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# The Difficulties Experienced in Teaching Proof to Prospective Mathematics Teachers: Academician Views

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**Abstract**

The aim of this study is to examine the difficulties prospective mathematics teachers experience in mathematical proving, the courses in which they have difficulties in proving, the importance of proof in mathematics education and its functions in their professional lives. The data of the study was collected via semi-structured interviews with fifteen academicians who volunteered to take part in the study. Content analysis method was used to analyze the data obtained. As a result of the study, based the views of the academicians, it was seen that prospective mathematics teachers experience four different difficulties in proving. Besides, in line with the views of the academicians the following categories were formed: the courses that prospective teachers experience difficulty, the importance of proof in mathematics education and its functions in prospective teachers' professional lives and these categories were presented with their subcategories.

**Keywords:** academician views, difficulties, mathematical proof, prospective mathematics teachers

## 1. Introduction

Proof and proving are regarded as the most important mathematical activities by mathematicians and mathematics educators (Baştürk, 2010) because in every case proof provides certainty for the results together with the solution of the problem and it provides opportunity to justify not only the correctness of the case but also to explain why it is correct (Hanna, 1995). The most important aspect in proof is that it states whether an explanation is correct or incorrect together its reasons (Ko, 2010; Yandell, 2002). Understanding of mathematical knowledge by individuals in the society enables social interaction between individuals and the group of people who conduct mathematical studies (Alibert & Thomas, 1991). In line with this view, it is seen that proof is a social process (Hanna, 1991). Harel and Sowder (2007) classified this process into two stages: the research process for individuals to persuade themselves and then the process of persuading others. Besides, De Villiers (1999) defines proof as exploration, analysis and inventing new results. In that vein, it is pointed out that proof plays an important role in exploring new mathematical knowledge through deduction (De Villiers, 1999).

Bell (1976) defines proof as the schematization of axioms, basic concepts and theorems via the deductive method. It is emphasized that in proving deduction is made via reasoning chains and ideas are revealed using logical structure and concepts, axioms, theorems and expressions are organized using the results reached (Davis, Hersh, & Marchisotto, 1995). Besides, proof is a means of mental challenge to help an individual achieve his/her self-realization (Tsamir et al., 2009). Mathematical proof enables the creation of acceptable argument and the presentation of these arguments in class (Stylianides, 2007) and provides individuals with logical explanations of why and how the justification underlying the expression is correct (Tall, 1989). Therefore, in advanced mathematics courses in universities all over the world, students' construction and understanding mathematical proof is emphasized. Especially, Prospective mathematics teachers, who are expected to teach mathematical proof to next generations, frequently come across with mathematical proof in their university education (Baştürk, 2010).

Although mathematicians regard proving as the most important ability in advanced mathematics courses, studies (Almeida, 2000, 2003; Dreyfus, 1999; Güler & Dikici, 2012; Güler, Özdemir, & Dikici, 2012; Güler, 2013; Güler & Dikici, 2014; Harel & Sowder, 1998; Moore, 1994) indicate that students from all levels have great difficulties in proving processes (Weber, 2001). In fact, the difficulties universities students have in proving are similar (Stylianou, Blanton, & Rotou, 2015). Most of the students think that mathematical realities were proven years ago and proving is an unnecessary activity (Coe & Ruthven, 1994). When it is considered that there is a significant correlation between students' success in mathematics and their beliefs about the importance of the topic to be learnt (Schoenfeld, 1989; Stylianou, Blanton, & Rotou, 2015), it can be argued that the basis of the problem students experience in proving is that they regard proof unnecessary (Furinghetti, Olivero, & Paola, 2001). In addition to this, another difficulty experienced especially by university students with regard to mathematical proof is that they suddenly encounter the concept of proof during their university education (Baştürk, 2010). Although the report issued by NCTM (2000) underscore that proof improves mathematical thinking and reasoning skills at all age levels, it is seen that mathematical proving is ignored before university education.

