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Mathematical Aspects for RNSS Constellation with IGSO Satellites

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Abstract

Besides from existing global navigation satellite system (GNSS), several countries such as France, Indian, Japan and China have being developing their regional navigation satellite system (RNSS) in recent years. The Inclined Geosynchronous Orbit (IGSO) satellites with a big inclination angle play an important role in these RNSS constellations. The mathematical aspects of RNSS constellations are studied in this work. Analysis shows obvious anomaly solutions happen several times in everyday positioning. Therefore redundant satellites are needed in constellation optimization design for RNSS.

Keywords: RNSS, IGSO satellite, GDOP, positioning anomaly

1. Introduction

Global Navigation Satellite System (GNSS) provides autonomous geo-spatial positioning with global coverage, which allows the user to determine their location within a few meters using signals transmitted from satellites. In recent years, the applications of GNSS become more and more widely (Ashby, 1996, 2002a, 2002b, 2003; Iorio, 2001, 2009, 2014; Iorio et al., 2004; Renzetti, 2012, 2013a, 2013b, 2014). At present, only the NAVSTAR Global Positioning System (GPS) of the United States and the GLONASS of Russian are fully operational GNSSs. China is in the process of expanding its BeiDou navigation system from regional to global by 2020. The European Union's Galileo positioning system is a GNSS in its initial deployment stage, and is scheduled to be fully operational by 2020. Several other countries are in the process of developing their own regional navigation satellite systems (RNSS). The Doppler Orbitography and Radio-positioning Integrated by Satellite (DORIS) is a French satellite system used for the determination of satellite orbits and for positioning purpose. A so-called beacon is installed on the ground and emits a radio signal which is received by the satellite. A frequency shift of the signal occurs that is caused by the movement of the satellite (Doppler effects). Then satellite orbits and ground positions can be derived (Seeber, 2003). The Indian Regional Navigational Satellite System (IRNSS) is an autonomous regional satellite navigation system whose constellation is composed of Geostationary Earth Orbit (GEO) and Inclined Geosynchronous Orbit (IGSO) satellites, being developed by the Indian Space Research Organization. The Quasi-Zenith Satellite System (QZSS) with GEO and IGSO satellites is a proposed regional time transfer system and Satellite Based Augmentation System (SBAS) for GPS which is available within Japan. The constellation of regional BeiDou system is also composed of GEO and IGSO satellites, which offers limited coverage and applications. It has been offering navigation services since 2012, mainly for users in China and neighboring regions. Additionally, the Chinese Area Positioning System (CAPS) has been developed in China since 2003. As an alternative satellite system for navigation, CAPS used commercial communication satellites to transmit navigation message and ranging signals. CAPS navigation and ranging signals are generated at a ground station. Extensive positioning experiments have been performed within China since 2005. Systematic descriptions about the CAPS principle were introduced in some references (Ai et al., 2008; Li & Dempster, 2010). Its constellation framework is composed of Geosynchronous Orbit (GSO) communication satellites including Slighted Inclined Geosynchronous Orbit (SIGSO), GEO and IGSO satellites (Ma, 2011; Ma et al., 2011). Considering that the constellations of IRNSS, QZSS, CAPS, and the regional BeiDou system is composed of IGSO satellites, mathematical aspects for these RNSS constellation is investigated in this paper. The RNSS constellation information is presented in Section 2, detailed mathematical aspects of geometrical dilution of precision (GDOP) are given in the following section. Section 3 displays positioning anomaly for these RNSS and a summary is presented in the end of this paper.

2. RNSS Description with IGSO Satellites

2.1 IRNSS

In May 2006, the Indian Government has approved a project to implement IRNSS in the next few years. The requirement of such a navigation system is driven by the fact that access to GNSS is not guaranteed in hostile situations with the GNSS owner. The IRNSS would provide two kinds of services, one is the Standard Positioning Service open for civilian use and the other is Restricted Service, encrypted one, for authorized users. Its constellation consists of seven satellites and a large ground segment. Three satellites in IRNSS constellation will be placed in GEO orbit (at 34°E, 83°E and 131.5°E) and four satellites be placed in IGSO orbit with an inclination angle 29° with their longitude crossings at 55°E and 111.5°E (two in each plane) (Majithiya et al., 2011). All the satellites will be continuously visible in the Indian region for 24 hours a day.

