# An application of Stochastic Models - Grading System 

# in Manpower Planning 

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#### Abstract

Manpower planning refers to the rather complex task of forecasting and planning for the right number and the right kind of people at the right places and the right time to perform activities that will benefit both the organization and the individuals in it. Human resource management is an important aspect of study and the manpower planning is to be done taking into consideration the dynamics of manpower availability and requirements. In any organization it usually happens that whenever the policy decisions regarding pay, perquisites, promotion and targets of work or sales to be achieved are revised there will be exit of personnel, which in other words is called the wastage. The expected time to cross the threshold level of cumulative loss of manpower is obtained under the assumption that an organization has two grades of personnel and that the loss of manpower in one category is compensated by utilization of manpower available in the order to sustain the activities intact. In this paper we made an attempt that the threshold follows the special case of Exponentiated exponential distribution (EE distribution) is used/depicted, to study the two grade system with numerical illustration.


Keywords: Expected time, Manpower planning, Recruitment, Threshold, Wastage, EE distribution

## 1. Introduction

In marketing organizations it is a usual phenomenon that exit of personnel takes place every time a policy decision regarding revision of wages, incentives and revised sales targets is announced. This in turn leads to depletion of manpower which can be conceptualized in terms of man hours. It would be uneconomical to go in for frequent recruitments in view of the cost involved in the same. A detailed discussion of the cost pertaining to recruitment is seen in (Poornachandra Rao, 1990). Hence the organization goes for recruitments as and when the cumulative loss of manpower crosses a random threshold level beyond which marketing activities would be adversely affected. Cumulative Damage Process (CDP) is related to the shock models in reliability theory. One can refer more on shock model approach by (Esary, Marshall and Proschan, 1973). The basic idea is that accumulating random amount of damages due to shocks in successive epoch leads to the breakdown of the system when the total damage crosses a random threshold level.

In this study grade represents technical and non-technical personnel. Suppose an organization faces the shortage of in technical personal that it can be compensates/shared by some of non-technical (they must have knowledge at the time of recruitment) personnel at certain period because the organization is not to recruit immediately. For example, the hierarchy of organization may have sales representatives, supervisors, managers say grades. So the shortage of manpower in higher grades can be compensating by the lower grades having some knowledge about other grades and the vice-versa to some extent. Consider two grades of personnel of an organization in which the transfer of personnel between the grades is permitted. At the time of decision epoch, if any one of two grades has more loss of manpower then manpower are transferred from one grade to the other in order to sustain the activities intact and for this kind of migration of individuals seen in (Sathiyamoorthi and Parthasarathy, 2002) the expected time to recruitment in a two grades marketing organization to find the expected time and variance for recruitment. The related work further modified suitably to determine the expected time to recruitment in manpower planning has been found (Parthasarathy and Vinoth, 2009).

In this paper instead of considering the usual distribution such as exponential, gamma, weibull etc., for threshold, an alternative distribution namely exponentiated exponential distribution which was introduced by (Gupta and Kundu, 2001) is depicted/ used. The three parameter Generalized Exponential distribution (location, scale, shape) discussed by (Gupta and Kundu, 1999) which has an increasing or decreasing failure rate depending on the shape parameter. The same authors (Gupta and Kundu, 2001) discussed about Exponentiated Exponential distribution with two parameter namely scale and shape parameter.

If $x$ follows Exponentiated Exponential distribution (EE distribution) then
The distribution function, $F_{B}(x, \alpha, \lambda)$

$$
F_{e}^{C}\left(x_{1} \alpha_{1} \lambda\right)=\left(1-z^{-\tau v}\right)^{\alpha} \quad \alpha, \lambda, x>0
$$

The density function

$$
A\left(x, \alpha_{i} \lambda\right)=\alpha \lambda\left(1-\epsilon^{-\tau x}\right)^{\alpha-1} e^{-\delta x}
$$

