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The Relation of Mathematics and Language Ability

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Abstract

How number is presented? Is it represented difference for different species? We first review behavioural and neuropsychological studies for the representation of different level of mathematical ability for animals, infants, lesion cases and images studies to demonstrate the relationship of mathematics and language. It can be included that 1) both humans and animals share an elemental number quantification system, which is without supporting by language ability; 2) the language ability is crucial to get higher mathematical ability. We then summarize the main research status for each line of studies. Finally, we outline recommendations for future research directions.

Keywords: mathematical ability, number representation, language ability

1. Introduction

To date, numbers of studies have been carried out to explore the relationship between number and language. And the basic debate for this line of studies is that whether the numerical competence was affected by the language or not (Morris, 2016). One view for this topic was that language affected the cognition (Whorf, 1953); accordingly, the numerical ability may be influenced by language. However, the opposite view for this topic held that the numerical ability was independent of the language; accordingly, the human begins who is without language and even the animal endowed the numerical ability (Socrates, cited from Feigenson, 2004).

Before review details, we need to know some knowledge of mathematical ability. The mathematical competence was divided into two classifies. One was quantity representation, the other was calculation ability. For the former one was further divided into a precise small number representation system and an approximate large number representation system; the later one was also divided into simple calculation and higher calculation processing (Izard et al., 2008; Bongard & Nieder, 2010; Cohen et al., 2008).

To figure out the last-mentioned debate, in the present paper, firstly, we will review the evidences for quantity representation and calculation ability for animals, preverbal human infants, primitive tribe people and lesion case for human adults in the following sections.

2. Review of quantity representation: exact representation of small numerosity and approximate representations of numerosity

Through the observation, we can find that animal have the number sense in some degree (Núñez, 2017). However, with the common sense, we know that animals cannot figure out the advanced math problem such as advanced algebra. What's the limitation of numerical ability for animals? Although nonhuman animals do not endow the language ability to label or enumerate different items, many studies have found that animals (such as guppies, bees, pigeons, rats, dogs, dolphins and monkeys) can differentiate different quantity of items (Bisazza et al., 2014; Pahl et al. 2013; Pepperberg, 2013; Piazza & Dehaene, 2004, for review; Boysen & Berntson, 1989). For example, when presented two boxes which contained different numbers of apple pieces to the semi-free monkeys, Hauser and his colleagues found that monkeys prefer to choice the box with more pieces of apple in it while the items number is not larger than 4 (the comparison number pairs of apple pieces included number pairs -1 vs. 2, 2 vs. 3, 3 vs. 4, 3 vs. 5). However, when the number of apple slices in each box was greater than 4 (such as the number pairs like 4 vs. 5, 4 vs. 6, 4 vs. 8, 3 vs. 8), monkeys choose the box randomly (Hauser et al., 2000). The result indicated that the monkeys can discriminate the number but with the limitation around 4. Indeed, recent studies showed that not only vertebrates but also in some invertebrate taxa. Furthermore, this result was supported by many animal studies which employed different paradigms and materials (Pahl et al., 2013; Agrillo & Bisazza, 2014; Bisazza et al., 2014; Santos, 2004; Hauser & Carey, 2003).

Similarly, the comparable numeral ability also found in human infants. By using the habituation paradigm to present different numbers of dots stimuli, Starkey and Cooper (1980) found that 4- to 7-month-old infants can discriminate between 2 and 3 dots, but failed to discriminate 4 dots from 6 dots. Moreover, older infants (6 to 12-month-old infants) even can differentiate numerosities in 3 vs. 4 (Van Loosbroek & Smitsman, 1990; Starkey et al., 1990).

Actually, by summary the previous mathematical related studies investigated from different kinds of participants, Feigenson (2004) pointed out when the quantities are smaller than 4, both animal and preverbal human infant endowed the numerical ability to precise represent the numerosity. This view means that animals and infants can absolutely discriminate the individuals within the limited number to 3 or 4.

Besides the ability of exact presentation of small numerosity, both animal and preverbal human infant also endow the ability for approximate representing large numerosity while the quantities is larger than 5 (Gebuis et al., 2016; Feigenson, 2004). Furthermore, large approximate number discrimination is an estimate number performance which is often influenced by the variable ration between different numerosities and become poorer and poorer as the number increase (Piazza, 2004). For animal, their variable estimate performance obeys the "Weber's Law". For example, Pigeons compared their numbers of pecks by pecking one of the two center keys, and they can discriminate 35 between 50 pecks above the 90% correct rate. However, their response correct rate decreased below 60% when the peck number comparison was 45 vs. 50 by the ration of 9:10 (Rillin & McDiarmid, 1965). More specifically, the response of participants became more accurate and faster when the ration of comparison number was relative lager (Gilmore et al., 2011; Gebuis & Van der Smagt, 2011).

