Computer and Information Science
www.ccsenet.org/journal.html
Vol. 2, No. 3
August 2009

Fuzzy Evaluation of Weapons System

Mahmod Othman (Corresponding author)
Faculty of Science Computer and Mathematics
Universiti Teknologi Mara, Perlis Branch, 02600 Arau, Malaysia
Tel: 60-4-987-5157 E-mail: mahmod135085@perlis.uitm.edu.my

Shaiful Annuar Khalid
Faculty of Business Management
Universiti Teknologi Mara (UiTM), 02600, Arau, Perlis, Malaysia
Tel:60-12-5140436 E-mail: shaiful@perlis.uitm.edu.my

Mohammad Ismail
Faculty of Business Management
Universiti Teknologi Mara (UiTM), 02600, Arau, Perlis, Malaysia
Tel:199-544-900 Tel: mohammadismail@perlis.uitm.edu.my

Norshimah Abdul Rahman
Faculty of Business Management
Universiti Teknologi Mara (UiTM), 02600, Arau, Perlis, Malaysia

Mohamad Fadhili Yahaya
Academy of Language Studies
Universiti Teknologi MARA, 02600 Arau, Perlis, Malaysia
Tel: 60-12-455-5164 E-mail: mohdfadhili@perlis.uitm.edu.my

Abstract

This paper proposed to apply fuzzy sets and approximate reasonings to evaluate the weapons system. The objective of the study is to determine the ranking of the weapons system in a subjective environment. The proposed model based on fuzzy sets has initiated the idea of membership set score value evaluation of each criterion alternative. This enables the inclusion of requirements which are incomplete and imprecise. The approximate reasonings of the method allows the decision maker to make the best choice in accordance to human thinking and reasoning processes. The proposed model is based on fuzzy multi-criteria decision-making that consists of fuzzy rules. The use of fuzzy rules, which are extracted directly from input data in making evaluation, contributes to a better decision in selecting the best choice and is less dependent on the domain of expert. The dataset from previous research was used to validate the fuzzy evaluation model. Results from numerical examples are comparable to other fuzzy evaluation approaches. Copyright © 2005

Keywords: Fuzzy sets, Multi-criteria decision making, Approximate reasoning

1. Introduction

A reliable evaluation method which is of quality is a necessary process in decision making environments. In practice, the evaluation of performance usually uses the subjective criteria. In doing so, one has to depend on one's wisdom, experience, professional knowledge and information, which is difficult to define and/or describe accurately. When analysing using incomplete data, a lot of uncertainties will arise and this will confuse not only decision-makers but also complicate decision-making as it is made under unknown situations. The application of fuzzy sets theory in evaluation systems can improve the evaluation results (Turban, et al. 2000). Several researchers have tried to solve this problem using analytical hierarchy process (AHP), for example in personnel selection (Liang & Wang, 1992; Sonja, 2001) and shipping performance evaluation (Chou & Liang, 2001), whereby evaluation was done by aggregating all the fuzzy sets. However, the presence of imprecision, vagueness and subjectivity at each level further accumulates greatly the undesired elements in aggregating the marks.

In the literature, various concepts have been proposed focusing on the combination of fuzzy logic model with multi objective decision that can assist in reducing errors in making judgment (Pedrycz & Gomide, 1982; Liang & Wang, 1992). The research provides approaches to judgment procedure on personnel selection through the development of AHP fuzzy multi criteria. It is cited as being able to minimise subjectivity. Some research in fuzzy evaluation methods is discussed in Othman (2004a; 2004b; 2004c). The authors have proposed algorithms based either on fuzzy similarity function or fuzzy synthetic decision and ranking procedures through satisfaction function. Fuzzy sets membership enables the interpretations of linguistic variables in a very natural and plausible way to formulate and solve various problems. However, expressing the linguistic variable using the singleton fuzzy sets such as in Capaldo and Weon (2001) could result in the loss of much important information and would additionally complicate the course of action. Although many evaluation methods for selecting or ranking have been suggested in the literature, there is yet a method which can give a satisfactory solution to every situation. For this reason, a fuzzy evaluation method is proposed by combining the concepts introduced by Othman (2003) and integrating it with a fuzzy rule (Othman, 2004a) that is derived automatically from input data. This research makes its contribution by introducing the bridging and linking of these two methods. Previous studies on fuzzy evaluation methods evaluate (Tseng & Yeh, 2000; Chang & Yeh, 2002; and Kuo & Chen, 2002) the use of the number of respondents who answered the survey questions to represent fuzzy set in forms of membership function. However, these methods have a drawback, whereby they are unable to produce a generalised fuzzy evaluation method to evaluate various types of data. Hence, this research introduced the membership set score where various input data are transformed and are not predetermined by the expert. This is important to ensure the consistency in generalising the proposed framework.

