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Applying Game Theory to Optimize Expenses for Employees' Remuneration

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Abstract

This article contains methodological aspects of expenses optimization for compensation of employees. An analytical evaluation of the funds allocated to the remuneration of personnel based on the application of game theory models is suggested. Calculations are recommended to regulate the expenses for remuneration in accordance with the objectives of the company.

Keywords: the expenses for remuneration, game theory, management analysis, managerial decisions

1. Introduction

Evolution of expenses for employees' compensation is not always a formal and predictable process which arises from the long-range business objectives since expenses optimization can be carried out during successive change of the elements states of the system. Expenses optimization for remuneration is characterized by flexibility and dynamics, orientation toward change, the implementation through actions, which is estimated during management analysis. Analytical system provides funds forecasting which are allocated to finance the total compensation of employees, determines the strategic interaction of the factors that contribute to the change in expenses, and generates information for the developing and making managerial decisions. These problems can be solved by applying the methods of study of optimal strategies, one of which is the games theory.

2. Method

Game theory is a quantitative analytical approach to modeling the probabilities of different behavior scenarios of the system studied which determines the possible actions of different parties, the outcomes and their implications. This theory of mathematical models facilitates making optimal decisions in the conditions of uncertainty or potential conflict of parties with different interests and which have potential opportunities to use diverse actions to achieve their goals.

Situations that arise in reality, as a rule, are quite complex, and their study is hampered by different circumstances. Therefore, to make the analysis of uncertain situation more possible, one should be distracted from the less significant factors, which allows us to construct a simplified formal model of an economic situation in the form of a mathematical game. Game theory models enable to estimate different scenarios and demonstrate possible solutions. In all unclear circumstances a careful application of game theory elements helps the parties to determine the correct position depending on the goals that were set. Because of this, when dealing with the issue of employees' adequate compensation and, consequently, when optimizing expenses for personnel remuneration, elements of game theory can be applied.

3. Results

When forecasting expenses for personnel remuneration, situations of uncertainty may occur rather often, because it affects employee' personal interests on the one hand, and the interests of the company to optimize the expenses for compensation - on the other, so that there are conflicts between the administration and the subordinate staff.

If to consider the situation of "management-employee" from the perspective of game theory, there are two parties involved: *A* - administration safeguarding the interests of the organization in effective use of funds for remuneration and *B* - worker. Player *A* (administration) has *m* behavioral strategies, player *B* - *n* strategies worker. Administration (player *A*), without knowing the employee's choice, selects the strategy A_i (i = 1,2, … m). Worker (player *B*) selects one of his possible strategies B_j (j =1,2, … n). For each pair of strategies (A_i, B_j), payment a_{ij} of the first player to the second one is defined, or Administration's (player *A*) gain, then there is a

figure reflecting the degree of satisfaction of the company's interests in this situation. Employee's gain (player *B*) will be ($-a_{ij}$) consequently. There is no discrimination in relation to the second player, as a_{ij} quantities may be negative. This condition shows that in the given circumstances one player's gain is equal to another's with the opposite sign. For example, a_{13} = - 2 player *A* gain, $-a_{13}$ = 2 - player *B* gain. Therefore, in the analysis of this kind of game it is possible to study the gain of only one of the parties.

If a_{ij} values of gain with each pair of strategies are known, they can be written in the form of the payoff matrix. Then a_{ij} values are payments which are the sum of reward or utility in a specific strategy together with the specific circumstances. When relating the strategies of the parties to payments, payoff matrix can be written as:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & \cdots & B_n \end{array} \\ \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \end{array} \quad (1)$$

where: A_i (i =1,2, … m) - strategy for player *A* (administration); B_j (j =1,2, … n) - strategy for player *B* (employee); a_{ij} - elements of the payoff matrix.

The use of game theory elements for optimizing the expenses for employees' remuneration is considered on the model of *Administration – employee* uncertain situation of *Shoe factory Sparta*k, an oldest enterprise in Russia.

The worker of shop №2 *Shoes assembly shop* (worker) has the third qualification grade, working in a shoe factory under the supervision of foreman who safeguards the interests of the enterprise administration. Providing that the production task is accomplished, the expenses for constant component of the worker's basic remuneration is 11 250 rubles. (According to the wage rate scale of the 3rd qualification grade).

In order to optimize the expenses for constant component of the worker's basic remuneration which satisfies both parties, elements of game theory are used. The administration in the person of the foreman (player *A*) has the following behavior strategies:

1) to accept employee's resignation - strategy A_1;

2) to upgrade the qualification category for multimachine service, with the growth of constant component of the employee's basic remuneration by 300 rubles - strategy A_2;

3) to transfer the worker to the shop № 4 *Cutting shop* where work is more labour-consuming, it is operated on machines with manual mode and the wage rate of the 3rd qualification grade is higher. As a result, the constant component of the employee's basic remuneration increases by 450 rubles - strategy A_3.

