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Approximate Methods of the Decision Differential the Equations for Continuous Models of Economy

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Abstract

The review of some models of economy based on application of the ordinary differential equations is provided in article linear and nonlinear, and also the review of approximate methods of their decision. Expediency of use of this or that numerical methods of the solution of the differential equations is shown. For some equations of continuous models of economy in work the approximate method is offered and reasonable. Estimates of convergence of approximate methods are given in the corresponding functional spaces. Results of the numerical solution of these equations in the form of tables and schedules are also received. The comparative analysis of the received results is carried out.

Keywords: differential equations, approximate methods, continuous models of economy

1. Introduction

The review of some models of economy based on application of the ordinary differential equations is provided in article linear and nonlinear, and also the review of approximate methods of their decision. Expediency of use of this or that numerical metods of the solution of the differential equations is shown. For some equations of continuous models of economy in work the approximate method is offered and reasonable. Estimates of convergence of approximate methods are given in the corresponding functional spaces. Results of the numerical solution of these equations in the form of tables and schedules are also received. The comparative analysis of the received results is carried out.

2. Theory

It is known (Crassus & Chuprynov, 2002; Ross, 2006) that some continuous models of economy, for example, investigating economic dynamics, defining evolutions of economic systems are closely connected with the differential equations, with systems of the differential equations of the first order which, in turn, at the decision lead to the equations of the highest orders. So, for example in the study of the dynamics of capital-labor ratio (in the neoclassical growth model) it is presented as a function of time t and obtain the following nonlinear first order differential equation with separable variables:

$$k' = lf(k) - (\propto + \beta)k \tag{1}$$

Productivity as the function of time determined through *Y = F (K, L)* — the national income, where F - uniform production function of the first order for which is fair F (TK,TL) = TF (K, L), To - the volume of capital investments (business assets), L — the volume of expenses of work. We will enter into consideration k capital-labor size = K/L, then labor productivity is expressed by a formula:

$$f(k) = \frac{F(K,L)}{L} = F(k,1) \tag{2}$$

The purpose of a task is the description of dynamics of a capital-labor or its representation as functions from t time. As any model is based on certain prerequisites, we need to make some assumptions and to enter a number of the defining parameters.

Suppose that the natural increase in temporary workforce:

$$L' = \alpha L \tag{3}$$

Investment spent on increasing production assets and depreciation, that is

$$I = K' + \beta K,$$

where β - the depreciation rate.

Then if L - the rate of investment, so $I = LY = K' + \beta K$ or

$$K' = lF(K, L) - \beta K \tag{4}$$

From the definition of capital-labor ratio K implies that

$$lnk = lnK - lnL.$$

Differentiating this equation with respect to t, we have

$$\frac{k'}{k} = \frac{K'}{K} - \frac{L'}{L}$$

Substituting this relation in the expression (3) and (4), we obtain the equation for the unknown function K

$$k' = lf(k) - (\propto + \beta)k,$$

Where the function is defined by the formula (2)

The resulting ratio is a non-linear first order differential equation with separable variables.

Consider the specific task. Find the integral curves of the equation (1) and the steady-state solution for the production function. From (2) it follows that

$$F(K, L) = \sqrt{KL},$$

and then equation (1) has the form

$$\frac{dk}{dt} = l\sqrt{k} - (\propto + \beta)k \tag{5}$$

Many models of economy lead to the differential equations, both linear, and nonlinear, the first and second orders.

In certain cases it isn't possible to find the exact solution of such equations therefore there is a need to apply numerical methods to finding of the approximate decision of these of the equation. In this regard it would be pertinent to mention approximate methods of the solution of the differential equations.

The choice of this or that numerical method is defined by a variety of reasons. Some of them are features of this class of tasks, requirements imposed to the numerical solution in the field of science and appendices, possibility of computer facilities, and also scientific traditions, qualification of developers.

At the same time, it should be noted also theoretical aspect of research. If Runge-Kutt's method used in the works (Vorontsova, 2013; Goncharova & Vorontsova, 2008; Vorontsova, 2011), despite the labor input, possesses considerable accuracy and is widely applied at computer calculation, however, it isn't possible to apply it to theoretical justification of the approximate decision for a wide class of tasks, in a type of the above. Whereas straight lines and projective methods possess this quality.

From a big set of direct methods (Samarsky & Gulin, 1989; Semushin & Algebras, 2006) it is possible to allocate group of projective methods, such as Galerkin's method (Bagoutdinova & Zolotonosov, 2007), collocations (Bagoutdinova & Zolotonosov, 2004), subareas, spline methods. They are more convenient for theoretical justification of existence and an assessment of an error of the approximate decision, however, difficult realized in practice.

