Asian Social Science; Vol. 11, No. 24; 2015
ISSN 1911-2017 E-ISSN 1911-2025
Published by Canadian Center of Science and Education

A Marking Scheme Rubric: To Assess Students' Mathematical Knowledge for Applied Algebra Test

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Received: March 25, 2014 Accepted: June 17, 2014 Online Published: August 18, 2015

doi:10.5539/ass.v11n24p18 URL: http://dx.doi.org/10.5539/ass.v11n24p18

Abstract

Students' ability in mathematics mainly relies on their performance in the assessment task such as tests, quizzes, assignments and final examinations. However, the grading process depends on the respective mathematics teacher who sets a marking scheme in assessing students' learning. How do these teachers assign grades to their students' problem solving work? What does it mean by five marks or ten marks for a mathematics problem? How does a teacher evaluate a student's mathematical knowledge and skills based on the grades? These questions address the vagueness of the grading process that gives no concrete evidence about a student's mathematical thinking. Hence, this paper aims to discover the effectiveness of using a marking scheme rubric to assess students' mathematical knowledge. The paper begins by reviewing different types of scoring rubrics in assessing mathematical problem solving tasks. A marking scheme rubric was proposed to assess samples of actual students' problem solving work in an applied algebra test. The rubric serves as an assessment instrument to gather information about students' achievement level in demonstrating both knowledge and skills in the test. Based on the findings, the score reflected the quality of the students' work rather than just a numerical representation. It showed the students' comprehension of adapting the mathematical concepts and problem solving strategies. In a nutshell, the implementation of rubric marking scheme has improved the consistency in grading and made the scoring points as a "meaningful figure" that describes the quality of a students' performance.

Keywords: marking scheme rubric, mathematical knowledge, assessment

1. Introduction

Students' exposure to mathematical thinking and problem solving begins from their primary education. The mathematics curriculum at the pre-tertiary education in Malaysia has been systematically structured to provide opportunities for students to develop mathematical knowledge and problem solving skills throughout their academic years (Malaysian New Integrated Mathematics Curriculum, 2003). Nevertheless, students are obliged to take part in a series of formal and informal mathematics assessments such as quizzes, assignments and tests at school, to evaluate their proficiency in mathematics learning. At the end of primary, lower secondary and upper secondary levels of education, students are obliged to sit for common public examinations under the jurisdiction of the Ministry of Education Malaysia. The final result of each examination indicates students' pre-requisite background knowledge before proceeding to the next level of education. The recognition as a talented mathematics student depends on the performance in mathematics subjects at these public national examinations. Thus, assessment is part of educational practices that provides evidence of students' achievement in mathematics. It cannot be separated from students' learning and plays a critical role in monitoring students' competency level when they have successfully completed a certain topic or module. Grades are often assigned to each assessment task that indicates how well a student has performed. The final grade determines the standard of students' learning attainment. In other word, students learning ability is normally judged based on how well they do in the assessment task.

2. Problem Statement

Assessment is an integral part of the learning and teaching process. However, the teacher is the key assessment grader who determines the score of a student's work. The grading process depends on the respective mathematics teacher who sets a marking scheme in assessing students' learning. Vasquez-Levy, Garofalo and Timmerman (2001) conducted assessment workshop with a group of teachers where they were assigned to grade samples of actual students' problem solving work. These teachers’ justifications for giving points in the evaluation of a student's work were not the same. For example, one teacher gave one point for student A's computation effort, while another teacher allocated five points. Some teachers expressed a belief that mathematics is about getting the right or wrong answer, and did not take account of the students' mathematical effort. However, some believed that credits should be given to students' attempt for solving the problem. This results in the inconsistency in the grading process. Besides the various marking habits, another issue of the grading system is the scoring point. What does it mean by five points or ten points for a mathematics problem? How does a teacher evaluate a student's mathematical knowledge and skills based on the final grades? How much mathematics is known by two students whose scores are comparatively the same? In general, the score assigned to each student simply describes how he or she did relative to other students in a class and carries little information about the students' strengths and weaknesses in learning mathematics (Romagnano, 2001). The grading process gives no concrete evidence about a student's mathematical thinking. It is important to consider ways to make the scoring points as a "meaningful figure" that describes the quality of students' performance in the mathematics assessment, and a rubric seems to be the perfect instrument to assist teachers in identifying the important scoring guides of a good solution. It consists of specific performance descriptions in a hierarchy order to analyze and grade students' work explicitly (Mertler, 2001). Hence, this paper aims to discover the effectiveness of using a marking scheme rubric to assess students' mathematical knowledge.

