

# Relating the deBroglie and Compton Wavelengths to the Velocity of Light?

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## Abstract

No report in the literature has directly described this relation. New constants for particles are presented. One relates to the Compton wavelength, called here the “mass-wave” constant for all particles. The other relates to the deBroglie wavelength, called here the “velocity-wave” constant for a particle. An equation is derived based on these two constants encapsulating a fundamental relation between the matter-states, particle and wave, to the velocity of light. New approaches to the Uncertainty relations are shown. The basic Schrodinger equation is derived from the perspective of a non-dimensional second-order differential equation free of any assumed empirical constants. The resulting time-dependent wave equation for a free particle was then expressed in terms of the particle velocity and deBroglie wavelength.

**Keywords:** deBroglie particle constant, matter waves, new physical constants, new fundamental equation, wave-state, particle-state

## 1. Introduction

There appears to be no study in the literature that has directly examined this relation. One possible reason is that certain constants relevant to the relation have generally gone unrecognized. This paper describes a constant dependent on the Compton wavelength (Compton, 1923) and another dependent on the deBroglie wavelength (deBroglie, 1925) The analysis relates these constants to the velocity of light, the Uncertainty relation (Heisenberg, 1927) and the wave equation (Schrodinger, 1926).

## 2. Definitions

$m_0$  = particle rest mass; for the electron =  $9.109 \cdot 10^{-28}$  g.

$\lambda_C$  = Compton wavelength for a particle of specific rest mass ( $m_0$ ); for the electron it is experimentally measured to be =  $2.426 \cdot 10^{-10}$  cm.

$\lambda_B$  = deBroglie wavelength of a particle of specific rest mass at a known velocity.

$V_p$  = velocity of particle.

$c$  = velocity of light =  $2.998 \cdot 10^{10}$  cm/s.

$h$  = Planck's constant =  $6.626 \cdot 10^{-27}$  g·cm<sup>2</sup>/s.

$p$  = particle momentum =  $[m_0 \cdot V_p]$  g·cm/s.

## 3. Results

### 3.1 Relation to $c$

From the well-known deBroglie equation,  $\lambda_B = h / p$ , and with  $p = m \cdot V_p$ , obtain the relation  $\lambda_B \cdot V_p = h / m$ . Introduce the constant,  $\kappa = [m \cdot \lambda_C]_{\text{particle}} = 2.21 \cdot 10^{-37}$  g·cm, which holds for every particle. Then  $\kappa \cdot c = h$  and  $\lambda_C \cdot c = h / m$ , so for any particle this gives  $[(\lambda_B / \lambda_C) \cdot V_p]_{\text{particle}} = c$ .

This result for  $c$  is independent of particle mass. In the relation  $\lambda_B = \lambda_C \cdot c / V_p$ , when  $V_p$  approaches  $c$ , then the magnitude of the matter-wave property of the particle,  $\lambda_B$ , decreases, more rapidly than  $\lambda_C$  and approaches the magnitude of the equivalent  $\lambda_C$  for that particle. When  $V_p$  approaches zero the magnitude of  $\lambda_B$  increases and  $\lambda_B / \lambda_C$  approaches infinity, as expected.

Introduce the new constant,  $\beta_p = \lambda_B \cdot V_p$ , called here the deBroglie particle constant, for non-relativistic velocities where mass =  $m_0$ , which varies from particle to particle because it depends on the particle's rest mass via  $\lambda_B$ . A plot of  $\lambda_B$  versus  $V_p$  gives a rectangular hyperbola that is specific for each particle. Thus,  $\lambda_B \cdot V_p = \lambda_C \cdot c = \beta_p = h / m_0$  offers different ways to calculate  $\beta_p$  for a particle of rest mass,  $m_0$ . For the electron,  $\beta_e = (2.426 \cdot 10^{-10} \text{ cm}) \cdot (2.998 \cdot 10^{10} \text{ cm/s}) = 7.27 \text{ cm}^2/\text{s}$ . It gives for the proton,  $\beta_{pr} = (1.321 \cdot 10^{-13} \text{ cm}) \cdot (2.998 \cdot 10^{10} \text{ cm/s}) = 3.96 \cdot 10^{-3} \text{ cm}^2/\text{s}$ . The value of  $\beta_p$  depends inversely on the value of the particle's mass because a larger mass means a smaller  $\lambda_B$ , as well as a smaller  $\lambda_C$ . The values of all three parameters  $\lambda_B$ ,  $V_p$  and  $\lambda_C$  are accessible experimentally only for the electron.

