurers want to purchase, they have attempted to find alternative financial instruments such as state and federal level catastrophe funds as well as new sources of funds from the capital markets. In the past few years, investment banks and brokerage firms have shown considerable interest in developing various financial instruments for providing protection against catastrophe risks. Academic researchers and practitioners expect that insurers could supplement traditional reinsurance with these new financial instruments when their losses exceed a certain level. This would relax solvency constraints and stimulate additional coverage in

[2] More a detailed discussion of this, see Gron (1999).

high-risk areas.[3]

Meanwhile, analysts expect that insurance companies will shift their buying habits from proportional reinsurance to excess-of-loss treaties. The key drivers of this trend are the consolidation of insurers due to excess capacity and competition as well as the move by insurers to increase their retentions. Several studies indicate that if current growth rates and market trends continue, the world reinsurance market will consist of 50 percent proportional reinsurance and 50 percent excess-of-loss reinsurance within 10 years, compared with today's 75 percent-25 percent split.[4]

3.1 Conceptual Framework for a Property Catastrophe Reinsurance Program

In today's property catastrophe reinsurance market, various insurance forms have been used within the range of non-proportional methods. There are three general types of property catastrophe reinsurance contracts: occurrence, aggregate, and loss-ratio or stop-loss covers. While occurrence and aggregate covers focus on events by size, the loss-ratio cover deals with total losses of the insurer. Sometimes, there are variations and combinations of these three types. When an excess-of-loss treaty is used for property catastrophe reinsurance, the usual processing of a catastrophe reinsurance program, including setting retentions and limits, is as shown in Figure 1.

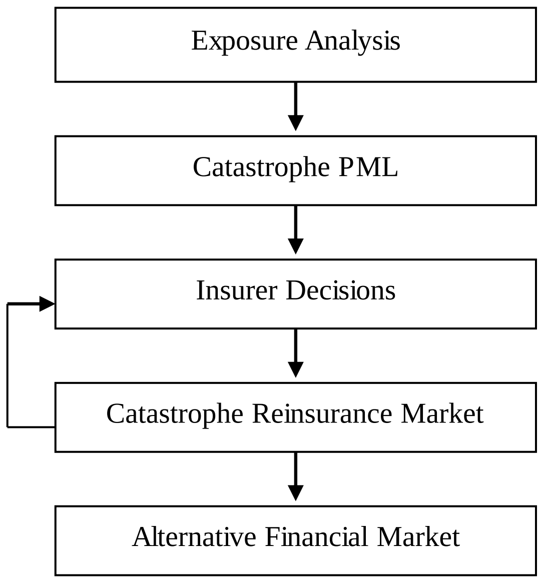

Figure 1. Framework for a Property Catastrophe Reinsurance Programming

Adapted from Kleindorfer and Kunreuther (1999)

3.1.1 Exposure Analysis

A primary insurer must first analyze catastrophe exposures to decide how much catastrophe exposure the company should retain. The exposures associated with catastrophic disasters such as hurricane, windstorm, and earthquakes fall into the class of low-frequency and high-severity (LF-HS) exposures.[5] With LF-HS exposures, the insurers are concerned with not only the uncertainty of the probability of loss, but also the magnitude of claims from a single catastrophe occurrence. In other words, the key characteristics of the catastrophe exposures are that it is difficult to predict both the likelihood and potential losses from specific catastrophic events.

3.1.2 Catastrophe Probable Maximum Loss (PML)

The concept of Probable Maximum Loss (PML) gives primary insurers guidance to their capital adequacy and their catastrophe reinsurance needs. Traditionally, there are two basic approaches to estimating PML: (1) written premium by state, and (2) exposure unit by zone. The first approach is used the Annual Statement of the insurer and extracts the extended coverage premium written in each state. This method has some limitations and the

[3]For a more detailed discussion of these financial instruments, see Powers (2007) and Kunreuther (1999).

[4]Howard S. (1999). Excess-of-loss Market on the Rise, *National Underwriter*, May.

[5]Kleindorfer and Kunreuther call these risks "low-probability and high-consequence" (LP-HC) risks.

most serious flaw of this approach is that it uses premiums as a measure of exposure. In recent years, the exposure unit by zone approach has been commonly used in practice. The analysis of the exposure unit method to calculate catastrophe PML is more complicated and costly since it requires the use of computer time and special computer programs. This method is based on an analysis of key elements such as location, damageability by construction type, and concentration. Many brokers, reinsurers, and consulting firms have developed advanced computer models to estimate catastrophe PML for their customers. Even though each catastrophe model adapts different assumptions, different methodology, and different data in generating its estimates, insurers can improve their exposure assessments and PML estimates by using appropriate catastrophe models.