Employee - player *B* has the following behavior strategies:

1) to accomplish a production task - strategy B_1;

2) to request the resignation due to the low constant component of basic remuneration - strategy B_2;

3) to request doing additional types of work in the profession along with the main functions. Consequently, the extra pay for the plurality of professions will make 400 rubles - strategy B_3.

To make up the payoff matrix, we analyzed each of the parties' behavior and considered the possible outcomes when administration and employees choose different strategies.

Strategy 1: Administration selects the first strategy A_1 and decides to fire the employee. However, if the employee does not want to be fired, and accomplishes production task (strategy B_1), then there are no grounds for dismissal, thus in selecting these strategies payment will be zero. If an employee decides to resign (strategy B_2), the administration carries out its decision and the expenses for constant component of the basic remuneration of production workers on the shop floor № 2 will be less than for the current month by the amount of employee benefits, i.e. payments are equal to -11 250 rubles. If an employee decides to request doing additional types of work along with his main functions in the profession (strategy B_3), the administration is required to increase the expenses for a constant component of the basic remuneration of the department production workers in the amount of a fee for the plurality of professions by 400 rubles. Therefore, with employee behaving so, administration keeps to its decision and fires the employee. In this case, the expenses for constant component of the basic remuneration of the department production workers in shop № 2 will be less for the current month in the amount of employee benefits and fee for plurality of professions (payment is -11,650 rubles).

Strategy 2: administration chooses its second strategy A_2 - to upgrade employee's qualification. If an employee chooses the accomplishment of production tasks (strategy B_1), the administration is not required to raise the expenses for constant component of the basic remuneration of the employee, due to changes in skills category (payment -300 rubles), because it is not necessary to motivate the employee (as he accomplished the plan with lower qualification grade). If an employee resigns (strategy B_2), the administration is not required to increase the qualification grade, and the expenses for a constant component of the basic remuneration for the period decrease (payment -11,550 rubles.). If an employee decides to request additional work in his profession (strategy B_3), due to the lack of additional amount of work, the administration will not make additional payments for the plurality of professions, but, alternatively, may increase the qualification grade. As a result, the expenses for a constant component of the basic remuneration will increase (payment of 300 rubles).

Strategy 3: administration chooses to transfer the employee to cutting shop - strategy A_3. Administration, on its own initiative, transfers employee in another shop for the same qualification grade, and, providing that he accomplished the production task (strategy B_1), the employee's constant component of the basic remuneration will increase by 450 rubles (payment -450 rubles.). If an employee decides to resign (strategy B_2), the administration does not have either to transfer an employee to another shop or to pay a constant component of the basic remuneration (payment -11 700 rubles.). If an employee asks for additional work to obtain additional payments for the plurality of professions, the administration will transfer the worker to another shop in order to optimize the expenses for remuneration, but only with the 2nd qualification grade (strategy A_3), i.e., without increasing the constant component of basic remuneration: payment will then be -450 rubles in favor of the administration.

As a result of the above strategies, payoff matrix (gains matrix) of "administration-worker" game is drawn:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{pmatrix} 0 & -11250 & -11650 \\ -300 & -11550 & 300 \\ -450 & -11700 & -450 \end{pmatrix} \end{array} \tag{2}$$

With the payoff matrix, we should define the rationale for the appropriateness degree of the methods known to solve this problem by modeling the actions of the parties. To exclude additional measurements and the inevitable simplification of the original matrix we will solve the problem as a whole - in the terms of moves by "nature" or statistical games. Since this approach addresses issues related to decision made only by one of the parties (the administration A strategy), the possible strategies for the employee (B) are determined independently of the administration actions and have at least the objective possibility to manifest themselves as a state of "nature", i.e. as a party with respect to which one can make assumptions k. Then the problem of one of the party A involved (player-administration) is to choose one of the m formed strategies, which is optimal for it, without taking into account the employee's interests (party B), but only considering its own interests. The fact that party B (employee) formulates his n proposals (uncoordinated with the administration actions) independently (neutrally) of the game outcome, can be considered, in this case, as a methodological assumption. Also, given the fact that player A (administration), according to our assumption, does not have both objective and expert information about the probabilities of possible states of "nature", and all information about the situation in which it is necessary to act, lies only in the payoff matrix, it is possible to use several options (criteria) for solutions optimality. Let us consider the most popular ones in statistical decision theory: Wald (W), Savage (S) and Hurwitz (H) criteria.

According to the Wald criterion (W) as optimal strategy it is defined the administration's (A_{opt}), in which the minimum gain is maximal, i.e. strategy to ensure gain is no less than maximin (Sarkisyan, 1977, p. 310):

$$W = \max_{i=1..m} \min_{j=1..n} a_{ij} \tag{3}$$

where: a_{ij} - elements of the payoff matrix. Solution by Wald equals -11 550 rubles (Table 1).

