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Assessing Students' Mathematical Problem-Solving and Problem-Posing Skills

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Abstract

Problem-solving and problem-posing have become important cognitive activities in teaching and learning mathematics. Many researchers argued that the traditional way of assessment cannot truly reveal what the students learnt and knew. Authentic assessment was used as an alternative method in assessing the students' mathematical learning. A performance rubric is an appropriate tool in examining students' ability to solve and pose mathematical problems.

Keywords: problem-solving, problem-posing, authentic assessment

1. Introduction

The shift in learning theory from behaviourism to constructivism has had an enormous impact on the teaching and learning of mathematics (Hatfield, Edwards, Bitter, & Morrow, 2003). According to von Glasersfeld (1989) students acquire knowledge by constructing and restructuring it over time which is similar to the experiential learning theory by Dewey (1938/1997). An individual learns by doing or experiencing, and teachers should facilitate students' learning for attaining knowledge developmentally (Dewey, 1938/1997). In order to make the learning of mathematics meaningful, teachers are responsible for choosing and posing tasks that engage students actively in building their understanding, mathematical thinking, and confidence (Kulm, 1994).

For many years, scholars have discussed the difficulty of assessing students' mathematical understanding using traditional assessment (Anderson, 1998). Kulm (1994) argued that traditional tests only focus on students' mathematical skills and procedures. Therefore, the use of authentic assessment tools to measure students' learning is critically needed in mathematics. In this paper, a discussion on appropriate tools is presented for assessing student performance in mathematical problem-solving and problem-posing. The paper begins with a brief summary of problem-solving and problem-posing tasks and activities that influence the need of authentic assessments. The focus will be on the use of performance rubrics for examining a student's ability to solve and pose mathematical problems. The advantages and disadvantages of using performance rubrics for mathematics assessment will be discussed, and then followed by some concluding remarks.

2. Mathematical Tasks

The National Council of Teacher of Mathematics' [NCTM] *Principles and Standards for School Mathematics* urges teachers to use authentic mathematical tasks in the classroom to facilitate knowledge construction (NCTM, 2000). The mathematical problems and activities should engage students with real-world contexts by using and applying mathematical content they have learned into their workplace (Lajoie, 1995). Teachers should consider choosing and creating tasks that utilize the application of mathematical procedures and concepts with a variety of solution approaches that can demonstrate students' understanding (Kulm, 1994). According to Cohen and Fowler (1998) and Kulm (1994), teachers can adapt some beneficial strategies to generate many new mathematical tasks by changing the way the tasks are presented. For example, "Calculate 2.4 x 5.3", teachers can reformulate this routine task into a variety of meaningful tasks as following:

(1) *How is 2.4 x 5.3 = 12.72?*

(2) *Generate a story problem with 2.4 x 5.3.*

(3) *Display 2.4 x 5.3 using pictorial representation.*

(4) *How much difference is there between 2.4 x 5.5 and 2.4 x 5.3?*

Tasks (1) through (4) are considered authentic that utilized the constructivist stance for stimulating students' mathematical learning (Silver, 1994). These types of task have become commonly used in schools of mathematics in addition to journals, portfolios, and class projects. The openness feature of the tasks allows students to communicate their thinking and reasoning mathematically through their writing (Berenson & Carter, 1995; Burns, 2007). When using this type of task, teachers can help students extend what they know for developing their mathematical fluency and engage them in higher-order thinking (NCTM, 2000). The use of open-ended tasks has led to a growing demand for more authentic forms of assessment for measuring students' learning based on the usage of problem-solving and problem-posing in school mathematics (Kulm, 1994; Lajoie, 1995; Wiggins, 1990).

3. The Need for a Better Assessment

In today's curriculum reformation, a variety of assessment tools and strategies are available to measure students' learning in school mathematics (Hatfield et al., 2003). Ironically, the traditional mathematics assessments that were predominately used to measure the correctness of a student's knowledge base are multiple choice, true or false, or recall questions (Watt, 2005). Cohen and Fowler (1998) argued that mathematical tasks used should be able to assess a student's depth of understanding of a particular concept. As in the case of a student who was asked to find 2.4 x 5.3, we could conclude that the student had a knowledge base of multiplying decimals if she or he answered 12.72 correctly.