At relativistic velocities  $m$  is increasing as  $V_p$  increases, so  $\beta_p$  now becomes a variable instead of a constant. The value of  $\lambda_B$  depends on  $\{[(c^2) - (V_p)^2] / (V_p)^2\}^{1/2}$ , whereas  $\lambda_C$  depends on  $\{[(c^2) - (V_p)^2] / c^2\}^{1/2}$ . Thus,  $\lambda_B$  must decrease faster than  $\lambda_C$  when  $V_p$  is increasing. This satisfies  $\lambda_B = \lambda_C \cdot c / V_p$ .

### 3.2 Uncertainty Relation

The Uncertainty relation (Heisenberg, 1927, Wheeler & Zurck, 1983) for pairs of non-commuting conjugate operators, such as position and momentum, is given by  $(\Delta x \cdot \Delta p) \geq h / 4\pi$ . It places a lower bound on the product of the standard deviations when considering measurements on non-commuting variables — particularly for preparation uncertain relations (Ma et al., 2016). The results obtained here offer another perspective on it. Rewriting in terms of  $V_p$ ,  $\lambda_B$ ,  $\lambda_C$  and  $\beta_p$  gives for any particle ( $V_p < c$ )

$$\Delta x \cdot \Delta p = \Delta x \cdot [\Delta(m_0 \cdot V_p)] \geq m_0 \cdot (\lambda_C \cdot c) / 4\pi = m_0 \cdot (\lambda_B \cdot V_p) / 4\pi$$

So,  $\Delta x \cdot \Delta V_p \geq (\beta_p / 4\pi) \text{ cm}^2/\text{s}$  and then dividing by  $\beta_p$  yields the product of the dimensionless percentage errors,  $(\Delta x / \lambda_B) \cdot (\Delta V_p / V_p) \geq 1 / 4\pi$ .

For kinetic energy and time (assuming  $\Delta t$  is seen as the time it takes for an observable to change by one standard deviation)

$$\Delta t \cdot \Delta E_{kin} = \Delta t \cdot (m_0 / 2) \cdot \Delta[(V_p)^2] \geq m_0 \cdot (\lambda_C \cdot c) / 4\pi = m_0 \cdot (\lambda_B \cdot V_p) / 4\pi$$

then  $\Delta t \cdot \Delta[(V_p)^2] \geq (\beta_p / 2\pi) \text{ cm}^2/\text{s}$  for any particle. For the lower bound (*lb*) case this leads to the relation  $[\Delta x / \Delta t] \cdot [\Delta V_p / \Delta(V_p)^2]_{lb} = 1 / 2$ .

One other relevant pair of operators does not commute. It requires different units than  $h$ . The relation is then  $\Delta x \cdot \Delta E_{kin} = \text{cm} \cdot \text{g} \cdot \text{cm}^2/\text{s}^2$ , which leads to

$$\Delta x \cdot \Delta[(V_p)^2] \geq [(\beta_p / 2\pi) \cdot < V_p >] \geq (\beta_p / 2\pi) \cdot V_p \text{ cm}^3/\text{s}^2$$

Dividing by  $\beta_p$  yields the product of the dimensionless percentage errors given by the relation  $(\Delta x / \lambda_B) \cdot \{\Delta[(V_p)^2] / (V_p)^2\} \geq 1 / 2\pi$ .