The paper proceeds as follows. The proposed model is introduced in Section 2. Section 4 presents the algorithm of the proposed model and numerical results. It is followed by the concluding in Section 5.)

2. The Fuzzy Evaluation Model

The algorithm for the evaluation model consists of 10 steps as listed below:

Step 1	**:**	Calculate membership set score
Step 2	**:**	Determine grade range, mid-points and mid-intervals
Step 3	**:**	Construct fuzzy set membership for each criterion
Step 4	**:**	Define fuzzy sets for the grades
Step 5	**:**	Calculate maximum similarity value and determine grade
Step 6	**:**	Construct the similarity curve and map the grade to mid-point or mid-interval mark
Step 7	**:**	Calculate the normalised synthetic score value
Step 8	**:**	Determine multi-criteria rules combination and calculate factor rule value
Step 9	**:**	Calculate appraisal fuzzy value and the appraisal product value
Step 10	**:**	Compute satisfaction value and ranking

The performance evaluations of weapons system (WS) datasets are taken from Mon et al. (1994). The objective of this evaluation is to choose the best weapon system among a finite number of alternatives. The weapons system has three tactical missile systems (TMS) alternatives. The model started with the calculation of the membership set of score. A fuzzy number was used to evaluate the fuzzy weapons system. The data in terms of fuzzy triangle number had to be transformed into degree of membership set. Degree of membership $\mu_A(x)$ was defined as the degree of belonging fuzzy set score grade to the universe of discourse X for each criterion, as in

$f_{ij} = \{ \mu_{f_{ij}}(x)/x, x \in X \}$

Where f_{ij} = the degree of membership of fuzzy evaluation mark (i = 1, 2, 3, weapon systems and j = 1, 2, …m, the criteria environment), $\mu_{f_{ij}}(x)$ = fuzzy set of average fuzzy performance rating of 3 TMS according to the criteria given by experts in terms of fuzzy number (for example $\tilde{N}$, where N = 1, …, 9). Fuzzy numbers $\tilde{1}$, $\tilde{x}$, $\tilde{9}$ are defined as (1, 1, 3), (x – 2, x, x + 2) for x = 3, 5, 7 and (7, 9, 9). The data in terms of fuzzy numbers could be transformed into membership set score by using the following membership function μ_A (T) defined as

$$\mu_A(x) = \begin{cases} 0 \quad , & x_i \le a_1 \\ \dfrac{x_i}{T - a_1} \quad , & a_1 < x_i \le T \\ \dfrac{a_3 - x_i}{a_3 - T} \quad , & T < x_i \le a_3 \end{cases} \tag{1}$$

$\mu_A(x)$ describes the degree of membership of x X in fuzzy set A. The generated fuzzy set characterises the membership values $\mu_A(x)$ E [0, 1]. Table 1 depicts part of the membership set score of the first criteria. For example, to obtain fuzzy set score C_1 in Table 1, the element in the third row and in the eighth column was computed by taking the input as a fuzzy number. Let the fuzzy number be $\tilde{5}$, expressed by three parameters of the symmetric triangular fuzzy number as, (x – 2, x, x + 2) = (5 – 3, 5, 5 +2) = (2, 5, 7). The fuzzy number was transformed into the fuzzy membership set score as in Table 1 using eqn. 1. The values μ_{13}, μ_{14} and μ_{15} were computed as $\mu_{13} = \frac{4-3}{5-3} = 0.5$; $\mu_{14} = \frac{5-3}{7-5} = 1$; $\mu_{15} = \frac{7-2}{7-5} = 0.5$ (using eqn. 1). The same method was applied to calculate all the fuzzy membership set scores for each alternative evaluated by the three experts, and the results were tabulated in Table 1.

The second step determined the range of marks for each grade and then calculates the mid-point or mid-interval mark for each criterion. The standard fuzzy sets grade had to be defined so as to feed the model with knowledge. The standard fuzzy sets grade for the weapons evaluation was then defined as shown in Table 2. The values are the enhancement of the values defined as practised in (Biswas, 1995).

