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The New Weakening Buffer Operator with Parameters

Rui Zhou[1] & Jun-jie Li[1]

[1] Lecturer of College of Mathematics and Information, China West Normal University, China

Correspondence: Rui Zhou, College of Mathematics and Information, China West Normal University, China. E-mail: 85605830@qq.com

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Abstract

Based on the monotonicity of elementary functions, this paper constructed a new kind of buffer operator with parameter. The new weakening buffer operator improved the prediction accuracy; and through an example, the effect is good.

Keywords: parameter, gray system, weakening, buffer operator

1. Introduction

The gray system theory that, despite the surface forms of the objective is complex and the data is disorderly, but they always have their own function as a whole and contain some kinds of internal law, the key is how to choose the proper methods to mine out and use them. Since the birth of the theory of grey system, many scholars have made a lot of work in data mining and utilization, and the theory is widely used in national product. But be disturbed by outside factor, the data will be distortion. To eliminate the disturbing factors, restore the data true colours, Mr. Liu Si-feng put forward the shock disturbed system, which make the data simulation more fit with reality. Based on this theroy, many scholars have constructed many kinds of buffer operator, which made the application of the grey system theory to get extension. Based on the existing research results and combined with the monotonicity of elementary functions, this paper constructed a kind of buffer operator with parameter.

2. The Basic Concepts and Axioms

Defintion 1: Let the system behavior data series is $X = (x(1), x(2), \cdots x(n))$,

1) $\forall k = 2,3\cdots, n, x(k) - x(k-1) > 0$, then X is a Monotone increasing series.

2) $\forall k = 2,3\cdots, n, x(k) - x(k-1) < 0$, then X is a Monotone decreasing series.

3) If $k, h \in \{2,3,\cdots n\}$ and $x(k) - x(k-1) > 0$, $x(h) - x(h-1) < 0$, then X is a oscillatory series.

Defintion 2 If $X = (x(1), x(2), \cdots x(n))$ is a system behavior data series, D is a operator about X , through operated by D, X turn into $XD = (x(1)d, x(2)d, \cdots x(n)d)$, then D is a series operator, XD is a first-order operator series.

Axiom 1 (Fixed Point Axiom) Let X is a system behavior data series, D is a series operator, then $x(n)d = x(n)$

Axiom 2(Full use of the information axiom) Each data $x(k), k = 1, 2, \cdots n$ in the system behavior data series X should be fully involved in the whole process of the series operator function.

Axiom 3(Analytical, normative axiom) Each data $x(k)d, (k = 1, 2, \cdots n)$ can be expressed by a unified analytical formula.

Meet the three axioms of the seies operator is a buffer operator. XD is a buffer series.

Characteristic 1 If $X = (x(1), x(2), \cdots x(n))$ is a system behavior data sequence, and $XD = (x(1)d, x(2)d \cdots x(n)d)$ is an operator sequence, then

(1) When X is a monotone increasing sequence, D is a strengthening buffer operator $\Leftrightarrow$ $x(k) \geq x(k)d, k = 1, 2, \cdots, n$.

(2) When X is a monotone decreasing sequence, D is a strengthening buffer operator $\Leftrightarrow$ $x(k) \leq x(k)d, k = 1, 2, \cdots, n$

Characteristic 2 If $X = (x(1), x(2), \cdots x(n))$ is a system behavior data sequence, and $XD = (x(1)d, x(2)d \cdots x(n)d)$ is an operator sequence, then

(1) When X is a monotone decreasing sequence, D is a weakening buffer operator $\Leftrightarrow$ $x(k) \leq x(k)d, k = 1, 2, \cdots, n$.

(2) When X is a monotone increasing sequence, D is a weakening buffer operator $\Leftrightarrow$ $x(k) \geq x(k)d, k = 1, 2, \cdots, n$.

3. The Construction of Weakening Buffer Sequences and Their Characteristics

If $X = \left(x(1), x(2), \cdots x(n)\right)$ is a system behavior data sequence, and $x(k)d = \sqrt[m]{\frac{x(k) + x(k+1) + \cdots x(n)}{n-k+1} \cdot x^{m-1}(n)}$, let $XD = (x(1)d, x(2)d, \cdots x(n)d)$, X is a monotone increasing(decreasing) sequence, then D is a weakening buffer operator.

Proof: (1) If $X = \left(x(1), x(2), \cdots x(n)\right)$ is a non-negative monotone increasing sequence, that means $0 < x(1) \leq \cdots \leq x(n)$, then $0 < x^m(1) \leq \cdots \leq x^m(n), m = 1, 2 \cdots n$

$0 < x(k) + \cdots + x(k) \leq x(k) + \cdots + x(n)$

And $\frac{x(k) + x(k) + \cdots x(k)}{n-k+1} \cdot x^{m-1}(k) \leq \frac{x(k) + x(k+1) + \cdots x(n)}{n-k+1} \cdot x^{m-1}(n)$ that is

$$x(k)d = \sqrt[m-1]{\frac{x(k) + x(k+1) + \cdots x(n)}{n-k+1} \cdot x^{m-1}(n)} \geq x(k)$$

So D is a weakening buffer operator.