3.1.3 Insurer Decisions

The insurer decision phase consists of a set of decision rules utilized by insurers in making choices regarding the purchase of reinsurance. With improved estimates of catastrophe exposures and more accurate PML estimates, the insurers can analyze the effect of different scenarios on their profitability and solvency. If possible, various catastrophe models should be used, and the largest resulting catastrophe PML estimate would be the minimum amount of upper limit purchased. In their recent study, Kleindorfer and Kunreuther (1999) suggest a theoretical model, which is named the PML rule, to choose the reinsurance amount and a portfolio size for insurers.

3.1.4 Catastrophe Reinsurance and an Alternative Financial Market

The final phase of the framework consists of finding catastrophe reinsurance providers and/or alternative financial markets. Most of the earlier discussions of retentions and limits have ignored the cost of reinsurance and the role of the reinsurer. However, those factors cannot be overlooked in actual practice. Most insurers purchasing excess treaties accept retentions higher than they would prefer, either to reduce reinsurance costs or because the reinsurer insists on it. The primary insurer should adjust its catastrophe reinsurance contracts through negotiation with reinsurer. Both primary insurers and reinsurers are interested primarily in the cost of reinsurance. Reinsurers make contracts with ceding companies at a price that is sufficient to cover both the expected losses transferred by ceding companies and additional loadings required to cover the costs of administration and a profit. As mentioned above, reinsurance programs are now being hit with rate hikes and higher retentions as a result of a series of catastrophes. Furthermore, primary insurers have experienced a difficult time in obtaining catastrophe coverage due to the capacity limits of the worldwide reinsurance market. Consequently, the primary insurer needing large catastrophe reinsurance coverage has to find alternative financial instruments such as state and federal level catastrophe funds as well as new sources of funds from the private capital markets. Studies suggest that reduced transaction costs and greater risk spreading are typical benefits from using these alternative financial instruments.

3.2 The Determinants of Setting Retentions and Limits

In the rest of this section, we will discuss the determinants of setting retentions and upper limits and develop theoretical hypotheses. The purposes of catastrophe reinsurance are to protect its policyholders' surplus and to lessen large fluctuations in operating results. The decisions on proper retentions and upper limits for primary insurers have been made on their own catastrophe exposures and financial conditions. For this reason, there is no formula that all insurers can apply to their own situation. However, there are basic considerations in setting retentions and upper limits. The determinants of retentions and upper limits for property catastrophe reinsurance will be examined.

3.2.1 Retention as a Function of the Upper Limit and the Co-Reinsurance

To adapt the PML model by Kleindorfer and Kunreuther (K-K model) to our analysis, we first need to change their assumptions. In practice, the catastrophe reinsurance contracts usually do not have an additional provision that requires the primary insurer to retain some fraction of the worst-case-scenario event. Instead, the reinsurance contracts have a percentage coverage provision. That is, the reinsurer makes the added stipulation that the insurance company retains some fraction $C \in [0,1]$ (co-reinsurance rate) of coverage limit $\Delta = L_1 - L_0$.

We will change the K-K model slightly with the following assumptions:

- Reinsurers offer a limited amount of coverage $\Delta = L_1 - L_0$, where L_0 is the retention and L_1 is the upper limit of the reinsurance contract. Therefore, if primary insurers want to increase the upper limit, they are required to increase the retention as well.
- Primary insurers usually choose the catastrophe PML as the upper limit. That is, the upper limit (L_1) equals the catastrophe PML.

- The primary insurers are assumed to vary their retentions and co-reinsurance rates under the upper limit (catastrophe PML) so that the estimated ruin probability is less than some predetermined value, p. A primary insurer will adjust the retention and the co-reinsurance rate to solve

$$1-F\left(A+\rho_0-(1+\theta_I)\int_{L_0}^{L_1}[1-F(L)]dL+(1-C^*)(L_1-L_0)\right)=p,$$

where:

ρ_0 = Primary Insurance Premium = $(1+\theta_I)$ x Expected Losses;

$(1+\theta_I)\int_{L_0}^{L_1}[1-F(L)]dL$ = Reinsurance Premium;

L_0 = Reinsurance Retention;

L_1 = Property Catastrophe PML;

$(1-C)(L_1-L_0)$ = Reinsurance Payment;

C = Co-Reinsurance Rate; and

p = Target Ruin Probability.