Likewise, for preverbal human infants, even newborn infants (11 to 105-hour-old) can differentiate between 6 dots and 18 dots (Izard et al., 2009) or 3 and 9 items (Coubart et al., 2014) by the ration of 1:3 (Izard et al., 2009). Moreover, with the increase of the age the infant's estimate ability can be improved. It was found that 6-month-old infants can discriminate between 8 dots and 16 dots by the ration of 1:2 (Xu & Spelke, 2000) and the appropriate number discrimination of 9-month-old infants reach the ration up to 2:3 (Feigenson, 2007). And what interesting was that only the studies which used the non-visual dots as experimental stimuli can detect the ration of 2:3 in infants' large number discrimination ability (Lipton & Spelke, 2004; Wood & Spelke, 2005). For detail, Lipton et al. (2004) applied auditory sounds as stimuli, and they found 9-month-old infants successfully discriminated 8 versus 12 sounds; while Wood et al used moving events (jump sequences of a puppet) as stimuli and they found 9-month-old infants successfully discriminated between the sequences presenting 4 versus 6 actions.

3. Review of the studies for arithmetic capability

The animals and preverbal infants not only endow the ability to represent of numerical quantities (Sasanguie, Defever, Maertens, & Reynvoet, 2014), but also possess the ability to solve simple mathematical problems (Starr, Libertus, & Brannon, 2013). Several studies have proposed that 5-month-old infants shown sensitive to the impossible outcomes of addition and subtraction operation on small sets of items (Wynn, 1992; Simon et al., 1995). By using the violation of expectation method, Wynn (1992) placed two toys, one after the other, into a box which amounts to an exercise in addition '1+1' (Similarly, by removing one of two toys from the box constitutes subtraction '2-1'); then masked the box ; lastly, shown the outcome to the infants. But the outcomes might be 1 or 2 for '1+1' (similarly, 2-1 = 1 or 2). She found that the infants stared longer at the box when it displayed the incorrect outcome (such as the operand 1+1=1 and 2-1=1). Later on, Simon et al. (1995) used video to present the similar operands to the infants. Their result further supported Wynn's outcome. The same simple mathematical ability continued to be evidenced in other animal species. Hauser and Carey (2003) carried out a series of experiments, which were similar to what have been done in Wynn's and Simon's studies, to the monkeys. The results revealed that monkeys' looking time were different in impossible/possible outcome in conditions such as 1 +1= 2 or 3, 2+ 1 =2 or 3, and 2 +1=3 or 4. These researches indicated that both infants' and animals' cognition system was not only limited in representing the number, but also can perform simple arithmetic functions.

By summary the above behavior studies, both animal and preverbal infant have the mathematical ability. Accordingly, it seems that the mathematical ability universal exists without language (Feigenson, Libertus &

Halberda, 2013; Smyth & Ansari, 2017). Also, this viewpoint has been proved by some special lesion cases from aphasia patient (Cohen et al., 2000; Klessinger et al., 2007; Urano et al., 2009), and we will review it in next section.

4. Review of the Role of Language for Mathematics Ability

4.1 Lesion Cases

Urana et al. (2009) reported a 68-year-old woman with severe global aphasia who cannot spontaneous speak, repeat, reading aloud and comprehend words in everyday life. However, she has the quantity represent competence by point out a visual Arabic number to auditory presented numbers and vice versa. Further tests shown that this patient even has the arithmetical ability to figure out simple addition, subtraction and multiplication (e.g. $5 + 6 = 11$ or 13, $9 – 3 = 6$ or 4) by choosing the correct answer from the alternative answers. With regard to the mathematical performance in previous aphasia cases, actually, it was found that the aphasia patients' mathematical performance was not only limited to quantity representation and simple calculation, but also up to the higher algebraic arithmetic. The most famous case (Klessinger et al., 2007) was that a patient called SO whose left hemisphere including the posterior aspect of the middle and inferior frontal gyri, the left temporal lobe and the anterior portion of the superior and inferior parietal lobe were seriously damaged. Due to the wide range of damage in left hemisphere, SO got a severe aphasia and an apraxia of speech, but even so he showed normal capacity to perform higher order algebraic arithmetic such as multiplication and division of algebraic terms (e.g., $3y \times 5a = ?$, or $y \div 2y=?$), simplify and transform expressions with alphanumeric terms (e.g.,$7a - 3b + 3a - 2c + 3b + 4c = 10a - 2c$) and so on; But then his performance was impaired in solving algebraic fractions and mixed expressions (e.g., ****). Conversely, some acalculia cases showed the reverse pattern of behavior and lesion location (Takahata et al., 2014; Cohen et al., 2008). Patients who got isolated calculation disturbances were due to the lesion in the left parietal region while without damaging any language related area (Takayama et al., 1994)

These cases added further empirical evidences to the views that some aspects of mathematical processing can be sustained despite severe destroy to the language competence and some kinds of mathematical disability can be occurred without language deficit.