When the literature related to the difficulties in mathematical proving is examined, it is seen that several researchers tried to specify the difficulties students' experience. Moore (1990) determined the difficulties students experience in proving as follows: understanding the concept of proof, mathematical language and notations and difficulties when starting proof. As a result of a similar study, Weber (2001) found that most of the students did not know how to construct proof and how to start proving, the concepts, their definitions and how to use them. Besides, it is seen that researchers categorize the difficulties in mathematical proving as difficulties in deciding how to start proving (Moore, 1990, 1994; Selden, A., & Selden, 2003, 2007), difficulties in expressing the definitions used in proving (Azrou, 2013; Bayazıt, 2009; Knapp, 2005), difficulties in expressing proof in their own words (Dubinsky, 2000), difficulties in using logical and proof methods (Harel & Sowder, 2007; Selden, A., & Selden, 2007; Knapp, 2005; Andreas, Gabriel, & Philippou, 2004; Gabriel, Andreas, & Philippou, 2007) and difficulties in using mathematical language and notation (Moore, 1994; Selden, A., & Selden, 2007; Biehler & Kempen, 2013). Moreover, in his study Güler (2013) determined that prospective mathematics teachers had difficulties in mathematical proof processes when learning algebra, creating concept image, problem solving, proving and assessing the accuracy of proof. Weber (2006) determined difficulties in mathematical proving as the inadequacy of information students have with regard to mathematical proof, misunderstanding and thus misapplication of a concept or a theorem and inadequacy in developing strategy for proving.

Another difficulty related to mathematical proof stems from the fact that academicians teach advanced mathematics courses as definition, theorem and proof (Weber, 2004) or as theorem and examples (Almeida, 2003). Similarly, Weber (2012) examined why and how mathematicians present proof in advanced mathematics courses. As a result of the study four important results were reached: (1) when proof presentations are made directly for correct theorems, they do not persuade students but they persuade when illustrated with drawings, (2) students cannot interpret proof, which includes a complex process and these processes often are not taught, (3) superficial methods are used to assess students' understanding of proof, (4) only some mathematicians question whether information is transferred or not. However, it was seen that there is not any study examining the views of academicians with regard to the importance of mathematical proving in advanced mathematics courses, possible difficulties prospective teachers experience and the reason why proof is taught. Therefore, this study aims to examine the views of academicians with regard to the difficulties prospective mathematics teachers experience in mathematical proving, the courses in which they have difficulties in proving, the importance of proof in mathematics education and its functions in prospective teachers' professional lives in the future. It is envisaged that this study will contribute to literature with regards to the following issues: the difficulties prospective mathematics teachers experience in proving in advanced mathematics courses and the reason why proof is taught.

## 2. Method

### *2.1 Participants*

The study was carried out in the spring semester of 2014-2015 academic year with 15 academicians (5 professor doctor, 5 associate professor, 5 assistant professor) of 7 different universities, who volunteered to take part in the study. Each academician who took part in this study has been teaching prospective mathematics teachers advanced mathematics courses at least for five years. To choose participants, maximum variation sampling method, which is one of the purposeful sampling method, has been used because the aim of maximum variation sampling is to find out what types of commonalities or similarities there is among diverse cases rather than making generalization by achieving diversification (Yıldırım & Şimşek, 2008). When choosing academicians, initially they were informed about the study via e-mail and then appointments with the academicians who volunteered to take part in the study were made. In doing so, the aim was to reach to all academicians working at the mathematics education departments in all universities all over the Turkey but only academicians working at seven universities responded. After the participants were informed about the study, semi-structured interviews with each one of the 15 academicians were made. Each academician taking part in this study was given a code name from A1, A2, … , A12 to keep their names confidential. Demographic information with regard to participants is presented in Table 1.

Table 1. Demographic information with regard to the academicians

| Academician (Gender) | Title | Length of Service (Year) | Courses Given |
|---|---|---|---|
| A1 (M) | Professor Doctor | 17 | Abstract Algebra, Analysis, Geometry |
| A2 (M) | Professor Doctor | 20 | Analysis, Linear Algebra, Abstract Mathematics |
| A3 (M) | Professor Doctor | 18 | General Mathematics, Analysis, Complex Analysis |
| A4 (M) | Professor Doctor | 15 | Differential Equations, Topology, Algebra |
| A5 (M) | Professor Doctor | 20 | Analytic Geometry, Linear Algebra |
| A6 (M) | Associate Professor | 14 | Geometry, Statistics, Analysis, Introduction to Algebra |
| A7 (F) | Associate Professor | 10 | Analysis, Number Theory, Introduction to Algebra |
| A8 (M) | Associate Professor | 15 | Geometry, Theory of Complex Numbers |
| A9 (M) | Associate Professor | 22 | Linear Algebra, Differential Geometry |
| A10 (F) | Associate Professor | 9 | General Mathematics, Abstract Mathematics |
| A11 (F) | Assistant Professor Doctor | 9 | Analysis, Complex Analysis, Topology |
| A12 (F) | Assistant Professor Doctor | 11 | Differential Equations, Analysis |
| A13 (F) | Assistant Professor Doctor | 13 | Number Theory, Logic, Analysis |
| A14 (M) | Assistant Professor Doctor | 22 | General Mathematics, Probability and Statistics |
| A15 (E) | Assistant Professor Doctor | 25 | Linear Algebra, Abstract Algebra, Number Theory |

