Is Hurst Exponent Value Useful in Forecasting Financial Time Series?

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Abstract

We estimated Hurst exponent of twelve stock index series from across the glove using daily values of for past ten years and found that the Hurst exponent value of the full series is around 0.50 confirming market efficiency. But the Hurst exponent value is found to vary widely when the full series is split into smaller series of 60 trading days. Later, we tried to find relationship between Hurst exponent value and profitable trading opportunity from these smaller series and found that periods displaying high Hurst exponent have potential to yield better trading profits from a moving average trading rule.

Keywords: technical analysis, detrended fluctuation analysis, moving average, Hurst exponent, forecasting financial series

1. Introduction

In this paper, we empirically examined the relationships between Hurst exponent and the predictability of financial time series. Capital markets theories are based on the assumption that security prices are martingales, implying the expected value of security price is the price in the previous period. Therefore, security prices follow random walks and returns from financial securities are unpredictable. This theory is popularly referred as Random Walk Hypothesis (RWH) and represents one of the variants of the broader Efficient Market Hypothesis (EMH).

The efficient market hypothesis (EMH) is based on whether any new information is instantly and sufficiently reflected in stock prices. Depending on time of availability, information can be classified into three types: historical information, public information, and future (or internal) information. Based on influence of information in stock prices, the EMH can be divided into three types: weak form (historical information), semi strong form (public information) and strong form (future information). The weak form of Efficient Markets Hypothesis assumes that current security prices impound all the information that can be gathered from their past values (Fama, 1970). The implication of the weak-form EMH is that security prices follow random walks and cannot be predicted from analysis of past prices.

However, many physical and biological systems show presence of long memory or trends in the time series. For example, the number of particles emitted by a radiation source in a unit time decrease over time as the source becomes weaker. The density of air because of gravity changes at a different altitude following a trend. The air temperature, rainfall and the water flow of rivers in different geographic locations show a periodic trend because of seasonal changes. Even the occurrence earthquakes show some kind of trend in certain areas.

In line with analysis of trends in physical science domain, financial analysts have also tried to detect trends in financial time series that seemingly look random. This type of analysis to detect trend based on analysis of past prices are classified as “Technical Analysis” and many mathematical and graphical techniques are proposed. Though the detection of trends by technical analysis methods are often criticized, there is no denying that a good number of market trade takes place in financial market are because of speculations.

Many studies in the field of econophysics have also examined properties and phenomena of financial time series through interdisciplinary studies and found notable deviations from random walk and presence of long memory in the time series. In this paper we compared the relationship between Hurst exponent and predictability of selected financial series.

2. Literature Survey

Harold Edwin Hurst (1880-1978) was a British hydrologist, who studied storage capacity of reservoirs in Nile river basin documented the presence of long-range dependence in hydrology. Hurst (1951) proposed a method for the quantification of long-term memory, which is based on estimating range of swings of the variable over time. According to the original proposition, Hurst exponent (H) = 0.5 would represent a self-determining process, in which the current value of the series would not dependent of past values of the series. When the value of H lie between the range $0 < H < 0.5$, the series becomes anti-persistent. Anti-persistent series display 'mean-reverting' characteristics. If a value in the time series was high in the previous period, it is likely to reduce in the following period towards the mean value. The strength of the mean reverting behavior increases as Hurst exponent approaches to zero. When the range of H exponent vary between $0.5 < H < 1$, the values of the series rise and fall in upward and downward direction in a broader range than likely by pure random walk. Such series display seeming trends for some time, but these seeming trends are erratically interrupted by abrupt discontinuities. The power of the trend-reinforcing behavior increases as the value of the Hurst exponent increase to the upper ceiling value of one.

When the observations from the series are independent of previous observations, the value of H is expected to be around 0.5. However, while examining Nile River overflow, Hurst found the value of H-exponent was much larger at 0.91. The higher value of H implies a larger variation of water flow than could be possible from a random walk.

Hurst's method of detecting long-term dependence in hydrology was later extended to other fields and methods of their identification were further generalized by Mandelbrot & Ness (1968), Mandelbrot (1982) among others. Application of Hurst exponent in financial time series was popularized by Peters (1991, 1994). In order to allow for non-Normality and autocorrelation in security returns, Peters introduced the Fractal Markets Hypothesis (FMH). The FMH is not based on a priori assumption that returns are lognormal and uncorrelated, it allows for a broader range of returns behavior. Several studies after that have documented relationship between Hurst exponent and return behavior in financial markets; some of them are discussed below.

