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The Real Options Model of Optimal Timing in Banks' Write-off Decisions under Dynamic Circumstance

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Abstract

This paper employs a real option approach to evaluate the value of the option to delay write-offs non-performing loans (NPLs) in commercial banks. On the assumption that the callback rate of NPLs follows the standard geometric Bronian and the reinvestment return follows jump-diffusion model, the partial differential equation which the value keep to is obtained using dynamic programming technique. With the condition of value-matching and smooth-pasting, the solution of the equation is obtained. The optimal timing in banks' writing off their NPLs is gained with the solution, along with the condition to put off disposal of NPLs.

Keywords: Non-performing loans, Real Options, Optimal Timing

1. Introduction

Since 1990s, the Japanese economy has been in a prolonged recession, with many banks weighed down by large-scale non-performing loans (NPLs), in particular to the real estate and construction corporations, when the bubble economy blow up, the land prices and real estate prices fall sharply. This brought serious impact on the development of Japanese economy and banks' operation. A lot of Japanese economist made deep research to the problem. Many authors like Cargill argued (Cargill, 2000), the failure to promptly solve the non-performing loan problem generated a credit crunch. It has contributed to stagnant for almost a decade and has interfered with the Bank of Japan's effort to stimulate the economy. The influence is embodiment in the damage of bank' asset collocation function. They figured out that the banks should write off their non-performing loans immediately. However, there are some authors, like Hoshi (2000), they argued, that purely from the banks' perspective, to put off writing off their NPLs until the economy anabiosis can be a rational choice. This is because under the stochastic circumstance with potentially large losses associate write-offs, the option to write should have some value. Hence, in deciding whether to write off the non-performing loans immediately, the banks shoud weight between the value of the option to wait and the (net) value of carrying out write-off immediately. Hoshi pointed out that under the condition of banks non required by the authorities to dispose the true magnitude of their non-performing loans, there is no incentive to dispose of their non-performing loans. Rather, they tend to increase their lending to riskier project. The true problem cause by non-performing loans is that banks lose their incentive to lend to corporations with prospective project, which might damage the intermediary function of banks, and the economy, meanwhile, the magnitude of non-performing loans enlarge continue. If the social cost caused by the damage of banks' financial intermediary function outweigh the subsidy costs, then in might justified to use funds to push self-helf efforts on banks to clean up their non-performing loans.

Presently, the largeness non-performing loans in bank system is also one of the most importance questions which influent the finance stabilization and secure in China. Now there are two ways to disposal of the NPLs, one is centralization another is decentralization. Centralization manners are done by financial Asset Management Company (AMC) formed by the government, with responsibility for purchase the NPLs produced in banks, then sale, auction, securitisation them and so on. Decentralization is a way to dispose of the NPLs which is done by each bank separately. Though centralization ways can write-off the mass NPLs immediately, improve the quality of the banks' assets, there are still a lot problem. The government seldom employs them. From the status of the banks in our country, it is an effective way to dispose of the NPLs continuative coming into being, which adjust measures to local condition. It can make use of the embranchment institutions all over the country of the commercial banks.

At the present time, there are a lot of researches on the issue of NPLs in China. But their research mainly focus on the definition and the causation of NPLs, the impact on economy and financial system of the country, and the mode of writing-off the NPLs, and so on. There are few researches on the issue of NPLs from the point of microcosmic view. Naohiko Baba

(2001), a scholar from Japan, researched the optimal timing in banks' write-off decisions to NPLs under the possible implementation of a government subsidy scheme, with a real option approach, according to the characteristic of Japanese banks' NPLs and the financial assistance policy of Japanese government. But what he researched was housing mortgage loans in Japan. This paper we apply the approach from Naohiko Baba to research the optimal timing in banks' write-off decisions of the commercial banks in general.

2. Basic Assumptions and Models

From the commercial banks' perspective, there are five main factors that affect the disposition of NPLs as following:

(1) The rate of return from non-performing loans.

(2) The value of the reinvestment return from the non-performing loans: we assume that the bank can invest the funds (collect from reinvestment) to prospective projects, and we consider the average return of the financial market as the reinvestment return.

(3) The value loss caused by carrying out write-offs: Namely, the loss because of non-performings being unable to be reclaimed completely.

(4) The loss of reputational repercussion from not writing off non-performing loans immediately: such as the loss caused by an upward jump in fund-raising costs when the high rate of non-performings leads to the credit rank's change.