The construction of the fuzzy set membership was undertaken in the third step. For example, the fuzzy set of Λ_1 for C_1 can be written as {0/0, 0/10, 0/20, 0.5/30, 1/40, 0.5/50, 0/60, 0/70, 0/80, 0/90, 0/100} to represent the degree of belonging of the score to each mark. The results of the calculated fuzzy set membership were tabulated in Table 3.

In the fourth step, the fuzzy sets grade was defined in Table 4. The fifth step involved the calculation of the maximum similarity value and determination of the grade for each criterion. The normalisation operation process used the fuzzy similarity function as discussed in Biswas (1995). The grade for each criterion of the three TMS was accorded by solving the fuzzy similarity function as in eqn. 2.

$$S(F,M) = \frac{\hat{F}.\hat{M}}{max\ (\hat{F}.\hat{F}, \hat{M}.\hat{M})}, \tag{2}$$

where $\hat{F}$ = ($\mu_F(x_1)$, ($\mu_F(x_2)$, …),$_\wedge$ $\hat{M}$ = ($\mu_M(x_1)$, ($\mu_M(x_2)$, …) were the vectors and $\hat{M}$ denoted the transpose vectors A^T, B^T, C^T, D^T and E^T. F represented the transpose vector of fuzzy set f_{ij} where i = 1, 2, 3 and j = 1,2 3, 4, 5. Set X = (x_1, x_2, …,x_n) symbolised the set of universe of discourse and '•' is the dot product. For example, if S(A, C_1) is the similarity value between grade A and criteria C_1, then the computation of $S(A, C_1)$ = ((0, 0, 0.5, 1, 0.5, 0, 0, 0, 0, 0) • (0, 0, 0, 0, 0, 0, 0, 0.33, 0.67, 1)^T)/ max((0, 0, 0.5, 1, 0.5, 0, 0, 0, 0, 0)^2) • (0, 0, 0, 0, 0, 0, 0, 0.33, 0.67, 1)^T) = 0.67. Next, the maximum similarity values for each factor were identified from the highest similarity values. Table 5 displays the similarity values obtained for the three TMS for criterion 1.

The sixth step was constructed based on the calculated similarity value. The similarity curve was developed for course Λ_1 from Table 5. The maximum similarity value was determined by identifying the maximum of the similarity values in Table 5. Next, the grade was mapped to the appropriate mid-interval mark. In this step, the similarity value and the similarity curve were used to map the mark. The results of allocating an appropriate mid-point and mid-interval mark to each criterion for the first course are shown in Table 6.

The normalised synthetic score was then calculated as shown in Table 7 using eqn. 3.

$$\text{Normalised synthetic score} = \frac{1}{N} r \tag{3}$$

where N = 100 and r is fuzzy mark.

Table 8 shows the fuzzy rules generated by the proposed model from the weapons system data in terms of rules properties, number of rules, maximum length, minimum length are 3, 3 and 3 respectively.

The computational results of factor rule values are shown in Table 9. For example, the value 0.4000 in row two and column two of Table 9 was obtained by using the antecedent of decision criteria DC_1, that is $C_2 \cap (C_3 \cup C_4)$ in Table 8. Therefore, when the values of C_2, C_3 and C_4 of Λ_1 from Table 7 were substituted into $C_2 \cap (C_3 \cup C_4)$ the result became $0.4000 \cap (0.8000 \cup 0.4000) = 0.4000$.

Then the appraisal fuzzy value, $(d_j(m,l))$, of Table 10 was computed as follows (Othman et al., 2004d):

$d_j(m,l) = 1 \wedge (1 - \widetilde{c}(u_m) + A_k(v_l))$, where j = 1, 2, 3, m = 1, 2, 3, l = 1, 2, ..., 11 and $\widetilde{c}(u_m)$ is the factor rule value.

The appraisal fuzzy values for decision criteria DC_1 was tabulated in Table 10.Therefore, the appraisal product value D was calculated by multiplying all elements of the appraisal fuzzy value, Dj obtained earlier. The formula is given in eqn. 5.

$$\text{D} = \left(\prod_{j=1}^{3} d_j(m,l)\right) = \left(\widetilde{E}_1, \widetilde{E}_2, ..., \widetilde{E}_F, ..., \widetilde{E}_M\right) \in M_{M\times 1} \tag{5}$$

Assuming that $E_{m\alpha}$ is the α level of $\widetilde{E}_m$, $\alpha \in [0, 1] = I$, it should be noted that the sets $E_{m\alpha}$ were ordinary subsets of V. For each $E_{m\alpha}$, $H_l(E_{m\alpha})$ = mid-point could be calculated. The calculated appraisal product value is shown in Table 11.