Does it mean that language have no effect on mathematical competence? The answer for this question must be 'no'. Next, we will review some studies to show how language influences the development of the mathematical ability both in animal and human.

4.2 The Speaking-animal and Less-number-word Primitive Tribes

Earlier experiments have revealed that a wonderful speaking bird, a parrot which was taught and trained by an animal psychologist, could speak out to recognize 50 objects, 7 colors, 5 shapes and represent quantities up to 6 as well as have the zero like concept (Pepperberg, 2005). Compared with other animal species with limitation of precise representation up to 3, this bird has higher numerical ability (up to 6) might be due to its brilliant language ability—he mastered the vocabulary of about 150 words. Thus, he might use the number words to label different quantities so that he can discriminate them better. The role of language in mathematical ability was also tested people who live in primitive tribe (Pica et al., 2004； Gordon, 2004). These people do have the language in their culture. However, their language systems have limited words to describe numbers. One of tested tribe groups called Munduruku whose language lacking words to describe numbers above 5 (Pica et al., 2004). The adult people in this tribe were tested for several mathematical tasks such as (a) larger number comparison for discriminate number dots which range from 20 to 80 and (b) exact subtraction performance to predict the outcome of a subtraction of a set of dots from an initial set which might comprise 1 to 8 dots. The results shown that (1), for task (a), participants were able to discriminate from 2 numbers by the ration of 1:1.2 and (2), for the task (b), participants always failed to answer correctly when the initial operand was larger than 5.

The other tested group was called Piraha, and there are 3 number words to represent '1', '2' and 'many' when numbers larger than 2 in its culture (Gordon, 2004). Similarly, a series of mathematical task were tested to Piraha people. For the first part of tasks, after watching experimenter placed numbers of items one by one in line, participants were asked to perform the same task or to place the same number items in different forms (e.g. cluster, orthogonal line and uneven line). As a result, Piraha participants' performance deteriorated to about 60% correct rate when the manipulated number of items was larger than 4 in most of tasks. In the second part of experiment, participants were also asked to do subtraction tasks, and the outcome of their performance was similar to what has been tested in Munduruku people.

In sum, these two primitive tribe cases shown that tribe people's limited number words language system depressed their mathematical ability in comparison with normal adults who live in modern society. It was reported that modern culture adults can approximate discriminate two sets of large number items up to the ration of 7:8 (Feigenson, 2004; 2007), easily manipulate the basic arithmetic operations and even to solve advanced mathematics.

Altogether, the cases of speaking parrot and primitive tribes provided rare and unique cases to underscore that the strong linguistic determinism view on numerical competence development.

4.3 Image Studies

Further clarifying the role of language in mathematical cognition was also been found in case studies of pathology and fMRI studies (Wang et al., 2015; Klessinger et al., 2007; Urano et al., 2009; Sandrini et al., 2003; Delazer & Bartha, 2001; Dehaene & Cohen, 1991). As mentioned earlier, several aphasia cases have shown that mathematics was independent of the language ability (Klessinger et al., 2007; Urano et al., 2009). However, it was extensive reported that aphasia, especially for Wernicke's aphasia, is often accompanied with impaired calculation ability (Sandrini et al., 2003; Delazer & Bartha, 2001; Delazer et al., 1999, Dehaene & Cohen, 1991). For example, a patient called DB with damage centered on the language areas in left hemisphere and got a severe aphasia (fluent aphasia with impaired auditory comprehension). The detailed investigation found that she preserved related intact ability in calculate the addition and subtraction (e.g. she could correctly perform expression like 236-41=195), but she was completely unable to solve simple multiplication and division (Sandrini et al. 2003).

Expect for the patients, normal people's mathematical ability was investigated by using the fMRI technology. In Simon et al.'s (2002) study, specially, one of their aims was to localize calculation and language-related activations in normal people. As a result, they found several common areas were activated in both calculation task and language related phoneme detection task. And the common activated areas included angular, the left inferior frontal gyrus in the region corresponding to Broca's area (BA 44/45), superior frontal gyrus, bilateral intraparietal sulcus in the parietal lobe. The overlap brain area of language and mathematical tasks implied that mathematical process might consult the substrate of language processing.