The traditional ways of assessing students were focused on incorrect or correct scores rather than pursuits of students' mathematical understanding (Berenson & Carter, 1995). In contrast, we would be able to assess the student's conceptual understanding of the placement of decimal point by modifying the task to "Given that 2.4 x 5.3 is 12.72, explain how you could find the product of 0.24 x 0.53 without recalculating" (Cohen & Fowler, 1998). With regard to this problem-solving task, an authentic assessment is significantly important for teachers to be able to infer the meaning of an understanding of mathematics (Romberg & Wilson, 1995).

4. Authentic Assessment in School Mathematics

For the past few decades, teaching and learning of mathematics has been focused on problem-solving activity (Schoenfeld, 1992) that is seen as a goal, a process, and a skill (Hatfield et al., 2003). During problem-solving activities, students often reflect on the mathematical ideas in the tasks, formulating ideas more likely to be assimilated with their prior knowledge during the activities (Van de Walle et al., 2009).While solving worthwhile problems, students are constructing and restructuring their own knowledge and will be actively engaged in all NTCM's process standards: problem-solving, reasoning, communication, connections, and representation (Van de Walle et al., 2009). When problem-solving is used effectively in classroom, it will eventually develop students' mathematical power as described by the Principles and Standards document (NCTM, 2000).

According to Kilpatrick (1987), problem-posing is an integral part of problem-solving as it can occur before, during, and after a student is solving a mathematical task. Problem-posing refers to the generation of new problems based on a context as well as the reformulation of new problems from the given ones (Silver, 1994). Similar to the problem-solving process, as students creatively pose new problems they are developing and acquiring mathematical ideas while exploring their curiosity within specific contexts (English, 1997; Silver, 1994).

Literature has shown that problem-posing is increasingly being used in classroom activities to enhance students' mathematical learning (Barlow, 2006; English, 1997, 1998). It has now become an important component of mathematics teaching and learning. Thus, along with problem-solving, problem-posing is part of the reform vision for promoting mathematics as a cognitive activity (NCTM, 2000). Indeed, the Principles and Standards document states, "Good problem solvers tend to naturally pose problems based on situations they see" (p. 53). Wiggins (1990) claimed that astonishing efforts have emerged in the usage of authentic assessment for examining; not only the product and the skill of problem-solving and problem-posing, but also the process when a student is doing the task (Wiggins, 1990).

To date, no standard definition of authentic assessment or also known as alternative assessment or performance-based assessment has been offered because of its complex nature (Terwilliger, 1997). Regardless of any informal definition, authentic assessment provides a better way to assess students' mathematical knowledge through broader types of tasks and diverse approaches (Kulm, 1994). According to Kulm, teachers can assess students' conceptions and misconceptions of mathematical ideas beyond the scope of traditional tests, which focus only on computational skills and procedures. In relation to the measure of students' performance on problem-solving and problem-posing tasks, a performance rubric can be designed or adapted. It can be used as a measure to assess mathematical concepts, procedures, processes, and students' disposition toward mathematics (Van de Walle et al., 2009). Rubrics are now widely used by a community of practice to assess students' learning in mathematics to complement the weakness of traditional tests.

5. Performance Rubrics

The openness feature of problem-solving and problem-posing tasks has increased the demand of alternative assessments (Wiggins, 1990). A performance rubric is more comprehensive for assessing students' mathematical concepts, procedures, processes, and disposition toward mathematics (Sanchez et al., 2002). It breaks the use of traditional assessments that only based on the correctness of students' answer (Van de Walle et al., 2009). A well-developed performance rubric offers teachers with valid and reliable information about students' progress of specific criteria, knowledge, and process (Van de Walle et al., 2009). Based on the information from the rubric, teachers can monitor students' learning and provide some feedbacks so that students can restructure their knowledge (Anderson & Puckett, 2003).