*2.2 Data Collection Tool and Analysis*

In the study, semi-structured interview form developed by the researcher was used as a data collection tool. The researcher initially referred to the studies in the relevant literature to develop the interview form. Later on, the researcher arranged the questions he prepared and those in the literature as an 8-item draft interview form in line with the aims of the study. The draft form was examined by 3 mathematics education experts, who had carried out studies on proving processes of Prospective Mathematics Teachers. Finally, the draft form, which was revised in line with expert views, was used to make semi-structured interviews with 2 academicians. Considering the responds provided by the academicians, it was decided that some questions in the draft form were to be removed as they were similar. The questions in the semi-structured interview form used in the study are given below:

- What are the difficulties in teaching and learning proof? Please explain.
- In what courses and in relation to what concepts do you think difficulties in teaching and learning proof are experienced? Please explain.
- What is the importance of teaching proof in mathematics education? Please explain.
- What kind of functions do you think teaching proof will have in your students' professional lives? Why?

Semi-structured interviews with academicians took 30 minutes on average and each interview was audio recorded. Later on, the voice recordings were transcribed and data were analyzed. In data analysis, content analysis method was used because the basic aim of content analysis is to reach concepts and relations that can explain data collected (Yıldırım & Şimşek, 2008). To this end, responds provided by the academician were coded according to their common characteristics and common codes were listed. Finally, the codes created were categorized according to their conceptual characteristics and presented as tables. Categories created by the researcher were controlled by an expert who had made studies in proving processes of Prospective Mathematics Teachers. As a result, it was seen that there is a congruence ranging between 88% and 100% between the codes and categories created by the two researchers. Goodness of fit index formula developed by Miles and Huberman (1994) (Agreement/(Agreement + Disagreement) x 100) was used to calculate agreement between coders. Besides, the direct quotations from academicians responds were used to support the codes and categories descriptively.

## 3. Results

In this section, the views of academicians with regard to the difficulties prospective mathematics teachers experience in mathematical proving, the courses in which difficulties in proving are experienced, the importance of proof in mathematics education and its functions in prospective teachers' future professional lives are presented in tables under different categories. According to the views of academicians, the difficulties prospective mathematics teachers experience with regard to proof stem from their lack of prior knowledge, rote learning of proof methods and biases against proof.

Table 2. Difficulties stemming from the lack of prior knowledge

| Difficulties stemming from the lack of prior knowledge |
|---|
| ✓ Mathematics education at high schools does not discourse on proving |
| ✓ Lack of questions about proof in university entrance exams |
| ✓ Focus on operational processes |
| ✓ Students getting used to short answers and to techniques for solving multiple choice questions |

When Table 2 is examined, according to the views of academicians the difficulties prospective mathematics teachers have because of the lack of prior knowledge and the subcategories related to these difficulties can be seen. According to the academicians, proof is not emphasized in mathematics education in high schools, lack of questions about proof in university entrance exams, overemphasis on operational processes, students getting used to short answers and to techniques for solving multiple choice questions are seen as difficulties stemming from the lack of prior knowledge. Academicians emphasize that this case poses a very big obstacle both for themselves and prospective mathematics teachers in advanced mathematics courses. In line with this, the views of academicians coded as A1, A6 and A11 are given below.

**A1:** … *The primary difficulty encountered in proving in mathematics courses is that students had not received education in this issue. Generally, in high schools when mathematics is mentioned, the habit of solving questions in test books with some knowledge is understood because when teaching fundamental mathematical information at university level, especially recently we encounter students' asking "Sir, Will you always be proving like this?" Therefore, the most important difficulty experienced at university level stems from the multiple-choice solving technique in high schools.*

**A6:** … *They are of the opinion that it has already been proven, why we should try more? Neither in KPSS (Public Personnel Selection Exam) nor in YGS (The Transition to Higher Education Examination) or in LYS (Undergraduate Placement Examination), there is no question about proof. Therefore, although students can construct proof and understand proof, they do not want to construct proof.*

**A11:** … *Among the difficulties that can be encountered can be that operation is perceived as the only procedure and the idea that only the relevant procedure should be performed. The student remains intellectually passive and performs what the operation necessitates but the procedure is inefficient as there is no conceptualization, making sense or necessary relations are not established.*

According to the views of academicians, another difficulty prospective mathematics teachers experience in proving is categorized as difficulties stemming from proof methods. In Table 3, this category is presented together with its subcategories.