The calculated values of the range of appraisal product value (α), the different of range of appraisal product value ($\Delta\alpha_l$), and mean value of $E_{m\alpha}$, ($H_l(E_{m\alpha})$) were tabulated in Table 12.

The calculated values of the range of α, $\Delta\alpha_l$, and $H_l(E_{m\alpha})$ were substituted in the following Equation (6) to calculate the satisfaction value in the final step of the method.

$$SV(m) = \frac{1}{\alpha_{\max}} \sum_{l=1}^{11} H_l(E_{m\alpha}) \Delta\alpha_l \tag{6}$$

where α = degree of appraisal product value D; $\Delta\alpha_l = \alpha_l - \alpha_{l-1}$; $\alpha_0 = 0$; $H_l(E_{m\alpha})$ = mid-point of V_l (l = 1,2,3…,L); and $\alpha_{\max}$ = maximum degree of appraisal product value.

3. Numerical Results

The results of evaluating the ranking of weapons system were tabulated in Table 13. Columns 2, 4 and 3, 5 of Table 13 illustrate the performance value and ranking order for measuring TMS alternatives Λ_1, Λ_2, and Λ_3 respectively. The satisfaction values calculated by using the fuzzy evaluation model represent the performance values which are used to rank the TMS alternatives. The satisfaction values in column 4 were 0.7006, 0.6100, 0.0.6043 and in column 5 the rankings were 1, 2, 3 respectively. The Mon *et al*. (1994) method produced the performance values and ranking as listed in columns 2 and 3 as 0.3392, 0.3368, 0.3241 and 1, 2, 3, respectively. Clearly, it shows that the satisfaction values were higher than the values obtained from Mon *et al*.'s method. The higher value indicates that the reliable experts are satisfied with the TMS alternatives offered. From these results, the fuzzy evaluation model shows outstanding performance when compared to Mon *et al*.'s method with 100% accuracy in ranking three TMS alternatives, Λ_1, Λ_2, and Λ_3. Again the subjective evaluation method showed advantage with simpler rules properties with a smaller number of rules and maximum and minimum length.

4. Conclusion

A new fuzzy evaluation model has been proposed for the evaluation of the weapons system. The model was implemented using C++ programming language and is suitable for various fuzzy environments. Experimental results produced are comparable to results obtained from the model by Mon et al. The main contribution of the research model was the use of a fuzzy expert system consisting of a set of rules in the form of IF (antecedent) THEN (Conclusion). The model could be used as an alternative approach in solving problems that involve uncertainties. The evaluation output would become more precise if the combination factors were accurately defined. The rule properties were also analysed to judge the strength of the subjective evaluation method. The results of the experiments showed remarkable ranking performance even with the use of small-sized rule properties.

References

Biswas, R. (1995). An Application of Fuzzy Sets in Student's Evaluation, *Fuzzy Set and Systems*, 74: 187-194.

Chou, T.Y and Liang, G.S., (2001). Application of a Fuzzy Multi-criteria Decision-making Model for Shipping Company Performance Evaluation, MARIT. POL. MGMT., 28(4): p. 375 – 392.

Chu, F. (1995). Fuzzy Multicriteria Decision-Making in Distribution of Factories: An Application of Approximate Reasoning, *Fuzzy Sets and Systems*, 71: 197-205.

Hong, T.P. and Lee, C.Y. (1996). Induction of Fuzzy Rules and Membership Functions from Training Examples, *Fuzzy Sets and Systems*, 84: 33-47.

Klir, G. J. and Folger, T. A. (1988). *Fuzzy Sets, Uncertainty, and Information,* New York: Prentice-Hall International Inc.

Laarhoven and Pedrycz. (1983). A Fuzzy Extension of Saaty's Priorty Theory, *Fuzzy Sets and Systems*, 11: 229-241.

Lee, K.M., Cho, C.H., and Kwang, H.L. (1994). Ranking Fuzzy Values with Satisfaction Function, *Fuzzy Sets and Systems*, 64: 295-309.