2.2 QZSS

The QZSS mainly with three Quasi-Zenith Satellites (QZSs) is targeted at mobile applications, to provide communications-based services and positioning information. With regards to its positioning service, QZSS can only provide limited accuracy on its own and is not currently required in its specifications to work in a stand-alone mode (Pullen, 2007). As such, it is viewed as a GNSS augmentation system. Besides of three QZS satellites, four GEO satellites will be launched to constitute the planned satellite constellation. Its positioning service could also collaborate with the GEO satellites. The QZS satellites would be placed in a periodic Highly Elliptical Orbit (HEO). These orbits allow the satellite to dwell for more than 12 hours a day with an elevation above 70° and give rise to the term "quasi-zenith" from which the system is named. Three satellites in QZSS constellation will be placed in HEO orbit with inclination angle 43° whose central longitude of ground trace is located at 130°E, 135°E and 140°E respectively and four GEO satellites will be placed in the orbit position 78°E, 116°E, 154°E, 168°E respectively. On September 11th 2010, the first QZS satellite was launched. Full operational status is expected after 2014 (Nishiguchi, 2011; Takasu et al., 2009).

2.3 Regional BeiDou Navigation System

The China hopes to develop an independent satellite navigation system which is called the BeiDou navigation system. The system finally will be a global satellite navigation system consisting of 35 satellites. Currently, it has launched 9 satellites and can cover Asia region. It has planned to offer services to customers in the Asia-Pacific region, and to global customers after its completion by 2020. Among these satellites, four GEO satellites are located at 59°E, 84°E, 140°E and 180°E. Five IGSO satellites with inclination angle 55° are located at 95°E (two satellites) and 118°E (three satellites) (Zhang et al., 2010) In the following section, the regional BeiDou navigation system is abbreviated as RBD.

2.4 CAPS

Based on transponders onboard commercial communication satellites, CAPS achieves navigation and positioning. Integration of communication and navigation is realized in this system. To develop navigation new technique of satellite navigation in the China 2nd Generation Satellite Navigation System, the constellation framework of CAPS is composed of two operational GEO satellites, four SIGSO satellites and one IGSO satellite of 55° inclination with its longitude crossings at 95°E (Ma et al., 2011).

3. Mathematical Aspects of GDOP

To determine the receiver's location, pseudoranges from $n(\geq 4)$ satellites must be used simultaneously. By linearizing the pseudorange equation with Taylor series expansion at the approximate receiver position, the relationship between pseudorange difference ($\delta\rho_i$) and positioning difference (δx_i) can be summarized as follows (Kaplan & Hegarty, 2006; Doong, 2007):

$$\begin{bmatrix} \delta\rho_1 \\ \delta\rho_2 \\ . \\ . \\ . \\ \delta\rho_n \end{bmatrix} = \begin{bmatrix} e_{11} & e_{12} & e_{13} & 1 \\ e_{21} & e_{22} & e_{23} & 1 \\ . & & & \\ . & & & \\ . & & & \\ e_{n1} & e_{n2} & e_{n3} & 1 \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \\ c\delta t \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ . \\ . \\ . \\ v_n \end{bmatrix} \tag{1}$$

where e_{i1}, e_{i2} and e_{i3}, are the direction cosines from the receiver to the ith satellite. Equation (1) can be written in a compact form:

$$\mathbf{z} = \mathbf{Hx} + \mathbf{v} \tag{2}$$

The matrix $\mathbf{H}_{(n\times4)}$ is formed with direction cosines e_{i1}, e_{i2} and e_{i3}, and $\mathbf{v}$ denotes a random noise with an expected value of 0. The difference between the estimated and true receiver position is given as

$$\tilde{x} = (\mathbf{H}^{\mathbf{T}}\mathbf{H})^{-1}\mathbf{H}^{\mathbf{T}}\mathbf{v} \tag{3}$$

where $\mathbf{H}^{\mathbf{T}}$ denotes the transpose of $\mathbf{H}$, and $\mathbf{Q} = (\mathbf{H}^{\mathbf{T}}\mathbf{H})^{-1}$ is called the precision dilution matrix.

