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Unbalanced Bidding Problem with Fuzzy Random Variables

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Abstract

Unbalanced bidding problem with mixed uncertainty of fuzziness and randomness is considered in this paper, where the bidding engineering quantities of each activity are assumed to be fuzzy random variables. Two types of fuzzy random models as expected value maximization model and maximax chance-constrained model are built to satisfy different optimization requirements. Then a hybrid intelligent algorithm integrating fuzzy random simulations, neural network and genetic algorithm is designed to solve these models. Finally, a numerical experiment is given to illustrate its effectiveness of the algorithm. The results show that the algorithm is feasible and effective.

Keywords: Unbalanced bidding, Fuzzy random variable, Fuzzy random programming, Hybrid intelligent algorithm

1. Introduction

With the further development of the bid mechanism, the engineering item invitation has already become more and more standardized. How to master the strategy of invitation, increasing the efficiency of bids is very important for the bidders. In order to make the bid price more competitive and obtain sufficient profits in engineering bids, they can use many kinds of methods to calculate the bid price, including unbalanced bidding, unexpected markdown, loss-first-profit-later quotation, multiple alternative quotation, etc. On the whole, unbalanced bidding is the most widely used tactic.

Unbalanced bids is opposite to the conventional balanced bids, which is a wide-used method in the optimization of unit price internationally. It can be described as follows: after the contractors carry on the resource distribution, the cost analysis and the research of bidding skills according to bills of quantities provided by the owners, they enhance some comprehensive unit price in bills of quantities consciously and reduce other comprehensive unit price simultaneously in order to obtain more economic benefits without raising the total price.

Unbalanced bids is divided into two types of “earlier receiving money” and “more receiving money”. However, how to realize these two types lacks quantitative optimization models and efficient algorithms, so it can bring randomness and blindness in process of bidding. In order to solve this problem, many researchers at home or abroad have carried on extensive research. Dayanand and Padman (1997, p.906 & 2001, p.197) initially established optimal models of the project payment scheduling problem from contractor′s and client′s viewpoints respectively, also presented a heuristic algorithm to find the payment schedule of maximizing project profits. Afterwards Ulusoy et al. (2000, p.262) introduced a method of seeking the equal payment scheduling from both sides of contractor and client. In China, the research of this problem is still on the beginning period, and most researches were presented theoretically. Xu (1990) ' s research result in the field of project cost management has laid a solid theoretical foundation for other researchers; After that, Zhang et al. (2005, p.595) studied bids strategy via the game theory, using the characteristics of engineering quality list in the market economy. Chen et al. (2005, p.118) built linear programming models with objective of maximizing surplus profits according to the fundamental principles and the conditions of unbalanced bids; He and Xu (2007, p.474) set up nonlinear mixed integer programming models from contractor′s and client′s viewpoints respectively, also designed the two-module simulated annealing heuristics algorithm to solve these models. Generally speaking, most of these researches were built in certain environments, and the results showed that it could be optimized to give the best present value of actual payment without raising the total price when the unit price of front activities were enhanced by 10% and the unit prices of following activities were reduced by 10%.

Nevertheless, in real world, unbalanced bids is a complex problem. If contractors overuse unbalaned unit price, it can not only bring the loss to contractors or influence winning a bid, but also result in the serious trouble and increase the investment risk to clients during the project management process. Due to the uncertainty of budget engineering quantities and substitution of main components in each activity, it is difficult for contractors to estimate bidding engineering quantities of each activity accurately. Therefore, there is a need to study unbalanced bidding problem in uncertain environments so as to make the contractors' bid price more competitive and improve the practical application value of unbalanced bids.

Furthermore, fuzziness and randomness sometimes may co-exist in unbalanced bidding problem. In a bidding project, some bidding activities may seldom or never be performed, of which bidding engineering quantities can be described by fuzzy variables, while some other bidding activities may have been processed many times before, of which bidding engineering quantities can be summarized by random variables. In this case, fuzzy random variable, which was initialized by Kwakernaak (1978, p.1), can be introduced as a useful tool for optimization problems with mixed uncertainty of fuzziness and randomness.