The primary insurers' decision making process in setting the retention, the upper limit, and the co-reinsurance rate is illustrated in Figure 2.

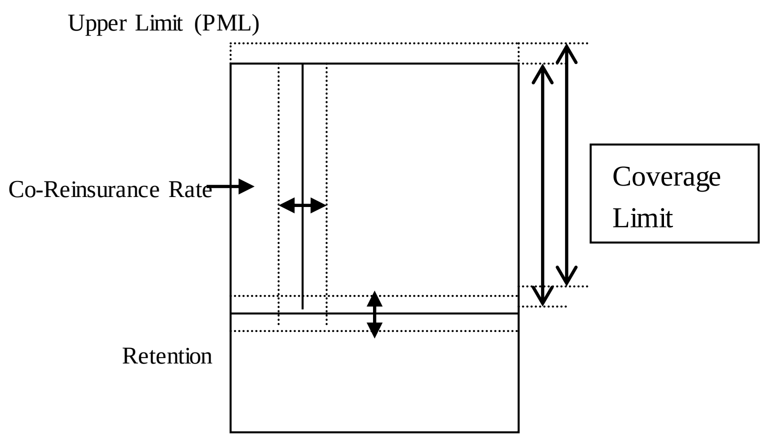

Figure 2. Retention and Limits Decision Making

Upper limits for property catastrophe reinsurance contracts are usually set based on the catastrophe PML. If possible, various catastrophe models should be used for each type of catastrophe reinsurance coverage, and the largest resulting catastrophe PML estimate would be the minimum upper limit to purchase. With respect to the upper limits based on the catastrophe PML estimate, it is apparent that retentions tend to move in the same direction with the upper limits.[6] In other words, the higher is the upper limit, the higher is the retention. Why is this happening? One possible answer is a size effect. Insurer size is likely to be associated with a higher probability of losses reflecting its greater exposure. In other words, the insurers with larger premiums from property catastrophe lines have usually more exposures to catastrophes and try to purchase greater upper limits. Also, larger insurers tend to have a higher retention because the insurers have a greater ability to sustain a large size loss. Therefore, we expect larger insurers to have higher retentions and upper limits. The insurer's budget

[6]Froot (1999) indicates that after a large event like Hurricane Andrew in 1992, retentions tend to increase. However, the total amount of coverage does not rise. He concludes that an insurer's window of coverage for catastrophic events shifts toward higher layers of protection.

constraint is another possible answer. Under the budget constraint, the upper limits for catastrophe reinsurance increases usually at the expense of coverage for retentions. That is, coverage for large events (upper limit) increases usually at the expense of coverage for small events (retention). It also means that the higher is the upper limit, the higher is the retention. All of these suggest the following hypothesis:

Hypothesis 1: *The amount of retentions and upper limits for property catastrophe reinsurance tend to move in the same direction, all else being equal. That is, the higher is the upper limit, the higher is the retention, and vice versa.*

In practice, reinsurance contracts usually have a percentage coverage provision. That is, the reinsurer makes the added stipulation that the insurance company retains some fraction of the coverage limit. For example, when the description of coverage is stated as "95 percent of $3,000,000 in excess of $1,000,000," it means that the ceding company has a 5 percent participation in the coverage limit ($2,000,000). Sometimes, the co-reinsurance rate is 10 percent rather than the more normal 5 percent, and no co-reinsurance contracts are also written by reinsurers but often these tend to be expensive. The basic reason for the provision is moral hazard consideration. Most reinsurers believe strongly that keeping the reinsured involved in excess-of-loss will encourage conservative loss settlements.[7] However, the moral hazard theory by itself is not adequate to explain existing variation in co-reinsurance rates among property catastrophe reinsurance contracts.