Lipka and Los (2002) measured the degrees of persistence of the daily returns of eight European stock market indices and found that the Hurst exponents measure the long-term dependence of the data series well. They found that the FTSE returns represent an ultra-efficient market with abnormally fast meanreversion, than that possible by a Geometric Brownian Motion.

Corazza and Malliaris (2002) analyzed returns of several foreign currency markets and found that Hurst exponent value to be statistically different from 0.5 in most of the samples. They also noted that the Hurst exponent is not fixed but it varies overtime.

Cajueiro and Tabak (2004) tested for long-range dependence and efficiency in stock indices for 11 emerging markets along with US and Japan. They adopted a "rolling sample" approach and calculated median Hurst exponents to assess relative efficiency of these equity markets. They suggested that Asian equity markets show greater inefficiency than those of Latin America and developed markets rank first in terms of efficiency.

Kyaw et al. (2006) analyzed the degree of long-term dependence of Latin American financial markets, measuring mono-fractal Hurst exponents from wavelet multi-resolution analysis (MRA) of Latin American stock and currency markets. They found that the financial rates of return from are non-normal, non-stationary, non-ergodic and long-term dependent.

Singh & Prabakaran (2008) examined return spectrum of the Indian stock markets using various statistical tests for the normality of data. They performed rescaled range analysis and estimated Hurst's exponent. They inferred that the Indian capital markets are not random and therefore, do not form a population that is normally distributed. Further, geometric Brownian motion cannot accurately model the stock prices, because of significant memory effects.

The Detrended Fluctuation Analysis (DFA) technique was first introduced to investigate long-range power-law correlations among DNA sequences by Peng et al. (1992) and Stanley et al. (1992). In the method, the whole data sequence was split into a number of smaller sized non-overlapping boxes, each containing equal number of data points. The method was found to be an efficient method for accurately calculating the Hurst exponent.

Few recent studies that used DFA method to analyze return behavior in financial markets are discussed below. Wang and Gu (2009) used Shenzhen stock market data and classified into two sub-series at the criterion of the date of a reform and their scale behaviors are examined using DFA. Using rolling window, they found that Shenzhen stock market was becoming more and more efficient by analyzing the change of Hurst exponent.

Yuan et al (2009) analyzed the Shanghai stock price index daily returns using DFA method, and found there are two different types of sources for multi-fractality in time series, namely, fat-tailed probability distributions and non-linear temporal correlations. It was found that when the stock price index rises and falls sharply, a strong variability is clearly characterized by the generalized Hurst exponents. They used measures based on generalized Hurst exponents to compare financial risks of the market.

Yue et al (2010) used detrended fluctuation analysis (DFA) method to detect the long-range correlation and scaling properties of daily precipitation series of Beijing from 1973 to 2004 before and after adding diverse trends to the original series. However, we could not come across studies that have empirically noted direct relationship between the Hurst exponent and predictability of security prices.

3. Measuring Hurst Exponent

Several methods are available in literature for estimating Hurst exponent of a time series. We discuss below two methods based on (1) Range to Standard deviation ratio and (ii) Detrended Fluctuation Analysis.

3.1 Range to Standard Deviation Ratio

One of the popular method to measure Hurst exponent is the Range to Standard deviation analysis. The method is based on a study of the average rescaled range of cumulative deviations of a series from its mean values.

A procedure to calculate the R/S statistic of financial data series is available in Peters (6). The method presented below is customized based on the work of Peters.

Step 1:

We begin with a time series having M observation. As financial time series display high degree of non-stationarity, it is a common practice to work with first differenced series than with the original series. Therefore in the first step, we reduced non-stationarity by converting the original series to a returns series taking logarithm returns from successive values of the series.

$$x_t = \log\left(M_t \Big/ M_{t-1} \right)$$

Step 2:

The entire data series was divided into several contiguous sub-periods each having n-observations and defined each sub-period as I_a. For each sub-period, the average value of the sub-period can be determined as:

$\mu_a = 1 \Big/ n \sum_{k=1}^{n} x_k$, π_a is the mean return of the sub-period I_a.

