

Theoretical Analysis of How to Resolve the Nursing Shortage in Japan

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Abstract

This study examines how to resolve the acute nursing shortage in Japan, by proposing a simple but useful model that helps provide a basis for the theoretical analysis of this issue. Although nurses in Japan need a lifetime license, only issued by the government after completing rigorous training, they still do not hesitate to leave their nursing positions. The fact that licensed nurses do not necessarily work in the health care market, and the question of how “inactive nurses” can be lured back to their former jobs, are significant social problems in Western countries as well as in Japan. Some inactive nurses desire to return to nursing work if the incentives offered were to meet their expectations. In that regard, we propose an approach for inducing inactive nurses to return to their former jobs. The presented theoretical analysis concludes that providing childcare support for nurses, a non-pecuniary driving force, is more effective than offering them a wage rise for increasing the supply of nursing services.

Keywords: childcare support, inactive nurse, nursing shortage, severity, vocation, wage rise

1. Introduction

Although nurses in Japan need a lifetime license, only issued by the government after completing rigorous training, a number of nurses are not beneficially employed for a variety of reasons (e.g., childcare responsibilities). The presence of so-called “inactive nurses,” who include not only individuals currently out of work but also those holding jobs not appropriate to their qualifications, can give rise to a severe nursing shortage. Therefore, the question of how inactive nurses can be lured back to their former jobs is a significant social problem in Western countries as well as in Japan. Some inactive nurses actually desire to return to nursing work if the incentives offered meet their expectations. According to the survey conducted by the Japan Federation of Medical Worker’s Unions (2010), nurses quit their jobs for the following main reasons: (a) over-work due to the shortage of nurses (46.1% of respondents), (b) very low wages (37.0%), (c) insufficient holidays (35.4%), (d) tough working conditions on night shifts (30.5%), and (e) dissatisfaction with nursing work (30.5%). These data suggest that nurses leave their jobs because of non-pecuniary factors as well as pecuniary ones (i.e., wage rates).

Based on the foregoing findings, this study examines how to resolve the acute nursing shortage in Japan while focusing on how nurses behave in the labor market. The presented theoretical analysis examines the policy options for inducing inactive nurses to return to their desired jobs and allows us to conclude that providing childcare support for nurses is more effective than a wage rise for increasing the supply of nursing services.

The majority of economic papers discussing nursing shortages have not clearly defined the term. The economic definition of nursing shortages used herein runs as follows. Given that the actual nursing wage rate is less than the equilibrium wage rate at which the supply and demand curves for nursing services intersect, excess demand reflects a shortage of nurses. The actual nursing wage rate does not correspond to the equilibrium wage rate because the market is controlled by the government. We thus propose an increase in the wage rate and an upward shift of the supply curve to resolve or decrease nursing shortages while keeping constant the current medical system controlled by the government.

The remainder of this article is organized as follows. Section 2 reviews the relevant literature on nursing shortages. Section 3 describes our model based on Heyes’ (2005) concept of vocation. (Note1) Section 4 proposes some effective policy solutions for addressing the nursing shortage in Japan. Finally, we present our concluding remarks.

2. Literature Review

2.1 Critique of Heyes' Model

We briefly survey previous studies of nursing shortages and describe the labor market for nurses in this section. Heyes (2005) believes that the main reason for the nursing shortage in Great Britain is the low wage rate, concluding that higher wages would attract newcomers with no vocation and thus reduce average nursing quality. Taylor (2007) criticizes Heyes' model by pointing out that the wage rate for nurses in a monopsonistic labor market such as the British National Health Service (NHS) has always been lower than that in a competitive market, which is why the shortage of nurses remains. He concludes that policy-makers should raise the wage rate to the competitive equilibrium point to increase the supply of nurses. Further, Nelson and Folbre (2006) argue that the quality of nursing care is irrelevant to nurses' motivations and assert that Heyes fails to distinguish between motivations and the job.