For the solution of the equations (methods of Ritz, Bubnova – Galerkina) the set of works is devoted to application of straight lines and projective methods (Gorskaya & Ojegova, 2013; Gorskaya, Zolotonosov, & Barmin, 2013) recently. It is connected with that questions of theoretical researches of these equations and possibility of finding of their optimum decision represent a great interest for modern science.

Despite the results received for research of the differential equations (Vorontsova, 2013; Goncharova & Vorontsova, 2008; Vorontsova, 2011; Beletsky & Vorontsova, 2000) now questions of stay and justification of approximate methods of their decision very are particularly acute.

3. Results

We will consider a task for the differential equation of a look (Beletsky & Vorontsova, 2000):

$$-\frac{d^2u}{dx^2}+\frac{du}{dx}+qu=f(x), \qquad x\in(0,1), \quad q>0, \tag{6}$$

$$u(0)=u(1)=0,$$

or in an operator form: $Au+Bu=f$, where $Au=-d^2u/dx^2$, $Bu=du/dx+qu$. As D (A) we will take a set twice continuously differentiable on (0,1) functions meeting the set regional conditions. The power space generated by the operator A: H_A −space of the functions belonging to W_2^1, meeting the set boundary conditions. It is possible to take system of functions in qualities of basic functions $\varphi_i = sini\pi x$, full in H_A space.

Then the approximate solution of the equation (6) is looked for in a look:

$$u_N=\sum_{i=1}^{N} a_i sini\pi x.$$

Unknown coefficients of a_i are defined according to Bubnov-Galerkina's method from the system of the linear algebraic equations which is written down in a matrix look: $La=f$, где $L=\{L_{ij}\}$, $f=(f_1,\dots,f_N)^{\mathrm{T}}$, where

$$L_{ij}=\int_0^1\left(\frac{d\varphi_i}{dx}\frac{d\varphi_j}{dx}+\frac{d\varphi_j}{dx}\varphi_i+q\varphi_i\varphi_j\right)dx, \quad f_i=(f,\varphi_i)=\int_0^1 f(x)\varphi_i(x)dx.$$

Then for the equation (6) the following theorem is fair.

Theorem. Let the operator $A^{-1}B$ be quite continuous in H_A.Then at enough big N method of Bubnov-Galerkina gives the only approximate solution of u_N, which meets to the generalized solution u of the equation (6) on norm of H_A.

The proof of the theorem is kept according to (Marchuk & Agoshkov, 1981).

Further by way of illustration we will give examples of solutions of the differential equations for some models of economy by various approximation methods. As a first example we present the solution of the nonlinear differential equation (5) by Runge-Kutta method.

We adopt in equation (5) α = 2; β = 1,5; L = 10, and the initial time Y = 2, then the matrix of Zr, the resulting solution of the nonlinear first order differential equation with separable variables has two columns: the first column contains the values of t, in which the solution is sought; the second column contains the values of the found solution K (t) at the corresponding points. The solution was prepared at 300 in a packet Mathcad.

Zr =

	0	1
0	0	2
1	0.033	2.238
2	0.067	2.475
3	0.1	2.709
4	0.133	2.94
5	0.167	3.166
6	0.2	3.387
7	0.233	3.602
8	0.267	3.811
9	0.3	4.014
10	0.333	4.21
11	0.367	4.4
12	0.4	4.582
13	0.433	4.758
14	0.467	4.926
15	0.5	5.088
16	0.533	5.243
17	0.567	5.391

Family of integral curves in Figure 1 converges to the top and bottom of the stationary solution

$K_{st}=\left(\frac{L}{\alpha+\beta}\right)^2$, so $K \to K_{St}$ when $T \to \infty$.

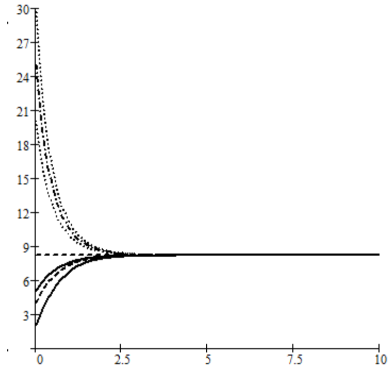

Figure 1.

Consequently, at constant input parameters of the problem L, α and β function of the assets-in this case tends to stable steady-state value, regardless of nachalnyh conditions. Such a stationary point $K = K_{St}$ is a point of stable equilibrium, which is consistent with the known results of economic theory (Zhilyakov, Perlov, & Revtova, 2004). As an illustration, consider the following approximate methods for linear differential equation for the model with the projected market prices.

$$P'' + 2P' + 5P = 15. \tag{7}$$

Consider the case 1. Assume that at P (0) = 4, and P'(0) = 1, then the matrix Z_w (8) obtained by solving the linear inhomogeneous second order differential equation for the function *P(T)* by the Runge-Kutta fourth-order accuracy has three columns: the first column contains the values of t, in which the solution is sought; the second column contains the values of the found solution P (t) at the corresponding points and the third value P'(t).