3. Literature Review

Mathematics assessment with rubric specification plays a significant role to both teachers and students as a descriptive feedback of performance levels across a scoring scale. Rubric can be accommodated into a variety of assessment scales that depends on the purpose of the assessment (Moskal, 2000). It is developed with its own design features that fit into a specific assessment instrument such as group work, peer or individual assessment (Tierney & Simon, 2004).

Step 1		Step 2		Step 3
State the assessment purpose and the objectives	⇒	Develop score criteria for each objective	⇒	Reflect on the following: 1. Are all of the objectives measured through the scoring criteria? 2. Is any of the scoring criteria unrelated to the objectives?

Figure 1. The process for developing a scoring rubric

Moskal and Leydens (2000) have prescribed three steps for developing a scoring rubric as illustrated in Figure 1. The most important feature in the process was to state clearly the learning objectives of a module. According to them, the instructor should be aware of the level of knowledge and skills that the students should attain and how they should demonstrate their proficiencies. The verbs used in the learning objectives serves as guidance in setting performance criteria for a scoring rubric, with appropriate assessment task. Thus, well-defined objectives ensure the fairness in both testing and grading process (Blumberg, 2009).

Some researchers use the phases of a problem solving model as the scaling criteria and guidelines for the rubric development to assess the quality of a student’s mathematical problem solving work (Szetela & Nicol, 1992; Gadanidis, 2003; Egodawatte, 2010). For example, Charles, Lester and O Daffer (1987, cited in Szetela & Nicol, 1992) devised an analytic scale that assigned separate scores to each of three stages in problem solving i.e. understanding the problem, solving the problem and answering the question. Such system allowed teachers to analyze students’ responses and identify their mathematical knowledge development in attempting a mathematics task. The hierarchical level of the rubric explained why the students received the score and what they should do in order to improve their performance in future (Szetela & Nicol, 1992; Moskal, 2000). Toh et al. (2009) developed a scoring rubric based on Polya’s model and Schoenfeld’s framework to assess students’ problem solving in a practical worksheets that highlighted the problem solving processes i.e. understand the problem, devise a plan, carry out the plan and, check as well extend. Student who did not obtain a correct answer but showed evidence of responses to the cycle of problem solving processes earned some marks.

Docktor and Heller (2009) designed a problem solving rubric to evaluate students' written solutions to physics problems. The scoring scale assessed five problem solving processes i.e. translating information into a useful description, selecting appropriate physics approaches, applying physics concepts, using mathematical procedures to execute toward the solution and giving logical reasoning of the solution. Approximately 300 students' written solution to the first semester introductory physics tests were analyzed and scored using the rubric. The grading process produced reasonably valid and reliable scores. Moreover, the numerical values provided meaningful information about students' work in key categories associated with the problem solving processes. Thus, a rubric exhibits the qualities that need to be displayed in a student's work and this transparency makes the assessment task as objective as possible (Moskal, 2000).

The focus of this study is to propose a marking scheme rubric to assess samples of actual students' problem solving work in an applied algebra test. The main objective of this exercise is to improve the consistency in grading and make the scoring points as a "meaningful figure". In other word, a final grade of a mathematics task is no more to reflect a quantity of students' work but to provide the information that helps to see the quality of students' work.

