# Evaluation of Accuracy of the Geodetic Reference Systems for the Modelling of Normal Gravity Fields of Nigeria

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**Abstract**

The normal gravity fields of Nigeria have been modeled exploiting the Geodetic Reference System 1930 (GRS-30), Geodetic reference system 1967 (GRS-1967) and the world geodetic reference system 1984 (WGS-84). The determination of the normal gravity fields from the three Geodetic Reference Systems were carried out to evaluate the accuracy of one reference datum with respect to another. Descriptive statistics of the normal gravity field values modeled on regional and local scales showed a large difference ($16.11 mGal$) between the 1930 reference earth model (GRS-30) and 1967 reference earth model (GRS-67). A large difference was also established between the 1930 and 1984 reference earth models. The large difference in the mean of normal gravity field is attributed to error inherent in the Potsdam value ($16 mGal$). However, the small difference in normal gravity field values between the 1967 and 1984 reference systems is a pointer that the choice of either application in geophysical exploration or geodetic applications is of minor importance. The trends in discrepancies between the 1930 and 1967 reference earth models and 1930 and 1984 reference earth models are reflected in the standard deviation and standard error (S.E) of the normal gravity field values.

**Keywords:** Ellipsoid, Geodetic, Reference, Gravity, Normal, Statistics, Earth

## 1. Introduction

One of the stages in the analyses of gravity data is the conversion of gravity measurements to gravity anomalies. To determine gravity anomaly the theoretical (normal) gravity values are required. Arafin (2004) defined gravity anomaly at a point on the physical surface of the earth as the deviation of the earth's normal gravity value at the point of latitude from the observed gravity value at the same point. The normal gravity constitutes a theoretical value representing the acceleration of gravity that would be generated by a uniform ellipsoidal earth. If the figure of the earth is assumed to be ellipspsoid of revolution, the normal gravity will vary with the latitude of the observation points. The ellipsoid of revolution can be accomplished by rotating an ellipse about its minor axis. Thus, at the poles, the earth is slightly flattened and bulges at the equator. Therefore, the ellipsoid of revolution is the geometrical figure used to approximate the shape of the earth. The undisturbed surface of the ocean (no wind, no current, no tides and averaged over time) perpendicular to the direction of the plumb line is called the geoid.

There are several factors that determine the value of normal gravity at a point on the ellipsoidal surface. They include the size and shape of the ellipsoid and a value computed from observational data which represents the value of gravity at the equator. If normal gravity is computed for a relatively small area, for example a city; this constitutes local modelling of normal gravity. This type of modeling gives a local ellipsoidal surface (Fig.1a) and does not account for the curvature of the entire earth. The local ellipsoidal surface only closely approximates the size and shape of the earth in the small area. This idea represents that of a flat earth model. If a gravity survey is done for such small area (plane – table survey) exact location can be established relative to each other without accounting for the size and shape of the total earth. The difference in gravity value between the measured gravity and normal gravity values computed for such a relatively small area constitutes gravity anomaly. Determination of different types of gravity anomalies require that the observed data be reduced to an equipotential surface called the geoid.

The computation of normal gravity for the entire earth for the determination of the ellipsoid constitutes global modeling (Fig.1a). Where the local ellipsoidal surface is large, the modelling of the normal gravity field for example, country/region constitutes regional modelling. Osazuwa (1993) determined the normal gravity values for Nigeria and evaluated the accuracy of the geodetic reference systems (GRS-30, GRS-67 and GRS-80). This study considered an additional reference earth model, the WGS-84 (World geodetic system1984) which represent the latest reference earth model for determination of normal gravity field. Therefore, the objective of this study is to compute the normal gravity field of Nigeria per $0.5^{\circ} \times 0.5^{\circ}$ change in latitude (regional modeling) and normal gravity field for a geological province (fig1b) in southeastern Nigeria (local modeling ) where gravity data exists. Thus, where gravity observation exists, normal gravity field values could be handy for gravity anomalies determination. Gravity anomalies are useful for tectonic studies, structural relations of deep and adjacent geologic features (Okiwelu et al., 2010) and basin analyses. It is also useful for anomaly transformation (Saheel et al., 2010) and reconnaissance tool for hydrocarbon exploration. Since normal gravity field data are indispensable in modelling gravity anomalies the reference earth models for determining normal gravity fields should be evaluated to ascertain their accuracy. This is the main objective of this study. Normal gravity fields are computed from International gravity formula (IGF) [Geodetic Reference Systems (GRS)]. The popular reference earth models are the GRS-67 (Geodetic Reference System 1967) and WGS-84 (World Geodetic Reference System 1984). The 1930 formula (GRS-30) was previously used. Other earlier Schemes include WGS-72, WGS-66 and WGS-60. These geodetic Systems e.g. WGS84 are standard for coordinate frame for the earth and a standard spheroidal reference ellipsoid for raw data and gravitational equipotential surface that defines the nominal sea level. The geodetic systems are therefore very suitable for computing the equipotential surfaces. The WGS-84 reference ellipsoid is more amenable to geophysical applications (Fairhead et al., 2003, Kuhn et al., 2009) while the GRS-80 is usually used for geodetic applications. The earth's physical surface comprising the mountains, valleys, rivers and surface of the sea is highly irregular and not suitable as a computational surface. The geoid represents a smooth surface while the ellipsoid is a better smooth mathematical surface that best fits the shape of the earth. Both equipotential surfaces do not coincide. The separation between the two is referred to as geoid undulation or geoid heights.