(2) If $X = \left(x(1), x(2), \cdots x(n)\right)$ is a non-negative monotone decreasing sequence, that means $x(1) \geq \cdots \geq x(n) > 0$, then $x^m(1) \geq \cdots \geq x^m(n) > 0, m = 1, 2 \cdots n$

$x(k) + \cdots + x(k) \geq x(k) + \cdots + x(n) > 0$

And $\frac{x(k) + x(k) + \cdots x(k)}{n-k+1} \cdot x^{m-1}(k) \geq \frac{x(k) + x(k+1) + \cdots x(n)}{n-k+1} \cdot x^{m-1}(n)$ that is

$$x(k)d = \sqrt[m]{\frac{x(k) + x(k+1) + \cdots x(n)}{n-k+1} \cdot x^{m-1}(n)} \leq x(k)$$

So D is a weakening buffer operator.

(3) If $X = \left(x(1), x(2), \cdots x(n)\right)$ is a non-negative oscillatory sequence, let $x(k) = \max x(i), 1 \leq i \leq n$, $x(h) = \min x(h), 1 \leq h \leq n$, for arbitrary $i \in \{1, 2, \cdots n\}$

And $x^m(k) \geq \frac{x(k) + \cdots + x(n)}{n-k+1} x^{m-1}(n)$ that means $x(k)d = \sqrt[m-1]{\frac{x(k) + \cdots + x(n)}{n-k+1} x^{m-1}(n)} \leq x(k)$

And $x^{m-1}(h) \le \dfrac{x(h)+\cdots+x^m(n)}{n-h+1} x^{m-1}(n)$ that means $x(h)d = \sqrt[m]{\dfrac{x(h)+\cdots+x^m(n)}{n-h+1} x^{m-1}(n)} \ge x(h)$

$$\max x(i) \ge \max x(i)d, i = 1, 2\cdots n$$
$$\min x(i) \le \min x(i)d, i = 1, 2\cdots n$$

So D is a weakening buffer operator.

Characteristic Weakening buffer sequence XD and system behavior data sequence X have the same monotonicity.

Proof: (1) If $X = \left(x(1), x(2), \cdots x(n)\right)$ is a non-negative monotone increasing sequence,

$$\begin{aligned}
&\left(x(k)+x(k+1)+\cdots x(n)\right)\cdot(n-k)\\
&=(n-k)\cdot x(k)+(n-k)\cdot\left(x(k+1)+\cdots x(n)\right)\\
&=\underbrace{\left[x(k)+x(k)\cdots+x(k)\right]}_{n-k}+(n-k)\cdot\left(x(k+1)+\cdots x(n)\right)\\
&<\left(x(k+1)+\cdots x(n)\right)+(n-k)\cdot\left(x(k+1)+\cdots x(n)\right)\\
&=\left(x(k+1)+\cdots x(n)\right)\cdot(n-k+1)
\end{aligned}$$

that is

$$\left(x(k)+x(k+1)+\cdots x(n)\right)\cdot(n-k) < \left(x(k+1)+\cdots x(n)\right)\cdot(n-k+1)$$

$$\frac{x(k+1)+\cdots x(n)}{n-k} > \frac{x(k)+x(k+1)+\cdots x(n)}{n-k+1}$$

And X is a non-negative monotone increasing sequence,

So $\sqrt[m]{\dfrac{x(k+1)+\cdots x(n)}{n-k} x^{m-1}(n)} > \sqrt[m]{\dfrac{x(k)+x(k+1)+\cdots x^m(n)}{n-k+1} x^{m-1}(n)}$

That is $x(k)d < x(k+1)d$

(2) By the same methods show, if X is a non-negative monotone decreasing sequence, XD is also a non-negative monotone decreasing sequence.

4. The Example Analysis

Take Shaanxi province as an example, table 1 is The total agricultural output value of Shaanxi Province (2000-2009), Takes the modeling data by 2000-2008 years data, and takes the simulation examination data by 2009 data. Next using the original series, the buffer operators D_1, D_2 in reference and buffer operator D in this paper (m=0.9) to forecast. The predicted results in Table 2.

Table 1. Total agricultural output value of Shaanxi Province

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Value	258.22	263.63	282.21	302.28	372.28	435.77	484.81	592.63	753.72	789.64

Table 2. The model GM (1,1) before and after the buffer operator weakening

Series	GM(1,1)	Forecast Value	Error Value
X	x(k+1)=1280.714377exp(0.162975k)-1022.494377	834.99	5.74
XD_1	x(k+1)=5073.959948exp(0.079522k)-4657.749948	793.42	0.48
XD_2	x(k+1)=14156.829328exp(0.039092k)-13596.729328	771.59	2.29
XD	x(k+1)=4246.2exp(0.083776k)-3868.5	788.67	0.12

From Table 2,using the original series ,the forecast error is 5.74%, and using the buffer operators D_1,D_2, the forecast error are 0.48%, 2.29%, whose forecast precision are improved. But using the buffer operator D in this paper, the forecast error is only 0.12%. So the buffer operator in this paper is best.

5. Conclusions

Combined with the monotonicity of elementary functions, this paper constructed a kind of buffer operator. Through the example, the prediction effect of buffer operator constructed in this paper is good. But in the process of constructing buffer operator, how to select the parameters M, which need a further research.

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