4. Proposed Marking Scheme Rubric

Applied algebra with the module code F40APA is one of the core mathematics subjects in the Engineering Foundation program of the University of Nottingham, Malaysia Campus. This module provides basic concepts of matrix algebra and vectors that require mainly solving practical linear systems of equations and the analytical geometry of space that appear in engineering and science. It gives a thorough grounding of mathematics and abilities to deal with real life application problems. Assessment tasks consist of take-home test, classroom written test and final examination with the weightage of 10%, 20% and 70% respectively. In this study, the rubric development is focused on the vector component of the classroom written test.

The test measures three learning objectives on vectors, which are:

- Perform standard operation on vectors in two-dimensional space and three-dimensional space.
- Apply the concept of dot product and cross product of vectors and interpret it geometrically.
- Derive parametric equations of lines and equations of planes in 3 spaces and use these equations to solve geometric problems.

The attainment of learning objectives can be identified through the use of Bloom's taxonomy revised in Anderson's et al. (2001, cited in Blumberg, 2009). The verbs used in the learning objectives determine the levels of cognitive processes such as remember, understand, apply, analyze, evaluate and create. For example, the cognitive level of "understand" is defined by the verbs "interpret", "exemplify", "classify", "summarize", "compare" and "explain" while "apply" describes the activities like "solve", "apply", "implement", "execute" etc. (Blumberg, 2009). Table 1 shows the revised taxonomy of the cognitive domain for categorizing the levels of learning.

Table 1. Structure of the cognitive process dimension of the revised taxonomy (cited in Krathwobl, 2002)

Categories	Cognitive Process
Remember	Retrieving relevant knowledge from long term memory (recognizing, recalling)
Understand	Determining the meaning of instructional messages, including oral, written and graphic communication (interpreting, exemplifying, classifying summarizing, inferring, comparing, explaining)
Apply	Carrying out or using a procedure in a given situation (executing, implementing)
Analyze	Breaking a material into its constituent parts and detecting how the parts relate to one another and to an overall structure or purpose (differentiating, organizing, attributing)
Evaluate	Making judgment based on criteria and standards (checking, critiquing)
Create	Putting elements together to form a novel, coherent whole or make an original product (generating, planning, producing)

By going through the phrasing of the learning objectives of F40APA, the role of assessment in this module is to examine students' cognitive skills of remembering, understanding and applying the concept of vectors at the end of their learning experiences. However, these cognitive skills demand a sequence of cognitive activities of problem solving, for example, reading and understanding the problem, planning, performing the plan and getting the answer.

Based on the cognitive skills and activities required to meet the learning objectives, four main criteria with each having four levels of performance from 0 to 3 is proposed, as shown in Table 2. A zero indicates the lowest level of performance while a three indicates the highest level.

Table 2. Scoring rubric for an applied algebra test

Characteristics	3	2	1	0
Understand information given (*Do I understand the problem? What information do I know*?)	Appropriate mathematics information (including, graphs or diagrams) is identified. Appropriate symbol or/and formula are defined.	Mathematics information identified but symbol or/and formula are partially defined.	Some mathematics information identified. Symbol or/and formula are not defined.	No mathematics information and no formula or symbol used.
Work out proper strategies (*Do I use the correct strategy to tackle the problem*?)	Can pick a good strategy and approach the problem systematically. No conceptual errors.	Can pick a good strategy but cannot approach the problem systematically. Minor conceptual error.	Can use somewhat good strategy but cannot approach the problem systematically. Major conceptual error.	Apply a strategy that does not work. Conceptual error.
Perform calculation accurately (*Are my working and calculation accurate*?)	Work shown is logical. Calculations are completely correct.	Work shown has gaps. Calculations are mostly correct, may contain minor errors.	Work is partially shown. Calculations contain major errors.	A limited amount of work shown. Calculations are completely incorrect.
Answer the problem (*Do I answer the question with proper working and give appropriate statement*?)	Can get a correct answer and give a perfect answer statement.	Can get a correct answer but cannot give an appropriate answer statement.	Can give an appropriate answer statement despite the incorrect answer.	Incorrect answer and no answer statement

The proposed rubric analyzes the students' responses to problems on the basis of four characteristics i.e. identifying the information given, working through proper strategies, performing a calculation accurately and answering the problem. Each characteristic is attached with a set of guiding questions to examine how well the students have responded.