## 2. Theory and Methodology

### *2.1 Theory*

The gravity on the ellipsoid can be derived from the gravitational potential U:

$$U = \frac{GM}{r} + \frac{GMa^2}{r^3} J_2 \left[ \frac{3}{2} Sin^2\phi - \frac{1}{2} \right] - \frac{1}{2} \omega^2 r^2 \cos^2 \phi \tag{1}$$

Where $r$ is the radius of the spheroid and varies with latitude, $\phi$ according to

$$r(\phi) = a(1 - f \sin^2 \phi) \tag{2}$$

$a$ is the radius of the earth at equator while G, M, $\omega$ are the gravitational constant, mass of the earth and the angular speed of the earth rotation respectively. The second term in equation (1) is due to spheroidal shape of the earth. $J_2$ is a constant determined by the distribution of mass and the term in bracket is the second degree harmonic giving the spheroidal shape. The third term is the centrifugal potential. $f$ in equation (2) is the flattering of the earth.

$f = \frac{a-c}{a} = \frac{1}{298.257223560}$ (Sandwell, 2002), where $c$ is the radius of the earth at the pole. Thus, the gravity on the ellipsoid is

$$g = -\nabla u \sqrt{\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial u}{r\partial \theta}\right)^2} \approx \frac{\partial u}{\partial r} = \frac{GM}{r^2} - \frac{3GMa^2}{r^4} J_2 \left[\frac{3}{2}\sin^2\phi - \frac{1}{2}\right] - \omega^2 r \cos^2\phi \quad (3)$$

The value of gravity on the ellipsoid is the normal gravity $\gamma$ given by

$$\gamma(\phi) = g_{eq}[1 + p\sin^2(\phi) + q\sin^2(2\phi)] \quad (4)$$

Where
$g_{eq} = 978.0327 gal (gravity\ at\ equator)$
$\rho = 0.0053024$
$q = -0.0000059$

The ellipsoidal model can now be written for GRS30, GRS1967, GRS80 and WGS84. Thus,

$$\gamma_{(\phi)1930} = 978049.0\left(1 + 0.0052884\sin^2\phi - 0.0000059\sin^2 2\phi\right) mGal \quad (5)$$

$$\gamma_{(\phi)1967} = 978031.8\left(1 + 0.0053024\sin^2\phi - 0.0000059\sin^2 2\phi\right) mGal \quad (6)$$

$$\gamma(\phi)1980 = 978032.7(1 + 0.0053024\sin^2\phi - 0.0000058\sin^2 2\phi) mGal \quad (7)$$

$$\gamma_{(\phi)1984} = 978031.85\left(1 + 0.00527889\sin^2\phi + 0.000023462\sin^4\phi\right) mGal \quad (8)$$

The expression for gravity anomaly can be written as

$$\Delta g = g_{obs} - \gamma_\phi \quad (9)$$

Where $g_{obs}$ is observed gravity

*2.2 Methodology*

The ellipsoidal model of the earth listed in equations (5, 6, 7 and 8) are utilized in geodesy for computing the shape of the earth and applied in exploration geophysics for determination of gravity anomalies. At the 1967 meeting of the International Union of Geodesy and Geophysics(IUGG) held in Lucerne Switzerland, the ellipsoid (GRS-67) was recommended for use to make up for the accuracy and precision lacking in GRS-30 which was adopted at the general assembly in Stockholm in 1930. The GRS67 was eventually approved and adopted at the 1971 meeting of the IUGG held in Moscow and therefore replaced the earlier Geodetic reference systems.