The corresponding survival function is

$$
S_{E}\left(x_{i} \alpha_{i} \lambda\right)=1-\left(1-\epsilon^{-\lambda x}\right)^{\alpha}
$$

When the shape parameter $\varepsilon=1$, it represents the exponential distribution. (Damodaran and Gopal, 2009) stated that, for the simplicity and for the single parameter distribution, the Generalized Exponential Distribution with shape parameter as $\alpha=2$. Then a random variable which has the density function defined as

$$
f\left(x, \alpha_{t} \lambda\right)=2 \lambda\left(1-e^{-\tau x}\right) e^{-\lambda x}
$$

In this paper, introducing two grades of marketing personnel in the organization, the expected time to recruitment is obtained allowing for the mobility of manpower from one category to the other where there is more of depletion. The threshold level of grade-1 and grade-2 are $Y_{1}$ and $Y_{2}$ respectively which follows exponentiated exponential distribution with $\kappa=2$. Obviously the breakdown occurs only when the total depletion crosses the maximum of the two threshold levels.

## 2. Assumptions of the model

a. The organization comprises two grades of personnel.
b. Mobility or transfer of manpower from one grade to the other is permitted.
c. Each grade has its individual random threshold and if the loss of manpower crosses the maximum of the two, recruitment becomes necessary; in other words, the time to recruitment is equal to the maximum of the time taken for each one of the two grades to cross the threshold.
d. The policy decisions are taken with inter arrival times which are i.i.d. random variables depending upon the market environment, production, etc.
e. The processes which give rise to policy revisions and the threshold random variables are statistically independent.

## 3. Notations

$X_{i}$ : a continuous random variable denoting the amount of loss of manpower caused to the system on the $i^{\text {th }}$ occasion of policy announcement (shock), $1,2, \ldots . \mathrm{k}$ and $\mathcal{X}_{1}^{\prime} s$ are i.i.d
$Y_{1}, Y_{2}$ : continuous random variable denoting the threshold levels for the two grades.
$8($.$) : The probability density function of X$.
$g^{\circ}()$ : Laplace transform of $g()$

$T_{1} \quad:$ Time to breakdown of the system due to depletion in the first grade.
$T_{2} \quad$ : Time to breakdown of the system due to depletion in the second grade.
$T=\max \left(T_{1}, T_{2}\right):$ Time to breakdown of the system or to recruitment.
$f($.$) : p.d.f. of random variable denoting between successive policy announcement with the orresponding c.d.f.$ $F(1)$.
$F_{R}(1):$ k-fold convolution of $\left.F^{( }\right)$.
$5($.$) : Survival function.$
F $F_{\mathrm{K}}(t)$ : Probability of exactly k policy announcements.
$L(t): 1-5(t)$.

## 4. Results

Let $Y$ be the random variable which has the cdf defined as

$$
F_{\bar{L}}\left(x, \alpha_{1}, \lambda\right)=\left(1-e^{-\Delta x}\right)^{2}
$$

$$
\alpha, d o x \geqslant 0
$$

Therefore it has the density function

$$
h_{\pi}(x, \alpha, \lambda)=2 \lambda\left(1-e^{-\lambda x}\right)
$$

The corresponding survival function is

$$
S_{E}\left(x_{i} \alpha_{t} A\right)=1-\left(1-e^{-\pi x}\right)^{2}
$$

Now,
$P\left(X_{1}+X_{2}+\cdots+X_{k}<Y^{r}\right)=\mathrm{P}$ [the system does not fail, after $k$ epochs of exits].
In general, assuming that the threshold $Y$ follows an Exponentiated exponential distribution with parameter $\lambda$, it can be proved that

$$
P\left(X_{1}<\gamma\right)=\int_{0}^{\infty} g_{n}(x) F(x) d x
$$

$$
\begin{equation*}
P\left(\sum_{n=1}^{k} X_{1} \alpha y\right)=\int_{0}^{\infty} g_{2}(x)\left[1-\left(1-e^{-I_{1} x}\right)^{2}\left(1-e^{-L_{2} x}\right)^{2}\right] d x \tag{1}
\end{equation*}
$$