5. Examining Students' Problem Solving Work

The test was conducted on the tenth week of the Spring Semester 2013. There were four questions in the test and two questions were regarding vector problems, with a total of 20 marks each. Each question consisted of two to three sub-questions. The students were given one hour to complete the test. However, the discussion in this section is restricted to question No.2 part (a), as it would be lengthy to discuss the students' responses for all the questions. This problem examines the students' understanding on applying the concept of scalar and vector product to write an equation for a plane in space, as shown below.

> *Question 2 (a): A plane contains the point A (-4, 9, -9) and B (5, -9, 6) and is perpendicular to the line which joins B and C (4, -6, q). Evaluate q and find the equation of the plane.*

5.1 Students' Responses for Question 2 Part (a)

Five students' problem solving works were considered in the following discussion.

Student P

The actual worksheet of student P attempting to solve the problem is shown in Figure 2.

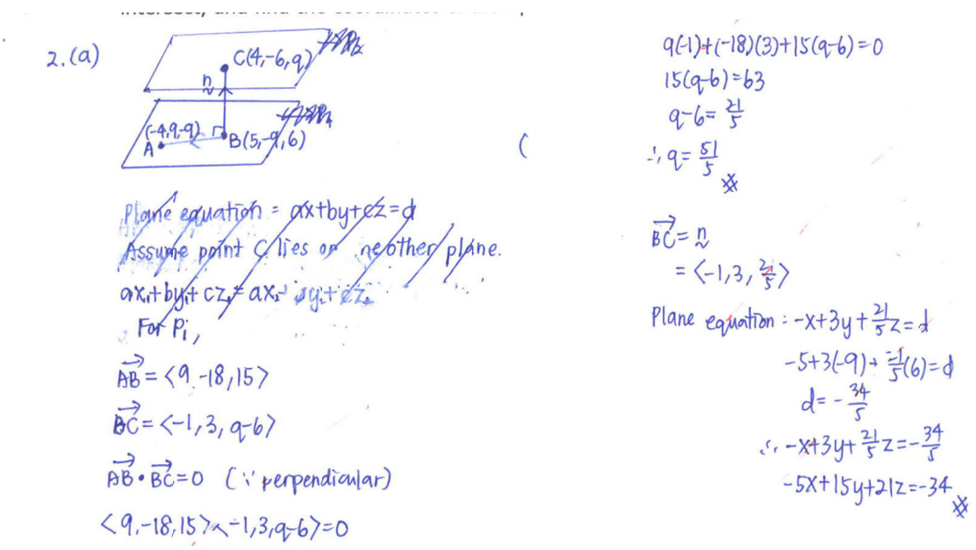


Figure 2. Student P's problem solving work

Student P's work showed a thorough understanding of the problem. The student managed to extend the concept of perpendicularity by drawing a diagram with two planes and two arrows to indicate the two orthogonal vectors. The student used appropriate symbols to label the vectors and utilized the concept of scalar products to determine the value of q. Student P approached the problem systematically with no error in his computation. He finally arrived at a correct answer although he did not supply an appropriate answer statement to the question.

Student Q

Figure 3 shows student Q's responses to the problem. The student took the symbol " $\perp$ " to indicate the perpendicularity of the two vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$, and compute the value of q by applying the concept of scalar product. Student Q seemed to spot the relevant information but the work showed conceptual error in finding the normal vector of the plane. The student could not visualize the problem situation and applied an invalid strategy. Student Q's calculation was incomplete and obtained an incorrect answer.