The framework used in this study employs the category described by Heyes (2005) as vocation. Although this does not necessarily mean we fully accept Heyes' model, vocation is a significant factor in our framework, as we focus on inactive nurses who have already obtained an assured level of skill and knowledge in this field and are temporarily out of work because of birth and childcare as opposed to newcomers without vocation.

2.2 Labor Market for Nurses

Hirsch and Schumacher (2005) explain that the market for registered nurses in the United States is an example of a monopsonistic labor market, where the larger the degree of monopsony, the lower the wage rate becomes. If monopsonistic power were stronger in this market, the wage rate would be lower than that of a competitive case, as pointed out by Hurd (1973) and Taylor (2007), meaning that nursing shortages would persist. Their analyses conclude that the labor market for registered nurses is not a monopsony because nurses have considerable mobility between hospitals. Hirsch and Schumacher (2005) also indicate that when many new individuals enter a market, the market might have high mobility for workers, demonstrating a lower level of monopsony. It is also suggested that if the entry of employees comes from the unemployed (including inactive individuals considered in our framework), it would lead to weaker mobility for nurses between hospitals. In the model presented herein, it is supposed that people can freely enter and leave the labor market for nurses, given that only the wage rate is decided by the government (Note 2).

Di Tommaso, Strøm, and Sæther (2009) empirically analyze Norwegian registered nurses and conclude that their labor supply is so inelastic that a rise in the wage rate would only be expected to yield a moderate increase in overall labor supply. They emphasize that registered nurses tend to prefer shift work to daytime work because the former offers flexible hours as well as job and childcare choices despite the relatively low rates of pay. Shields (2004) similarly shows that labor supply for nurses is inelastic with respect to wage rates.

2.3 Non-pecuniary Factors that Affect the Retention of Nurses

Ahlburg and Mahoney (1996) find that a 10% increase in the wage rate for a nurse relative to the expected wage rate of the next best job would provide a 2 percentage point higher probability of retaining the nurse in the U.S. health care market. This result implies that the decision to remain working as a nurse is affected by the wage rate although by a less-than-expected degree. Ahlburg and Mahoney (1996) conclude that nurses leave the profession for non-pecuniary factors such as an undesirable work schedule and a lack of professional autonomy. The model proposed herein follows this conclusion by using childcare support as an incentive for a nurse to stay in the job.

Phillips (1995) emphasizes that one should distinguish the decision to work as a nurse from that on how many hours to work. He estimates that the elasticity of the participation as a nurse with respect to the accompanied working cost is -0.64 for married nurses in Britain. This finding means that a subsidy that decreases the working cost by 10% would increase the participation of married nurses by 6.4%. The childcare support cost we consider in this study might be a significant category included in the working cost for married nurses.

Shields and Ward (2001) focus on the determinants of job satisfaction for qualified nurses working in the NHS. They insist that a policy of increasing wages would not necessarily succeed without considering non-pecuniary factors such as promotion and training opportunities, because they find that dissatisfied nurses have a 65% higher probability of intending to quit than satisfied nurses and that dissatisfaction with non-pecuniary conditions has a stronger impact than that with wages.

3. Model

The model proposed in this study introduces the vocation of each individual as well as the severity of nursing labor into the utility function of nurses. Severity is measured by the number of nurses per patient. The vocation

factor, emphasized by Heyes (2005), plays a crucial role in our model. Society imposes social norms of behavior on nurses (see Young, 2007 for a discussion on social norms). Such norms provide the basis on which nurses practice, despite the high pressure applied by society. Nurses with a positive vocation feel the pressure affirmatively and convert it into their own power to work. On the contrary, nurses with a negative vocation unwillingly endure the social pressure because of the better wages than those in other sectors. Nurses who do not react to this pressure have a zero vocation level. However, rather than considering either the absence or presence of vocation, we presume that every nurse has a certain level of vocation. In other words, vocation takes a continuous value between a negative and a positive level. We represent our framework next.