Li, H.C. and Ilacqua, J.A. (1994). Job Search and Employment, *Fuzzy Sets and Systems*, 68: 335-342.

Liang, G. and Wang, A. (1992). Personnel Placement in a Fuzzy Environment, *Computers Operations Research*, 19(2): 107-121.

Mon, D.L., Cheng, C.H. and Lin, J.C. (1994). Evaluation Weapon System Using Fuzzy Analytical Hierarchy Process Based on Entropy Weight, *Fuzzy Sets and Systems*, 62: 27-134.

Othman, M., Ku-Mahamud, K.R. and Bakar, A.A. (2003). Quality Evaluation Using Two Phase Fuzzy Approach, *Proceeding DSI 7TH International Conference*, Shanghai, China.

Othman, M., Ku-Mahamud, K.R. and Bakar, A.A. (2004a). Fuzzy Rule Generation Using Membership Function, *Proceeding The 4th Information and Computer Engineering Postgraduate Workshop, Phuket Thailand*, p. 115 – 120.

Othman, M., Ku-Mahamud, K.R. and Bakar, A.A. (2004b). Fuzzy Similarity Function For Ranking River Basin Planning Alternatives, *Preprints Fifth International Workshop on Artificial Intelligence in Agricultural*, Cairo, Egpyt, p. 95 – 100.

Othman, M., Ku-Mahamud, K.R. and Bakar, A.A. (2004c). Ranking The River Basin Planning Alternatives Using Fuzzy Approximate Reasoning, Chiang Mai Journal of Science.

Othman, M., Ku-Mahamud, K.R. and Bakar, A.A. (2004d). Subjective Evaluation Using Fuzzy Rule Approach, Journal of Institute of Mathematics & Computer Sciences (Comp. Sc. Series), 15(1).

Pedrycz,W. and Gomide F. (1998). *An Introduction to Fuzzy Sets Analysis and Design*, England: The MIT Press.

Saaty, T.L. (1995). *The Analytic Hierarchy Process*, Pittsburgh: RWS Publications.

Sonja P. L. (2001). Personnel Selection Fuzzy Model, *International Transactions in Operational Research*, 8(1): 89-105.

Turban, E., Zhou, D. and Ma, J. (2000), A Methodology for Grades of Journals: A Fuzzy Set-Based Group Decision Support System, *Proceedings of the 33rd Hawaii International Conference on System Science*.

Turksen, I.B. (1992). Fuzzy Expert Systems for IE/OR/MS, *Fuzzy Sets and Systems*, 51:1-27.

Turksen, I.B. and Wilson, I.A. (1994). A Fuzzy Sets Preference Model for Consumer Choice, *Fuzzy Sets and Systems*, 68: 253-266.

Yamasita, T. (1997). On a Support System for Human Decision Making By the Combination of Fuzzy Reasoning and Fuzzy Structural Modeling, *Fuzzy sets and Systems*, 87: 257-263.

Table 1. Membership Set Score

TMS	C_1										
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Λ_1	0	0	0	0.5	1	0.5	0	0	0	0	0
Λ_2	0	0	0	0	0	0.5	1	0.5	0	0	0
Λ_3	0	0	1	0	0	0	0	0	0	0	0

Table 2. Grade Mid-Point and Mid-Interval Mark

Grade	**Mid-Point Mid-interval (X)**				
A	90.0	92.5	95.0	97.5	100.0
B	70.0	75.0	80.0	85.0	90.0
C	50.0	55.0	60.0	65.0	70.0
D	30.0	35.0	40.0	45.0	50.0
E	0.00	7.5	15.0	22.5	30.0

Table 3. Fuzzy Set Membership

TMS	C_1										
Λ_1	0/0	0/10	0/20	0.5/30	1/40	0.5/50	0/60	0/70	0/80	0/90	0/100
Λ_2	0/0	0/10	0/20	0/30	0/40	0.5/50	1/60	0.5/70	0/80	0/90	0/100
Λ_3	0/0	0/10	1/20	0/30	0/40	0/50	0/60	0/70	0/80	0/90	0/100

Table 4. Grade Fuzzy Set

Fuzzy Set											
Grade	0	10	20	30	40	50	60	70	80	90	100
E	1	0.67	0.33	0	0	0	0	0	0	0	0
D	0	0	0.5	1	0.5	0	0	0	0	0	0
C	0	0	0	0	0	0.5	1	0.5	0	0	0
B	0	0	0	0	0	0	0.5	1	0.5	0	0
A	0	0	0	0	0	0	0	0	0.33	0.67	1