(5) The implementation of a government subsidy policy: The government subsidies the banks' write-off through the government's rescue scheme.

The government subsidy police are temporarily not considered in this article. Only the optimal timing in banks' write-off decisions under the first four factors is considered in this article.

2.1 Basic Assumptions

Suppose that at the time of t = 0, banks' asset &debt situation is as follows:

Assets	Liabilities
loans（$L_G^0 + L_B^0$）	Funds（F_U）
Good Loans（L_G^0）	Net worth（N）
Bad Loans（L_B^0）	

Figure 1. A Simplified Balance Sheet

Assumption 1: The book value of banks' non-performing assets is L_B^0 , after becoming non-performing assets, the interest rate is as the same as before, For the convenience, we employ the continuous compound interest. At the time t, $L_B^0 e^{r_0 t}$ denotes the book value of non-performing assets, α_t denotes the ratio of loss from writing-off non-performing assets, $L_B^0 e^{r_0 t}(1-\alpha_t)$ denotes the writing-off income, and $L_B^0 e^{r_0 t}\alpha_t$ denotes the write-off loss.

Assumption 2: The rate of the reinvestment return of write-off NPLs is $\bar{R}_t$, which is defined as the proportion of closing value to opening value. For the convenience to set up the analyzing model and to solve the model at the last, we set R_t denoting the reinvestment return rate of book-value NPLs, namely, the yield coming from writing-off one book- value' NPLs. The relation between $\bar{R}_t$ and R_t is:

$$L_B^0 e^{r_0 t} R = L_B^0 e^{r_0 t}(1-\alpha)\bar{R}$$

Assumption 3: α_t , the ratio of loss from writing off NPLs, and R_t , the return coming from writing-off one book-value' NPLs, follows the geometry Brownian motion:

$$d\alpha = \alpha\mu_\alpha dt + \alpha\sigma_\alpha dw_1 \quad (1)$$

$$dR = R(\mu_R dt + \sigma_R dw_2 - dq) \quad (2)$$

Where w_1, w_2 denotes the standard Brownian motion. $\mu_\alpha\left(\mu_R\right)$ denotes the expected growth rate of $\alpha(R)$,$\sigma_\alpha\left(\sigma_R\right)$ the standard deviation parameter of $\alpha(R)$, $dw_1(dw_2)$ the increment of a wiener process of $\alpha(R)$, dq the increment of a Poisson (jump) process with the probability λdt, the paper assumes $E[dRdq] = E[d\alpha dq] = 0$, that is, dw_2 and dq are assumed to be independent of each other, so as to dw_1 and dq. Also, it is assumed $E\left[(dw_1)^2\right] = E\left[(dw_2)^2\right] = dt$ and $E[d\alpha dR] = \rho dt$, implying the correlation ρ between R and L can be considered in the following analysis. Lastly, equation (2) states that when a jump occurs, R falls by sum fixed ratio ϕ ($0 < \phi < 1$). For computational facility, this paper assumes $\phi = 1$ throughout the paper.

2.2 The Basic Model

Then at the time t, the value of write-offs is given by considering only the part of the standard geometric Brownian motion of equation (1) such that

$$V_t(R_t) = E_t\left[\int_t^\infty L_B^0 e^{r_0 t} R_s e^{-\mu(s-t)} ds\right] = \int_t^\infty L_B^0 e^{r_0 t} R_t e^{-(\mu-\mu_R)(s-t)} ds = \frac{L_B^0 e^{r_0 t} R_t}{\mu - \mu_R} = \frac{L_B^0 e^{r_0 t} R_t}{\delta_R} \tag{3}$$

Here $\mu = \mu_R + \delta_R = r_f + v\rho(R, M)\sigma_R$ is assumed to be hold as in Dixit and Pindyck(1994), which is derived from the CAPM(Capital Asset Pricing Model). Here, μ denotes the risk-adjusted discount rate, δ_R the rate of return shortfall in R, r_f the risk-free interest rate, v the market price of risk, $\rho(R, M)$ the coefficient of correlation between R and the market return M. For the value of write-off $V(R)$ to be bounded, the condition $\delta_R > 0$ must hold. Otherwise, the bank would never carry out write-off irrespective of uncertainty and sunk cost.