3.1 Definitions

We start our theoretical discussion by providing the following definitions:

N : number of nurses with a nursing license (we take the number as given);

n : number of individuals working as a nurse (hereafter working nurses);

P : number of patients;

u_i : utility of nurse(i);

w : wage rate for a nurse (a policy variable controlled by the government);

v_i : vocation level of nurse(i), (or utility corresponding to it);

S : degree of severity of nursing labor ($S > 0$);

u : utility of the provider of general products;

\bar{w} : wage rate for the provider of general products (given);

\bar{S} : degree of severity of labor for general products (given).

3.2 Basic Framework

The relationship between the number of nurses with nursing qualification N and the number of working nurses n is as follows:

$$n \leq N \quad (1)$$

If both sides of (1) are equal, the number of inactive nurses is zero. For simplicity, suppose that all inactive nurses provide general products. Each individual involved in this sector has the following utility function:

$$u = \bar{w} - \bar{S} \quad (2)$$

The utility function of nurse(i) is:

$$u_i = w - S + v_i \quad (3)$$

As shown in (2) and (3), we assume for analytical convenience that both equations are linear. The degree of the severity of nursing labor S in (3) is measured by:

$$S(\gamma; A) = A\gamma \left(\frac{n}{P}\right) \quad (4)$$

where γ depends on the “nurse-to-patient ratio,” n/P , which indicates the number of nurses involved in nursing service per patient. It is apparent that γ varies inversely with n/P , since the larger the number of patients per nurse, the higher γ becomes. Furthermore, as the number of patients per nurse increases, the level of severity of nursing labor rises rapidly. Thus, we suppose:

$$\gamma' \left(\frac{n}{P}\right) < 0, \quad \gamma'' \left(\frac{n}{P}\right) > 0 \quad (5)$$

and then

$$\lim_{n \rightarrow 0} \gamma' \left(\frac{n}{P}\right) = -\infty \quad (6)$$

$A(> 0)$ in (4) depends on the stress caused by childcare responsibilities while working or on the pressure of the risk of making medical errors or nursing mistakes. v_i in (3) implies the utility associated with the vocation of nurse(i).

As shown in Figure 1, $f(v)$ constitutes a density function distributed with respect to v between $-a(< 0)$ and $b(> 0)$ involving a global maximum c . Thus,

$$\int_{-a}^b f(v) = 1 \tag{7}$$

and

$$\begin{cases} f'(v) \geq 0 & \text{if } -a \leq v \leq c (< 0) \\ f'(v) < 0 & \text{if } c < v \leq b (> 0). \end{cases} \tag{8}$$

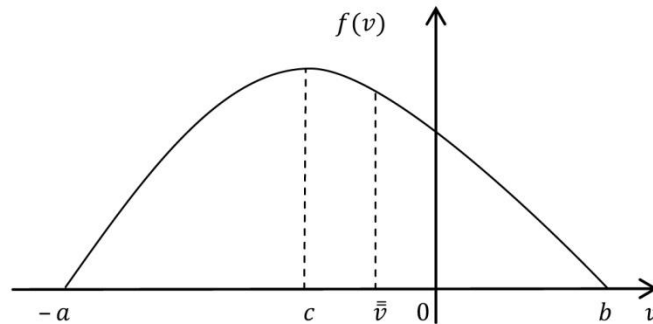


Figure 1. Density function of vocation level

3.3 Behavior of Nurses

We consider a case in which nurses are indifferent between working as nurses and providing general products. In this case, the utility in (2) and (3) must be equivalent. Let \bar{v} be the “threshold level” of vocation, which is assumed to satisfy the following condition:

$$\bar{w} - \bar{S} = w - A\gamma \left(\frac{n}{P}\right) + \bar{v} \tag{9}$$

This can be rearranged to obtain

$$\bar{w} - w = \bar{S} - A\gamma \left(\frac{n}{P}\right) + \bar{v} \tag{10}$$

where the number of working nurses n is represented by

$$n = N \int_{\bar{v}}^b f(v)dv \tag{11}$$

The left-hand side of (10) denotes the difference between the wage rates for providing general products and those for nursing services. In Figure 1, the feasible area to the right divided by the threshold vocation level \bar{v} represents the proportion of working nurses, while the feasible area to the left of \bar{v} measures the latency rate.