To the best of our knowledge, there is little research for unbalanced bidding problem with the uncertainty of combining fuzziness and randomness. In this paper, we try to consider the unbalanced bidding optimization problem when the bidding engineering quantities of each activity cannot be known precisely. We regard the bidding engineering quantities of each activity as fuzzy random variables. This paper will effectively solve unbalanced bidding problem with mixed uncertainty of fuzziness and randomness. In Section 2 we recalls some basic concepts of fuzzy random variable which are necessary to understand the rest of the paper. Then some assumptions are given and the problem is described in detail in Section 3. According to the analysis of the problem, Section 4 builds two types of fuzzy random models as expected value maximization model and maximax chance-constrained model. Furthermore, in order to deal with fuzzy random models in Section 4, Section 5 integrates fuzzy random simulations, neural network and genetic algorithm to design a hybrid intelligent algorithm. To reveal the effectiveness of the hybrid intelligent algorithm, Section 6 gives a numerical experiment. Finally, some conclusions are drawn in Section 7.

2. Fuzzy random variable

In many cases, fuzziness and randomness simultaneously appear in an optimization framework. In order to describe this phenomena, the concept of fuzzy random variable was introduced by Kwakernaak (1978, p.1). Since then, this concept was developed by other researchers such as Colubi et al. (2001, p.3), Kruse and Meyer (1987), and Liu and Liu (2003, p.143) according to different requirements of measurability. In this paper, we adopt the defnition of fuzzy random variable given in Liu and Liu (2003, p.143) for fuzzy random optimization. For convenience, we shall recall briefly some basic concepts and results on fuzzy random variables.

Definition 2.1 (Liu and Liu (2003, p.143)). Let $(\Omega,\sum,\Pr)$ be a probability space. A fuzzy random vector is a map $\xi=(\xi_1,\xi_{2,},...,\xi_m):\Omega\rightarrow F_V^m$ such that for any closed subset C of R^m,

$$\xi^*(C)(\omega)=Pos\{\xi(\omega)\in C\}$$

is a measurable function of ω, where F_V^m be a collection of fuzzy vectors defined on a possibility space, and each element X of F_V^m is characterized by a possibility distribution function μ_X of the fuzzy vector X. If $m=1$, then ξ is called a fuzzy random variable.

Theorem 2.1 (Liu and Liu (2003, p.143)). If ξ be a fuzzy random variable, then the expected value $E[\xi(\omega)]$ of fuzzy variable $\xi(\omega)$ is a random variable.

Definition 2.2 (Liu and Liu (2003, p.143)). Let ξ be a fuzzy random variable defined on the probability space $(\Omega, \Sigma, \Pr)$. The expected value of the fuzzy random variable is defined by

$$E[\xi] = \int_{\Omega} [\int_0^{\infty} Cr\{\xi(\omega) \geq r\} dr - \int_{-\infty}^{0} Cr\{\xi(\omega) \leq r\} dr] \Pr(d\omega).$$

Definition 2.3(Liu (2001, p.713))**.** Let $\xi = (\xi_1, \xi_2, ..., \xi_n)$ be a fuzzy random vector, and $f: R^n \to R^m$ be real-valued continuous functions. Then the primitive chance of fuzzy random event characterized by $f(\xi) \leq 0$ is a function from $[0,1]$ to $[0,1]$ such that for any given $\alpha \in [0,1]$, we have

$$Ch\{f(\xi) \leq 0\}(\alpha) = \sup\{\beta \mid \Pr\{\omega \in \Omega \mid Cr\{f(\xi(\omega)) \leq 0\} \geq \beta\} \geq \alpha\}$$

Where α is a prescribed probability level. The value of primitive chance at α is called α -chance.