Assuming $E\{\mathbf{v}\mathbf{v}^{\mathbf{T}}\} = \sigma^2\mathbf{I}$, then

$$E\{\tilde{x}\tilde{x}^{\mathbf{T}}\} = \sigma^2(\mathbf{H}^{\mathbf{T}}\mathbf{H})^{-1} \tag{4}$$

The GDOP factor is defined as the square root of the trace of the matrix $\mathbf{Q}$:

$$GDOP = \sqrt{\mathrm{trace}(\mathrm{Q})} = \sqrt{\mathrm{trace}(\mathbf{H}^{\mathbf{T}}\mathbf{H})^{-1}} \tag{5}$$

It is desirable to choose the combination of satellites with GDOP as small as possible for the GDOP provides a simple interpretation of how much positioning precision can be diluted by a unit of measurement error. As for the case of four-satellite positioning, the four receiver-satellite unit vectors form a tetrahedron structure. The volume of the tetrahedron is inversely proportional to the GDOP value (Phillips, 1984; Langley, 1999).

If omitting measurement noise and atmospheric effects, one can achieve best performance after selecting the four-satellite navigation constellation with a maximum volume of tetrahedron. When the ends of unit vectors from available satellites are approximate co-planar, GDOP becomes infinite, and a position is unreachable.

4. Positioning Fix Anomaly for RNSS with IGSO Satellites

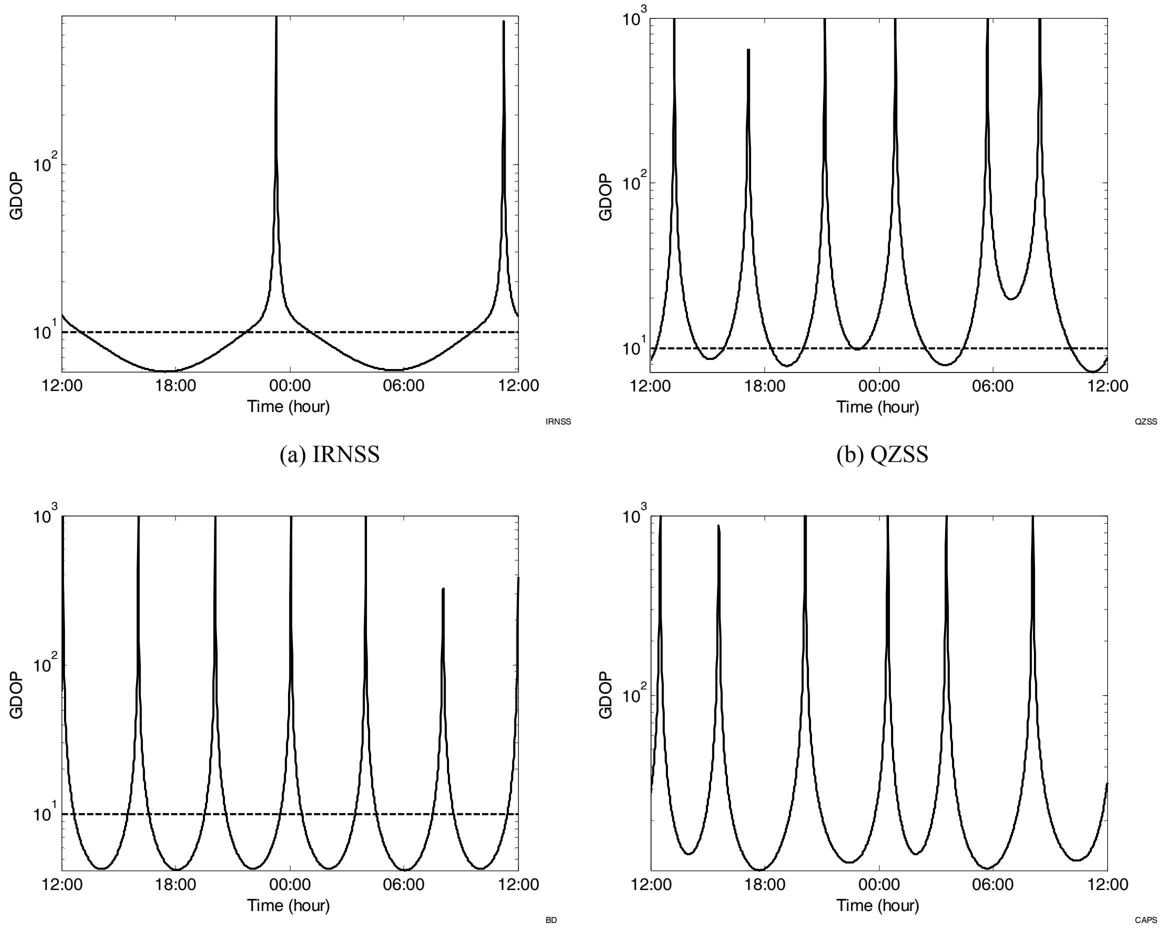
The main source of perturbation force for satellite orbit is combined gravitational attractions of the sun and the moon. This is countered on north-south station-keeping maneuvers to keep the GSO satellite moves within a small scale near the equatorial plane which conformances to the International Telecommunication Union (ITU) regulations. Additionally, the irregular profile of the earth causes a longitudinal drift, which is compensated by east-west station-keeping maneuvers. Solar radiation pressure is caused by the transfer of momentum from the sunlight and infrared radiation, which both flattens the orbit and disturbs the attitude of the satellite. The orbit is compensated by an eccentricity control maneuver (Agrawal, 1986; Ma, 2011). During the time period of control maneuvers, the satellite broadcast ephemeris is inaccurate, after maneuvers, it takes a long time for the ground stations to determine the satellite orbit again. Then location estimations of navigation satellites are calculated and uploaded to the satellites with other navigation data for rebroadcast to the user. Additionally, users can't receive the signals from all satellites for the possible of surrounding obstructions. Here we analyze positioning fix with four satellites including one GEO satellite and three IGSO satellites.