In December, 1979 at Canberra the general assembly of the IUGG adopted the Geodetic Reference System 1980. The body recognized the fact that the Geodetic reference System 1967 no longer represent the size, shape and gravity field of the earth to an accuracy adequate for many geodetic, geophysical, astronomical and hydrographic applications (Moritz,1980). The GRS-67 was then replaced by a new GRS-80 based on the theory of the geocentric equipotential ellipsoid defined by the following Conventional Constants (Moritz, 1980): equatorial radius of the earth (a) = 6378137m, geocentric gravitational constant of the earth (including atmosphere, $GM = 3986005 \times 10^8 m^3 s^{-2}$, dynamical form factor of the Earth, excluding the permanent tidal deformation: $J_2 = 108263 \times 10^{-8}$, angular velocity of the Earth: $\omega = 7292115 \times 10^{-11} rad/s$

Due to improved data, increasing data coverage, new data types and improved techniques, the idea of new WGS emerged. Using the GRS-80 parameters in addition to available doppler, satellite laser ranging and very long Baseline interferometry (VLBI), the WGS-84 (World Geodetic System 1984) was conceptualized. Additional data from satellite radar altimetry and sophisticated technique such as least squares method (Collocation) presented a uniform combination solution from different types of measurements; all relative to the earth's gravity field; including geoid and gravity anomalies. This latest earth gravitational model is currently the reference System being used by the Global Positioning Systems. At present the WGS84 exploits the 1996 Earth's Gravitational model (EGM96) geoid which was revised in 2004. This geoid defines the nominal sea level surface by means of spherical harmonics series of degree 360.

The normal gravity fields for Nigeria $(lat.2^{\circ} - lat.15^{\circ})$ have been determined based on the GRS-30, GRS-67 and WGS-84 using QCTools Software. An example of local modeling of normal gravity field is demonstrated using geological province (Calabar Flank) [fig.1b] in the Southeastern Sector of Nigeria. To evaluate the accuracy of the computed normal gravity fields, the values were subjected to descriptive statistical analyses using MINITAB 14 software.

## 3. Results and Discussion

The results of the normal gravity fields computed from the GRS-30, GRS-67 and WGS-84 on a regional and local scale are presented in *Tables 1 and 2* respectively. It is important to look at the descriptive statistics (e.g. mean, standard deviation) [table3, table4, fig.2, fig.3, fig.4, fig.5, fig.6 and fig.7]. The local modeling, for example, gives a mean difference of $16.79mGal$ between the GRS-30 and GRS-67. This result is consistent with the changes in theoretical gravity values with a difference of about $17.00mGal$ at the equator. However, there is a minimal difference of $1.3603mGal$ between WGS-84 and GRS-67. The conversion from 1967 reference model to 1984 reference model (Osazuwa, 1993) gives a value of -1.3603mGal. The large difference in values between GRS-30/GRS-67 and GRS-30/GRS-84 is a manifestation of the error in the Potsdam value (Dryden, 1942; Jeffreys, 1949; Morelli, 1959 and Wollard, 1950). This error necessitated a change from 1930 reference model to 1967 reference model. There was a recognition that the absolute gravity values at Potsdam System (1930 model) base station were in error and that allowing for an improved knowledge of the shape of the earth required a corresponding change in formula (Milson, 2003).

The mean values of normal gravity field for Nigeria (regional modeling) are $978167 \pm 20.4mGal$ and $978169 \pm 20.4mGal$. ; the values were computed from GRS-67 and WGS84 respectively. The closeness in values suggests that the choice of either approach is of minor importance for most applications in geophysical exploration and geodesy. The change from 1967 reference model to 1984 world geodetic system is progressing gradually since the actual changes implied in theoretical gravity are often smaller than the errors in absolute gravity of individual gravity stations and no changes in base station values are required (Milsom, 2003). Despite the change from GRS-67 to WGS-80 and now WGRS-84, the 1967 International Gravity formula is still very popular because of its compatibility with the network of international gravity base stations known as IGSN71[International Gravity Standardization Network(1971)]. While the International gravity formula (geodetic reference systems) serve as a means of computing the normal gravity at the reference ellipsoid based on the latitude of the point of consideration, the International standardization network serves as a reference datum by means of which absolute gravity values are extrapolated to another area through relative gravity measurements (Osazuwa, 2004). The changes in geodetic reference system in computing the normal gravity fields is due to changes in some geodetic parameters such as Newtonian gravitational Constant(G), Geocentric gravitational Constant(GM), mean equatorial gravity in the Zero-frequency tide system $(g_{eq})$ and equatorial radius of the reference ellipsoid(a). The WGS-84 was computed by considering the variation of atmospheric density. This is done by applying correction to measured values (Moritz, 1980). The reference ellipsoid is defined to enclose the whole mass of the earth including the atmosphere. The 1967 geodetic reference system computation is, however, based on the theory of the equipotential ellipsoid without an atmosphere.