In some cases, greater co-reinsurance rates stem from the placement problem. When the coverage limit was not placed completely in the reinsurance market, the ceding companies are forced to assume the remaining percentage of the coverage limit. In other cases, even more usual, the co-reinsurance rates vary with retentions and upper limits. Again, the insurer's budget constraint is a possible answer as to why co-insurance rates vary among primary insurers. Under the budget constraint, to buy property catastrophe reinsurance with the lower co-reinsurance rate, the insurer should decrease its retention.[8] That is, the insurers have to increase their retention to get low co-reinsurance rates or no co-reinsurance subject to the budget constraint. In Figure 3, we would expect that no co-reinsurance cases form the highest line, normal cases (5 percent co-reinsurance) form the middle, and greater co-reinsurance cases (usually greater than and equal to 10 percent) form the lowest. This suggests the following hypothesis:

Hypothesis 2: *A decrease in the co-reinsurance rate will result in an increase in the retention, all else being equal.*

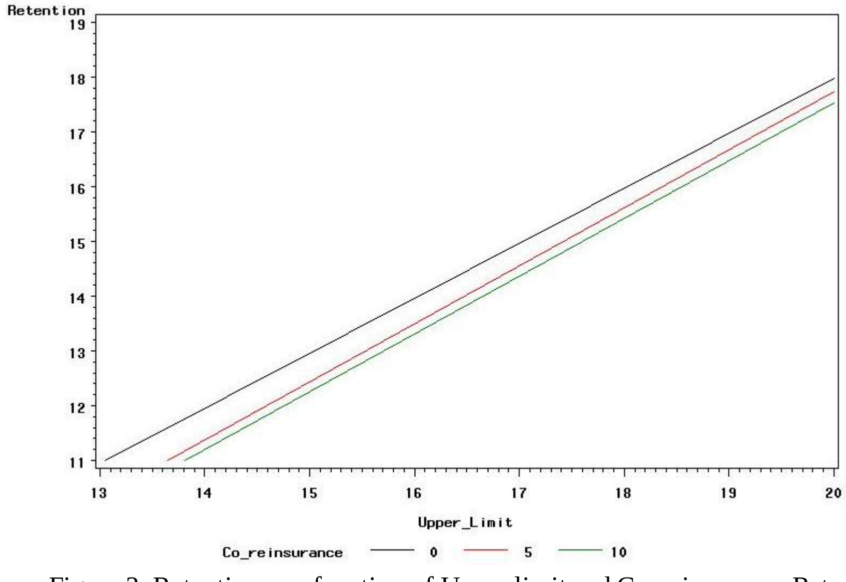

Figure 3. Retention as a function of Upper limit and Co-reinsurance Rate

3.2.2 Retention as a Function of the Surplus and the Premiums

Evaluating the effectiveness of the various forms of reinsurance and establishing a suitable level of retention are among the most important applications of risk theory. Especially the examination of the proper size of the safety

[7]Gilliam, Robert, 1981, The Property Catastrophe Reinsurance Contract, *Reinsurance*, p. 201.

[8]A recent study by Gron (1999) demonstrates that insurers are expected to respond to higher price with higher retentions and greater co-reinsurance rates from perspective of catastrophe reinsurance demand.

loading as a function of the excess-of-loss retention is one of the principal topics of traditional risk theory. After introducing several simple rules for the optimal retention, European risk-theorists developed more difficult mathematical models depending on the optimality criteria used and assumptions on the random variables involved. For example, Daykin, et al. (1994) introduce the following formula in the compound mixed Poisson case, other things being equal:

$$M \approx \frac{\left(\lambda^2 - y_\varepsilon{}^2 \sigma_q{}^2\right)P^2 + 2\lambda UP + U^2}{\sigma_X{}^2 y_\varepsilon{}^2 / n},$$

where:

M = Excess-of-Loss Retention;

λ = Safety Loading;

y_ε = $1-\varepsilon$ Percentile of Standard Normal Probability Distribution;

σ_q = Standard Deviation of Claim Frequency;

σ_X = Standard Deviation of Claim Severity;

P = Premium; and

U = Surplus.