A performance rubric is a key component in many authentic assessments. This rubric is grounded on criterion-referenced measures (Reynolds, Livingston, & Willson, 2009). With criterion-referenced interpretations, Reynolds et al. affirmed that students are assessed to a specified level of performance in a rubric based on what they know or what they can do. Holistic, analytic, process, and anaholistic rubrics are some of the scoring rubrics that are commonly used by a community of practice (Kulm, 1994). Each of these rubrics serves different ways of evaluating students' level of understanding. For instance, anaholistics is a combination of analytic and holistic rubric, which is based on several criterion-referenced that covers students' overall conceptual and procedural knowledge (Kulm, 1994; Reynolds et al., 2009). The anaholistic rubric is an appropriate assessment tool for assessing problem-solving and problem-posing because it provides a comprehensive rating of broad range of mathematical learning (Kulm, 1994). It is an assessment approach for scoring a mathematical task based on more than one criterion in terms of students' overall conceptual and procedural knowledge as well as problem-solving processes. Scores in each criterion can be summed up to obtain a total point as the performance indicator.

5.1 Problem-Solving Rubric

Problem-solving has a long history in the research practice of mathematics education community. Based on George Polya's work, NCTM documents suggest teachers to assess students' complex thinking processes through problem-solving tasks that includes understanding the problem, planning a strategy, carrying out the solution, and looking back at the solution (NCTM, 1989, 1991, 1995, 2000). As teaching and learning mathematics has been focused on the problem-solving approach, many scholars have contributed their works on the development of innovative instructional strategies and assessment tools (e.g., Charles, Lester, & O'Daffer, 1987; Hatfield et al., 2003, Kulm, 1994; Schoenfeld, 1992; Van de Walle et al., 2009).

Malloy and Jones (1998) showed a practical use of a problem-solving rubric in a study of 24 African American eighth graders. Five open-ended tasks that have multiple strategies and varied difficulty levels were presented in this study to assess students' mathematical understanding through problem-solving. The students had to solve the problems using the holistic or analytic approaches. Also, they needed to provide inferential, deductive, or inductive reasoning when solving the problems. A conceptual scoring rubric was adapted from Charles et al. (Figure 1) to assess students' levels of success in solving problems. The criteria of the rubric emphasized the process of getting the solution that reflect the student's conceptual understanding based on 0 through 4 points. The researchers have categorized students as successful if their total conceptual scores were 13 or above and unsuccessful if the scores were below 13. The results showed African American students were marginally successful in solving the open-ended problems. However, students' problem-solving actions matched the characteristics of good problem solvers reported in previous studies such as the ability to use diverse strategies, flexibility approaches, verification actions, and the ability to deal with irrelevant information in problem tasks.

Understanding the problem	• 2	Complete understanding of the problem
	• 1	Part of the problem misunderstood or misinterpreted
	• 0	Complete misunderstanding of the problem
Planning a solution	• 2	Plan could have led to a correct solution if implemented correctly
	• 1	Partially correct plan based on part of the problem being interpreted correctly
	• 0	No attempt, or totally inappropriate plan
Getting an answer	• 2	Correct answer
	• 1	Copying error; computational error; partial answer for a problem with multiple answers
	• 0	No answer, or wrong answer based on an inappropriate plan

Figure 1. The analytic scoring scale from Charles, R., Lester, F., & O'Daffer, P. (1987). *How to evaluate progress in problem-solving*, Reston, VA: National Council of Teachers of Mathematics

5.2 Problem-posing Rubric

To date, many scholars have used several different frameworks to assess student's understanding on problem-posing tasks (Chen, Dooren, Chen, & Verschaffel, 2010; Kulm, 1994; Leung & Silver, 1997; Silver & Cai, 2005; Yuan & Sriraman, 2011). For instance, Silver and Cai suggested general criteria that focused on quantity, originality, and complexity of posed problems; these criteria were similar to Yuan & Sriraman's classification of problems: fluency, flexibility, and originality. However, in both studies, no specific scores were assigned to each of the problems posed. Instead, these researchers used descriptive statistics such as frequencies and percentages to measure the tasks in general. Similarly, in a recent study by Chen et al., the tasks were examined through distinctive categories: a realistic problem, a contextually inappropriate problem, and a mathematically inappropriate problem. Even though the distribution of 0, ½, and 1 scores correspond with the categories assigned, the procedures were not clearly described in the paper. Almost no previous research studies had any holistic scoring rubric for assessing problem-posing tasks; the researchers only classified different kind of problems posed into similar categories. Much work remains to be undertaken in the assessment of problem-posing because of its complexity. Kulm (1994) has also contributed a piece of scholarly work from his teaching experience of using problem-posing activities in a high school. Kulm noted many students lacked the opportunity to formulate problems; however, he believed that the students could write a problem if they understand the mathematical concepts profoundly. On a weekly basis, the students in his class were asked to pose a problem for their partners and solve their own. A process-based scoring rubric (in Figure 2) was utilized to examine his students' problem-posing tasks based on understanding the concept, solution, creativity, and solution of partner's problem. Students were awarded 1, 2, or 4 points for each of category being assessed. Though this simple rubric to assess the basic elements of students' learning, there is still much room for improvement to expand it into a more comprehensive anaholistic rubric that can effectively measure the range of students' mathematical abilities (Watt, 2005).