Definition 2.4(Liu (2001, p.713))**.** Let ξ be a fuzzy random variable, and $\gamma, \delta \in (0,1]$ .Then

$$\xi_{\sup}(\gamma, \delta) = \sup\{r \mid Ch\{\xi \geq r\}(\gamma) \geq \delta\}$$

Is called the $(\gamma, \delta) - optimistic$ value to ξ, and

$$\xi_{\inf}(\gamma, \delta) = \inf\{r \mid Ch\{\xi \leq r\}(\gamma) \geq \delta\}$$

Is called the $(\gamma, \delta) - pessimistic$ value to ξ.

Example 2.1 (Liu (2007))**.** Let $\bar{a}_1, \bar{a}_2, \cdots, \bar{a}_m$ be fuzzy variables defined on the credibility space $(\Theta, P(\Theta), Cr)$, and $p_1, p_2, \cdots, p_m$ nonnegative numbers with $p_1 + p_2 + \cdots + p_m = 1$ .Then

$$\xi = \begin{cases} \bar{a}_1 & with \quad probability \quad p_1 \\ \bar{a}_2 & with \quad probability \quad p_2 \\ \cdots & \\ \bar{a}_m & with \quad probability \quad p_m \end{cases}$$

is clearly a fuzzy random variable.

Example 2.2 Let η be a fuzzy random variable defined as

$$\eta = \begin{cases} (-6,-4,-2) & with \quad probability \quad 0.25, \\ (-2,0,2) & with \quad probability \quad 0.5, \\ (2,4,6) & with \quad probability \quad 0.25. \end{cases}$$

3. Problem Description

Before we begin to study unbalanced bidding problem with fuzzy random bidding engineering quantities, we first make some assumptions as:

(a) The contractor's anticipated starting time and duration time of each activity are the same as owner's;

(b) The interest rate doesn’t change during the period of the project;

(c) Each activity should be processed without interruption;

(d) The owner decides payment according to the construction schedule of each activity;

(e) The last payment must be arranged when the total project is finished.

For simplicity, we assume that bill of quantity (BOQ) consists of n bidding activities, and the ith bidding activities have m main components, $i = 1,2,\cdots,n$, respectively.

Next, in order to model unbalanced bids problem, we must introduce the following indices and parameters:

t_{si}: The starting time of the ith activities;

t_i: The duration time of the ith activities;

r: The interest rate;

k_i : The discounting coefficient of construction cost needed for the ith bidding activities, and it can be calculated by the following equation, $k_i = \frac{(1+r)^{t_i} - 1}{t_i \cdot r \cdot (1+r)^{t_{si}+t_i}}$ (1)

P_i : The budget price for the ith activities;

$\overline{P_i}$: The bidding price for the ith activities;

q_{ij} : The budget engineering quantities for the jth components in the ith activities, $j = 1,2,\cdots,m$;

ξ_{ij} : The fuzzy random bidding engineering quantities for the jth components in the ith activities;

p_{ij} : The budget unit price of the client for the jth components in the ith activities;

x_{ij} : The bidding unit price of the contractor for the jth components in the ith activities;

According to the assumptions, the budget price of the client for the total project should be $P = \sum_{i=1}^{n} P_i$. (2)

The bidding price of the contractor for the total project is $\overline{P} = \sum_{i=1}^{n} \overline{P_i}$. (3)

The budget engineering quantities of the client for the ith activities should be $q_i = \sum_{j=1}^{m} q_{ij}$. (4)

The fuzzy random bidding engineering quantities of the contractor for the ith activities should be

$$\xi_i = \sum_{j=1}^{m} \xi_{ij} . \quad (5)$$

The budget price for the total project is $P_i = \sum_{j=1}^{m} q_{ij} p_{ij}$. (6)

The bidding price for the total project is $\overline{P_i} = \sum_{j=1}^{m} \xi_{ij} x_{ij}$. (7)

Therefore, the present value of the client's budget price for the total project can be written as

$$f = \sum_{i=1}^{n} k_i P_i . \quad (8)$$

The present value of the contractor's bidding price for the total project can be written as

$$\overline{f} = \sum_{i=1}^{n} k_i \overline{P_i} . \quad (9)$$

As these parameters and basic formulas have been given in the above section, we can establish different fuzzy random programming models to satisfy different optimization goals.