This formula shows that the retention is a function of the safety loading, the premiums, and the surplus.[9]

In general, given sound rating and reserving practices, the higher the premium income, the higher is the retention of the primary insurer because the relative variance of large claims experience will be less and the need for reinsurance will be less, all else being equal. Also, insurer size measured as the policyholders' surplus (surplus) is likely to be associated with a higher retention because the insurer has a greater ability to sustain a large loss. Meanwhile, a general rule that regulators use in measuring the maximum retention of the company on a single risk is that it must not exceed 10 percent of the policyholders' surplus. As a whole, the retention forms an increasing function with the surplus and the premiums, suggesting the following hypothesis:

Hypothesis 3: *Larger insurers as measured by their policyholders 'surplus and premiums have larger retentions than smaller ones, all else being equal.*

3.2.3 Firm Exposure to Catastrophe Risk

A primary insurer's exposure to catastrophe risk can be measured by two variables: the percentage of premiums that are in property lines in catastrophe-prone regions (geographic concentration) and the percentage of premiums in property lines (property-line concentration). For property-liability insurers, the geographic location of a book of business can have a great impact upon its exposure to natural hazards, such as hurricanes, tornadoes, and earthquakes. Risk management theory suggests that insurers with larger exposure to catastrophe risk are likely to purchase more catastrophe coverage than insurers with smaller exposure. This suggests the following hypothesis:

Hypothesis 4: *An increase in property-line and/or geographic concentration will result in lower retention, all else being equal.*

3.2.4 The Price of Catastrophe Reinsurance

The price of the catastrophe reinsurance will depend upon the amount of reinsurance (retention, upper limit, and co-reinsurance rate), perils reinsured against, the individual insurer's exposures to the perils, prior catastrophe loss experience of the insurer, and reinsurance market conditions. The price of catastrophe reinsurance is quoted as the subject premium divided by the coverage limit, called the "rate on line" (ROL). Since 1989, the price of

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[9]They also introduce the rules of thumb to estimate appropriate retention level. Some of them suggest that M should be a small portion of the premium ($M = \gamma P$); others that it should be proportional to the surplus ($M = \delta U$) or proportional to the safety loading and surplus ($M = \zeta \lambda U$).

property catastrophe reinsurance in the United States has fluctuated markedly. For example, during the period 1992 to 1994, prices on property catastrophe reinsurance more than doubled and then began to decline thereafter. Froot and O'Connell (1999) and Froot (2000) indicate that reinsurance prices are high relative to some natural benchmark. Demand theory suggests that insurer demand for catastrophe reinsurance is likely to increase with higher prices. That is, in the demand curve, the estimated coefficients on retention and co-reinsurance are positive while the estimated effect of limit should be negative. In a recent study, using Guy Carpenter's property catastrophe reinsurance data from 1987 to 1993, Gron (1999) found that as prices increase, insurers increase their retention and co-reinsurance rates, and decrease total limits. This suggests the following hypothesis:

Hypothesis 5: *Insurers respond to higher prices with higher retentions.*

3.2.5 Operating Results

The catastrophe protection is not generally designed to handle the routine losses that occur each year, but to take care of the large ones that are expected to occur every five to twenty years or more. That is, one of the main purposes of catastrophe reinsurance is to level operating results. Therefore, the principal consideration in setting the retention of catastrophe contract is the predetermined operating results for the property lines covered by the treaty. According to risk management theory, the insurers with worse operating results are likely to purchase more catastrophe coverage than the insurers with better operating results, suggesting the following hypothesis:

Hypothesis 6: *Insurers are likely to respond to worse operating results with higher retentions.*

3.2.6 Control Variables

The control variables are measured in the same manner as in the Mayers and Smith (1990), Garven and Lamm-Tennant (1999), and Gron (1999) papers. Corporate risk management theory indicates that purchasing catastrophe reinsurance is likely to reduce the probability of financial distress and related costs by reducing the probability of insolvency resulting from catastrophic events. The probability of financial distress is usually measured by several variables: leverage, default risk, and liquidity. Most of the previous theoretical literature suggests that insurers with greater leverage, greater default risk, and lower liquidity are more likely to suffer from the costs of financial distress since they are less likely to have required funds. Note that as a proxy for default risk, this study employs Best's ratings since insurers with the lower Best's ratings will tend to be more highly leveraged than insurers with the higher Best's ratings. We expect that insurers with greater leverage, greater default risk, and lower liquidity have a greater willingness to purchase lower retentions.

Many studies indicate that mutual insurers are more likely to operate in types of insurance with lower risk and less managerial discretion. Hence we expect that mutual insurers tend to keep lower retentions than stock insurers since the owners (policyholders) are less diversified and managers will benefit from the risk reduction.[10]

Since the catastrophe reinsurance contract is typically written for a whole insurance group, it is interesting to examine the difference between group and unaffiliated companies' decisions for setting catastrophe reinsurance retentions. We expect that firms in groups are likely to have higher retentions.