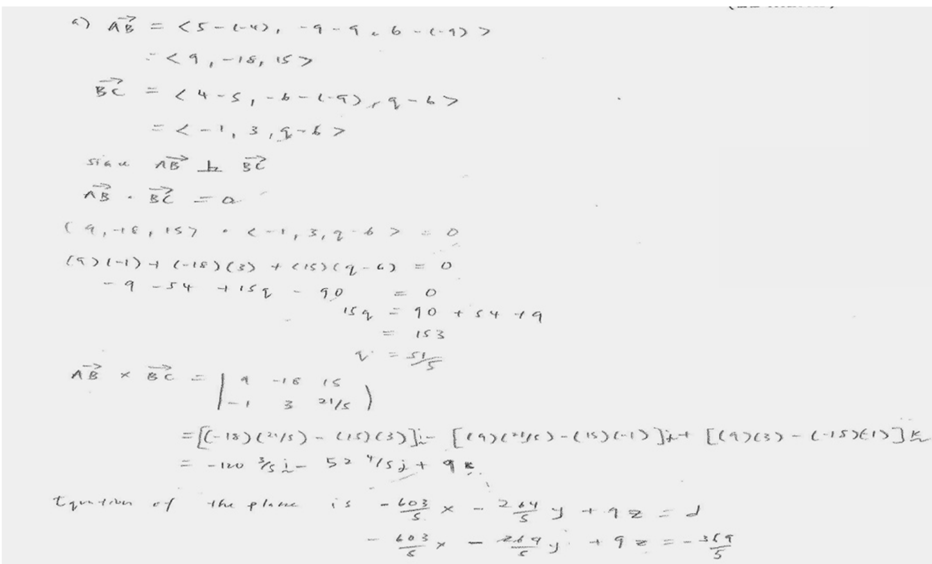


Figure 3. Student Q's problem solving work

Student R

This student has correctly revealed the important aspects of the problem by sketching a simple diagram. The diagram enabled student R to see the situation more easily. The student took the same strategy as student P to compute the value of q and obtained the normal vector of the plane, as presented in Figure 4. However, student

R’s work was not clearly organized and has done minor computation errors in finding the plane equation.

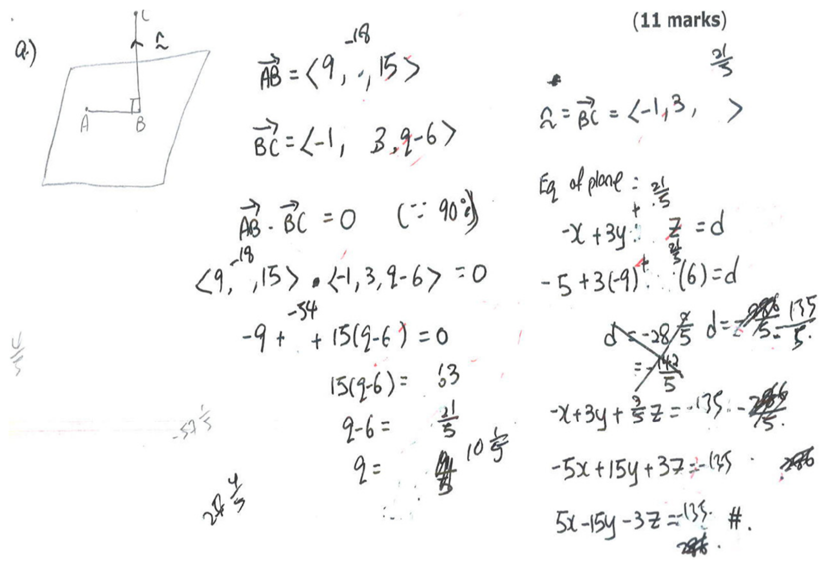


Figure 4. Student R’s problem solving work

Student T

This student applied different approach to solve the problem and Figure 5 shows his style of working towards the final solution. Student T could recognize the vector $\overrightarrow{BC}$ as a normal vector of the plane. But he skipped the work of applying the perpendicularity concept of the two vectors. Instead, he transformed the problem into a system of linear equations with two unknown variables q and d. Student T solved the system algebraically and managed to gain the value of d. However, he made a minor computation error in getting the value of q.