Table 3. Difficulties stemming from proof methods

| Difficulties stemming from proof methods |
|---|
| ✓ Not being able to decide how to start proving |
| ✓ Not being able to establish relation between hypothesis and judgment |
| ✓ Inexperience with regard to proof methodologies |
| ✓ Trying to make generalizations via trial and error method |

According to the views of the academicians, not being able to decide how to start proof, not being able to establish relation between hypothesis and judgment, inexperience with regard to proof methodologies, trying to make generalizations via trial and error method were determined as difficulties stemming from proof methods. According to academicians, the reason of this difficulty is that prospective mathematics teachers have difficulty to understand types of proof and the logic of proofs. With regard to this issue, the views of academicians coded as A2, A8 and A14 are as follows:

**A2:** … *Students generally have difficulties in distinguishing hypothesis and judgment, in understanding necessary and adequate conditioned theorems and in deciding which proof method to use and using notation.*

**A8:** … *It is almost impossible to teach students who have not completely learnt proof method and who do not keep their prior knowledge refresh and study only before exams. Therefore, I frequently see that especially on exam papers students try to reach results using trail error method.*

**A14:** … *The most important difficulty in teaching proof is that they do not know the method they will follow when starting proof. When students start proof, they cannot estimate how they can relate the steps they take with the result. I think besides the difficulties students experience with regard to mathematical concepts, this case results from students' being inexperienced with regard to types of proof and the methodology they (proof types) require because propositional logic is used to express types of proof. Students cannot completely grasp these cases. Besides, students do not regard proving as an enjoyable process like solving a problem but rather as case approached with more fear. This causes them to make errors.*

In Table 4, according to academicians' views, the difficulties categorized as the difficulties stemming from rote learning of proofs and its subcategories are given all together.

Table 4. The difficulties stemming from rote learning of proofs

| The difficulties stemming from rote learning of proofs |
|---|
| ✓ Memorization of definitions used in proving |
| ✓ Not questioning the logic of proof made |
| ✓ Not being able to develop abstract thinking |
| ✓ Not being able to understand the expression of theorem |

As it can be seen in Table 4, academicians express that memorization of definitions used in proof by Prospective Mathematics Teachers, not questioning the logic of proof made, not being able to develop abstract thinking and not being able to understand theorem expression as difficulties stemming from rote learning of proof. It is emphasized that prospective teachers choose memorization, which they regard as easier, rather than using mental processes. With regard to this issue, excerpts from interviews with academicians coded as A5, A7 and A12 are presented below.

**A5:** … *We should prevent proof from being a burden on memory, and tell them proof is different from memorizing a poem. I have difficulty to make them understand especially that logical ways are to be followed in the course of proof and it is all about this. A number of students try to memorize rather than probing the logic behind the methods used in proving.*

**A7:** … *When students are learning proof, rather than understanding the transitions between the steps, they tend to memorize them without questioning, which prevents development of their abstract thinking skills. Therefore, proof is regarded to be solely composed of memorization and copying what is written on the board.*

**A12:** *…Students mostly regard learning proof as memorization. Rather than following the steps taken logically, they tend to directly memorize them. Therefore, errors occur amply. They do not notice these errors.*

In line with the views of academicians, another type of difficulty prospective teachers experience in proving are difficulties that stem from their biases against proof. In Table 5, this category is presented together with its subcategories.

Table 5. Difficulties stemming from biases against proof

| Difficulties stemming from biases against proof |
|---|
| ✓ Proof is regarded as unnecessary by the students |
| ✓ Anxiety for proving |
| ✓ Perception of proving as a complex process |

According to Table 5, from the perspectives of academicians, students' regarding proof as unnecessary, their anxiety for proving and perceiving proving as a complex process are difficulties stemming from biases against proof. According to academicians, prospective mathematics teachers regard proofs made in classes as unnecessary. Therefore, they perceive proofs as difficult. This case increases Prospective Mathematics Teachers' level of anxiety; thus they develop biases against proving. With regard to this issue, excerpts from interviews with academicians coded as A3, A9 and A13 are presented below.