4. Model

The decision of property catastrophe reinsurance retentions will be based on the upper limits (*UL*), the co-reinsurance rates (*CORE*), the firm sizes (*SIZE*), the firm catastrophe exposures (*EXPO*), the prices of reinsurance (*P*), the operating results (*O_R*), and other financial conditions (*FIN*), such as the probability of financial distress, ownership structure, and group membership. More formally, we can say that the retentions are a function of the following variables:

$$R_i = \alpha + \beta_1 UL + \beta_2 CORE_i + \beta_3 EXPO_i + \beta_4 P_i + \beta_5 FIN_i + \varepsilon_i ,$$

where:

R_i = Property Catastrophe Retention from Company i, and

$\varepsilon_i \sim \text{i.i.d.}\, N(0, \sigma_i^2)$.

The size variables include property direct premiums written (subject premium) and policyholders' surplus (*PHS*). The catastrophe exposure variables include geographic concentration for earthquake (*E_CON*), hurricane (*H-CON*), tornado (*T_CON*), and property-line concentration (*PL_CON*). Geographic concentration is the ratio

[10] Gron (1999) does not include the indicator variable for ownership structure in the estimation because the variable was highly collinear with the regional indicator variable.

of property direct premiums written from catastrophe prone states to total direct premiums written. The ratio of property direct premiums written to total direct premiums written is used as a property-line concentration variable. For the price variable, this study uses the ratio of property excess-of-loss ceded premiums to the coverage limit. Various operating ratios are used to evaluate the operating results for the insurance companies. Finally, additional control variables include financial distress (leverage, default risk, liquidity), ownership structure (*OWNER*), and group membership (*GROUP*). The dependent and independent variables used are summarized in Table 2.

Table 2. Dependent and Independent Variables

Variable	Definition	Expected Sign
Dependent Variables		
R	retention in property catastrophe reinsurance	
Independent Variables		
UL	upper limit	+
CORE	co-reinsurance rate	-
PHS	policyholders' surplus	+
E_CON	earthquake premiums / total direct premiums written	-
H_CON	hurricane premiums / total direct premiums written	-
T_CON	tornado premium / total direct premiums written	-
PL_CON	subject premiums / total direct premiums written	-
P	excess-of-loss ceded premium / coverage limit	+
O_R	(loss + expense) / premiums earned	-
LEV	liabilities / policyholders' surplus	-
LIQ	total admitted assets / total liability	+

Our basic data for dependent variables are from *Best's Insurance Reports, Property-Casualty, United States*, 2015 Edition. This database contains information on 3,054 insurance companies representing virtually all significant and active insurance companies operating in the United States. The file also contains each company's various "Reinsurance Programs," including property catastrophe reinsurance. Sometimes, the reinsurance programs are supplemented by "Reinsurance Utilization." (Sample summaries are shown in Table 3.) In addition, the file identifies each firm's ownership structure (stock, mutual, or reciprocal), group membership, and other firm characteristics.

The data for independent variables are from the NAIC Annual Statement. The line-of-business file contains data on premiums, losses and expenses categorized into 24 insurance lines for a large sample of property-liability insurance firms. This 2015 edition presents data for the year ending December 31, 2014.[11] Summary statistics for the variables included in the regression are presented in Table 4.

[11]Retentions, limits, co-reinsurance rates, and other firm characteristics are from *Best's Insurance Reports* and NAIC tapes. Firm characteristics are lagged one year and therefore represent for the year prior to the property catastrophe reinsurance program since the primary insurer make a reinsurance contract with reinsurer based on previous year's business results.

Table 3. Sample Reinsurance Program and Utilization Summaries

Reinsurance Program
"The largest net aggregate amount insured in any one risk, excluding workers' compensation, was $42,500,000. The numbers of the pool maintain a general property per risk treaty that limits the group's net loss for HPR/B&M business to $10 million and other property business to $5 million. In addition, the group maintains a casualty clash excess of loss contract. Property catastrophe reinsurance is also maintained to protect the group from any abnormally large losses. The companies have a multi-layer property catastrophe contract providing $200 million excess $50 million per occurrence effective January 1, 2014. Reinsurance contracts are in place with various domestic and foreign reinsurers."
Reinsurance Utilization
"The group maintains moderate reinsurance utilization, demonstrated by a business retention of 95% and reinsurance recoverable leverage of 2%. Reinsurance is primarily utilizes on an excess of loss basis as well as for catastrophe protection. In 2014, management increased its catastrophe reinsurance limits by $45 million in excess of a $5 million per occurrence."

