

3D Localization Algorithm Based on Linear Regression and Least Squares in NLOS Environments

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Abstract

Based on the positive bias property of the time of arrival (TOA) measurement error caused by the non-line-of-sight (NLOS) propagation, a simple and effective three dimensional (3D) geometrical localization algorithm was proposed, the algorithm needs no prior knowledge of time delay distribution of TOA, and only linear regression was used to estimate the parameters of the relationship between the NLOS distance error and the true distance, thus, the approximate real distance between mobile terminal (MT) and base station (BS) was reduced, then, the 3D geometric localization of mobile terminal was carried out by the least square method. The experimental results shows the effectiveness of the algorithm, and the positional accuracy is far higher than the required accuracy by E-911 in NLOS environments.

Keywords: wireless localization, non-line-of-sight, time of arrival, linear regression, the least square method

1. Introduction

Since the Federal Communications Commission (FCC) proposed the E-911 location requirement in 1996, the wireless location has attracted extensive attention (CHEN, 2014; QU, XI, & REED, 1998). The wireless location technology is used to determine the distance and location information of the mobile terminal or its holder, and to realize the functions of locating, tracking and monitoring items. It has an extensive application value in wireless communication, mobile computing, sensor network and the Internet of things (XIAO, CHEN, WANG, LI, & LI, 2015; XIAO, WANG, LI, & YI, 2013; ZHAO, XI, & HE, 2013). Based on the measurement information of the base station, it is considered that the three-dimensional location of the three-dimensional position coordinates of the handheld terminal in the building and the underground parking area is considered to be a challenge to the technical difficulty of the modern commercial communication network. High precision 3D positioning under this scenario is expected to provide greater value for customers, such as intelligent warehousing, intelligent factories, fixed asset tracking, and so on, which are sensitive to three-dimensional coordinates information, as well as the basic technology for services such as location based indoor navigation, crowd flow analysis, and service pushing based on accurate 3D geographic location information in a shopping mall or office building. But in these environments, the radio signal will pass through the multiple reflection of the wall, the refraction and absorption of the interior objects. These physical factors will lead to the noise of the information, such as the distance, angle and so on, which are measured by the communication base station. How to accurately estimate localization information based on these noisy measurements is a problem that the communication base station needs to solve for terminal localization. From a technical point of view, a good wireless 3D localization algorithm requires as few base stations as possible to accomplish the localization of terminal, fast convergence speed, and robustness to interference and noise. In the mobile communication system, the TOA based positioning method, the measurement accuracy of TOA determines the positioning accuracy of the mobile terminal, the measurement error of 1ns corresponds to the distance error of 0.3m. The measurement error of TOA is mainly composed of two parts, namely, the error of system measurement and multipath fading, multiple access interference, NLOS propagating and far and near effect (CHANG, & LV, 2007; TIAN, & LIAO, 2003). The system measurement error obeys the Gauss distribution, with the continuous development of technology, it will gradually decrease, but other error factors will

always exist under NLOS propagation conditions. In order to reduce the influence of the NLOS propagation on the positioning accuracy of the TOA method, a large number of research were carried out. These localization algorithms in NLOS environments can basically be reduced to three types, namely, the accurate modeling of non sight distance error(CHAN, HANG, & CHING, 2006; CHANG, LV, & WANG, 2007; LI, & PAHLAVAN, 2007; LIU, & WNAG, 2000; LU, ZHANG, & HAN, 2006; XU, DING, & DASGUPTA, 2011), the identification of the NLOS base station(CHAN, TSUI, & CHING, 2006; GUO, & CHEN, 2016; TSALOLIKHIN, BILIK, & BLAUNSTEIN, 2011; YAN, CHEN, & WU, 2009), the weighting the measured distance or the intermediate estimator(CHEN, 1999; CUI, et al., 2014; HUA, et al., 2014; YU, DUTKIEWICZ, 2012). These algorithms have their own advantages, but their shortcomings are obvious. For example, it is particularly difficult to obtain the prior knowledge of the statistical characteristics of signals in actual environments, the estimating accuracy is not high, and it is difficult to be widely used. Therefore, the suppression of NLOS error has become the key to the practicability of wireless localization algorithm. Most of the work is focused on the research and optimization of the two-dimensional localization algorithm, such as the study of the TOA localization algorithm and the dynamic localization algorithm based on the mobile station position. These algorithms makes full use of the resources of the third generation mobile communication network, and achieved basically the E-911 positioning performance under the typical channel environment. However, a number of two-dimensional localization algorithms suitable for NLOS environment can not be applied directly to 3D scenes, and there are few literature to study the problem of 3D localization in NLOS environments(XIAO, CHEN, WANG, LI, & LI, 2015).