4. Fuzzy random models

4.1 Expected value maximization model

The first type of programming is fuzzy random expected value model(EVM), which optimizes the expected objective function subject to a set of expected constraints. It was introduced by Liu and Liu (2003, p.89). The fuzzy random expected value model is widely used to model practical problems with uncertain factors. In fuzzy random environments, objective functions and constraint functions always cannot be calculated directly. In practice, many decision-makers may tend to optimize expected objectives. In this case, we can optimize expected objective function under some expected constraints by the method of EVM. Hence, in fuzzy random unbalanced bidding problem, the present value of the contractor's bidding price for the total project can be required to be maximized under some expected constraints. To satisfy this type of requirement, we can build an expected value maximization mode as:

$$\begin{cases} \max\ E[\overline{f}] = E[\sum_{i=1}^{n} k_i \overline{P_i}] \\ subject \quad to: \\ \qquad E[\sum_{i=1}^{n} \overline{P_i} - \sum_{i=1}^{n} P_i] \le 0 \qquad (10) \\ \qquad E[\sum_{i=1}^{n}\sum_{j=1}^{m} \xi_{ij} - \sum_{i=1}^{n}\sum_{j=1}^{m} q_{ij}] \le 0 \qquad (11) \\ \qquad 0.9 p_{ij} \le x_{ij} \le 1.1 p_{ij}, \quad i = 1,2,\cdots,n; j = 1,2,\cdots,m. \qquad (12) \end{cases}$$

where x_{ij} is the decision variable, ξ_{ij} is the fuzzy random variable, the form (10) and the form (11) express expected constraints, P_i and $\overline{P_i}$ are defined by (6) and (7), respectively, the form (12) expresses the limits of contractor's bidding unit price in order to avoid suspicion of unbalanced bidding according to many practical cases.

4.2 Maximax chance-constrained model

The second type of programming is fuzzy random chance-constrained programming (CCP), which was initialized by Liu (2001, p.713). Its outstanding feature characteristic is that the chance constraints should hold at least some given confidence levels. In fuzzy random unbalanced bidding problem, contractors may just want to obtain the optimization goals with fuzzy random constraints holding at least some given confidence levels. In this case, we assume that x_{ij} is the decision variable and ξ_{ij} is the fuzzy random variable for the same reason. In order to maximize the present value of the contractor's bidding price with some confidence levels subject to some chance constraints, we can establish the following maximax chance-constrained model based on fuzzy random CCP:

$$\begin{cases} \max\ \overline{f} \\ subject \quad to: \\ \qquad Ch\{\sum_{i=1}^{n} k_i \overline{P_i} \ge \overline{f}\}(\gamma) \ge \delta \qquad (13) \\ \qquad Ch\{\sum_{i=1}^{n} \overline{P_i} \le \sum_{i=1}^{n} P_i\}(\alpha_1) \ge \beta_1 \qquad (14) \\ \qquad Ch\{\sum_{i=1}^{n}\sum_{j=1}^{m} \xi_{ij} \le \sum_{i=1}^{n}\sum_{j=1}^{m} q_{ij}\}(\alpha_2) \ge \beta_2 \qquad (15) \\ \qquad 0.9 p_{ij} \le x_{ij} \le 1.1 p_{ij}, \quad i = 1,2,\cdots,n; j = 1,2,\cdots,m. \qquad (16) \end{cases}$$

where $\gamma, \delta, \alpha_1, \beta_1, \alpha_2, \beta_2$ are the predetermined confidence levels, the form (14) and the form (15) express chance constraints, the other forms' meanings are similar to the analysis of the expected value maximization mode.