4.4 Summary

To sum up, the results of these studies, especially those came from the lesion studies might induce a controversial debate on the relationship between mathematics and language (Yang & Meng, 2016). However, over the past decades of investigation, researchers have drawn a clear line for this debate.

Firstly, the results of comparative psychological studies have shown that the quantity representation ability existed in animals, infants and primitive tribe cultures. And the neuroimaging and animal lesion studies also found quantity (both precise and approximate systems) representation was localize in intraparietal sulcus (IPS) for all of monkeys', infants' and human adults' brain. It does not depend on the language and education (Izard et al., 2008; Bongard & Nieder, 2010; Cohen et al., 2008; Nieder & Dehaene, 2009; Piazza & Izard, 2009).

Secondly, studies have shown that not only the adult but also infants and several species of animals can perform the rudimentary arithmetic operations. But only educated human being can perform the complicate calculation with intraparietal sulcus was systematically activated and a large range of left frontal cortex (L-PFC) as well as angular were also involved (Dehaene et al., 2004). And researchers suggested that complex, advance mathematics calculation needed participants to retrieve the mathematical facts, calculation rules and compute strategies which have been coded and stored as semantic language related format in human brain, so that this kind of calculation processing activated the language related area -- L-PFC and angular. (Bongard & Nieder, 2010; Schneider et al., 2009)

Thirdly, some forms of calculation such as geometry, algebra might be considered as a special calculation format. For example, algebra, some time, only needs to transform or simplify the expression (e.g., $2a + a + 4 + 3 = ?$) and does not need to retrieve language involved information during the calculation; thus this kind of calculation might activated the quantities represented area-IPS only (Klessinger et al., 2007).

5. Discussion and Possible Future Directions

As shown by above examples, the understanding of the relation between mathematics and language was largely clear: it can be included that both humans and non-human primates share an elemental quantification system that exists in a dedicated brain neural network (Nieder, 2016), only except for some details about the relationship between the language and complex calculation. And the evidence from a meta-analysis review study also demonstrated that there was high correlation between language proficiency and mathematics outcomes (He, 2016);

Firstly, the concept of mathematical ability is so complex that it includes all kinds of math problem. Even for the 'advance mathematics', it also needs to be clarified carefully. For the reason that the problem like algebra can also evolve to complex calculation; but the studies showed that some language deficit patients were able to solve this kind of problem (Klessinger et al., 2007). And study also found that Munkudura people can also solve the geometry problem which was considered as more advanced (Dehaene et al., 2006). Thus, two points should be shed light on, (1) more studies should be carried out to make sure which kinds of calculation may need or not need the support of language system.; (2) studies should be distinguished from different types of aphasia to make sure whether their corresponding locations are contributive to calculation or not (this has been explored by Sandrini et al., 2003).

Secondly, a following question is even if we confirm language related brain areas active during the state of mathematic solving. However, the question that what's the function of language in math processing is still leave to open. Is it only for retrieving the mathematical facts and calculation rules? In other words, can we say that the language processing was involved as long as the language related areas, such as LPFC, angular cortex, were active in a certain task? Many animal studies found monkey's LPFC neurons activate during the number related tasks (Nieder et al., 2003; 2004; 2006；Bongard & Nieder, 2010; Diester, 2007). According to the hypothesis, we should deduce that monkeys can speak or understand or have language, in which sounds ridiculously. Thus, we must consider the study logic seriously in the research of the function of language in mathematics.

Thirdly, is it the language ability crucial to get higher mathematical ability? For this question, the animal studies have shown that both monkeys and pigeons can sort the absolute numberrosities from 1 to 8 (Hirai & JitSuMori, 2009) or even up to 9 (Brannon et al., 1998) in an ascending numeral order. These cases implied that language might not be the only way to improve the mathematical ability while training may be one of the important ways. Furthermore, the results of limitation ability for animals and infants may be just because that we did not investigate their ability in a best suitable way. For example, when tested for the approximate representation of large number, the infant studies, which were mentioned previously, applied sounds or moving events as stimulus often found infants' got a higher ability than that used static dots as stimuli. Similarly, the same proposal can also be applied in the future animal studies. For example, dots' smell is keener than their vision. If testing their mathematical ability by using a suitable smell form of stimuli, we may find their truly ability is beyond what we have known. Here, what we want to emphasize is to explore more paradigms which are associate with the advantage of participants' physical development may help us well-understood truly limitation of mathematical ability cross species.

Fourthly, it was reported that there were neurophysiological and anatomical differences between the brains of male and female adults (Zell et al., 2015). So further investigations may try to trace back the critical development point that when this difference has been settled down and its influence to the mathematical ability. Thus, we can get more knowledge to improve our mathematical learning.

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