The survival function $S(t)$ is

$$
\begin{aligned}
& P(T>t)=\sum_{\hbar=0}^{\text {win }} P[\text { there are exactly } \mathrm{k} \text { metante of exitin }(0, t)] * P[\text { the eytem does not fall in }(0, t)] \\
& P(T>t)=\sum_{k=1}^{\infty} F_{t}(t) P\left[\sum_{i=1}^{k} X_{1} \alpha \max \left(y_{1}, k_{2}\right)\right]
\end{aligned}
$$

It is also known from renewal theory that

$$
\begin{aligned}
& P(\text { exactly k policy dececione in }(0, t])=F_{k}(t)-F_{k+1}(t) \quad \text { with } \quad F_{0}(t)=1 \\
& =\sum_{k=0}^{\infty} F_{n}(t) P\left(X_{i} \tau Y^{n}\right)
\end{aligned}
$$

$$
\begin{align*}
& +2 A^{-\left[\lambda_{1}+2 \lambda_{2} 3 x\right.}-4 \pi^{-\left[\lambda_{1} \Delta_{2} \lambda x\right]} \tag{3}
\end{align*}
$$

Now

$$
L(T)=1-S(t)
$$

Taking Laplace transform of $L(T)$, we get

$$
\begin{aligned}
& =1-\left\{2\left[1-\left[1-g^{p}\left(\lambda_{2}\right)\right]_{k=1}^{\infty} F_{k}(t)\left[g^{p}\left(\lambda_{2}\right)\right]^{k-1}\right]-\left[1-\left[1-g^{p}\left(2 \lambda_{2}\right) \sum_{k=1}^{\infty} F_{k}\left(t^{n}\right)\left[g^{p}\left(\lambda_{2}\right)\right]^{k-1}\right]\right]\right. \\
& +2\left[1-\left[1-g^{*}\left(\lambda_{1}\right) \sum_{k=1}^{\infty} F_{k}(i)\left[g^{*}\left(\lambda_{1}\right)\right]^{k-1}\right]-\left[1-\left[1-g^{*}\left(2 \lambda_{1}\right) \sum_{k=1}^{\infty} F_{k}(t)\left[g^{*}\left(2 \lambda_{1}\right)\right]^{n-1}\right]\right]\right. \\
& -2\left[1-\left[1-g^{*}\left(\lambda_{1}+\lambda_{2}\right)\right] \sum_{k=1}^{\infty} F_{k}(c)\left[g^{*}\left(\lambda_{1}+\lambda_{2}\right)\right]^{n-1}\right] \\
& +\left[1-\left[1-g^{s}\left(2 \lambda_{1}+\lambda_{2}\right)\right] \sum_{k=1}^{\infty} F_{k}(t)\left[g^{s}\left(2 \lambda_{1}+\lambda_{2}\right)\right]^{k-1}\right] \\
& +\left[1-\left[1-g^{*}\left(\lambda_{1}+2 \lambda_{2}\right)\right] \sum_{n=1}^{\infty} F_{k}(\varepsilon)\left[g^{*}\left(\lambda_{1}+2 \lambda_{2}\right)\right]^{k-1}\right] \\
& \left.-4\left[1-\left[1-g^{\theta}\left(\lambda_{1}+\lambda_{2}\right)\right] \sum_{k=1}^{\infty} F_{k}(c)\left[g^{s}\left(\lambda_{1}+\lambda_{2}\right)\right]^{k-1}\right]\right\}
\end{aligned}
$$