The returns are trend adjusted by subtracting the mean return from daily returns: $r_{t,a} = x_{t,a} - \mu_a$

Step 3:

A cumulative trend adjusted return series for each sub-period is created

$$c_{t,a} = \sum_{i=1}^{t} r_{(t,a),i}$$

The range of the cumulative trend adjusted return series for each sub-period is measured by taking differences of maximum and minimum values of $c_{t,a}$:

$$R_{t,a} = \max(c_{1,a}, c_{2,a} \ldots c_{t,a}) - \min(c_{1,a}, c_{2,a} \ldots c_{t,a})$$

The standard deviation (S) of the sub-period is: $S_{t,a} = \sqrt{\frac{1}{t} \sum_{i=1}^{t} (x_{t,a} - \mu_a)^2}$

Step 4:

In the next step, rescaled range to standard deviation of each sub-period is obtained by taking the ratio: $\left(R/S\right)_{t,a} = R_{t,a}/S_{t,a}$

As there are many contiguous sub-periods, the average R/S value of full series is calculated by averageing R/S values of all individual sub-periods.

$$\left(R/S\right)_t = 1/A \sum_{a=1}^{A} \left(R/S\right)_{t,a}$$

Step 5:

According to Hurst's proposition, H-value can be obtained by solving the equation $\left(R/S\right)_t = C.n^H$.

One of the ways to estimate Hurst Exponent is to run an OLS regression taking logarithim values of the series.

$$\log_{10}\left[\left(R/S\right)_t\right] = \log_{10}[C] + H.\log_{10}[t].$$

However, the easier way to find the H-value is to plot $\left\{\log_{10}\left[R/S\right]_t\right\}$ versus $\log_{10}[t]$ in a graph and fit a regression line on it. The slope of the regression line is the H-value for the series.

3.2 Detrended Fluctuation Analysis

Similar to analysis procedure described in section 3.1, the impact of non-stationarity is first reduced by subtracting mean return from daily returns. This removes trend between end points of the time series. The detrended series is

$y_t = \sum_{i=1}^{N} [(x_i - \mu]$, where $\mu = 1/N \sum_{i=1}^{N} x_i$.

In the next step, we divided the entire data series into many contiguous sub-periods each having n-observations and defined each sub-period as I_a. Within each subperiod we tried to find out the local trend by fitting a straight line within the subperiod: $z_t = a.y_t + b$. In place of a straight line, a polynomial of higher order can also be fitted. The use of other detrending functions may improve the accuracy of the DFA technique but we kept this outside the scope of the present paper.

For each I_a of length n, the corresponding variance between y_t and its fitted value z_t is obtained by

$$F(n) = \sqrt{\frac{1}{n}\sum_{t=1}^{n}[y_t - z_t]^2}$$

The process mentioned above was repeated for various time scales n, and the scaling relationship is defined by $F(n) = c.n^H$, where c is the constant and H is the Hurst exponent.

4. Data

We have considered the stock index series of twelve stock indices across the globe for about N= 2560 data points. Because of different weekends and holidays of different countries, it was not possible to match a common date-wise series. We therefore collected data of past 2560 daily observations backwards from April 30, 2010.

The needed data was downloaded from Yahoo Finance website. Brief particular of each index series are given in Annexure 1.

Stock Index series and symbols used in the study are tabulated below:

Sl. No	Symbol	Index
1	AORD	The Australian All Ordinaries Index
2	BSE30	The Bombay Stock Exchange Sensitive Index
3	CAC40	The CAC-40 Index
4	DAX	The German Stock Index
5	DJI	The Dow Jones Industrial Average
6	FTSE	The FTSE 100 Index
7	HSI	The Hang Seng Index
8	KSE	The Karachi Stock Exchange
9	N225	The Nikkei-225 Stock Average
10	NDX	The NASDAQ Composite Index
11	SP500	Standard and Poor's 500 Index
12	STI	The Straits Times Index

5. Analysis

Standard descriptive statistics applicable to the selected index series are produced in Table 1. It shows means, medians, maximum and minimum values of the original series and standard deviations, skewness, and kurtosis of the daily returns obtained from the first difference of the series.