Table 5. Similarity Value

TMS	C_1				
	E	**D**	**C**	**B**	**A**
Λ_1	0	0.17	0.67	0.17	0
Λ_2	0	0	0.67	1	0.11
Λ_3	0	0.67	0	0	0

Table 6. Maximum Similarity Value

TMS	Factor	Max Similarity Value	Grade	Fuzzy Mark
Λ_1	C_1	0.67	C	60
	C_2	0.67	D	40
	C_3	1.00	B	80
	C_4	1.00	D	40
	C_5	0.67	D	40
Λ_2	C_1	1.00	B	80
	C_2	0.67	C	60
	C_3	0.67	C	60
	C_4	0.67	C	60
	C_5	1.00	B	80

Λ_3	C_1		0.47	D	35
	C_2		1.00	D	40
	C_3		0.67	D	40
	C_4		0.67	D	40
	C_5		0.67	C	60

Table 7. Normalised Synthetic Score Value

TMS	**Factor**				
	C_1	C_2	C_3	C_4	C_5
Λ_1	0.6000	0. 4000	0.8000	0.4000	0.4000
Λ_2	0. 8000	0.6000	0.6000	0.6000	0.8000
Λ_3	0.4000	0.3500	0.4000	0.6000	0.6000

Table 8. Multi-criteria Rules Combination

Decision Criteria	**Factor Rule**	**Linguistic Variable**	**Description**	**Appraisal Set**
1	$C_2 \cap (C_3 \cup C_4)$	A_1	Satisfactory	v
2	$C_3 \cap (C_2 \cup C_4)$	A_1	Satisfactory	v
3	$C_4 \cap (C_2 \cup C_3)$	A_1	Satisfactory	v

Table 9. Factor Rule Value

	C_1	C_2	C_3
Λ_1	0.4000	0.4000	0.4000
Λ_2	0.6000	0.6000	0.6000
Λ_3	0.3500	0.4000	0.4000

Table 10. Appraisal Fuzzy Value for Decision Criteria DC_1

	Appraisal Set										
Λ_1	0.6000	0.7000	0.8000	0.9000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Λ_2	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	1.0000	1.0000	1.0000	1.0000	1.0000
Λ_3	0.6500	0.7500	0.8500	0.9500	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 11. Appraisal Product Value

	Appraisal Set										
Λ_1	0.2160	0.3430	0.5120	0.7290	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Λ_2	0.0640	0.1250	0.2160	03430	0.5120	0.7290	1.0000	1.0000	1.0000	1.0000	1.0000
Λ_3	0.2340	0.3675	0.5440	0.7695	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 12. Calculated Range of α , $\Delta\alpha_l$, and $H_l(E_{m\alpha})$

l	Range α	$E_{m\alpha}$	$H_l(E_{m\alpha})$	$\Delta\alpha_l$
1.	0.0000 < α ≤ 0.2160	{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.50	0.2160
2.	0.2160 < α ≤ 0.3430	{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.55	0.1270
3.	0.3430 < α ≤ 0.5120	{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.60	0.1690
4.	0.5120 < α ≤ 0.7290	{0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.65	0.2170
5.	0.7290 < α ≤ 1.0000	{0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.70	0.2710
6.	1.0000 < α ≤ 1.0000	{0.5, 0.6, 0.7, 0.8, 0.9, 1}	0.75	0.0000
7.	1.0000 < α ≤ 1.0000	{0.6, 0.7, 0.8, 0.9, 1}	0.80	0.0000
8.	1.0000 < α ≤ 1.0000	{0.7, 0.8, 0.9, 1}	0.85	0.0000
9.	1.0000 < α ≤ 1.0000	{0.8, 0.9, 1}	0.90	0.0000
10.	1.0000 < α ≤ 1.0000	{ 0.8, 1}	0.95	0.0000
11.	1.0000 < α ≤ 1.0000	{1}	1	0.0000

Table 13. Results of WS

Method	**Mon *et al.***		**Subjective evaluation**	
	Performance	**Ranking**	**Performance**	**Ranking**
Λ_1	0.3368	2	0.6100	2
Λ_2	0.3392	1	0.7006	1
Λ_3	0.3241	3	0.6043	3
Acc %				100%