The results of local modeling in *Table 4* from the three reference models show that the standard deviations $(2.54mGal)$ are low. This suggests that the set of data were computed at close interval. This is approximately at $0.01^{0}$ intervals. The computation of data point at $0.5^{\circ} \times 0.5^{\circ}$ change in latitude for the regional modeling increased the standard deviation to $106mGal$. The similarity in standard deviations of $2.54mGal$ *and* $106mGal$ for the local and regional modeling respectively for the three reference models is a pointer that the data sets were computed at equal intervals. This interpretation is compatible with the standard errors in *Tables 3 and 4*. The standard errors $(0.158mGal)$ for the three reference models are similar for the locally modeled normal gravity values and $20.4mGal$ for the regional modeled values.

The relationship between the three reference models is demonstrated in *Figs.5 and 6* (normal curve superimposed in histogram). The curve is skewed more to the left in the 1930/1967 and 1930/1984 reference earth models than the 1967/1984 reference earth models. These relationships are revealed also in the box plot. Medians for normal gravity fields for Nigeria are $978145mGal$ *and* $978146mGal$ for 1967 and 1984 geodetic reference systems respectively. This implies a minimal difference and close association between the two models. The median value for the 1930 model is however $978162mGal$ suggesting a large difference from the other models. This relationship is propagated to the locally modeled values Figs. *6 and 7* which are not unconnected with the error in Potsdam value.

## 4. Conclusion

Evaluation of the normal gravity field determined from the geodetic reference systems (1930, 1967 and 1984) for Nigeria revealed a large difference between the 1930 and 1967 reference earth models and 1930 and 1984 reference earth models. The large difference in the means of normal gravity fields manifests as constant bias (16mGal).The large difference is consistent with the error in Potsdam value. The minor difference between the values computed from the GRS-67 and WGS-84 suggests that the choice of either approach is of minor importance for most applications in geophysical exploration and geodetic applications. The change in geodetic reference systems in computing normal gravity fields is due to changes in some geodetic parameters. The results of the normal gravity fields are consistent because the values are dependent only on latitude of the computation point. The standard deviation and standard errors are also consistent indicating that the data points were computed at equal interval.

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Table 1. Regional modeling of normal gravity fields from the geodetic reference systems

| Station No. | Latitude (Deg.) | GRS-30 (mGal) | GRS-67 (mGal) | WGS-84 (mGal) |
|---|---|---|---|---|
| 1 | 2.000 | 978055.272 | 978038.138 | 978039.502 |
| 2 | 2.500 | 978058.797 | 978041.673 | 978043.045 |
| 3 | 3.000 | 978063.104 | 978045.992 | 978047.368 |
| 4 | 3.500 | 978068.191 | 978051.092 | 978052.456 |
| 5 | 4.000 | 978074.057 | 978056.973 | 978058.345 |
| 6 | 4.500 | 978080.699 | 978063.633 | 978065.010 |
| 7 | 5.000 | 978088.116 | 978071.070 | 978072.434 |
| 8 | 5.500 | 978096.305 | 978079.281 | 978080.620 |
| 9 | 6.000 | 978105.264 | 978088.264 | 978089.612 |
| 10 | 6.500 | 978114.991 | 978098.017 | 978099.367 |
| 11 | 7.000 | 978125.482 | 978108.536 | 978109.894 |
| 12 | 7.500 | 978136.735 | 978119.818 | 978121.163 |
| 13 | 8.000 | 978148.745 | 978131.861 | 978133.185 |
| 14 | 8.500 | 978161.510 | 978144.659 | 978146.004 |
| 15 | 9.000 | 978175.025 | 978158.210 | 978159.556 |
| 16 | 9.500 | 978189.286 | 978172.510 | 978173.859 |
| 17 | 10.000 | 978204.289 | 978187.553 | 978188.915 |
| 18 | 10.500 | 978220.030 | 978203.336 | 978204.703 |
| 19 | 11.000 | 978236.504 | 978219.854 | 978221.206 |
| 20 | 11.500 | 978253.706 | 978237.101 | 978238.460 |
| 21 | 12.000 | 978271.630 | 978255.073 | 978256.438 |
| 22 | 12.500 | 978290.272 | 978273.765 | 978275.135 |
| 23 | 13.000 | 978309.626 | 978293.170 | 978294.549 |
| 24 | 13.500 | 978329.685 | 978313.282 | 978314.658 |
| 25 | 14.000 | 978350.444 | 978334.097 | 978335.464 |
| 26 | 14.500 | 978371.897 | 978355.606 | 978356.972 |
| 27 | 15.000 | 978394.037 | 978377.805 | 978379.177 |