Table 1. Descriptive statistics of the index series

	Original Series			Daily Return Series				
Index	Maximum Value	Minimum Value	Average	Median	Mean	Standard Deviation	Skewness	Kurtosis
AORD	6854	2673	4139	0.049%	0.014%	1.020%	(0.70)	7.72
BSE30	20873	2600	8587	0.132%	0.048%	1.774%	(0.18)	5.86
CAC	6922	2403	4391	0.014%	-0.022%	1.564%	0.03	5.15
DAX	8106	2203	5293	0.068%	-0.009%	1.654%	0.07	4.33
DJI	14165	6547	10472	0.042%	0.001%	1.303%	0.01	7.79
FTSE	6798	3287	5235	0.039%	-0.009%	1.328%	(0.12)	6.33
HSI	31638	8409	15786	0.028%	0.011%	1.688%	0.00)	7.67
KSE	15676	1075	6534	0.137%	0.084%	1.585%	(0.22)	2.47
N225	20833	7055	12685	0.003%	-0.022%	1.623%	(0.30)	6.28
NDX	4705	805	1726	0.072%	-0.033%	2.173%	(0.25)	4.87
SP500	1565	677	1182	0.054%	-0.009%	1.389%	(0.11)	7.82
STI	3876	1214	2203	0.039%	0.011%	1.331%	(0.39)	5.64

All daily return series reported in Table 1, qualitatively show to some degree of departure from normal distribution. This is evidenced by the difference between mean returns from their matching medians and higher kurtosis values.

5.1 Estimation of Hurst Exponent

We estimated Hurst exponent values of each index series using DFA method mentioned in Section 3.2. For estimation of Hurst exponent, each series was split into several smaller series each having n observations and $F(n)$ value of each short series were obtained. The process were repeated by taking several values of n, n={10, 20, 30, 50, 100, 200}. After that, we plotted $\log_{10}[F(n)]$ versus $\log_{10}[n]$ and fit a straight regression line on it. The slope of the regression line is the H-value for the Index series. To get $F(n)$ value of the full series, the $F(n)$ values of 60-period series are averaged. The H values found as above are given in Chart 1 and Table 2.

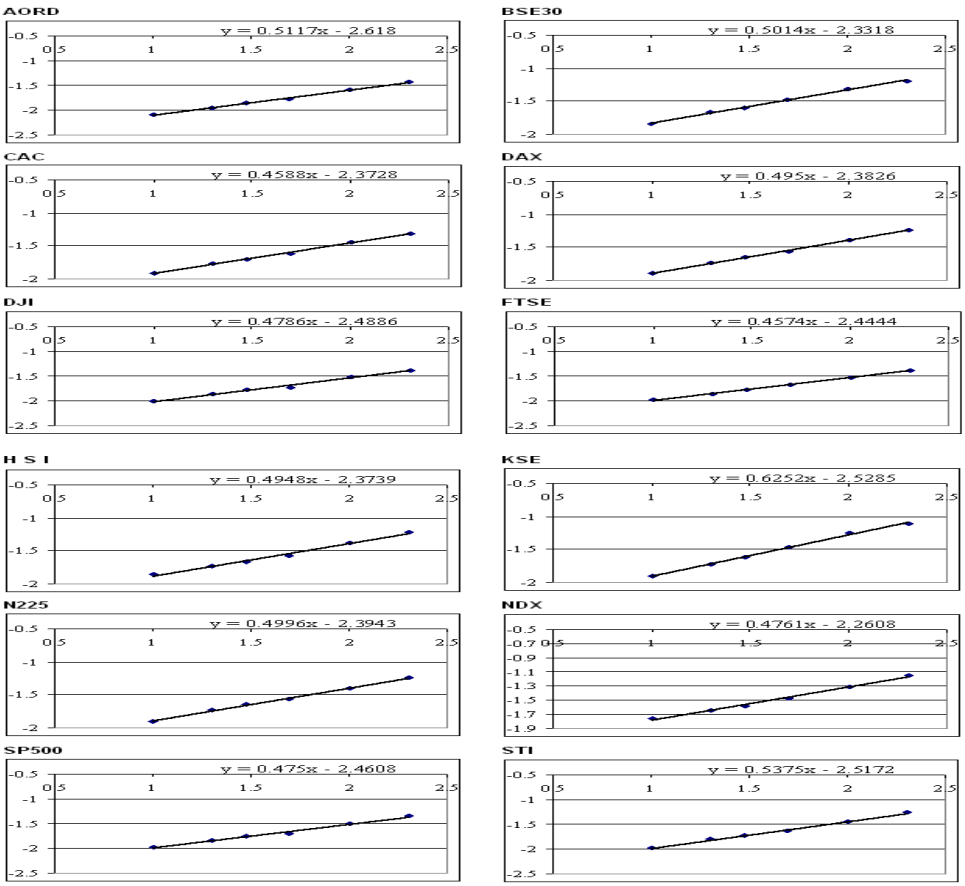

Chart 1. Hurst exponent of the full index series

To estimate H exponent value of the series, the average values of $\log_{10}[F(n)]$ are plotted in y-axis versus $\log_{10}[n]$ in x-axis for n={10, 20, 30, 50, 100, 200}. Refer section 3.2 and 5.1 for details. A fitted straight line and equation of the line are also displayed. The slope of the regression line is the H-value for the Index series.