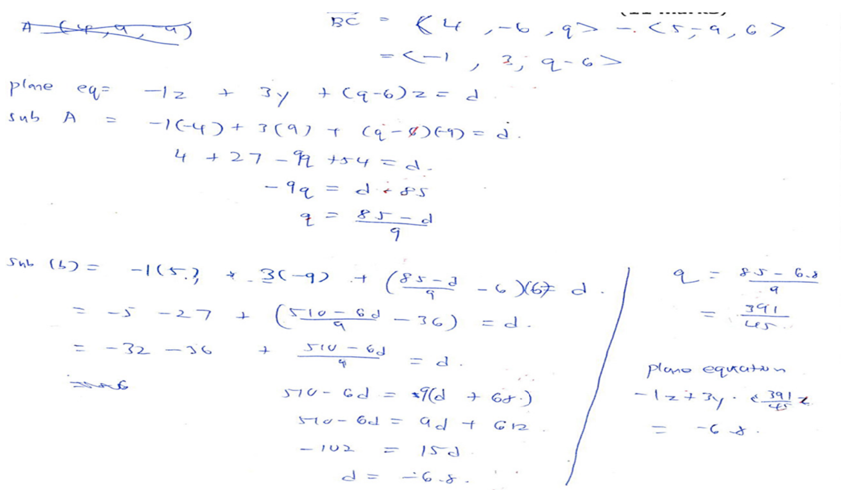
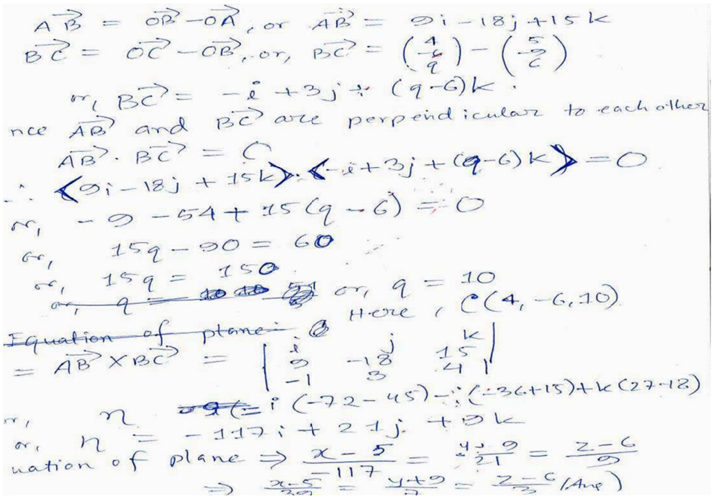


Figure 5. Student T’s problem solving work

Student U

Figure 6 demonstrates student U’s problem solving procedures. The student noticed the relation of the two vectors at the right angles to one another and applied the concept of dot product to compute the value of q. However, he has mistakenly assumed that all the points A, B and C were located on the same plane. He employed the tactic of cross product to work out the normal vector of the plane. This has jeopardized his efforts of getting the equation of the plane and worst of all, he has mistakenly adopted the Cartesian equation of a line as the plane equation.