If the banks can write their own ticket to the write-off time when the NPLs come into being, the banks will dispose of the NPLs immediately or wait behind in order to choose optimal timing under the uncertainy of rate of callback and return on assets in the future write-off. The decision-making of commercial banks aims to maximize the value of the disposal:

$$\max\left\{E\left[V_T(\alpha_T, R) - L_B^0 e^{r_0 T}\alpha_T\right] \cdot e^{-\rho T}, 0\right\}$$

Here ρ is discounted rate, T the time to make decision. The condition $r_0 < \rho$ must hold, because r_0 is the lending rate which the borrowing firm operates in gear, and ρ is the rate of discount on cashflow coming from the reinventment of NPLs. The former is risker than latter under the uncertainty of rate of callback and return on assets.

Let $F(\alpha_t, R_t)$ denote the value of keeping the option to write-off alive in the future. The Bellman equation can be written as

$$\rho F(\alpha_t, R_t)dt = E_t[dF] \tag{4}$$

$$\frac{1}{2}\left(\sigma_\alpha^2\alpha^2 F_{\alpha\alpha} + \sigma_R^2 R^2 F_{RR} + 2\rho\sigma_\alpha\sigma_R\alpha R F_{\alpha R}\right) + \mu_\alpha\alpha F_\alpha + \mu_R R F_R + F_t - \mu F + \lambda\left\{F(t+, 0, \alpha) - F(t, R, \alpha)\right\} = 0 \tag{5}$$

$$F\left(t,\alpha^{*},R^{*}\right)=V_{t}-L_{B}^{0}e^{r_{0}t}\alpha^{*} \tag{6}$$

$$F_{R}\left(t,\alpha^{*},R^{*}\right)=V_{R} \tag{7}$$

and
$$F_{\alpha}\left(t,\alpha^{*},R*\right)=-L_{B}^{0}e^{r_{0}t} \tag{8}$$

Where condition (6) is the value-matching condition and condition (7) and (8) are both the smooth-pasting conditions. At the value R^{*} and α^{*}, the value of $F(R,\alpha)$ would equate to the value of $V(R_{t})-L_{B}^{0}e^{r_{0}t}\alpha_{t}$, which is the income from disposing of the NPLs.

In fact the value of R^{*} and α^{*} are unknown, hence, the condition (6), (7) and (8) are unknown too. The problem is called a "free boundary" problem (Dixit and pindyck, 1994). In such a case, it is very difficult to obtain clear-cut analytical solutions even numerical solutions. Nevertheless, fortunately, the property of homogeneity of the net value function $V_{t}-L_{B}^{0}e^{r_{0}t}\alpha_{t}$ allows one to reduce the problem to one dimension. Thus the optimal decision only depends on the ratio $r_{R}=R/\alpha$, rather than the value of R and α itself. The waiting area and writing-off area can be parted by the radial $r_{R}=R/\alpha$. This implies that the value of $F\left(t,\alpha,R\right)$ should also be homogeneous of degree one with respect to R and α. That is, the following set of relationships holds:

$$F\left(t,\alpha,R\right)=\alpha\cdot f\left(t,\tfrac{R}{\alpha}\right)=\alpha\cdot f\left(t,r_{R}\right) \tag{9}$$

$$F_{\alpha}=F_{\alpha}\left(t,\alpha,R\right)=f\left(t,r_{R}\right)-r_{R}f_{r_{R}}\left(t,r_{R}\right) \tag{10}$$

$$F_{\alpha\alpha}=F_{\alpha\alpha}\left(t,\alpha,R\right)=\frac{\left(r_{R}\right)^{2}}{\alpha}f_{r_{R}r_{R}}\left(t,r_{R}\right) \tag{11}$$

$$F_{R}=F_{R}\left(t,\alpha,R\right)=f_{r_{R}}\left(t,r_{R}\right) \tag{12}$$

$$F_{RR}=F_{RR}\left(t,\alpha,R\right)=\frac{f_{r_{R}r_{R}}\left(t,r_{R}\right)}{\alpha} \tag{13}$$

$$F_{\alpha R}=F_{\alpha R}\left(t,\alpha,R\right)=\frac{-\left(r_{R}\right)}{\alpha}f_{r_{R}r_{R}}\left(t,r_{R}\right) \tag{14}$$