Zw =

	0	1	2
0	0	4	1
1	0.05	4.041	0.662
2	0.1	4.067	0.348
3	0.15	4.077	0.059
4	0.2	4.073	-0.202
5	0.25	4.057	-0.437
6	0.3	4.03	-0.643
7	0.35	3.993	-0.823
8	0.4	3.948	-0.976
9	0.45	3.896	-1.102
10	0.5	3.838	-1.203
11	0.55	3.776	-1.281
12	0.6	3.71	-1.336
13	0.65	3.643	-1.369
14	0.7	3.574	-1.384
15	0.75	3.505	-1.38
16	0.8	3.436	-1.361
17	0.85	3.369	-1.327

(8)

Solutions were prepared in 200 points in the package Mathcad. Analyzing the results, it was found that all prices tend to the steady price P_{St}=3 with fluctuations around it, and the amplitude of these oscillations decays with time.

Consider the case 2. Take the initial moment of P (0) = 4, and P'(0) = - 3, then the matrix Z_{w1} (9), the resulting solution is as follows:

Zw1 =

	0	1	2
0	0	4	-3
1	0.05	3.852	-2.934
2	0.1	3.707	-2.84
3	0.15	3.568	-2.721
4	0.2	3.435	-2.581
5	0.25	3.31	-2.424
6	0.3	3.193	-2.253
7	0.35	3.085	-2.071
8	0.4	2.986	-1.882
9	0.45	2.897	-1.689
10	0.5	2.817	-1.494
11	0.55	2.748	-1.299
12	0.6	2.687	-1.108
13	0.65	2.637	-0.922
14	0.7	2.595	-0.743
15	0.75	2.562	-0.571
16	0.8	2.538	-0.41
17	0.85	2.521	-0.259

(9)

The results of the solution of equation (7), have been found to prices from time to time in these two cases are shown in the graph (Figure 2).

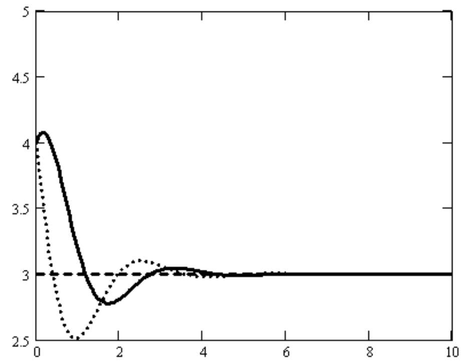

Figure 2. The family of integral curves of equation (7) for the cases 1 and 2

This resulting family of integral curves of equation (7) for the cases 1 and 2 completely coincides with the family of integral curves for the exact solution.

Next, perform the computation of approximate solutions using computational scheme of the Bubnov-Galerkin method proposed for the equation (6), according to which the solution is reduced to a system of linear algebraic equations $Au = F$. The elements of the matrix A (10), the column vector of free terms F (11) explicitly written below and calculated in the package Mathcad. To solve the system used the method of inverse matrix, the elements of which are also represented (13). As a result of an approximate solution u (x), where the coefficients are the elements of the vector Xi (12), whose graph is shown in Figure 3.

A =

	1	2	3	4	5	6
1	2.551	0.135	0	0.027	0	0.012
2	-0.135	2.513	0.081	0	0.019	0
3	0	-0.081	2.506	0.058	0	0.015
4	-0.027	0	-0.058	2.503	0.045	0
5	0	-0.019	0	-0.045	2.502	0.037
6	-0.012	0	-0.015	0	-0.037	2.501
7	0	$-9.006\cdot10^{-3}$	0	-0.012	0	-0.031
8	$-6.433\cdot10^{-3}$	0	$-7.369\cdot10^{-3}$	0	-0.01	0
9	0	$-5.263\cdot10^{-3}$	0	$-6.235\cdot10^{-3}$	0	$-9.006\cdot10^{-3}$
10	$-4.094\cdot10^{-3}$	0	$-4.454\cdot10^{-3}$	0	$-5.404\cdot10^{-3}$	0

(10)

$$F_i := \int_0^1 15\sin(i\cdot\pi x)\,dx \tag{11}$$

X =

	1
1	3.735
2	0.151
3	1.274
4	0.052
5	0.765
6	0.028
7	0.546
8	0.018
9	0.425
10	0.015

(12)