The paper transformed the TOA measured in NLOS environments into the distance between the terminal and the base station. Based on the positive bias of the measurement error of the TOA in the mobile communication environment, the measurement error caused by NLOS was suppressed by the linear regression method which estimated the linear relationship between the error and the real distance, then located the terminal's three-dimensional position based on the least square principle, the performance of the algorithm was verified by the experimental results. The proposed algorithm can overcome the influence of the NLOS propagation error to a great extent, and it has high localization accuracy.

2. TOA Time Measurement Model and Localization Principle

In the mobile communication system, due to the influence of measuring equipment and signal propagation environment, there is error in TOA measurement of base station. Assuming that the measurement results of each base station are independent of each other, then the TOA of mobile terminal MT to the i th base station BS_i is

$$t_{i,m} = t_{i,LOS} + t_{i,\varepsilon} + t_{i,N}, i = 1, 2, \dots, M \quad (1)$$

where $t_{i,LOS}$ is the line of sight(LOS) propagation time of signal between MT and BS_i , $t_{i,LOS}$ is system measurement error that obeys the $N(0, \sigma_i^2)$ distribution, it can be reduced with the improvement of timing technology and signal detection technology. and only accounts for a small part of the TOA error, it is generally a Gauss random variable of zero mean, $t_{i,N}$ is the error introduced by NLOS propagation and is the main component of TOA error, it can be expressed by random variables of exponential distribution, uniform distribution or delta distribution(CHANG, & LV, 2007), M is the number of base stations. Due to the existence of systematic measurement error and NLOS propagation error,

$$r_{i,m} = ct_{i,m} = ct_{i,LOS} + ct_{i,\varepsilon} + ct_{i,N} = r_{i,LOS} + r_{i,\varepsilon} + r_{i,N}, i = 1, 2, \dots, M \quad (2)$$

where c is the speed of light, taking $3 \times 10^8 m/s$, $r_{i,m}$ is the distance between BS_i and MT obtained by measuring $t_{i,m}$, $r_{i,LOS}$ is the real distance between the them, $r_{i,\varepsilon}$ is the distance error caused by measuring, $r_{i,N}$ is the distance error caused by the NLOS propagation. There are usually $t_{i,N} \gg |t_{i,\varepsilon}|$ in NLOS propagation environments, therefore, the $r_{i,m} - r_{i,LOS} = r_{i,\varepsilon} + r_{i,N}$ has a positive bias. Establishing a series of equations by the measured distance $r_{i,m}$, taking each base station as the center of the ball, the $r_{i,m}$ as the radius of the sphere, calculating all the intersection lines of any two spherical surfaces, then, the intersection point of all intersection lines is the position of MT, as shown in figure 1.

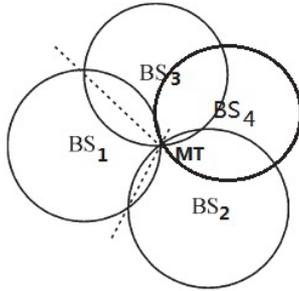


Figure 1. TOA localization schematic

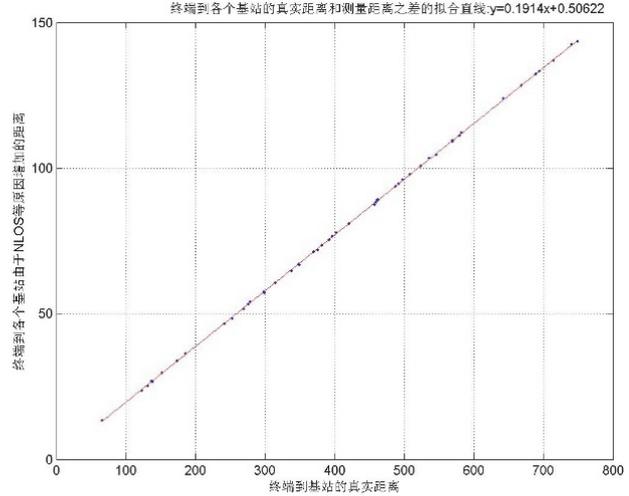


Figure 2. Relation between real distance and measured distance error

Assuming the three-dimensional position coordinates of the MT in figure 1 is (x, y, z) , the coordinates of BS_i is (x_i, y_i, z_i) , then, the distance between the MT and BS_i is