Table 4. Summary Statistics

Variable	N	Mean	Std Dev	Sum	Minimum	Maximum
R	302	14.17361	1.70298	4280	11.22524	19.33697
UL	302	16.75410	1.42969	5060	12.89922	20.36659
CORE	302	0.03998	0.03237	12.07500	0	0.20000
PHS	302	17.31614	1.90173	5229	13.72959	22.35445
E_CON	302	-0.85475	1.67841	-258.13363	-9.15530	0
H_CON	302	-1.22500	1.73198	-369.94890	-8.77887	0
T_CON	302	-0.98953	1.24485	-298.83858	-5.85152	0
PL_CON	302	0.94901	0.07028	286.60089	0	1.00000
P	302	6.44171	2.93520	1945	0	23.32199
O_R	302	-0.04529	0.20882	-13.67857	-1.42536	0.53030
LEV	302	-0.31272	1.02164	-94.44056	-4.90616	1.09557
LIQ	302	0.50425	0.30713	152.28345	0	1.93442

5. Empirical Results

Before fitting the model, we need to deal with one major estimation problem. This problem arises from the potential simultaneous-equation bias that will occur if there is interaction among the variables. That is, some independent variables such as the upper limit, the co-reinsurance rates, and the price that effect the retentions will generally be affected by the retentions as well. To eliminate the possible endogeneity of the upper limit, the co-reinsurance, and the price, the two-stage least square method is used.

To adapt the two-stage least square method, instrumental variables (uncorrelated with the error term but highly correlated with the endogenous variables) are created. One instrument that might affect the endogenous variables is the coverage limit, the upper limit minus the retention. Other firm characteristics, such as premiums, assets, and liability might produce acceptable instruments. To find the predicted values, the upper limit, the co-reinsurance rates, and the price are regressed on the instrumental variables. Then, the predicted values, UL, $CORE$, and P instead of the original *UL*, *CORE*, and *P* are used to regress the model.

In addition, in a cross-sectional study, heteroscedasticity is often present. To test for heteroscedasticity, the White test was used. The chi-squared statistic was computed as the sample size N times the R^2 from a regression of the squares of the Ordinary Least Square (OLS) residuals on a constant, the regressors from the equation being estimated, their squares and their cross-products. The statistic was 61.4, which is insignificant.

Table 5 reports the results from estimation of the model. The logarithm of retention was regressed on the explanatory variables for variance stabilization. Model 1 presents the results from OLS estimation to compare with the instrumental variables. Model 2 displays results from instrumental variables (IV) estimation of the model. The estimated coefficients in Models 3 and 4 include the default variables (co-reinsurance, Best's rating, ownership, and group) entering as discrete categories rather than as continuous variables.

Table 5. Estimation of Catastrophe Reinsurance Retention

Variable	Model 1	Model 2	Model 3	Model 4
Intercept	-0.33233 (-0.76)	-7.34529 (-11.05)	-7.56406 (-10.27)	-7.40621 (-9.91)
UL	0.77775 (21.73)	0.92585 (20.26)	0.91451 (19.69)	0.90855 (19.46)
CORE	1.02940 (1.50)	-4.54048 (-3.37)	-3.85287 (-4.65)	
CORE_10				-0.32359 (-3.32)
CORE_5				-0.22907 (-4.46)
PHS	0.11508 (3.80)	0.12913 (4.04)	0.14649 (4.13)	0.14507 (4.09)
E_CON	-0.00893 (-0.57)	-0.00449 (-0.27)	-0.01129 (-0.70)	-0.01199 (-0.74)
H_CON	-0.01644 (-1.04)	-0.01545 (-0.92)	-0.01137 (-0.69)	-0.01062 (-0.65)
T_CON	-0.02983 (-1.35)	-0.03668 (-1.57)	-0.03368 (-1.47)	-0.03237 (-1.41)
PL_CON	-1.99209 (-5.67)	-2.19911 (-5.86)	-2.07794 (-5.56)	-2.08168 (-5.58)
P	0.18236 (19.71)	0.90996 (19.89)	0.89033 (19.45)	0.88789 (19.39)
O_R	-0.16432 (-1.48)	-0.16709 (-1.42)	-0.18343 (-1.56)	-0.18993 (-1.61)
LEV	-0.00761 (-0.24)	-0.00384 (-0.11)	-0.01464 (-0.44)	-0.01505 (-0.45)
LIQ	0.17876 (1.90)	0.21829 (2.18)	0.24782 (2.38)	0.24481 (2.35)
RATING	-0.06022 (-1.79)	-0.05997 (-1.69)		
RATING3			0.16987 (1.48)	0.16804 (1.47)
RATING4			0.02611 (0.28)	0.03260 (0.35)
RATING5			-0.01761 (-0.26)	-0.01209 (-0.18)
OWNER			-0.01735 (-0.35)	-0.01278 (-0.26)
GROUP			0.15093 (2.42)	0.15354 (2.46)
Adjusted R^2	0.9510	0.9448	0.9481	0.9482
No. of Observations	302	302	302	302