Table 2. Hurst exponent of full index series

Index	H Exponent
AORD	0.51
BSE30	0.50
CAC	0.46
DAX	0.49
DJI	0.48
FTSE	0.46
HSI	0.49
KSE	0.63
N225	0.50
NDX	0.48
SP500	0.48
STI	0.54

It is found from the tables that Hurst exponent value for most of the series lie within the range of 0.46 to 0.54. Since H-values are very close to the value of 0.50, it can be said that the series are efficient and their movement resembles random walk, in general. However, it is a common observation in marketplace that despite market

efficiency, there are times when these series depart from normal behavior and show signs of trends. In order to detect presence of trends in a smaller period of time, the full series is divided into smaller series containing observation of 60 consecutive trading days. For each such series of 60 observations, the H-exponent is estimated using the DFA method mentioned in Section-3.2. The number of observation in these smaller series being 60, the $F(n)$ value for each of these series are estimated for n=10, 20 and 30. The H-value of each series was obtained by measuring slope of the trend line formed by plotting $\log_{10}[F(n)]$ versus $\log_{10}[n]$. The H-value for each of the intervening 60-day period is plotted in Chart 2.

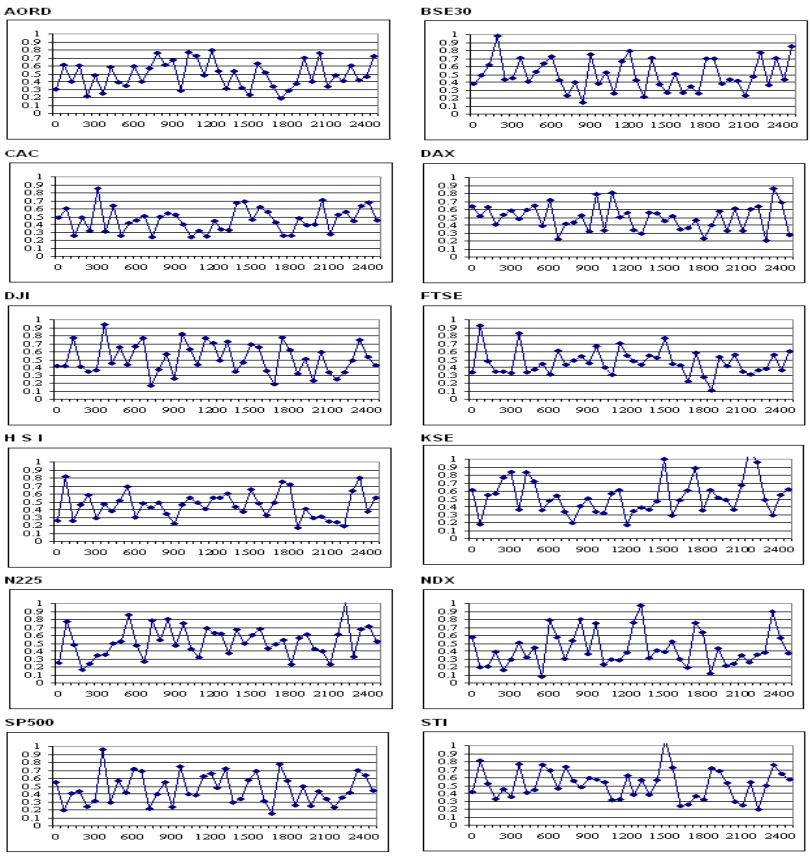

Chart 2. Variation of hurst exponent over time

The full series is divided into shorter series of 60 observations and H exponent of each short series is estimated. H-values of the short series are plotted in y-axis against days in x-axis. H-values are found to vary widely from period to period.

It can be watched from Chart-II that though the H-value for the full series is close to 0.5, during the intermediate periods it varies widely between 0 to 1. Whenever it is more than 0.5, it is expected show presence of long memory and display some kind of trending behavior. In addition, if trends can be detected, the same can be used for trading purposes.