$$r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} \tag{3}$$

the square of both sides (3) is

$$\begin{cases} r_1^2 = x_1^2 + y_1^2 + z_1^2 - 2x_1x - 2y_1y - 2z_1z + x^2 + y^2 + z^2 \\ r_2^2 = x_2^2 + y_2^2 + z_2^2 - 2x_2x - 2y_2y - 2z_2z + x^2 + y^2 + z^2 \\ \dots\dots \\ r_M^2 = x_M^2 + y_M^2 + z_M^2 - 2x_Mx - 2y_My - 2z_Mz + x^2 + y^2 + z^2 \end{cases} \tag{4}$$

Let $K_i = x_i^2 + y_i^2 + z_i^2$, subtracting the first equation from the second to the M th equations in (4), obtained:

$$\begin{cases} r_2^2 - r_1^2 + K_1 - K_2 = -2(x_2 - x_1)x - 2(y_2 - y_1)y - 2(z_2 - z_1)z \\ r_3^2 - r_1^2 + K_1 - K_3 = -2(x_3 - x_1)x - 2(y_3 - y_1)y - 2(z_3 - z_1)z \\ \dots\dots \\ r_M^2 - r_1^2 + K_1 - K_M = -2(x_M - x_1)x - 2(y_M - y_1)y - 2(z_M - z_1)z \end{cases} \tag{5}$$

let

$$b = \begin{bmatrix} r_2^2 - r_1^2 + K_1 - K_2 \\ r_3^2 - r_1^2 + K_1 - K_3 \\ \dots\dots \\ r_M^2 - r_1^2 + K_1 - K_M \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, A = -2 \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ \dots & \dots & \dots \\ x_M - x_1 & y_M - y_1 & z_M - z_1 \end{bmatrix},$$

then the matrix form of the (5) is

$$b = AX \tag{6}$$

therefore, in order to locate MT, at least four base stations are required for TOA measurement, and the 3D spatial position coordinates of MT are obtained by least square method,

$$\hat{X} = (A^T A)^{-1} A^T b \tag{7}$$

In practice, each r_i in (4) is replaced by $r_{i,m}$ of (2) respectively, therefore, in LOS propagation environments with only the system error, the precision of the least square method is higher, however, in NLOS propagation

environments, the error of the least square solution is larger due to the positive bias error.

3. Linear Regression Estimation Between the Measured Error and the True Distance

In order to avoid using TOA's prior know-ledge of time delay distribution, the $t_{i,m}$ between a position determined terminal and several base stations were measured, calculating out the true distance of the test terminal to these base station $r_{i,LOS}$, $i = 1, 2, \dots, M$, then, the distance error caused by NLOS propagation and so on is

$$r_{i,N} = ct_{i,m} - r_{i,LOS} \quad (8)$$

The relation between the real distance and the measured distance error of a test terminal is shown in figure 2, the cross coordinates of each point in the graph is the true distance between the test terminal and different base station, and the ordinate is the difference between the measured distance and the real distance, i.e. the measured distance error. Drawing and analyzing the real distance and the measured distance error of different terminal in the same conditions, the result was basically similar to that in figure 2, and the linear regression significance test showed that there was a highly significant linear relationship between the measured distance error and the real distance of the terminal to the base stations, i.e.,

$$r_{i,N} = k * r_{i,LOS} + a \quad (9)$$

The formula (9) shows that in the same environment, the distance error from the same terminal to different base stations is positively related to the true distance, and the degree of correlation is basically consistent, then, substituting (9) into (8), obtained

$$r_{i,LOS} = \frac{ct_{i,m} - a}{k + 1} \quad (10)$$

therefore, so as the appropriate value of k and a can be solved, the error caused by NLOS propagation can be effectively eliminated, and the approximate real distance between the terminal and the base stations can be calculated out by (10).