Table 1. Solution of the game *administration-employee* under the criterion of Wald

Strategy	*Nature* state B_j			W
A_i	B_1	B_2	B_3	
A_1	0	-11250	-11650	-11650
A_2	-300	-11550	300	**-11550**
A_3	-450	-11700	-450	-11700

Solution of the game *administration-employee* under the criterion of Wald

The optimal strategy, according to Wald test, is $A_{opt} = A_2$. This criterion aims administration at not most favorable conditions created for the employee, and recommends choosing the strategy, in which in the worst conditions the gain is maximum.

In order to verify the results obtained by Wald test (extreme pessimism criterion) Savage test is used which, in contrast to Wald test, in conditions of stochastic uncertainty chooses the strategy in which the risk is minimum in the worst case (minimum risk guarantee) (Sarkisyan, 1977, p. 310):

$$S = \min_{i=1..m} \max_{j=1..n} r_{ij} \tag{4}$$

where: r_{ij} - the elements of risk in size $m \times n$; $r_{ij} = b_j - a_{ij}$, wherein $r_{ij} > 0$, since $b_j = \max a_{ij}$.

According to Savage criterion b_j is the maximum possible gain of player-administration at the state of *nature* B_j. This value is some measure of favorability for the player of j-th state of *nature*.

Solution by Savage within the resulting matrix of risk will be equal to 600 rubles (Table 2).

The optimal strategy according to Savage criterion is $A_{opt} = A_1$. This criterion focuses the administration on the entire set of conditions imposed by the employee, and recommends to choose the strategy in which the risk is minimal.

Table 2. Solution of the game *administration-employee* by Savage criterion

Strategy A_i	*Nature* state B_j			S
	B_1	B_2	B_3	
A_1	0	600	0	**600**
A_2	11250	11850	11250	11850
A_3	11650	0	0	11650

Thus, by varying the input data both in a gains matrix and in a risk matrix, we have obtained different outcomes, these are strategy A_1 and A_2. To complement the picture obtained, Hurwitz criterion is used.

As it is known, Hurwitz criterion (H) is constructed directly, as well as Wald criterion is, on the payoff matrix and has the form (Sarkisyan, 1977, p. 311):

$$H = \max_{i=1..m} \left[\alpha \min_{j=1..n} a_{ij} + (1-\alpha) \max_{j=1..n} a_{ij} \right] \tag{5}$$

where: $\alpha \in [0;1]$ - a factor which expresses the measure of pessimism / optimism of the player which makes decision.

The outcomes of the solution by Hurwitz criterion (H) are summarized in Table 3 for values of α, strictly different for reasons from 1 and 0. The expert selected value α = 0,75 - moderate pessimism, α = 0,25 - moderate optimism and α = 0, 50 – average between them.

Table 3. Solution of the game *administration-employee* by Hurwitz criterion

Strategy	"Nature" state B_j			min a_{ij}	max a_{ij}	H when	H when	H when
A_i	B_1	B_2	B_3			α=0,75	α=0,5	α=0,25
A_1	0	-300	450	-300	450	-8737,5	-5825,0	-2912,5
A_2	-11250	-11550	-11700	-11700	-11250	-8587,5	-5625,0	**-2662,5**
A_3	-11250	300	-450	-11250	300	-8887,5	-6075,0	-3262,5

According to Table 3, the total variation of pessimism α measure for specific values of the payoff matrix influences the choice of the optimal strategy: $A_{opt} = A_2$. We should note that this solution is achieved both in choosing a strategy of extreme pessimism by Wald test (α = 1 by the expression for Hurwitz criteria), and in the situation of quite moderate level of pessimism, for which α = 0,75. From this we can conclude that the use of extreme and moderate pessimism criteria (Wald test, W and Hurwitz criterion, H), even with a moderate level of pessimism leads to the conclusion that the best strategy is A_2. If to follow the strategy of minimizing the risk (Savage, S), then the optimal strategy is A_1, but in this case, the administration should dismiss the employee that may not fully meet the interests of the organization.

4. Conclusions

Thus, these results demonstrate that the proposed approach to the solution of the problem allows management to organize its solution in the terms of moves by "nature" when the managed unit of the closed system studied (administration-worker) has the ability to simulate its possible actions. That is, it can implement management strategies A_i under neutral interest of workers taking into account full source information in the form of the generated payoff matrix in size ($m = 3$ x $n = 3$) obtaining the outcomes and suggested ways of their mutual verification within separate criteria (Wald, Savage, Hurwitz).

Consequently, game theory is applicable for finding optimal solutions in disagreement or uncertainty because game theory is not only the science about the content of the strategic thinking, but also methodological and practical means of solving management problems, in particular, tasks to optimize the expenses for staff remuneration.

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