**A3:** … *Students cannot get motivated as they do not feel proof is necessary. Either in teaching proof or another topic, teaching gets challenging when students are not motivated. Another difficulty is that students do not think proof is necessary. As students do not believe the necessity of proving, students either memorize them before an exam then forget it in a short time or attempt to cheat.*

**A9:** … *Proving process might seem meaningless for students. That is, it can be seen only showing the accuracy of existing theoretical information. As a result, the solution produced or obtained can be regarded as analysis of an existing case again. Therefore, it can be considered that it is necessary to emphasize the importance of proving in mathematics teaching and the meaningfulness of what students do beyond proving. It must be shown that proving is not so complex process as thought by emphasizing that proving is not only reaching to a result by following a certain method but also explanations of cause effect relations by students in this process.*

**A13:** … *As students approach proof with bias, proving attempts remain limited. They either memorize the proof of a theorem or they completely give up proving. I observe that students who exhibit such behaviors feel more anxious about proving.*

According to the views of academicians, the findings with regard to the courses in which difficulties are experienced in teaching proof to prospective teachers are presented in Table 6.

Table 6. The courses in which difficulty in teaching proof is experienced

| The courses in which difficulty in teaching proof is experienced |
|---|
| ✓ The courses which have strong prerequisite relations |
| ✓ The courses in which symbolic and notational representations are predominant |
| ✓ The courses which require abstract thinking |

When Table 6 is examined, it can be seen that academicians express that prospective mathematics teachers have difficulty in learning proof in courses which have strong prerequisite relations, the courses in which symbolic and notational representations are predominant and in courses which require abstract thinking. It has been observed that academicians emphasize the courses which are perceived to be abstract and the courses in which symbolic and notational representations are predominant. It has been emphasized that prospective teachers have difficulties in these courses as conceptual knowledge and prerequisite relations rather than numerical operations are used in them. With regard to this issue, quotations from the interviews with the academicians coded as A4, A10 and A12 are presented below.

**A4:** … *Generally, in courses related with numbers, teaching how to construct proof is easier. But, proofs related to teaching theoretical concept are very difficult. Algebra, Analysis and Topology courses which require abstract thinking are the best examples of such cases. These courses are based on definition, when definitions are not understood completely, difficulty is experienced with proofs. In some proofs, more than one definition are to be used side by side in appropriate order. In this case, it can turn into a difficult and boring activity for students.*

**A10:** … *It is more difficult to teach proof in courses like Abstract Mathematics, Algebra, Topology, Differential Geometry, Functional Analysis, Real Analysis, which involve abstract concepts and require very much prior knowledge compared to other courses.*

**A12:** … *To me, in Algebra and Abstract Mathematics courses, such difficulties become a little more apparent. As for courses like Analysis, these difficulties are fewer as visual explanations can be used. The reason for this is that symbols and signs are used a lot in courses like algebra and abstract mathematics and thus students perceive them as abstract concepts. That is, students experience more difficulty in cases where abstract thinking predominates and where visual explanation is not possible, more difficulty is experienced.*

The views of academicians with regard to the importance of proof in mathematics education are presented in Table 7.

Table 7. The importance of proof in mathematics education

| The importance of proof in mathematics education |
|---|
| Proof… |
| ✓ improves problem solving skill |
| ✓ develops reasoning skill |
| ✓ improves mathematical thinking skill |
| ✓ is the basis of mathematics |
| ✓ enables mathematical communication |
| ✓ develops creativity skill |
| ✓ creates persuading arguments for the accuracy or inaccuracy of concepts |
| ✓ prevents rote learning of information |

With regard to importance of proof in mathematics education, academicians emphasized that proving improves problem solving skill, develop reasoning skill, improves mathematical thinking skill, forms the basis of mathematics, enables mathematical communication, develops creativity skill, creates persuading arguments for the accuracy or inaccuracy of concepts and prevent rote learning of information. It is seen that with regard to importance of proof in mathematics education, academicians stated that proof develops higher order thinking skills and creates persuading arguments with regard to concepts. With regard to this issue, excerpts from the interviews with academicians coded as A1, A8 and A15.

**A1:** … *We can define mathematics as follows: mathematics "is the art of expressing the existing relations between objects during the construction of any model in the wisest and clear cut way". Therefore, here the ability to show existing relations between objects is proof. Another definition of mathematics is that it is "the art of thinking properly". In this definition, showing the accuracy of an idea is the proof aspect of mathematics.*

**A8:** … *In a sense, proof is presenting logical explanations whose accuracy everyone would be sure about. In this case, you persuade everyone about the reality of the situation revealed. Besides, it provides students with opportunities to see the aesthetical aspect of mathematics and relations between concepts. At the same time, it helps developing problem solving skill, using different strategies and determining their accuracy because when students are asked to prove a theorem and discuss different results obtained, you create a kind of problem based learning environment. This milieu enables students to criticize each other's thoughts and improve their reasoning skills. That is, it contributes to reasoning and creativity skills. At the same time, such behaviors increase students' motivation as well.*