$$
\begin{align*}
&=1-2+2\left[1-g^{*}\left(\lambda_{2}\right)\right] \sum_{k=1}^{\infty} F_{k}(t)\left[g^{*}\left(\lambda_{2}\right)\right]^{k-1}+1+\left[1-g^{*}\left(2 \lambda_{2}\right)\right] \sum_{k=1}^{\infty} F_{k}(t)\left[g^{*}\left(\lambda_{2}\right)\right]^{k-1}-2 \\
&+2\left[1-g^{*}\left(\lambda_{1}\right)\right] \sum_{k=1}^{\infty} F_{k}(t)\left[g^{*}\left(\lambda_{1}\right)\right]^{k-1}+1+\left[1-g^{*}\left(2 \lambda_{1}\right)\right] \sum_{k=1}^{\infty} F_{k}(t)\left[g^{*}\left(2 \lambda_{1}\right)\right]^{k-1}+2 \\
&+2\left[1-g^{*}\left(\lambda_{1}+\lambda_{2}\right)\right] \sum_{k=1}^{\infty} F_{k}(t)\left[g^{*}\left(\lambda_{1}+\lambda_{2}\right)\right]^{k-1}-2 \\
&+2\left[1-g^{*}\left(2 \lambda_{1}+\lambda_{2}\right)\right] \sum_{k=1}^{\infty} F_{k}(t)\left[g^{*}\left(2 \lambda_{1}+\lambda_{2}\right)\right]^{k-1}-2 \\
&+2\left[1-g^{*}\left(\lambda_{1}+2 \lambda_{2}\right)\right] \sum_{k=1}^{\infty} F_{k}(t)\left[g^{*}\left(\lambda_{1}+2 \lambda_{2}\right)\right]^{k-1}+4 \\
&+4\left[1-g^{*}\left(\lambda_{1}+\lambda_{2}\right)\right] \sum_{k=1}^{\infty} F_{k}(t)\left[g^{*}\left(a_{1}+\lambda_{2}\right)\right]^{k-1} \tag{4}
\end{align*}
$$

Taking Laplace transform of $L()$, we get

$$
\begin{align*}
& L^{*}(s)=1+\frac{2\left[1-g^{*}\left(\lambda_{2}\right)\right] f^{*}(s)}{\left[1-g^{*}\left(\lambda_{2}\right) f^{*}(s)\right]}+\frac{\left[1-s^{*}\left(2 \lambda_{2}\right)\right] f^{*}(s)}{\left[1-g^{*}\left(2 \lambda_{2}\right) f^{\prime}(s)\right]}+\frac{2\left[1-g^{*}\left(\lambda_{1}\right)\right] f^{*}(s)}{\left[1-g^{*}\left(\lambda_{1}\right) f^{*}(s)\right]}+\frac{\left[1-s^{*}\left(2 \lambda_{1}\right)\right] f^{*}(s)}{\left[1-g^{*}\left(2 \lambda_{1}\right) f^{*}(s)\right]} \\
& +\frac{2\left[1-g^{*}\left(\lambda_{1}+\lambda_{2}\right) f^{*}(s)\right.}{\left[1-g^{\circ}\left(\lambda_{1}+\lambda_{2}\right) f^{*}(s)\right]}+\frac{2\left[1-g^{*}\left(2 \lambda_{1}+\lambda_{2}\right)\right] f^{*}(s)}{\left[1-g^{*}\left(2 \lambda_{1}+\lambda_{2}\right) f^{*}(s)\right]}+\frac{2\left[1-g^{*}\left(\lambda_{1}+2 \lambda_{2}\right)\right] f^{*}(s)}{\left[1-g^{*}\left(\lambda_{1}+2 \lambda_{2}\right) f^{*}(s)\right]} \\
& +\frac{\left.4\left[1-g^{*}\left(\lambda_{1}+\lambda_{2}\right)\right] f^{*}(s)\right]}{\left[11-\varepsilon^{*}\left(\lambda_{1} \| \lambda_{2}\right) f^{*}(s)\right]} \tag{5}
\end{align*}
$$