5. Hybrid intelligent algorithm

Generally speaking, it is difficult to solve uncertain programming models. In order to solve fuzzy random models, a hybrid intelligent algorithm integrating fuzzy random simulations, neural network and genetic algorithm was designed by Liu (2002). In this paper, we take the chance-constrained model as the example to introduce the hybrid intelligent algorithm.

Firstly, we apply fuzzy random simulations to estimate the uncertain functions with fuzzy random variables. The fuzzy random simulation is one of the most widely used techniques in fuzzy random system modeling, which has been applied in a wide variety of real problems. Although the fuzzy random simulation can't give the accurate results and it is also a time-consuming process, it is possibly the only effective method for complex problems.

In order to solve the model, we generate training input-output data for the uncertain function $U : x \to (U_1(x), U_2(x), U_3(x))$, where

$$U_1(x) = \max \left\{ \overline{f} \middle| Ch\{\sum_{i=1}^{n} k_i \overline{P_i} \geq \overline{f}\}(\gamma) \geq \delta \right\},$$

$$U_2(x) = Ch\{\sum_{i=1}^{n} \overline{P_i} \leq \sum_{i=1}^{n} P_i\}(\alpha_1),$$

$$U_3(x) = Ch\{\sum_{i=1}^{n}\sum_{j=1}^{m} \xi_{ij} \leq \sum_{i=1}^{n}\sum_{j=1}^{m} q_{ij}\}(\alpha_2),$$

by the fuzzy random simulation. Then we train a neural network to approximate the uncertain function U. Finally, the trained neural network is embedded into a genetic algorithm to produce a hybrid intelligent algorithm. The procedure can be summarized generally as follows:

Step 1. Generate training intput-output data for the above uncertain function U by the fuzzy random simulation and then train a neural network to approximate the uncertain function U.

Step 2. Initialize pop_size chromosomes whose feasibility may be checked by the trained neural work.

Step 3. Update the chromosomes by crossover and mutation operations.

Step 4. Calculate the objective values for all chromosomes by the trained neural network and compute the fitness of each chromosome according to the objective values.

Step 5. Select the chromosomes by spinning the roulette wheel according to the different fitness values.

Step 6. Repeat the third to fifth steps for a given number of cycles.

Step 7. Report the best chromosome as the optimal solution of the problem.

6. Numerical experiment

Here we will give an example to show the fuzzy random chance-constrained model that we have just discussed and how the hybrid intelligent algorithm works. Let us consider the following unbalanced bidding problem. Assume that the client's BOQ consists of five bidding activities and each activity has six components which are man-power cost, material cost, mechanical cost, administrative charge, profit, risk cost in turn. The unit prices of man-power cost and administrative charge remain unchanged, but others change during the period of total project according to some practical cases. The budget unit prices and engineering quantities of the client for activities are presented in Table 1 and Table 2, respectively. The monthly interest rate r is given as 1%.

In the bidding project, we assume that the bidding engineering quantities of each activity as fuzzy random variables, denoted by a form of triangular fuzzy variable $(\rho, \rho + a, r_3 + b)$, where a, b are given crisp numbers and ρ is a random variable with uniform distribution given in Table 3.

6.1 Maximax chance-constrained model

For example, the contractor decides to bid the project, in which the starting times and the duration times of activities are given in Table 4.

With the idea of maximizing the present value of contractor's bidding price for the total project at predetermined confidence levels subject to some chance constraints, we consider the following chance-constrained model:

$$\begin{cases} \max \overline{f} \\ subject \quad to: \\ \quad Ch\{\sum_{i=1}^{3} k_i \overline{P_i} \geq \overline{f}\}(0.9) \geq 0.9 & (17) \\ \quad Ch\{\sum_{i=1}^{3} \overline{P_i} \leq \sum_{i=1}^{3} P_i\}(0.9) \geq 0.9 & (18) \\ \quad Ch\{\sum_{i=1}^{3}\sum_{j=1}^{5} \xi_{ij} \leq \sum_{i=1}^{3}\sum_{j=1}^{5} q_{ij}\}(0.9) \geq 0.9 & (19) \\ 0.9 p_{ij} \leq x_{ij} \leq 1.1 p_{ij}, \quad i = 1,2,3; j = 1,2,3,4,5. & (20) \end{cases}$$

6.2 Model solution

We use Visual C++ software to realize the hybrid intelligent algorithm of fuzzy random maximax chance-constrained model with the following parameters: the pop_size is 30, the $p\_crossover$ is 0.3, the $p\_mutation$ is 0.2.