According to (11), the number of nurses in jobs appropriate to their qualification depends on the threshold level \bar{v} . Other things being constant, the larger \bar{v} , the lower is the number of these nurses, so that

$$n'(\bar{v}) < 0 \tag{12}$$

We define the right-hand side of (10) as follows:

$$\Gamma(\bar{v}) \equiv \bar{S} - A\gamma \left(\frac{n}{P}\right) + \bar{v} \tag{13}$$

Given the wage rates for working nurses w and for individuals providing general products \bar{w} , we examine whether a solution for the threshold level of vocation \bar{v} exists and what kind of effect the solution has. We introduce the first derivative with respect to $\Gamma(\bar{v})$:

$$\Gamma'(\bar{v}) = A\gamma' \left(\frac{n}{P}\right) \frac{Nf(\bar{v})}{P} + 1 \tag{14}$$

We present and prove the following lemma.

Lemma1. Equation $\Gamma'(\bar{v}) = 0$ has at least one solution \bar{v}_T with $\Gamma''(\bar{v}_T) < 0$ at the interval $[-a, b)$.

Proof. As $\Gamma'(-a) = 1$ and $\Gamma(\bar{v}) \rightarrow -\infty$ is the case near b from (13), $\Gamma'(\bar{v}) < 0$ must always be the case near b . It follows from this explanation that equation $\Gamma'(\bar{v}) = 0$ has at least one solution \bar{v}_T with $\Gamma''(\bar{v}_T) < 0$ at the interval $[-a, b)$, according to the mean value theorem, because $\Gamma'(\bar{v})$ is continuous at the interval $[-a, b)$. See Figure 2.

Lemma 2. Suppose that Lemma1 is met and the equation $\Gamma'(\bar{v}) = 0$ has a unique solution \bar{v}_T . It follows that (13) has two real roots under an appropriate $\bar{w} - w$ that satisfies the following inequality:

$$\Gamma(-a) < \bar{w} - w < \Gamma(\bar{v}_T) \tag{15}$$

Proof. It is obvious that $\Gamma'(-a) = 1 (> 0)$. Since the sign of \bar{v} changes from plus to minus at the boundary \bar{v}_T with an increase in \bar{v} , we can easily see that $\Gamma(\bar{v})$ attains the maximum at $\bar{v} = \bar{v}_T$ and obtains a local concave function with $\Gamma(\bar{v}_T)$ as the maximum. Further, under a condition for (15), a straight line $\bar{w} - w$ parallel to the horizontal axis always crosses over the concave function at two points. It immediately follows that (13) has two real roots. See Figure 3.