$$
K(T)=-\frac{d}{d s} E^{0}(C) \text { givent } s=0
$$

Let the random variable U denoting inter arrival time which follows exponential with parameter c .
Now $f^{\bullet}(0)=\left(\frac{\varepsilon}{\varepsilon+8}\right)$, substituting in the above equation (5) we get,

$$
\begin{align*}
& =1+\frac{2\left[1-\varepsilon^{*}\left(a_{2}\right)\right] \frac{c}{c+g}}{\left[\begin{array}{ll}
1 & \varepsilon^{*}\left(a_{2}\right) \frac{c}{c+s}
\end{array}\right]}+\frac{\left[1-\varepsilon^{*}\left(2 \lambda_{2}\right)\right] \frac{c}{c+g}}{\left[\begin{array}{ll}
1 & \varepsilon^{*}\left(2 \lambda_{2}\right) \\
\frac{c}{c}+s
\end{array}\right]}+\frac{2\left[1-\varepsilon^{*}\left(\lambda_{2}\right) \frac{c}{c+g}\right.}{\left[\begin{array}{ll}
1 & \varepsilon^{*}\left(\lambda_{1}\right) \frac{c}{c+s}
\end{array}\right]}+\frac{\left[1-\varepsilon^{*}\left(2 \lambda_{2}\right)\right] \frac{c}{c+g}}{\left[\begin{array}{ll}
1 & \varepsilon^{*}\left(2 \lambda_{1}\right) \frac{c}{c+s}
\end{array}\right]} \\
& +\frac{2\left[1-s^{*}\left(\lambda_{1}+\lambda_{2}\right) \frac{c}{c+g}\right.}{\left[1-\varepsilon^{*}\left(\lambda_{1}+\lambda_{2}\right) \frac{c}{c+s}\right]}+\frac{2\left[1-s^{*}\left(2 \lambda_{1}+\lambda_{2}\right)\right] \frac{c}{c+g}}{\left[1-\varepsilon^{*}\left(2 \lambda_{1}+\lambda_{2}\right) \frac{c}{c+s}\right]}+\frac{2\left[1-s^{*}\left(\lambda_{1}+2 \lambda_{2}\right) \frac{c}{c+g}\right.}{\left[1-\varepsilon^{*}\left(\lambda_{1}+2 \lambda_{2}\right) \frac{c}{c+s}\right]} \\
& +\frac{4\left[1-s^{\circ}\left(\lambda_{1}+\lambda_{2}\right)\right] \frac{c}{c+g}}{\left[1-s^{\circ}\left(\lambda_{2}+\lambda_{2}\right) \frac{c}{c+s}\right]} \\
& L^{*}(s)=1+\frac{2 c\left[1-g^{*}\left(\lambda_{2}\right)\right]}{\left[c+\varepsilon-g^{*}\left(\lambda_{2}\right) c\right]}+\frac{c\left[1-g^{*}\left(2 \lambda_{2}\right)\right]}{\left[c+s-g^{*}\left(2 \lambda_{2}\right) c\right]}+\frac{2 c\left[1-g^{*}\left(\lambda_{1}\right)\right]}{\left[\varepsilon+\varepsilon-g^{*}\left(\lambda_{1}\right) c\right]}+\frac{c\left[1-g^{*}\left(2 \lambda_{2}\right)\right]}{\left[c+s-g^{*}\left(2 \lambda_{1}\right) c\right]} \\
& +\frac{2 c\left[1-g^{*}\left(\lambda_{1}+\lambda_{2}\right)\right]}{\left[c+s-g^{*}\left(\lambda_{1}+\lambda_{2}\right) c\right]}+\frac{2 c\left[1-g^{*}\left(2 \lambda_{1}+\lambda_{2}\right)\right]}{\left[c+s-g^{*}\left(2 \lambda_{1}+\lambda_{2}\right) c\right]}+\frac{2 c \cdot\left[1-g^{*}\left(\lambda_{1}+2 \lambda_{2}\right)\right]}{\left[c+s-g^{*}\left(\lambda_{1}+2 \lambda_{2}\right) c\right]} \\
& +\frac{4 c\left[1-g^{*}\left(\lambda_{1}+\lambda_{2}\right)\right]}{\left[c+s-g^{*}\left(\lambda_{2}+\lambda_{2}\right) c\right]} \tag{fi}
\end{align*}
$$