After a run of the hybrid intelligent algorithm (1000 cycles in simulation, 2000 training data in neural network, 600 generations in genetic algorithm), a quasi-optimal solution of the bidding project is presented in Table 5, whose objective value $\overline{f}$ =2780642.2.

The result shows that the present value of bidding price for the total project is 2780642.2 Yuan when the contractor uses the unbalanced bidding strategy with fuzzy random bidding engineering quantities. But the budget price for the total project is 3168000 Yuan when the owner adopts the method of linear programming. Then the present value of the budget price is 2742887.43 Yuan via conversion. In this way, the contractor can obtain surplus profit 37754.77 Yuan. The proof-test proves that this bidding strategy tallies with the actual situation completely. Therefore, the numerical result is persuasive and successful.

7. Conclusion

In this paper, we attempted to solve unbalanced bidding problem with mixed uncertainty of fuzziness and randomness. Fuzzy random variable was introduced and unbalanced bidding problem with fuzzy random bidding engineering quantities was dealt with. Two types of fuzzy random models, including the expected value maximization model and the maximax chance-constrained model were built to meet different optimization goals after some concepts of fuzzy random variable were recalled. Then hybrid intelligent algorithm integrating fuzzy random simulations, neural network and genetic algorithm was designed and a numerical example was given. From the numerical result, we could clearly see that the hybrid intelligent algorithm could effectively solve the fuzzy random unbalanced bidding problem. Furthermore, we could make the unbalanced bidding price more reasonable and applicable. In addition, this paper provided a good applied case for the practice of uncertain programming, and it also put forward a new approach for the promotion of uncertainty theory.

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Table 1. The budget unit price of the client (unit: RMB)

p_{ij}	1	2	3	4	5
1	60	55	70	80	35
2	60	120	110	80	30
3	60	160	80	80	20

Table 2. The budget engineering quantities of the client

q_{ij}	1	2	3	4	5
1	3000	5200	4000	1800	2000
2	4400	3700	4500	500	3900
3	4100	2000	2700	500	1300

Table 3. The bidding engineering quantities of the contractor

ξ_{ij}	1	2	3	4	5
1	$(\rho, \rho+150, \rho+300)$	$(\rho, \rho+250, \rho+450)$	$(\rho, \rho+200, \rho+350)$	$(\rho, \rho+150, \rho+300)$	$(\rho, \rho+100, \rho+200)$
2	$(\rho, \rho+150, \rho+350)$	$(\rho, \rho+150, \rho+300)$	$(\rho, \rho+300, \rho+500)$	$(\rho, \rho+20, \rho+50)$	$(\rho, \rho+150, \rho+300)$
3	$(\rho, \rho+200, \rho+400)$	$(\rho, \rho+100, \rho+250)$	$(\rho, \rho+100, \rho+180)$	$(\rho, \rho+25, \rho+45)$	$(\rho, \rho+60, \rho+120)$

Table 4. The starting times and the duration times of activities

the ith activity	t_{si}	$t_i(month)$
1	Jan. 2009	13
2	Oct.2009	11
3	Feb.2010	11

Table 5. The bidding unit price of the contractor (unit: RMB)

x_{ij}	1	2	3	4	5
1	59.161	56.187	70.042	81.348	36.813
2	58.834	123.63	111.50	78.967	30.424
3	60.782	158.05	79.791	76.102	21.871