Using equations (9)-(14), equation (5) can be rewritten as

$$\frac{1}{2}\left(\sigma_{\alpha}^{2}+\sigma_{R}^{2}-2\rho\sigma_{\alpha}\sigma_{R}\right)\left(r_{R}\right)^{2}f_{r_{R}r_{R}}+(\mu_{R}-\mu_{\alpha})r_{R}f_{r_{R}}+f_{t}+\left(\mu_{\alpha}-\mu\right)f=0 \tag{15}$$

Let $G=\sigma_{\alpha}^{2}+\sigma_{R}^{2}-2\rho\sigma_{\alpha}\sigma_{R}$; $H=\mu_{R}-\mu_{\alpha}$

Equation (15) is an ordinary second-order differential equation. The solution to the equation takes the form:

$$f\left(t,r_{R}\right)=Ae^{r_{0}t}\left(r_{R}\right)^{\beta} \tag{16}$$

Where A and β are coefficients to be determined. Direct substitution of solution (16) into equation (15) yields

$$\frac{1}{2}G\beta\left(\beta-1\right)+H\beta+\left(\mu_{\alpha}+r_{0}-\mu\right)=0 \tag{17}$$

Thus β can be solved analytically as

$$\beta=\frac{\frac{1}{2}G-H+\sqrt{\left(\frac{1}{2}G-H\right)^{2}+2G\left(\mu+\lambda-\mu_{\alpha}-r_{0}\right)}}{G} \tag{18}$$

$\beta > 1$ must hold. Now boundary conditions (6)-(8)can be rewritten as

$$Ae^{r_0 t}(r_R^*)^{\beta} = \frac{L_B^0 e^{r_0 t} r_R^{\ *}}{\delta_R} - L_B^0 e^{r_0 t} \tag{19}$$

$$\beta A e^{r_0 t}\left(r_R^*\right)^{\beta-1} = \frac{L_B^0 e^{r_0 t}}{\delta_R} \tag{20}$$

and

$$f\left(t, r_R^*\right) - r_R f_{r_R}\left(t, r_R^*\right) = -L_B^0 e^{r_0 t} \tag{21}$$

Where r_R^* denotes the threshold ratio. Note that of these three boundary conditions, no single one is independent of the other two.

Equation (19) and (20) jointly imply

$$r_R^{\ *} = \frac{\beta}{\beta - 1}\delta_R \ ; \ A = \frac{(\beta-1)^{\beta-1}}{\delta^{\beta}\beta^{\beta}} L_B^0 \tag{22}$$

The solution of Equation (15) is

$$f(t, r_R) = Ae^{r_0 t}\left(\frac{\beta}{\beta-1}\delta_R\right)^{\beta} = L_B^0 e^{r_0 t} / (\beta - 1)$$

And the optimal timing in banks' write-off decision can be obtained as follow:

$$T = \inf\{t \geq 0 : r_R = \frac{R_t}{\alpha_t} = \frac{\beta}{\beta-1}\delta_R\}$$

Figure 2 illustrates a free boundary r_R^* of the ratio of reinvestment return to write-off losses. In regime ¢ñ, the current value of r_R is below the threshold value r_R^*, so that the bank prefer waiting to writing off now. Also, Figure 2 depicts boundary conditions for $f(t, r_R)$ and the determination of r_R^*. At the threshold ratio r_R^*, the value from write-offs meets the value of waiting tangentially.