5.2 Detecting Trends

Trends in the financial time series are usually detected using various tools developed under the guise of "Technical Analysis". Technical analysis is a discipline for forecasting the future direction of prices by analyzing past market data. The oldest known application of technical analysis can be traced Joseph de la Vega's accounts of the Dutch markets in the 17th century. Technical analysts use several techniques based on both numerical and graphical analysis. A popular class of numerical analysis is based on moving averages of time series. Moving averages smooth the price data to form a trend following indicator.

There are few different types of moving averages depending on how they are calculated. The calculations only differ in regards to the weighting that they place on the price data, shifting from equal weighting of each price point to more weight being placed on recent data. The most common types of moving averages is Simple Moving Average (SMA)

A simple moving average is formed by computing the average price of a security over a specific number of periods. Most moving averages are based on the closing prices. It takes the sum of the past closing prices over the chosen time period and divides the result by the number of data used in the calculation. For example, in a 10-day simple moving average, the last 10 closing prices are added together and then divided by 10. As its name imply, the moving average is an average that moves with new data. The oldest data is dropped as a new data is available. For example, on day 11 of a 10-day SMA, the data of the first day would be deleted from the calculation and data for the eleventh day would be added.

Moving averages of daily data can be used to create signals with simple price crossovers. A buy signal is produced when index value move above the moving average value. Similarly, a sell signal is produced when index value come below the moving average value. In this study, we followed the buy and sell signal produced by a simple 10-day moving average using the above-mentioned rule. Though actual trading in the marketplace entails trading costs, in this study, for simplicity, we ignored trading costs.

5.3 Relationship between H-value and Trading Profit

Since Hurst coefficient is supposed to provide a measure of trending characteristic in a time series, we tried to analyze correlation between Hurst coefficient and profits from the trend based trading rule. To calculate H-value for intervening period, each index series were split into smaller series of 60 trading days. For each of these 60-trading day period we estimated trading profit using 10-day moving average rule mentioned in previous section. We compared correlation coefficient between Hurst exponent values and trading profits of these 60-day smaller series. The correlation coefficient and the p-values are presented in Table 3.

Table 3. Correlation coefficient between H-value and trading profits

Index	Correlation Coefficient	p value
AORD	0.24	0.060
BSE30	0.21	0.088
CAC	0.33	0.017
DAX	0.32	0.018
DJI	0.51	0.000
FTSE	0.42	0.003
HSI	-0.07	0.323
KSE	0.18	0.125
N225	0.44	0.002
NDX	0.35	0.011
SP500	0.42	0.003
STI	0.20	0.097

It is interesting to note that correlation coefficients of most of the series are positive and in some cases, it is statistically significant. These observations support the view that Hurst exponent can be used to measure trending characteristics in a financial time series. To find out direct impact of Hurst exponent on trading profit, we further classified each 60-day smaller series based on their Hurst exponent values. The 60-day series are classified into following three categories.

- H-value less than 0.45
- H-value between 0.45 and 0.55
- H-value greater than 0.55

The trading profit obtained vis-à-vis Hurst exponent classifications of the 60-period series are presented in Table 4. Though results in few cases are not very obvious, in general, series showing higher H value has given higher trading profits.

Table 4. Average profit and hurst exponent in 60-day trading windows

	Number of Smaller Series			Average Profit in 60-trading days		
Series	H < 0.45	0.45 < H < 0.55	H > 0.55	H < 0.45	0.45 < H < 0.55	H > 0.55
AORD	19	7	16	0.71%	2.71%	2.02%
BSE30	22	6	14	5.33%	14.66%	11.12%
CAC	20	11	11	-3.89%	-8.08%	4.80%
DAX	17	9	16	-1.05%	0.03%	4.11%
DJI	19	6	17	-6.25%	-4.83%	1.16%
FTSE	23	9	10	-4.70%	-1.74%	1.67%
HSI	20	11	11	1.48%	4.07%	3.54%
KSE	16	9	17	12.52%	4.05%	13.81%
N225	15	10	17	-5.33%	2.49%	3.03%
NDX	28	3	11	-6.59%	4.23%	-0.77%
SP500	24	3	15	-4.65%	-1.83%	0.51%
STI	16	8	18	1.51%	0.95%	6.66%
Average				-0.91%	1.39%	4.30%

6. Conclusion

Hurst exponent, which was originally used to study natural phenomena of river water flows are now extended to study behavior of financial markets. We used daily data of twelve stock index series for past ten years (n=2560) and sub-divided the data into shorter series of 60 contiguous trading periods.