**A15:** … *I think proof is to be the basic principle of mathematics education. Just as laboratory and experiments are inevitable for physics, chemistry, biology etc., proof is inevitable for mathematics. A mathematician should* *not only be interested in the result of theorem as in other disciplines which use mathematics as a means. Where did this result come from? What did this theorem facilitate? What is the difference between the stage before theorem and the stage after theorem? For which theorems will this theorem lay foundation after it? etc.. This scheme is to be understood well. When teaching students, their attentions are to be drawn with questions and the place of theorem is to be consolidated. When we can do these, rote learning can be prevented using mathematical proofs.*

According to the views of academicians, the findings with regard to the functions of proof in professional lives of prospective mathematics teachers are presented in Table 8.

Table 8. The function of proof in professional lives of prospective mathematics teachers

| The function of proof in professional lives of prospective mathematics teachers |
|---|
| Proof |
| ✓ develops different points of views for problems |
| ✓ develops problem solving skill |
| ✓ develops analytical thinking skill |
| ✓ contribution to mathematical content knowledge |
| ✓ contribution to beliefs about the nature of mathematics |

When Table 8 is examined, it is seen academicians stated that in professional lives of prospective mathematics teachers proof has the function of bringing in different points of views to problems, developing problem solving skills, improving analytical thinking skill, contributing to mathematical content knowledge and to beliefs about the nature of mathematics. It is seen that with regard to the functions of proof in professional life, academicians are of the opinion that it brings in different points of views to problems, improve higher order thinking skills and contribute to content knowledge. With regard to this issue, excerpts from interviews with academicians coded as A5, A9 and A11 are presented below:

**A5:** … *Mathematical proof develops reasoning and questioning skills because when proving, students' founds each step on the previous one. Agreement between them and possible inconsistencies are checked. In this respect, it enables students to make practice with how to behave in their professional lives to produce appropriate results and solution methods according to data at hand because deduction has an important place in proof. The culture acquired for this case is also important for professional life because in real life we always make inferences based on existing data. We reason. Therefore, proof culture provides practice opportunities for the acquisition of these behaviors.*

**A9:** … *Proof education is an important means necessary for the acquisition of basic and vital processing skills which teachers and educators expect their students to acquire. Acquisition of these skills by individuals enables them to make intellectual differences. Individuals, who research, question and think about the accuracy or inaccuracy of information rather than directly accepting it and who can use mathematical language are trained. Analytical thinking skills of individuals who acquire these skills develop as well. In other words, they bring up as individuals who are aware of how they think, and propose solutions for appropriate cases and conditions.*

**A11:** … *I think everybody accepts that there is a very high relation between mathematics and proof. Thanks to proof, mathematical information is constructed and developed. It is highly probable that a teacher candidate whose mathematical knowledge is well-developed will be more beneficial for his students because a teacher candidate who wants to be beneficial for his/her students in the future should first realize his/her own conceptual and meaningful learning. Proofs are very useful for conceptual and meaningful learning.*

## 4. Discussion

The results of the study which aims to examine the difficulties prospective teachers experience with regard to mathematical proof, the courses in which they experience difficulties in proving and the importance of proof in mathematics education and its functions in their prospective professional lives based on the views of academicians are discussed in this section. As a result of the study, it has been determined that according to the views of academicians the difficulties prospective mathematics teachers experience with regard to proof stem from the lack of prior knowledge, proof methods, students' memorization of proofs and biases against proof.

Academicians point out difficulties stemming from lack of prior knowledge as the main reason why prospective teachers experience difficulty in advanced mathematics courses. It is emphasized that differences in mathematics education at high schools and lack of mathematical proof instruction at high schools constitute an impediment for advanced courses at universities. Academicians attribute this case to study habits students develop when preparing for university entrance exams. The fact that university entrance exams include questions which require short answers and operational knowledge rather than questions about mathematical proofs causes proof to be ignored at high school education. Therefore, prospective mathematics teachers suddenly encounter proofs during their university education and thus experience difficulties with regard to proof. Baştürk (2010), who reached similar results, points out that the main reason for difficulties prospective teachers have in mathematical proving is the difference between high school and university mathematics education. Besides, Baştürk (2010) in parallel with the results of this study, states that the courses in high schools and private courses are taught considering the university entrance exams and so prospective teachers do not encounter the concept of proof before university education. In addition, this result obtained from the study supports many studies (Weber, 2001; Selden, A., & Selden, 2003; Healy & Hoyles, 2000) in the literature which indicate that prospective teachers have difficulties as they suddenly encounter proof at university level. In the report issued by NCTM (2000), it is emphasized that proof should be at the center of mathematics starting from very early levels. Besides, the studies in the literature suggest that proof is to be integrated to mathematics education at high school and earlier to prevent the difficulties experienced in advanced mathematics courses (Baştürk, 2010; Hanna, 2000; Öçal & Güler, 2010; Schoenfeld, 1994). Asking questions about mathematical proof in university entrance exams can increase interest in proof at high school and at earlier levels of education. In this way, the difficulties prospective mathematics teachers experience in advanced mathematics courses can be prevented.