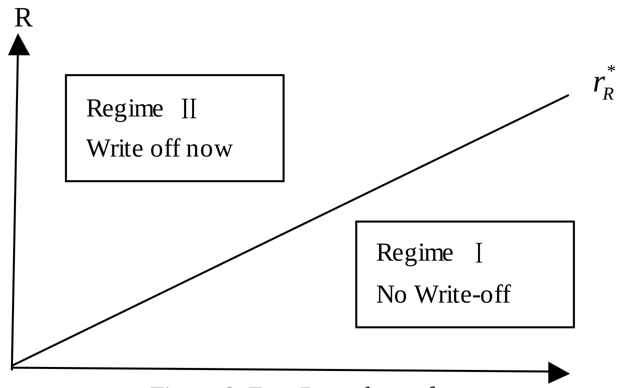

Figure 2. Free Boundary of r_R

Now we consider the influence coming from the change of the parameter λ. How does an increase in downward jump risk of the reinvestment return influence the optimal decision-making of rational banks? Generally, the effects of a positive value of the probability of the downward jump risk λ can be states as follows. First, it reduces the expected reinvestment return of capital gain on R, which decreases the value of waiting. On the other hand, it increases the variance of changes in R and thus raises the value of waiting. It turns out by numerical analysis that under normal circumstances, the former effect is more dominant. Figure 4 lay out the result that the former effect is much larger than the latter, thereby reducing the threshold ratio r_R^*. Further, notice that a small increase in λ lead to a substantial decline in r_R^*, prompting the bank to immediately write off. Based on the fact above, the government can takes some action to urge the banks to write off their NPLs by the believable way of threaten, for example, to reduce or cancel the assistance to the loss of disposal of the NPLs.

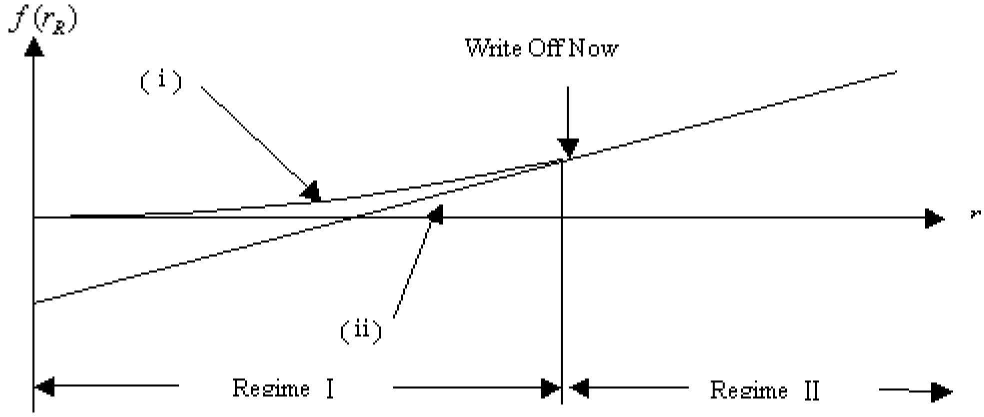

Figure 3. Boundary Conditions for $f(r_R)$ and the determination of r_R

(i) $f(r_R) = Ae^{\gamma t}(r_R)^{\beta}$, (ii) $f(r_R) = \dfrac{L_B^0 e^{\gamma t} r_R}{\delta_R} - L_B^0 e^{\gamma t}$

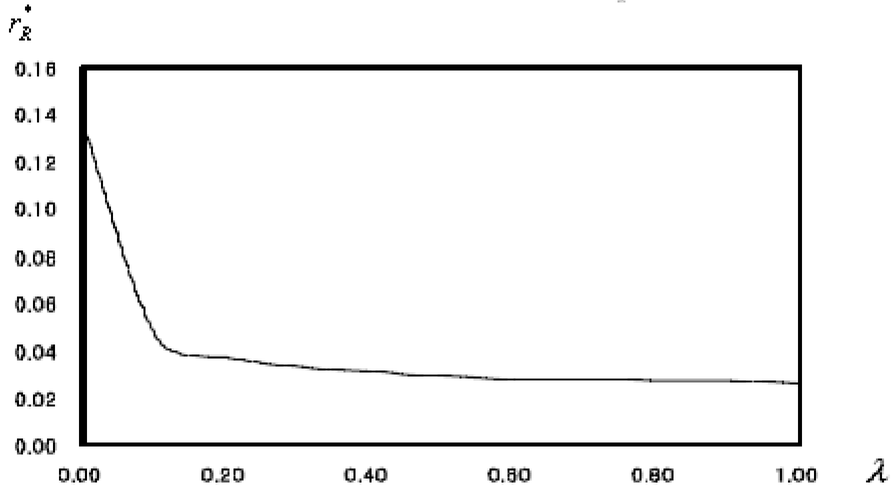

Figure 4. Dependence of r_R^* on λ

3. Conclusion

This paper has investigated how rational banks' optimal timing of write-offs is influenced by uncertainty stemming from the ratio of loss and return from writing-off NPLs. A real option approach was employed to evaluate the value of option to delay write-offs. Under normal circumstances, only when the rate of reinvestment return is very large, it is rational for the banks immediately to write off their NPLs. The uncertainty gives banks incentive to wait until the circumstance is favorableness to them to write off their NPLs.

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