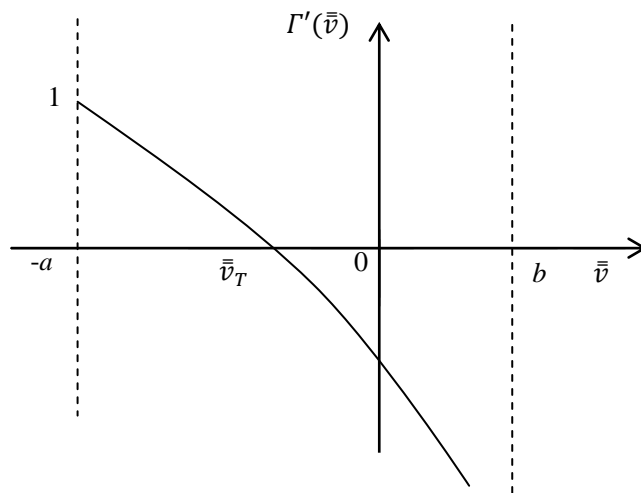


Figure 2. Solution for $\Gamma'(\bar{v}) = 0$

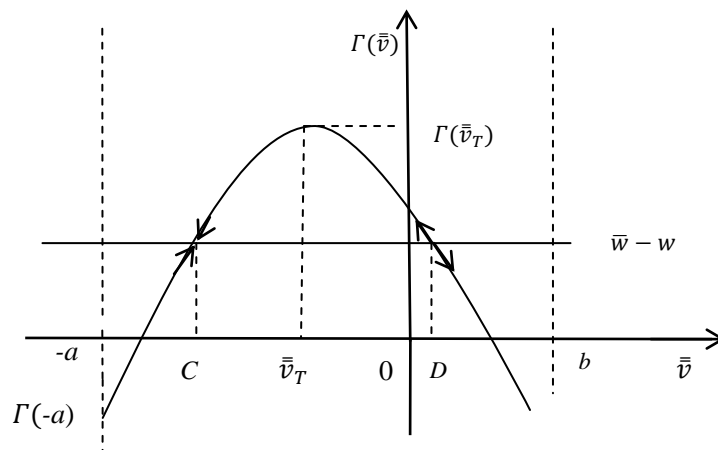


Figure 3. Shape of $\Gamma(\bar{v})$ and its solution

Lemma 3. When (13) has two real roots, the smaller root shown by C in Figure 3 is stable and the larger one given by D is unstable.

Proof. (a): As $\Gamma(\bar{v}) < \bar{w} - w$ in Figure 3 is valid for the case of $\bar{v} < C$, the utility of individuals providing general products is larger than that of nurses, meaning that the proportion of nurses at work will decrease. This results in a movement of \bar{v} toward C . (b): The opposite is the case for $C < \bar{v} < D$, meaning that it yields the same movement of \bar{v} as in (a). From cases (a) and (b), it is apparent that \bar{v} converges to C . (c): By contrast, the proportion of working nurses falls in the case of $\bar{v} > D$ because $\Gamma(\bar{v}) < \bar{w} - w$. Therefore, \bar{v} moves away from D .

4. Childcare Support or a Wage Rise?

In this section, we examine how to resolve the nursing shortage in Japan. Nurses are categorized into two groups under the following assumption.

Assumption 1: Let all nurses (including inactive nurses) with childcare responsibilities be αN (group 1) and all nurses without childcare responsibilities be $(1 - \alpha)N$ (group 2), where $0 < \alpha < 1$. We denote the co-efficient A of group 1 by A_1 and that of group 2 by A_2 , where $A_1 > A_2$.

Thus, at the equilibrium condition of (10), nurses are divided into two groups. That is,

$$\bar{w}_1 - w = \bar{S}_1 - A_1 \gamma \left(\frac{n}{P}\right) + \bar{v}_1 \tag{18}$$

$$\bar{w}_2 - w = \bar{S}_2 - A_2 \gamma \left(\frac{n}{P}\right) + \bar{v}_2 \tag{19}$$

where \bar{v}_1 indicates the threshold of v in group 1 and \bar{v}_2 in group 2. Further, \bar{w}_1 represents the wage rate and \bar{S}_1 the severity of labor for inactive nurses with childcare responsibilities. Similarly, \bar{w}_2 and \bar{S}_2 are the corresponding values for inactive nurses without childcare responsibilities. It is easily understood that the wage rates for the two groups of inactive nurses differ because people with childcare duties have limited opportunity to work. Therefore, the wage rates are lower for nurses with childcare responsibilities compared with those without. By contrast, the severity of labor might be smaller for people with childcare responsibilities than for those without in this case. Thus, we make the following assumption:

Assumption 2: $\bar{w}_1 - \bar{S}_1 = \bar{w}_2 - \bar{S}_2$.

This assumption implies that the utility of group 1 is identical to that of group 2. We, further, assume as follows:

Assumption 3: The density function $f(v)$ of group 1 with respect to v is the same as that of group 2.

The number of working nurses n is calculated as the total of these two groups. Thus,

$$n = N \left\{ \alpha \int_{\bar{v}_1}^b f(v)dv + (1 - \alpha) \int_{\bar{v}_2}^b f(v)dv \right\} \tag{20}$$

We now present the following lemma.

Lemma 4. $\bar{v}_1 > \bar{v}_2$.

Proof. According to Assumption 1, it is clear that $A_1 > A_2$. In addition, from Assumption 2, we know that $\bar{w}_1 - \bar{S}_1 = \bar{w}_2 - \bar{S}_2$. Then, from (18) and (19), it can be shown that $\bar{v}_1 > \bar{v}_2$.