All these Uncertainty relations are expressible in terms of the deBroglie particle constant introduced here, which is specific to each particle and therefore governs the magnitude of the Uncertainty for that particle.

### 3.3 Wave Equation

The deBoglie idea of a matter-wave that characterized a moving particle was established experimentally by electron diffraction on crystals (Davisson & Germer, 1928) shortly after the serendipitous creation of the particle wave equation (Schrodinger, 1926). Given the concept of matter-waves, it seems reasonable to seek a differential equation (D.E.) that describes the relation between the relevant variables governing the behavior of the matter-wave for a particle in motion, *without assuming any empirical constants*. This suggests a non-dimensional D.E. that employs the fractional change form,  $\Delta_f = dy / y$ , the % change in the variable  $y$ . Define:

$dy / dt$  = velocity of  $y$  with respect to  $t$ .

$d^2y / dt^2$  = acceleration of  $y$  with respect to  $t$ .

$dy / dx$  = slope of the  $y$  versus  $x$  plot.

$(d / dx) (dy / dx)$  = gradient with respect to  $x$  of the slope =  $d^2y / dx^2$  = curvature.

Illustrations:

— Hooke's law for the position of the moving mass ( $x_m$ ) on the end of the coiled spring is defined by the relation, the % change acceleration = the % change distance, so  $\Delta_f (d^2x_m / dt^2) = \Delta_f (x_m)$ .

— Maxwell's second-order D.E. for  $E$  is defined by the % change curvature = the % change acceleration, so  $\Delta_f (\partial^2 E / \partial x^2) = \Delta_f (\partial^2 E / \partial t^2)$ .

Such non-dimensional D.E. equations are free of assumptions about empirical constants. These emerge directly from the integration and consideration of the units.

Assume the matter-wave for a free particle (moving with fixed velocity,  $V_p \ll c$ ), in one dimension and not influenced by any external field) exhibits a periodic behavior. Assume a function  $A(x,t)$  describes this behavior, where  $A$  is related to the amplitude of the particle’s matter-wave. For  $A(x,t)$ , the second-order partial D.E. follows from assuming that the behavior of  $A(x,t)$  is governed by the following simple relation

% change curvature of  $A$  = % change velocity of  $A$  giving

$$\Delta_f (\partial^2 A / \partial x^2) = \Delta_f (\partial A / \partial t)$$

This defines the basic physical relationship for the variables governing the particle’s matter-wave behavior under the given conditions. Integrating gives

$$d^2 A / dx^2 = C_1 \cdot dA / dt$$

where  $C_1$  has the units of  $s/cm^2$ . Thus, set  $C_1 = (-i / \beta_p)$  in terms of the deBroglie particle constant. Separating variables gives,  $A(x,t) = \Psi(x) \Phi(t)$ . Then for  $\Phi(t)$ ,

$$d\Phi(t) / \Phi(t) = (K / C_1) \cdot dt = -i \cdot \beta_p / (\lambda_B)^2$$

where the separation constant,  $K$ , has units of  $(1 / cm^2)$  giving  $K = (i / \lambda_B)^2$ , and so  $K / C_1 = (i / \lambda_B)^2 / (-i / \beta_p) = -i \cdot V_p / \lambda_B$ , for fixed velocity. After integrating,

$$\Phi(t) = C_2 \cdot \exp [t \cdot K / C_1] = C_2 \cdot \exp [-i \cdot t \cdot V_p / \lambda_B]$$

The time-independent part of the equation has a solution,

$$\Psi(x) = C_3 \cdot \exp [K^{1/2} \cdot x] = C_3 \cdot \exp [i \cdot x / \lambda_B]$$

Then, with  $C_2 \cdot C_3 = A$ ,

$$A(x,t) = \Psi(x) \Phi(t) = A \cdot \exp [(i \cdot x - V_p \cdot t) / \lambda_B] = A \cdot \exp [i \cdot (x - x_t) / \lambda_B]$$

expressed in terms of  $\lambda_B$ , the deBroglie wavelength of the particle’s matter-wave, and  $V_p$ , its velocity, where  $(V_p \cdot t) = x_t$ , the expected “classical” distance traveled. Thus,  $(x - x_t)$  is the dispersion around this expected result, which is a direct consequence of the quantum mechanical nature of the wave function’s prediction for the particle’s location. This dispersion becomes a % change when expressed as  $(x - x_t) / \lambda_B$ .