4. 3D Localization Algorithm Based on the Least Squares Method

Substituting $r_{i,LOS}$ of (10) for r_i of (5) respectively, obtained

$$\begin{aligned} \left(\frac{ct_{2,m} - a}{k + 1} \right)^2 - \left(\frac{ct_{1,m} - a}{k + 1} \right)^2 + K_1 - K_2 &= -2(x_2 - x_1)x - 2(y_2 - y_1)y - 2(z_2 - z_1)z \\ \left(\frac{ct_{i,m} - a}{k + 1} \right)^2 - \left(\frac{ct_{1,m} - a}{k + 1} \right)^2 + K_1 - K_3 &= -2(x_3 - x_1)x - 2(y_3 - y_1)y - 2(z_3 - z_1)z \\ &\dots\dots \\ \left(\frac{ct_{M,m} - a}{k + 1} \right)^2 - \left(\frac{ct_{1,m} - a}{k + 1} \right)^2 + K_1 - K_M &= -2(x_M - x_1)x - 2(y_M - y_1)y - 2(z_M - z_1)z \end{aligned} \quad (11)$$

here,

$$b = \begin{bmatrix} \left(\frac{ct_{2,m} - a}{k + 1} \right)^2 - \left(\frac{ct_{1,m} - a}{k + 1} \right)^2 + K_1 - K_2 \\ \left(\frac{ct_{i,m} - a}{k + 1} \right)^2 - \left(\frac{ct_{1,m} - a}{k + 1} \right)^2 + K_1 - K_3 \\ \dots\dots \\ \left(\frac{ct_{M,m} - a}{k + 1} \right)^2 - \left(\frac{ct_{1,m} - a}{k + 1} \right)^2 + K_1 - K_M \end{bmatrix}. \quad (12)$$

Substituting b into (6), then, using the least square method of (7) to solve out the 3D coordinate of MT. Synthesizing the above analysis, the 3D geometric localization algorithm based on linear regression and least square principle was designed:

Step1: Calculating out the measured distance error $r_{i,N}$, $i = 1, 2, \dots, M$ by the (8);

Step2: Calculating out the linear regression coefficient k and a of (9);

Step3: Calculating out $r_{i,LOS}$ between the MT and BS_i , $i = 1, 2, \dots, M$ by (10), then, substituting all $r_{i,LOS}$ into (5), obtained b of (12);

Step4: Substituting b into (7) to solve out the 3D spatial position coordinates of MT.

5. Experimental Results

Selecting 10 location base stations, their 3D geometric coordinates are shown in Table 1, the 3D coordinates of the two test terminals are shown in Table 2, the TOA of the two test terminals to base stations are shown in Table 3, and the calculated values of k and a are shown in Table 4.

Table 1. 3D coordinates of base stations (unit:m)

BS	x	y	z	BS	x	y	z
1	402.19	380.52	3.26	6	-67.41	157.58	4.90
2	-56.01	258.54	4.93	7	172.00	277.76	2.71
3	-126.38	76.09	4.96	8	-321.66	253.43	4.90
4	75.41	375.90	4.43	9	-261.95	136.74	5.48
5	29.48	112.22	4.67	10	-373.51	391.84	4.75

Table 2. 3D coordinates of the test terminals (unit:m)

MT	x	y	z	MT	x	y	z
1	-204.97	170.3	1.83	2	-31.12	290.04	1.17

Table 3. TOA of of the test terminals (unit:ns)

BS	MT ₁	MT ₂	BS	MT ₁	MT ₂
1	0.00000255468006	0.00000176086634	6	0.00000054986021	0.00000054808902
2	0.00000069024533	0.00000016045844	7	0.00000155832397	0.00000081074662
3	0.00000048804554	0.00000093262072	8	0.00000054335029	0.00000112968572
4	0.00000138249438	0.00000054542706	9	0.00000026562882	0.00000110128115
5	0.0000009607022	0.00000074667053	10	0.00000110803434	0.00000141981560

Table 4. k and a of the test terminals

MT	k	a	MT	k	a
1	0.19140	0.50622	2	0.19135	0.57071

Choosing TOA from the test terminal MT₁ and MT₂ to the base stations of four to nine smaller to locate MT₁ and MT₂ respectively, the relationship between the mean positional error and the number of chosen TOA is shown in figure 3 and Table 5, therefore, locating with five TOA can not only reduce computational complexity, but also reduce measurement data and achieve higher positional accuracy in practice. Let values of k and a are the average values of the two groups of k and a in table 4, respectively, for each of the 100 terminals, choosing the TOA from the terminal to the base stations of five smaller to locate the terminal, the statistical results of the localization are shown in Table 6, the comparison of the located position of part of the terminals and their real position is shown in Table 7.