Lemma 4 means that the latency rate of group 1 is larger than that of group 2. Lemma 5 follows immediately from Lemma 4 and (20).

Lemma 5. The larger α , the lower n/N becomes.

Proof. By rearranging (20), we get

$$\frac{n}{N} = \alpha \left\{ \int_{\bar{v}_1}^b f(v)dv - \int_{\bar{v}_2}^b f(v)dv \right\} + \int_{\bar{v}_2}^b f(v)dv \tag{21}$$

As \bar{v}_1 and \bar{v}_2 are calculated from (18) and (19), these values do not depend on α . From Lemma 4, the sign of the co-efficient (shown by $\{ \}$) of parameter α in (21) is certainly negative.

By denoting the services supplied by nurses (but not inactive nurses) with childcare responsibilities as Y_1 and those provided by other nurses as Y_2 , we define total nursing services Y as follows:

$$Y = Y_1 + Y_2 = N \left\{ \alpha \int_{\bar{v}_1}^b y(v)f(v)dv + (1 - \alpha) \int_{\bar{v}_2}^b y(v)f(v)dv \right\} \tag{22}$$

where $y(v)$ indicates the nursing services provided by one nurse with vocation v and $y(v) > 0, y'(v) > 0$.

Thus, Lemma 6 is as follows.

Lemma 6. When dn caused by dY_1 (the increasing effect of Y from a fall in A_1) is identical to that caused by dY_2 (the increasing effect of Y from a rise in w), it follows that $dY_1 > dY_2$.

See Appendix A for the mathematical proof of Lemma 6. We summarize the arguments as follows.

Proposition 1. Provided that the rate of reinstatement to their original employment is the same for both a rise in the average wage rates for nurses and the expansion of childcare support for nurses, the increase in nursing services will be larger for the latter than for the former.

This proposition is derived because group 1 consists of more inactive nurses with a high level of vocation compared with group 2. From the cost/benefit analysis perspective, we must emphasize that our focus is not on the cost side but rather on the benefit side. Therefore, our model cannot compare the expenditure required to increase the wage rates of nurses with the cost of providing childcare services. However, assuming that the nursing and childcare reinstatement rates are identical, we can conclude that a policy of supporting working nurses with childcare responsibilities might be desirable from a cost/benefit analysis point of view.

5. Conclusion

The large number of inactive nurses is one factor behind the social issue of the nursing shortage in Japan. In this study, we explored policy options for resolving this problem by using a simple but useful model based on the findings of previous studies. One characteristic of the model proposed herein is the introduction, as non-pecuniary factors, of the vocation level of each nurse and the severity of nursing labor into the utility function of nurses.

The foregoing analysis suggests one policy implication. Reinstating inactive nurses by providing childcare support may be much more desirable than offering them a wage rise, concurring with the findings of Phillips (1995). Indeed, it is likely that this policy has already been implemented regionally in Japan. For example, a “childcare in hospital” program is currently underway in the Japanese nursing sector, particularly in small hospitals, through the support of central or local governments. The presented theoretical analysis therefore confirms the direction taken by policymakers to address this pressing issue.

Finally, this study has a number of limitations. We implicitly assumed that resolving the nursing shortage would improve social welfare. However, one cannot argue that a rise in the wage rate for nursing services through public intervention always increase social welfare. This must be argued along with the demand side of nursing services. Furthermore, our theoretical analysis must be fully confirmed by empirical research. These issues will be addressed in our future research.