$A^{-1} =$

	1	2	3	4	5	6
1	0.391	-0.021	$5.665\cdot10^{-4}$	$-4.228\cdot10^{-3}$	$2.058\cdot10^{-4}$	$-1.814\cdot10^{-3}$
2	0.021	0.396	-0.013	$5.892\cdot10^{-6}$	$-3.058\cdot10^{-3}$	$4.001\cdot10^{-6}$
3	$5.665\cdot10^{-4}$	0.013	0.398	$-9.222\cdot10^{-3}$	$2.539\cdot10^{-5}$	$-2.394\cdot10^{-3}$
4	$4.228\cdot10^{-3}$	$5.892\cdot10^{-6}$	$9.222\cdot10^{-3}$	0.399	$-7.183\cdot10^{-3}$	$2.881\cdot10^{-6}$
5	$2.058\cdot10^{-4}$	$3.058\cdot10^{-3}$	$2.539\cdot10^{-5}$	$7.183\cdot10^{-3}$	0.399	$-5.884\cdot10^{-3}$
6	$1.814\cdot10^{-3}$	$4.001\cdot10^{-6}$	$2.394\cdot10^{-3}$	$2.881\cdot10^{-6}$	$5.884\cdot10^{-3}$	0.4
7	$1.06\cdot10^{-4}$	$1.428\cdot10^{-3}$	$1.398\cdot10^{-5}$	$1.96\cdot10^{-3}$	$6.384\cdot10^{-6}$	$4.981\cdot10^{-3}$
8	$1.009\cdot10^{-3}$	$3.951\cdot10^{-6}$	$1.176\cdot10^{-3}$	$3.486\cdot10^{-6}$	$1.66\cdot10^{-3}$	$3.923\cdot10^{-6}$
9	$6.541\cdot10^{-5}$	$8.344\cdot10^{-4}$	$9.769\cdot10^{-6}$	$9.951\cdot10^{-4}$	$5.31\cdot10^{-6}$	$1.439\cdot10^{-3}$
10	$6.423\cdot10^{-4}$	$6.729\cdot10^{-6}$	$7.108\cdot10^{-4}$	$6.893\cdot10^{-6}$	$8.636\cdot10^{-4}$	$8.156\cdot10^{-6}$

(13)

$$U(x) := -\sum_{i=1}^{1} X_i \cdot \sin(i\cdot\pi\cdot x) \tag{14}$$

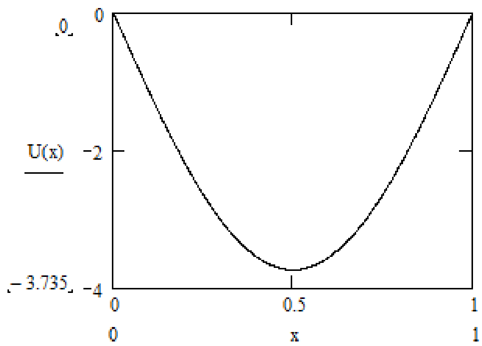

Figure 3.

We compare the approximate solution of equation (7) with the exact solution of the given initial conditions for the case of 2. We have an exact solution in the form of:

$$u1(x) := -3\cdot e^{-x}\cdot\cos(2\cdot x) + \left(3\cdot\frac{1}{\tan(2)} - \frac{3\cdot e}{\sin(2)}\right)\cdot e^{-x}\cdot\sin(2\cdot x) + 3 \tag{15}$$

Graph exact function (15) is shown in Figure 4 and is fully consistent with the schedule of the approximate solution (14) in Fig. 3 obtained by the Bubnov-Galerkin method.

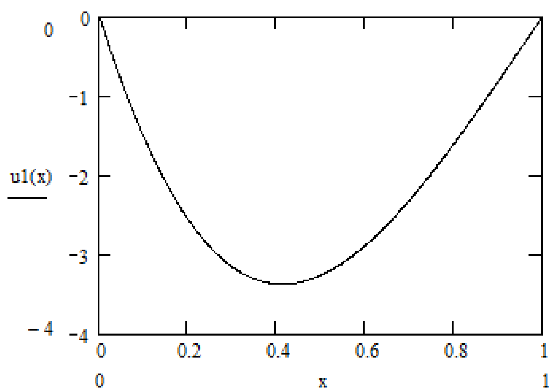

Figure 4.

4. Conclusions

Investigating various differential equations for continuous models of economy it is necessary to understand that Runge-Kutt's method of the fourth order is the most common for the ordinary differential equations. The formulation "the most common" is connected with the developed tendency in use of numerical methods. At formulas of an identical order of accuracy the main members of an error on a step often are disproportionate. For one equations one method, and for others – another gives a smaller error. In a similar situation of the recommendation in favor of this or that method have to be based on the "strong-willed decision" made taking into account traditions and practice of use of methods. The concept of practice of computing work is quite uncertain. However, despite such uncertainty, criterion of practice often bears in itself certain positive information which often at this stage of development of science can't be formalized or proved.

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