#### 4. Discussion

The equation  $[\lambda_B \cdot V_p = \lambda_C \cdot c]_{\text{particle}} = \beta_p$  encapsulates a fundamental relation for the two matter-states, particle and wave. This result for  $c$  has not appeared previously in the literature. The analysis revealed the unique connection of  $c$  to  $\lambda_B$ ,  $\lambda_C$  and  $V_p$  via the new constants  $\kappa$  and  $\beta_p$ . Recognize  $\kappa$  as the “mass-wave” constant for all particles and  $\beta_p$  as the matter-wave constant for any particular particle. It was essential to employ  $\kappa \cdot c = h$  in order to obtain this result for  $c$ .

The one-dimensional Schrodinger equation for the free non-relativistic electron was described in terms of  $\lambda_B$  and  $V_p$ . This result reopens the question of what exactly is waving in the wave equation? It suggests  $A(x,t)$  could be equated to the amplitude of the deBroglie matter-wave characterizing each solution of the wave function. The square of the amplitude of a classical wave is a measure of its relative intensity, which correlates with a relative probability. For relativistic velocities,  $\lambda_B \cdot V_p = \lambda_C \cdot c = \beta_p = h / m$ , a variable.

Looking at  $h$  in terms of  $h = \kappa \cdot c = m \cdot \lambda_C \cdot c = m \cdot \lambda_B \cdot V_p = m \cdot \beta_p$  reveals a different perspective on the Uncertainty principle, depending on the formulation of the specific variables involved. The role played by  $h$  in the Uncertainty principle is also reconsidered in terms of the appropriate parameters,  $\lambda_B$ ,  $\lambda_C$  and  $V_p$  as well as  $\beta_p$  and  $\kappa$ . This gave new expressions for the standard forms of the Uncertainty relation that were particle specific. Thus,  $\beta_p = h / m = h_p$  might also be interpreted as the “rationalized” Planck constant for any particle at velocity  $V_p$ .

The equation  $[\lambda_B \cdot V_p = \lambda_C \cdot c]_{\text{particle}}$  shows the close relationship between the wave-like behavior of a particle,  $\lambda_B$ , and the electromagnetic wave equivalent of that particle’s rest mass,  $\lambda_C$ . As  $V_p$  approaches  $c$ , these wavelengths approach ever more closely. This suggests that the wave character of a particle might approximate the wave character of an electromagnetic wave in this case.

## 5. Conclusions

The equation  $[\lambda_B \cdot V_p = \lambda_C \cdot c]_{\text{particle}}$  encapsulates a fundamental relation for the two matter-states, particle and wave. This result for  $c$  has not appeared previously in the literature. It revealed the unique connection of  $c$  to  $\lambda_B$ ,  $\lambda_C$  and  $V_p$  via the new constants  $\kappa$  and  $\beta_p$ . The utility of calculating  $\beta_p$  for the particle from  $\lambda_C \cdot c = \lambda_B \cdot V_p$  was shown. Looking at  $h$  in terms of  $h = \kappa \cdot c = m \cdot \lambda_C \cdot c = m \cdot \lambda_B \cdot V_p = m \cdot \beta_p$  reveals a different perspective on the Uncertainty principle, depending on the formulation of the specific relations involved. The time-dependent wave equation for a free particle was derived in terms of  $\lambda_B$  and  $V_p$ .

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