$$
\begin{gather*}
E(T)=\frac{2}{c\left[1-s^{*}\left(\lambda_{2}\right)\right]}+\frac{1}{c\left[1-s^{*}\left(2 \lambda_{2}\right)\right]}+\frac{2}{c\left[1-s^{*}\left(\lambda_{1}\right)\right]}+\frac{1}{c\left[1-s^{*}\left(2 \lambda_{1}\right)\right]}+\frac{6}{c\left[1-s^{*}\left(\lambda_{1}+\lambda_{2}\right)\right]} \\
+\frac{2}{c\left[1-s^{*}\left(2 \lambda_{1}+\lambda_{2}\right)\right]}+\frac{2}{c\left[1-s^{*}\left(\lambda_{1}+2 \lambda_{2}\right)\right]} \text { on elmplifflcation } \tag{7}
\end{gather*}
$$

$g^{\circ}(\beta) \sim \exp (\mu): g^{\circ}\left(\lambda_{1}\right)=\frac{\mu}{\mu+\lambda_{1}}: g^{\circ}\left(2 \lambda_{1}\right)=\frac{\mu}{\mu+2 \lambda_{1}}: g^{*}\left(\lambda_{2}\right)=\frac{\mu}{\mu+\lambda_{2}}: g^{*}\left(2 \lambda_{2}\right)=\frac{\mu}{\mu+2 \lambda_{2}}$

$$
\begin{aligned}
& E(T)=\frac{2}{c\left[1-\frac{\beta^{K}}{\beta+\lambda_{2}}\right]}+\frac{1}{c\left[1-\frac{\beta}{\beta+2 \lambda_{2}}\right]}+\frac{2}{c\left[1-\frac{\beta^{\mu}}{\beta+\lambda_{2}}\right]}+\frac{1}{c\left[1-\frac{\beta}{\beta+2 \lambda_{2}}\right]}+\frac{6}{c\left[1-\frac{\beta}{\mu+\left(\lambda_{1}+\lambda_{2}\right)}\right]} \\
& +\frac{2}{c\left[1-\frac{\beta \pi}{\mu+\left(2 \lambda_{2}+\lambda_{2}\right)}\right]}+\frac{2}{c\left[1-\frac{\beta}{\mu+\left(\lambda_{1}+2 \lambda_{2}\right)}\right]} \\
& =\frac{1}{c}\left[\frac{2 \mu+2 \lambda_{2}}{\lambda_{2}}+\frac{\mu+2 \lambda_{2}}{2 \lambda_{2}}+\frac{2_{\mu}+2 \lambda_{1}}{\lambda_{1}}+\frac{\mu+2 \lambda_{2}}{2 \lambda_{1}}+\frac{6 \mu+6 \lambda_{1}+\sigma \lambda_{2}}{\lambda_{1}+\lambda_{2}}+\frac{2 \mu+2 \lambda_{1}+2 \lambda_{2}}{2 \lambda_{1}+\lambda_{2}}\right. \\
& \left.+\frac{2 \mu+2 \lambda_{1}+4 \lambda_{2}}{\lambda_{1}+2 \lambda_{2}}\right] \quad \text { on elmplification }
\end{aligned}
$$

## 5. Numerical Illustration

The Expected time to recruitment in an organization when the amount of loss/breakdown of manpower crosses the maximum of the two grades, where manpower from one grade to other is permitted, the overall behavior of the system follows Exponentiated exponential distribution has threshold level observed which shown in the given tables ( $1,2,3,4,5,6$ ) and figures ( $1,2,3,4,5,6$ ).