Process-based Rubric			
Understanding the Concept	*Solution of the Problem*	*Creativity of the Problem*	*Solution of Partner's Problem*
• 4 points - Complete understanding • 2 points - Some understanding • 1 point - Poor understanding	• 4 points - All correct • 2 points - Partially correct • 1 point - Attempted to solve	• 4 points - Completely different from text • 2 points - Somewhat different from text • 1 point - Comparable to types in text	• 4 points - All correct • 2 points - Partially correct • 1 point - Attempted to solve

Figure 2. The process-based scoring rubric from Kulm, G. (1994). *Mathematics assessment: What works in the classroom*. San Francisco, CA: Jossey Bass Inc. Permission pending

5.3 Benefits of Performance Rubric

The scoring procedures based on a rubric break the traditional way of grading, which is merely based on correct or incorrect items (Kulm, 1994). Students' grades can be based on scores on their content and process abilities rather than solely rely on the percent of correct responses (Wiggins, 1990).When rubrics are used regularly in the classroom, scores provide rich information about students' learning development for both teachers and students. A well-designed rubric can provide teachers with valid and reliable scores in order to monitor and to provide feedbacks on students' progress of specific criteria on the rubrics (Van de Walle et al., 2009). In conjunction, students should receive the feedback as an opportunity to reconstruct and restructure their knowledge and work much better on the next task (Anderson & Puckett, 2003).

5.4 Drawbacks of Performance Rubric

The problem-solving and problem-posing rubrics are multifaceted. Even if it is proven beneficial by education research community, there are many opponents to implementing this type of assessment in classroom. For instance, Watt (2005) reported a study that involved 60 mathematics teachers who were asked to answer a survey about the use of authentic assessment in their secondary school classroom. The results showed that though experienced teachers had positive attitudes towards authentic assessments, most teachers in the study did not favour the authentic assessment because of its subjectivity (too unstructured/biased/inequitable). In fact, a majority of teachers were satisfied with traditional tests for their strong validity-reliability for measuring mathematical ability. In addition, many teachers believed they had insufficient time to implement authentic assessment in classroom and thought it was unsuitable and unreliable for mathematics.

In relation to the weekly problem-posing activities previously described in this paper, Kulm (1994) asserted that many students were not able to pose problems well and average grades taken from the process-based scoring were low. Many students complained and were concerned about the part of scores taken from the rubric. In addition, Kulm found students who did not perform well on the traditional tests had similar scores on the process-based scoring system. Findings from Watt (2005) and Kulm (1994) showed good authentic assessments are important as a catalyst of change because the education community is still reluctant to accept and adapt the new method of assessment.

6. Conclusion

From the perspective of teaching and learning mathematics, problem-solving and problem-posing have become imperative instructional approaches in classroom. These activities can be used to uncover how individuals learn mathematics based on the constructivist learning theory. However, adaptive, meaningful, and unique assessments are still underdeveloped for assessing students' learning, particularly for problem-posing. The community of practice should take a step ahead to improve the assessment tools and should move from using traditional to more authentic assessment to measure students' learning holistically. In the context of research, large number of significant works has been published in the interim on problem-solving, but still very little on problem-posing has been written. Future studies should be exploring the complex link between problem-solving and problem-posing and should find authentic ways to assess students' learning based on these activities as a whole. Finally, as these activities have emerged as a significant component in mathematics education community, a more scholarly piece of work remains to be undertaken in the near future to improve the quality of education, particularly the teaching and learning of mathematics.

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