Note: Model 1 gives OLS estimation. Model 2 gives instrumental variables (IV) estimation. Models 3 and 4 give IV with the default variables (co-reinsurance, Best's ratings, ownership, and group) entering as discrete categories rather than as continuous variables. The dependent variable is the natural logarithm of retention, and t-statistics are presented in parentheses. Each regression has 302 observations.

In Model 1, the results from OLS , the coefficients on upper limit (*UL*), policyholders' surplus (*PHS*), property-line concentration (*PL_CON*), operating results (*OR*), and liquidity (*LIQ*) are significant and they correspond to the hypothesized sign. However, OLS regression of retention on co-reinsurance rates, geographic concentration, and leverage does not perform well. Contrary to expectation, the estimated effect of co-reinsurance is positive, and not statistically significant at the 10 percent level. This means that primary insurers with greater co-reinsurance rates are likely to have greater retentions. The coefficients of geographic concentration and leverage have expected signs, but are not significantly different from zero. Also, the coefficient on Best's rating (*RATING*), as a measure of default risk, has a reversed sign and is statistically significant. This means that insurers with lower rating have greater retentions, contrary to expectation. The variable for subject premium was not included in the estimation because the variable was highly collinear with the policyholders' surplus.

In Model 2, the coefficients on upper limit, policyholders' surplus, property-line concentration, operating results, and liquidity generally increase in magnitude and statistical significance using instrumental variables. Estimating using instrumental variables causes the estimated coefficients on co-reinsurance to be statistically significant and to correspond to the hypothesized sign. One of the geographic concentration measures, tornado exposure (*T_CON*), is also statistically significant. However, the coefficients of other geographic concentrations (*E_CON*, *H_CON*) and leverage (*LEV*) are statistically insignificant, even though they correspond to the expected signs. The sign on Best's rating does not change.

In Models 3 and 4, the co-reinsurance rates and Best's ratings are entered as discrete categories rather than as continuous variables. Furthermore, ownership structure (*OWNER*) and group membership (*GROUP*) as a dummy variable are added to the original model. For co-reinsurance, two categories are included in Model 3: co-reinsurance rate and no co-reinsurance (the excluded category). In Model 4, the co-reinsurance case is decomposed into two parts, i.e., 10 percent co-reinsurance rates or high, 5 percent co-reinsurance or low (with no co-reinsurance again the excluded category). In both cases, the coefficients of co-reinsurance rates are statistically significant and they correspond to the expected sign. That is, the primary insurers are likely to respond to higher co-reinsurance rates with lower retentions. For the Best's ratings, four categories are included: rating of B or below (*RATING3*), rating of B+ or B++ (*RATING4*), rating of A- or A (*RATING5*), and rating A+ or A++ (*RATING6*, the excluded category). The coefficients of all variables are statistically insignificant. The coefficient estimation of group membership is statistically significant and it corresponds to the hypothesized sign, while the coefficient estimation of ownership structure is statistically insignificant. The estimated effects of other variables are qualitatively similar to those in Model 2.

6. Conclusion

This study is the first research that provides clear evidence to support the relationship among retentions, upper limits, and co-reinsurance rates. To keep the model simple and empirically tractable, several assumptions have been imposed. Several modifications to the retention model can be adapted to improve the results. For example, the catastrophe excess-of-loss reinsurance contracts usually have additional required provisions, such as "ultimate net loss," "layers", and "reinstatement". In this study, those variables are excluded. The inclusion of these provisions and other important variables would likely improve the results of the retention model, causing more variables to become significant and to possess the hypothesized sign.

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