$\vec{AB} = \vec{OB} - \vec{OA}$, or $\vec{AB} = 9i - 18j + 15k$

$\vec{BC} = \vec{OC} - \vec{OB}$, or, $\vec{BC} = \begin{pmatrix} 4 \\ -6 \\ q \end{pmatrix} - \begin{pmatrix} 5 \\ -9 \\ 6 \end{pmatrix}$

or, $\vec{BC} = -i + 3j + (q-6)k$

nce $\vec{AB}$ and $\vec{BC}$ are perpendicular to each other

$\vec{AB} \cdot \vec{BC} = 0$

$\therefore \langle 9i - 18j + 15k \rangle \cdot \langle -i + 3j + (q-6)k \rangle = 0$

or, $-9 - 54 + 15(q-6) = 0$

or, $15q - 90 = 60$

or, $15q = 150$

or, ~~$q = $~~ or, $q = 10$

Here, $C(4, -6, 10)$

~~Equation of plane~~

$= \vec{AB} \times \vec{BC} = \begin{vmatrix} i & j & k \\ 9 & -18 & 15 \\ -1 & 3 & 4 \end{vmatrix}$

$n = i(-72-45) - j(-36+15) + k(27-18)$

or, $n = -117i + 21j + 9k$

uation of plane $\Rightarrow \frac{x-5}{-117} = \frac{y+9}{21} = \frac{z-6}{9}$

$\Rightarrow \frac{x-5}{-117} = \frac{y+9}{21} = \frac{z-6}{9}$ (Ans)

Figure 6. Student U's problem solving work

5.2 The Students' Score Using the Proposed Rubric

After each student's response has been examined, students' works were rated by means of the proposed scoring rubric. Reliability for the grading process was achieved through an additional reviewer who analyzed ten samples of students' work in problem solving. He played the role of cross-checking the scoring process in order to ensure fairness, accuracy and consistency in grading. Table 3 demonstrates the performance of each student in solving the problem at the respective criteria.

Table 3. The students' score using the proposed marking scheme rubric

	Identify information given	Work through strategies	Perform calculation accurately	Answering the problem	Total score
Student P	3	3	3	2	11
Student Q	2	1	2	0	5
Student R	3	3	2	2	10
Student T	3	3	2	2	10
Student U	1	1	1	0	3

Student P, R and T demonstrate high level of performances in their problem solving works. Their responses revealed excellent implementation of a good plan. Student P and R used the drawing tactics to visualize and understand the problem situation. The diagram assisted them to discover the essential features of the problem easily. They extended the problem by applying the proper concept and worked on an appropriate strategy. On the other hand, student T presents a different solution strategy in his problem solving work. The student demonstrated his ability of exploring other successful mode of solving the problem. His work displayed his flexible thinking in interpreting information more than one way and looking for alternative approach rather than rote execution of mathematical procedures. However, student T did not reread and recheck his solution plan. He overlooked the verifying stage of problem solving process which plays an important role to increase one's mathematical problem solving experience (Polya, 1957).

Students Q and U performances were graded low due to incomplete representation of the problem explicitly. Although they recognized the principle of perpendicularity of two vectors and the concept of scalar product, they failed to integrate the given information and what was requested. They demonstrated conceptual error in their calculation that leaded them to an incorrect answer. Hence, their low level of performances indicate inadequate understanding of the problem and limited use of problem solving strategies such as drawing diagrams, identifying patterns, working backward and etc.

6. Conclusion

The above exercise exhibits the practical value of a rubric for learning and assessment. The score becomes a meaningful figure that describes the students' strength in comprehension of mathematics. With each assessment, it shows whether the students have learned thoroughly regarding a concept or theory. The rubric also targets to minimize the discrepancy of different graders' beliefs about the grading practices. This improves the consistency in grading and gives a fair judgment about students' problem solving works. Thus, a well-designed rubric offers more than an assessment of "right" and "wrong" answer. It makes the standard performance and expectation clear to instructors and students. It facilitates instructors' and students' awareness of the learning objectives and helps them to be thoughtful about the quality of teaching and learning. This will change their conception entirely from valuing a letter grade as an end product to a significant grade that values the learning outcomes achievement.

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