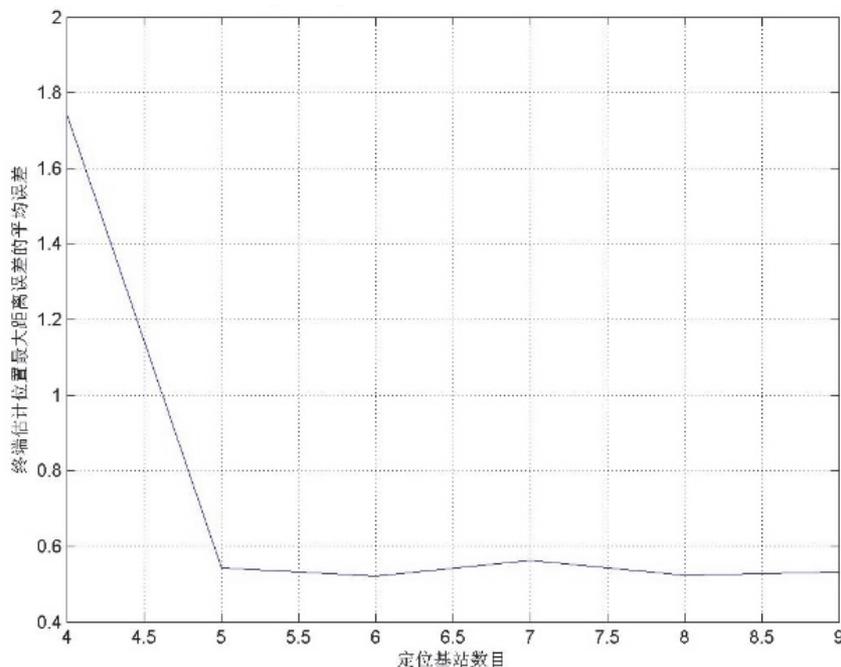


Figure 3. The relationship between the number of chosen TOA and the mean positional error

Table 5. The number of selected TOA and the mean positional error

BS	Mean error	Variance	BS	Mean error	Variance	BS	Mean error	Variance
4	1.7381	0.6909	6	0.5213	0.0444	8	0.5352	0.0566
5	0.5584	0.0442	7	0.5963	0.0644	9	0.5700	0.0571

Table 6. Analysis of positional error

Minimum Error	Maximum Error	Mean Error	Variance
0.0423	1.9960	1.1328	0.2428

Table 7. The real and located position of part of terminals (unit:m)

NO.	Real coordinates			Located coordinates			NO.	Real coordinates			Located coordinates		
1	-0.28	-194.22	1.25	-0.52	-	1.27	6	81.84	85.48	1.92	81.44	85.18	1.77
2	377.07	-310.91	1.69	377.76	-311.59	1.75	7	-	-140.12	1.54	-	-139.96	1.58
3	110.24	72.73	1.27	110.16	72.62	1.14	8	-	345.4	1.6	-	345.32	1.66
4	43.02	-319.59	1.58	-43.11	-	1.68	9	-56.88	123	1.61	-56.73	122.92	1.63
5	317.56	-368.33	1.45	318.94	-	1.39	10	-40.6	-19.87	1.86	-41.08	-19.55	1.96

6. Conclusion

The proposed 3D geometric locating algorithm based on the linear regression and the least square principle in NLOS environments not only does not need to obtain the prior statistical characteristic knowledge of the TOA, but also requires only five TOA to achieve more accurate positional results in practice, and the located accuracy can reach less than 2 meters. Basically, it can overcome the influence of NLOS propagation error, and the positional results basically meet the accuracy requirements of various localization mentioned in the introduction.

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