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References

- Ahlburg, D. A., & Mahoney, C. B. (1996). The effect of wages on the retention of nurses. *Canadian Journal of Economics*, 29, S126-125. <http://dx.doi.org/10.2307/135973>
- Di Tommaso, M. L., Strøm, S., & Sæther, E. M. (2009). Nurses wanted: Is the job too harsh or is the wage too low? *Journal of Health Economics*, 28(3), 748-757. <http://dx.doi.org/10.1016/j.jhealeco.2009.01.003>
- Heyes, A. (2005). The economics of vocation or why is a badly paid nurse a good nurse? *Journal of Health Economics*, 24(3), 561- 569. <http://dx.doi.org/10.1016/j.jhealeco.2004.09.002>
- Hirsch, B. T., & Schumacher, E. T. (2005). Classic or new monopsony? Searching for evidence in nursing labor markets. *Journal of Health Economics*, 24(5), 969-989. <http://dx.doi.org/10.1016/j.jhealeco.2005.03.006>
- Hurd, R. W. (1973). Equilibrium vacancies in a labor market dominated by non-profit firms: The “shortage” of nurses. *Review of Economics and Statistics*, 55(2), 234-240. <http://dx.doi.org/10.2307/1926998>
- Japan Federation of Medical Worker’s Unions. (2010). *Survey on Nursing Labor* [in Japanese].
- Nelson, J. A., & Folbre, N. (2006). Why a well-paid nurse is a better nurse. *Nursing Economics*, 24(3), 127-130. Retrieved from <http://www.medscape.com/viewarticle/541775>
- Phillips, V. L. (1995). Nurses’ labor supply: Participation, hours of work, and discontinuities in the supply function. *Journal of Health Economics*, 14(5), 567-582. [http://dx.doi.org/10.1016/0167-6296\(95\)00023-2](http://dx.doi.org/10.1016/0167-6296(95)00023-2)
- Shields, M. A. (2004). Addressing nurse shortages: What can policy makers learn from the econometric evidence of nurse labor supply. *The Economic Journal*, 114, F464-F498.

<http://dx.doi.org/10.1111/j.1468-0297.2004.00255.x>

Shields, M. A., & Ward, M. (2001). Improving nurse retention in the national health service in England: The impact of job satisfaction on intentions to quit. *Journal of Health Economics*, 20(5), 677-701. [http://dx.doi.org/10.1016/S0167-6296\(01\)00092-3](http://dx.doi.org/10.1016/S0167-6296(01)00092-3)

Taylor, L. J. (2007). Optimal wages in the market for nurses: An analysis based on Heyes' model. *Journal of Health Economics*, 26(5), 1027-1030. <http://dx.doi.org/10.1016/j.jhealeco.2007.01.005>

Young, H. P. (2007). "Social norms". University of Oxford, Department of Economics, Economics Series Working Papers: 307. Retrieved September 15, 2013, from http://www.economics.ox.ac.uk/materials/working_papers/paper307.pdf

Notes

Note 1. According to Heyes (2005), Longman's Dictionary of Contemporary English (1995) explains "vocation" as follows: "1[C] a job you do because you have a strong feeling that doing this job is a purpose of your life, especially because you want to help other people: *Teaching isn't just a job it's a vocation.* 2[C, U] a special ability for or devotion to a particular job or activity, especially one that gives service to other people:[+for] *He has a vocation for teaching*" (p. 562).

Note 2. We are convinced that whether the labor market for nurses is a monopsony is a significant issue, but our concern in this paper is mainly directed to how to resolve the nursing shortage. Therefore, in the presented analysis we ignore the various aspects of monopsony including the mobility of nurses, particularly the so-called "one nurse for every seven patients" problem in Japan.

Appendix A

Proof of Lemma 6: From (20),

$$dn = -N\{\alpha f(\bar{v}_1)d\bar{v}_1 + (1 - \alpha)f(\bar{v}_2)d\bar{v}_2\} \quad (\text{A.1})$$

From (18) and (19),

$$0 = -A_1\gamma' \left(\frac{n}{P}\right) \frac{dn}{P} - \gamma dA_1 + d\bar{v}_1 \quad (\text{A.2})$$

$$0 = -A_2\gamma' \left(\frac{n}{P}\right) \frac{dn}{P} + d\bar{v}_2 \quad (\text{A.3})$$

when $dA_1 < 0$.