When c increases [ $\mathrm{c}>0$ ] and amount of damage is fixed $(\mu)$ then the expected time to recruitment decreases. When the interval between policy changing times increases with fixed $\mu$ there trend to be decreases in expected time to recruitment which is natural.
When $c$ is fixed and the amount of wastage increases $[\mu>0]$ there exist an increase in the expected time to recruitment. When the interval is fixed in the policy changing time and with the amount of wastage increases there exists an increasing in the expected time to recruitment.
Since Exponentiated exponential distribution consider here and also variation of $\lambda_{1}, \lambda_{2}$ we get the expected time to recruitment decreases when the inter arrival increases whereas when the expected time to recruitment increases when the inter arrival time is fixed.

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Table 1. $\lambda_{1}=1: \lambda_{2}=1$

|  | 1 | 3 | 5 | 7 | 9 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\mu=1$ | 25.33 | 8.44 | 5.07 | 3.62 | 2.81 |
| $\mu=4$ | 53.33 | 17.78 | 10.67 | 7.62 | 5.93 |
| $\mu=7$ | 81.33 | 27.11 | 16.27 | 11.62 | 9.04 |
| $\mu=10$ | 109.33 | 36.44 | 21.87 | 15.62 | 12.15 |
| $\mu=13$ | 137.33 | 45.78 | 27.47 | 19.62 | 15.26 |
| $\mu=16$ | 165.33 | 55.11 | 33.07 | 23.62 | 18.37 |
| $\mu=19$ | 193.33 | 64.44 | 38.67 | 27.62 | 21.48 |
| $\mu=22$ | 221.33 | 73.78 | 44.27 | 31.62 | 24.59 |
| $\mu=25$ | 249.33 | 83.11 | 49.87 | 35.62 | 27.70 |
| $\mu=28$ | 277.33 | 92.44 | 55.47 | 39.62 | 30.81 |

Table 2. $\lambda_{1}=3: \lambda_{2}=1$

|  | 1 | 3 | 5 | 7 | 9 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\mu=1$ | 21.52 | 7.17 | 4.30 | 3.07 | 2.39 |
| $\mu=4$ | 38.08 | 12.69 | 7.62 | 5.44 | 4.23 |
| $\mu=7$ | 54.63 | 18.21 | 10.93 | 7.80 | 6.07 |
| $\mu=10$ | 71.19 | 23.73 | 14.24 | 10.17 | 7.91 |
| $\mu=13$ | 87.75 | 29.25 | 17.55 | 12.54 | 9.75 |
| $\mu=16$ | 104.30 | 34.77 | 20.86 | 14.90 | 11.59 |
| $\mu=19$ | 120.86 | 40.29 | 24.17 | 17.27 | 13.43 |
| $\mu=22$ | 137.42 | 45.81 | 27.48 | 19.63 | 15.27 |
| $\mu=25$ | 153.98 | 51.33 | 30.80 | 22.00 | 17.11 |
| $\mu=28$ | 170.53 | 56.84 | 34.11 | 24.36 | 18.95 |

Table 3. $\lambda_{1}=5: \lambda_{2}=1$

|  | 1 | 3 | 5 | 7 | 9 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\mu=1$ | 20.47 | 6.82 | 4.09 | 2.92 | 2.27 |
| $\mu=4$ | 33.87 | 11.29 | 6.77 | 4.84 | 3.76 |
| $\mu=7$ | 47.27 | 15.76 | 9.45 | 6.75 | 5.25 |
| $\mu=10$ | 60.68 | 20.23 | 12.14 | 8.67 | 6.74 |
| $\mu=13$ | 74.08 | 24.69 | 14.82 | 10.58 | 8.23 |
| $\mu=16$ | 87.48 | 29.16 | 17.50 | 12.50 | 9.72 |
| $\mu=19$ | 100.88 | 33.63 | 20.18 | 14.41 | 11.21 |
| $\mu=22$ | 114.29 | 38.10 | 22.86 | 16.33 | 12.70 |
| $\mu=25$ | 127.69 | 42.56 | 25.54 | 18.24 | 14.19 |
| $\mu=28$ | 141.09 | 47.03 | 28.22 | 20.16 | 15.68 |