It was found that H exponent value of most of the index series are close to 0.50 as expected from an independent process. But when the H exponent is estimated over smaller window size of 60 days, the value is found to vary widely, reflecting departure from normality. A high value of H exponent is indicative of long memory in the time series, in which case future value will depend partially on past values of the series. Various tools developed by Technical Analysts tries to capture this dependency of future value on past values. To find relationship between Hurst exponent and Technical analysis tools, we employed a 10-day simple moving average indicator in 60-period trading windows and compared trading results with Hurst exponent of those windows. We noted that H-value and returns from a trading rule are correlated and therefore, H-value can be used as a measure to find appropriateness of using Technical Analysis. H-value themselves may not have any power to find out direction of a trend but trend-detecting rules are supposed to produce better results during the periods of high H-value. Therefore knowledge of H- value of the series is important.

The 10-period moving average used in the study is merely indicative; the analysis can be extended to other trading rules as well. However, in the original interpretation of Hurst exponent, a low H-value show means reversion behavior. Whether contrarian-trading rules work better during those periods, need further examination.

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Annexure 1. Brief particulars of the selected index series

Sl.	Symbol	Index	Brief Particulars
1	AORD	The Australian All Ordinaries Index	It is a capitalization weighted index. The index is made up of the largest 500 companies as measured by market cap that arelisted on the ASX. The index was developed with a base value of 500 as of 1979.
2	BSE30	The Bombay Stock Exchange Sensitive Index	It is a cap-weighted index. The selection of the index members has been made on the basis of liquidity, depth, and floating-stock-adjustment depth and industry representation. Sensex has a base date and value of 100 in 1978-1979. The Index shifted to Free-float methodology since 2003.
3	CAC40	The CAC-40 Index	It is a narrow-based, modified capitalization-weighted index of 40 companies listed on the Paris Bourse. The index was developed with a base level of 1,000 as of December 31, 1987. As of December 1, 2003 the index has become a free float weighted index.
4	DAX	The German Stock Index	It is a total return index of 30 selected German blue chip stocks traded on the Frankfurt Stock Exchange. The equities use free float shares in the index calculation. The DAX has a base value of 1,000 as of December 31, 1987.
5	DJI	The Dow Jones Industrial Average	It is a price-weighted average of 30 blue-chip stocks that are generally the leaders in their industry. It has been a widely followed indicator of the stock market since October 1, 1928.
6	FTSE	The FTSE 100 Index	It is a capitalization-weighted index of the 100 most highly capitalized companies traded on the London Stock Exchange. The equities use an investibility weighting in the index calculation. The index was developed with a base level of 1000 as of January 3, 1984.
7	HSI	The Hang Seng Index	It is a free-float capitalization-weighted index of selection of companies from the Stock Exchange of Hong Kong. The components of the index are divided into four subindexes: Commerce and Industry, Finance, Utilities, andProperties. The index was developed with a base level of 100 as of July 31, 1964.
8	KSE	The Karachi Stock Exchange	KSE Index comprises the top company from each of the 34 sectors on the KSE, in terms of market capitalization. The rest of the companies are picked on market cap ranking, without any consideration for the sector to make a sample of 100 common stocks with base value 1,000.
9	N225	The Nikkei-225 Stock Average	It is a price-weighted average of 225 top-rated Japanese companies listed in the First Section of the Tokyo Stock Exchange. The Nikkei Stock Average was first published on May 16, 1949, where the average price was ¥176.21 with a divisor of 225.
10	NDX	The NASDAQ Composite Index	It is a broad-based capitalization-weighted index of stocks in all three NASDAQ tiers: Global Select, Global Market and Capital Market. The index was developed with a base level of 100 as of February 5, 1971.
11	SP500	Standard and Poor's 500 Index	It is a capitalization-weighted index of 500 stocks. The index is designed to measure performance of the broad domestic economy through changes in the aggregate market value of 500 stocks representing all major industries. The index was developed with a base level of 10 for the 1941- 43 base period.
12	STI	The Straits Times Index	It is calculated and disseminated by FTSE, comprises the top 30 SGX Main board listed companies selected by full market capitalization. The index was revamped effective January 10, 2008.

(Source: www.bloomberg.com)