Considering the above RNSS systems, several special observation stations are chosen. The station location and positioning constellation are listed in Table 1. With the location of receivers and satellites, the GDOP value can be calculated at every epoch and are drawn in Figure 1. To clearly display the results, here we set GDOP value as 1000 when the GDOP value exceeds 1000.

It is clearly to see that anomaly positioning occurs six times in one day. The time point is corresponding to the worst GDOP value of QZSS, RBD and CAPS. Further analysis shows that every positioning fix anomaly take place the nearly co-planar ends of unit vectors from receiver to satellite, at that time better-precision positioning fix is impossible. Time period with the GDOP value exceeding 10 is listed in Table 2. If navigation constellation with GDOP value exceeding 10 is unavailable, here the unavailable time of period is more than 28% in above RNSS systems.

Table 1. Positioning constellation parameter

System Name	Station Name	GEO	IGSO-1		IGSO-2		IGSO-3	
		λ	λ	i	λ	i	λ	i
IRNSS	New Delhi	83°E	55°E	29°	55°E	29°	111.5°E	29°
QZSS	Toyko	116°E	130°E	43°	135°E	43°	140°E	43°
RBD	Sanya	84°E	118°E	55°	118°E	55°	118°E	55°
CAPS	Sanya	71.7°E	95°E	55°	115°E	20°	115°E	20°



(a) IRNSS (b) QZSS

(c) RBD (d) CAPS

Figure 1. The daily GDOP variations from observation stations

Table 2. Time period with GDOP exceeding 10 in one day

System Name	Time period with GDOP exceeding 10		
	minutes	hours	%
IRNSS	410	6.8	28.5
QZSS	983	16.4	68.3
RBD	414	6.9	28.7
CAPS	1440	24.0	100

5. Summary and Discussions

Positioning solution is not obtainable when the ends of unit vectors from available satellites are co-planar. Analysis results from IRNSS, QZSS, RBD and CAPS confirm this point. There are many anomaly positioning fix taking place in one day even IGSO satellites with a big inclination angle. Considering that there are enough satellites this case isn't appear in GNSS constellation. However redundancy satellites are needed in RNSS constellation design in order to overcome the condition where ends of unit vectors from receiver to satellite are nearly co-planar. A well navigation constellation should be there will almost always be at least five visible satellites everywhere on the earth.

Since transponders of communication satellites are adopted to transmitted navigation signals, CAPS signal frequency and bandwidth can be set flexibly. The system can achieve ultrawide band, tri-frequency and multi-frequency navigation, as well as system embedded difference technique (Ai et al., 2011). Even if the

GDOP value is large in CAPS constellation, navigation performance which meets the need of common user can be achieved.

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