Table 4. $\lambda_{1}=9: \lambda_{2}=1$

|  | 1 | 3 | 5 | 7 | 9 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\mu=1$ | 19.66 | 6.55 | 3.93 | 2.81 | 2.18 |
| $\mu=4$ | 30.66 | 10.22 | 6.13 | 4.38 | 3.41 |
| $\mu=7$ | 41.65 | 13.88 | 8.33 | 5.95 | 4.63 |
| $\mu=10$ | 52.65 | 17.55 | 10.53 | 7.52 | 5.85 |
| $\mu=13$ | 63.64 | 21.21 | 12.73 | 9.09 | 7.07 |
| $\mu=16$ | 74.64 | 24.88 | 14.93 | 10.66 | 8.29 |
| $\mu=19$ | 85.63 | 28.54 | 17.13 | 12.23 | 9.51 |
| $\mu=22$ | 96.63 | 32.21 | 19.33 | 13.80 | 10.74 |
| $\mu=25$ | 107.62 | 35.87 | 21.52 | 15.37 | 11.96 |
| $\mu=28$ | 118.62 | 39.54 | 23.72 | 16.95 | 13.18 |

Table 5. $\lambda_{1}=11: \lambda_{2}=1$

|  | 1 | 3 | 5 | 7 | 9 |
| :--- | :--- | ---: | ---: | ---: | ---: |
| $\mu=1$ | 19.47 | 6.49 | 3.89 | 2.78 | 2.16 |
| $\mu=4$ | 29.87 | 9.96 | 5.97 | 4.27 | 3.32 |
| $\mu=7$ | 40.28 | 13.43 | 8.06 | 5.75 | 4.48 |
| $\mu=10$ | 50.68 | 16.89 | 10.14 | 7.24 | 5.63 |
| $\mu=13$ | 61.08 | 20.36 | 12.22 | 8.73 | 6.79 |
| $\mu=16$ | 71.49 | 23.83 | 14.30 | 10.21 | 7.94 |
| $\mu=19$ | 81.89 | 27.30 | 16.38 | 11.70 | 9.10 |
| $\mu=22$ | 92.30 | 30.77 | 18.46 | 13.19 | 10.26 |
| $\mu=25$ | 102.70 | 34.23 | 20.54 | 14.67 | 11.41 |
| $\mu=28$ | 113.11 | 37.70 | 22.62 | 16.16 | 12.57 |

Table 6. $\lambda_{1}=13: \lambda_{2}=1$

|  | 1 | 3 | 5 | 7 | 9 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\mu=1$ | 19.33 | 6.44 | 3.87 | 2.76 | 2.15 |
| $\mu=4$ | 29.31 | 9.77 | 5.86 | 4.19 | 3.26 |
| $\mu=7$ | 39.30 | 13.10 | 7.86 | 5.61 | 4.37 |
| $\mu=10$ | 49.28 | 16.43 | 9.86 | 7.04 | 5.48 |
| $\mu=13$ | 59.27 | 19.76 | 11.85 | 8.47 | 6.59 |
| $\mu=16$ | 69.25 | 23.08 | 13.85 | 9.89 | 7.69 |
| $\mu=19$ | 79.24 | 26.41 | 15.85 | 11.32 | 8.80 |
| $\mu=22$ | 89.22 | 29.74 | 17.84 | 12.75 | 9.91 |
| $\mu=25$ | 99.21 | 33.07 | 19.84 | 14.17 | 11.02 |
| $\mu=28$ | 109.19 | 36.40 | 21.84 | 15.60 | 12.13 |



Figure 1.


Figure 2.


Figure 3.


Figure 4.


Figure 5.


Figure 6.