Substituting these two equations into (A.1) yields

$$dn = -N \left[\alpha f(\bar{v}_1) \left\{ A_1\gamma' \left(\frac{n}{P}\right) \frac{dn}{P} + \gamma dA_1 \right\} + (1 - \alpha)f(\bar{v}_2)A_2\gamma' \left(\frac{n}{P}\right) \frac{dn}{P} \right] \quad (\text{A.4})$$

Rearranging (A.4) gives

$$Xdn = N\alpha f(\bar{v}_1)\{-\gamma dA_1\} \quad (\text{A.5})$$

where

$$X \equiv 1 + \alpha f(\bar{v}_1)A_1\gamma' \left(\frac{n}{P}\right) \frac{N}{P} + (1 - \alpha)f(\bar{v}_2)A_2\gamma' \left(\frac{n}{P}\right) \frac{N}{P} \quad (\text{A.6})$$

From (18),

$$-dw = -A_1\gamma' \left(\frac{n}{P}\right) \frac{dn}{P} + d\bar{v}_1 \quad (\text{A.7})$$

and from (19),

$$-dw = -A_2\gamma' \left(\frac{n}{P}\right) \frac{dn}{P} + d\bar{v}_2 \quad (\text{A.8})$$

Substituting (A.7) and (A.8) into (A.1), when $dw > 0$, gives

$$dn = -N \left[\alpha f(\bar{v}_1) \left\{ A_1\gamma' \left(\frac{n}{P}\right) \frac{dn}{P} - dw \right\} + (1 - \alpha)f(\bar{v}_2) \left\{ A_2\gamma' \left(\frac{n}{P}\right) \frac{dn}{P} - dw \right\} \right] \quad (\text{A.9})$$

Rearranging (A.9) indicates that

$$Xdn = N\{\alpha f(\bar{v}_1) + (1 - \alpha)f(\bar{v}_2)\}dw \quad (\text{A.10})$$

Thus, from (A.5) and (A.10), the following equation is given:

$$-\gamma dA_1 = \frac{\alpha f(\bar{v}_1) + (1 - \alpha)f(\bar{v}_2)}{\alpha f(\bar{v}_1)} dw \quad (\text{A.11})$$

We now get the following from (22)

$$dY = -N\{\alpha y(\bar{v}_1)f(\bar{v}_1)d\bar{v}_1 + (1 - \alpha)y(\bar{v}_2)f(\bar{v}_2)d\bar{v}_2\} \quad (\text{A.12})$$

Therefore, substituting (A.2) and (A.3) into (A.12) and calculating dY_1 (the increasing effect of Y from a fall in A_1) yields

$$dY_1 = -N\{Z + \alpha y(\bar{v}_1)f(\bar{v}_1)\gamma dA_1\} \quad (\text{A.13})$$

where Z is as follows:

$$Z \equiv \alpha y(\bar{v}_1)f(\bar{v}_1)A_1\gamma' \left(\frac{n}{p}\right) \frac{dn}{p} + (1 - \alpha)y(\bar{v}_2)f(\bar{v}_2)A_2\gamma' \left(\frac{n}{p}\right) \frac{dn}{p} \quad (\text{A.14})$$

By substituting (A.11) into (A.13), we obtain

$$dY_1 = -N[Z - \{\alpha y(\bar{v}_1)f(\bar{v}_1) + (1 - \alpha)y(\bar{v}_2)f(\bar{v}_2)\}dw] \quad (\text{A.15})$$

When we substitute (A.7) and (A.8) into (A.12) to draw out dY_2 (the increasing effect of Y from a rise in w), dY_2 is as follows:

$$dY_2 = -N[Z - \{\alpha y(\bar{v}_1)f(\bar{v}_1) + (1 - \alpha)y(\bar{v}_2)f(\bar{v}_2)\}dw] \quad (\text{A.16})$$

Consequently, subtracting (A.16) from (A.15) gives us

$$dY_1 - dY_2 = N(1 - \alpha)f(\bar{v}_2)\{y(\bar{v}_1) - y(\bar{v}_2)\} > 0 \quad (\text{A.17})$$

where the difference is positive according to Lemma 4. This completes the proof of Lemma 6.

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