Mathematical proof involves following a logical way to explain a hypothesis and explanations of why and how the result have been reached (Tall, 1998). According to academicians, another type of difficulty prospective mathematics teachers experience in learning proof is difficulties stemming from proof methods. It is emphasized that the methodology academicians use in proving in advanced mathematics courses is not understood by prospective teachers. Prospective teachers determine the hypothesis and judgment made in a theorem, but experience difficulties in establishing relations among them; therefore, they cannot decide how to start proving. Thereby, they experience difficulty in deciding how to fictionalize proof. The results of the study support the results of the studies which indicate that students have difficulties in deciding how to start proving at university level (Güler, 2013; Moore, 1990, 1994; Selden, A., & Selden, 2003, 2007; Weber, 2001). However, academicians marked that not understanding the logic of proof methods might cause prospective mathematics teachers to mistakenly think that proofs can be solved via trial and error method and examples. Therefore, types of proofs are to be understood and especially logical representation of proof methods are to be internalized by the students. With regard to difficulties experienced in advanced mathematics courses, similar results were also found in studies emphasizing logic and proof methods (Harel & Sowder, 2007; Selden, A., & Selden, 2007; Knapp, 2005; Andreas, Gabriel, & Philippou, 2004; Gabriel, Andreas, & Philippou, 2007).

In the study, another difficulty academicians emphasize is that prospective mathematics teachers tend to directly memorize proofs made without questioning them. Academicians express this by stating that prospective teachers do not question the logic of a theorem and memorize the proof of an expression in the way it is made in class. According to academicians, prospective teachers memorize the definitions used in proof rather than understanding proof and thus they try to write the same answer exactly when asked. Therefore, when prospective teachers are asked the proof of exactly the same theorem, they can construct proofs but when an application related to the theorem and proof they have memorized is asked, they cannot transfer their information as expression of theorem and proof is not understood. This difficulty revealed according to academicians' views as a result of the study is also seen in studies in the literature which examine the meaning and aim of proofs according to prospective mathematics teachers (Güler, 2013; Güler & Dikici, 2012; İmamoğlu, 2010; Öçal & Güler, 2010). When the results of these studies are examined, it is seen that prospective mathematics teachers memorize mathematical proofs. Therefore, it can be argued that the beliefs of prospective teachers with regard to proof cause this difficulty revealed as a result of the study in line with academicians' views. Thus, prospective teachers' awareness about the aim and meaning of proving should be raised especially in advanced mathematics courses.

According to the views of academicians, another type of difficulty experienced by prospective mathematics teachers with regard to proof is the difficulties that stem from bias against proof. Studies indicate that teachers' perceptions and experiences with regard to proof are influential when they are teaching proof skills (Almeida, 2000; Furinghetti & Morselli, 2009). Besides, students regard proving as an activity which is difficult to understand, meaningless, unnecessary and performed involuntarily (Almeida, 2003; De Villiers, 1990; Knuth, 2002). Therefore, this difficulty revealed as a result of this study supports the findings in the literature that prospective mathematics teachers regard proof as an unnecessary activity. Besides, especially when it is considered that prospective mathematics teachers will be teaching proof to students at different age groups, their bias against proof will affect their use of proof in their courses. Therefore, it is important to inform prospective teachers about proving and reasoning skills, which are given place in the revised high school mathematics education program, because proving is considered by MNE (Ministry of National Education) (2013) as a skill to be emphasized in mathematics courses. In this context, academicians, who are aware of this difficulty, can inform prospective mathematics teachers about education programs and prevent proving from being regarded as an unnecessary activity.

It has been found that with regard to courses in which academicians have difficulties teaching proof, their ideas mostly focus on the view that mathematics is incremental, has an abstract structure and heavily involves symbolic representations. Academicians stated that prospective mathematics teachers have difficulty understanding and proving especially in abstract courses in which notations are used frequently. This case is supported by the results of studies (Güler, 2013; Güler & Dikici, 2014) which examined Prospective Mathematics Teachers' proving processes in learning algebra. In both studies, it was seen that prospective teachers have difficulties using mathematical language and notations in proofs in abstract algebra courses. Similarly, studies by Moore (1990, 1994) on introduction to proof courses, it has been determined that students have difficulties in understanding mathematical language and notations. Besides, in a study by Weber (2012) on the effectiveness of the proofs academicians made in their courses, it was found that the proofs academicians made using symbolic and notational proofs were understood less compared to proofs supported by visual drawings. In this study, academicians emphasized that proofs made especially in abstract courses like abstract algebra, topology etc. could not be supported by visual representations but proofs made in analysis, analytic geometry courses etc. could be supported by different representations. The results of both studies are consistent in this respect. Therefore, such difficulties can be minimized if academicians support the proofs they construct in advanced mathematics courses, which are considered to be impossible to be supported by visual representations, with drawings by preserving the meanings of concepts.

Mathematical proof can be used to relate mathematical information, develop strategies and as a means which is necessary to solve the problems students encounter in mathematics (Hanna & Barbeau, 2008; Mariotti & Balacheff, 2008). It is seen that with regard to importance of mathematical proof, academicians emphasize its contribution to the development of higher order thinking skills, its being the basis of mathematics, its verification of results, and its contribution to communication and meaningful learning functions. Studies on the importance of proof (Güler, 2013; Shipley, 1999; Weber, 2001, 2012) frequently emphasize its contributions to mathematical thinking and problem solving skills. Besides, Harel and Sowder (1998, 2007) underscore that proof is a process of communication and persuasion aiming to persuade others in the accuracy or inaccuracy of mathematical concepts. It has been observed that with regard to the importance of proof, academicians often emphasize mathematical communication and persuasion processes. Similarly, Hanna (1991) states that mathematical proof is made to show the accuracy of a result with different justifications, to inform and persuade others of this issue and to incorporate a result reached into a system. It is considered that academicians' frequent emphases on the importance of proof in advanced mathematics courses can motivate prospective mathematics teachers to construct proof because it is seen that academicians views with regard to proving involves almost every meaning of mathematics.

With regard to the functions of proof in prospective professional lives of prospective mathematics teachers, it has been ascertained that academicians think that proof will improve their mathematical thinking skills. Researchers who examined the meaning and functions of proof argue that proof can improve students' mathematical thinking skills if integrated with in-class mathematics education activities (Hanna & Barbeau, 2002; Rav, 1999). Similarly, in a study by Ko (2010) it was stated that functions of proof instruction are verification, explanation, communication, exploration, systematization and mental change. Therefore, it can be argued that with regard to the functions of proof, academicians think that proving contributes to higher order thinking skills and problem solving skills of prospective mathematics teachers, which is also well-established finding in the literature. Besides, it was seen that with regards to the functions of proof in professional lives of mathematics teachers, academicians emphasize the contributions of proof to content knowledge of prospective teachers. Especially, when literature with regard to teacher education is considered, it is seen that one type of knowledge a teacher should have is content knowledge (Ball, Thames, & Phelps, 2008). In the same vein, when it is considered that teachers are to have perfect knowledge about basic mathematical concepts so that they can teach mathematical concepts (Ma, 2010), it can be argued that academicians think that prospective mathematics teachers can acquire this skill by proving.

The results of this study revealed the views of academicians with regard to difficulties prospective teachers experience when mathematical proving, the courses in which they have difficulties, the importance of proof in mathematics education and the functions of proof in prospective teachers' professional lives in the future. When it is considered that teachers who do not possess knowledge and skills with regard to the concepts they will teach can face challenges when teaching these concepts, it becomes more important that academicians be aware of possible difficulties prospective mathematics teachers can face with regard to proof and about the courses in which they have difficulties, emphasize the importance of proof and inform prospective teachers about the functions of proof in their professional lives. In this context, it is considered that the results of this study will contribute to the literature about teaching proof in advanced mathematics courses as it reflects academicians' views. The results of this study cannot clearly reveal whether views of academicians with regard to difficulties prospective mathematics teachers experience in proving and course they have difficulties are based on their personal experiences or on the results of the studies they have carried out. Therefore, further studies supported by in-class observations of the difficulties and the courses in which difficulties are experienced can make important contributions for the explanations and removal of these difficulties.

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