Table 2. Locally modeled normal gravity field values from the geodetic reference systems

| Station No | Longitude (Deg) | Latitude (Deg) | GRS- 30 (mGal) | GRS- 67 (mGal) | WGS- 84 (mGal) |
|---|---|---|---|---|---|
| 1 | 8.3215 | 4.9600 | 978087.494 | 978070.446 | 978072.001 |
| 2 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 3 | 8.3215 | 4.9600 | 978087.494 | 978070.446 | 978072.001 |
| 4 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 5 | 8.3330 | 5.1166 | 978089.956 | 978072.915 | 978074.470 |
| 6 | 8.3483 | 5.1466 | 978090.437 | 978073.397 | 978074.952 |
| 7 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 8 | 8.3446 | 5.1550 | 978090.572 | 978073.532 | 978075.087 |
| 9 | 8.3446 | 5.1616 | 978090.678 | 978073.639 | 978075.194 |
| 10 | 8.3600 | 5.1700 | 978090.813 | 978073.775 | 978075.329 |
| 11 | 8.5160 | 5.1783 | 978090.947 | 978073.909 | 978075.464 |
| 12 | 8.3216 | 4.9600 | 978087.494 | 978070.446 | 978072.001 |
| 13 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 14 | 8.3416 | 5.1333 | 978090.223 | 978073.183 | 978074.738 |
| 15 | 8.3366 | 5.0400 | 978088.742 | 978071.698 | 978073.253 |
| 16 | 8.3140 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 17 | 8.3366 | 5.1050 | 978089.771 | 978072.730 | 978074.284 |
| 18 | 8.3433 | 5.0966 | 978089.638 | 978072.596 | 978074.150 |
| 19 | 8.3450 | 5.0866 | 978089.479 | 978072.437 | 978073.991 |
| 20 | 8.3466 | 5.0783 | 978089.347 | 978072.305 | 978073.859 |
| 21 | 8.3498 | 5.0703 | 978089.220 | 978072.177 | 978073.732 |
| 22 | 8.3528 | 5.0626 | 978089.099 | 978072.055 | 978073.610 |
| 23 | 8.3548 | 5.0546 | 978088.972 | 978071.929 | 978073.483 |
| 24 | 8.3566 | 5.0633 | 978089.110 | 978072.067 | 978073.621 |
| 25 | 8.3666 | 5.0370 | 978088.695 | 978071.651 | 978073.205 |
| 26 | 8.3450 | 5.0350 | 978088.664 | 978071.619 | 978073.174 |
| 27 | 8.3466 | 5.0360 | 978088.679 | 978071.635 | 978073.190 |
| 28 | 8.3366 | 5.0333 | 978088.637 | 978071.593 | 978073.147 |
| 29 | 8.3140 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 30 | 8.3140 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 31 | 8.3588 | 5.1996 | 978091.292 | 978074.255 | 978075.809 |
| 32 | 8.3613 | 5.2138 | 978091.523 | 978074.486 | 978076.041 |
| 33 | 8.3596 | 5.2088 | 978091.442 | 978074.405 | 978075.959 |
| 34 | 8.3610 | 5.2380 | 978091.917 | 978074.882 | 978076.436 |
| 35 | 8.3595 | 5.2490 | 978092.097 | 978075.062 | 978076.617 |
| 36 | 8.3576 | 5.2610 | 978092.294 | 978075.259 | 978076.814 |
| 37 | 8.3593 | 5.2710 | 978092.458 | 978075.424 | 978076.979 |
| 38 | 8.3583 | 5.2696 | 978092.435 | 978075.401 | 978076.956 |
| 39 | 8.3610 | 5.2920 | 978092.804 | 978075.771 | 978077.326 |
| 40 | 8.3598 | 5.2980 | 978092.904 | 978075.871 | 978077.425 |
| 41 | 8.3561 | 5.3060 | 978093.036 | 978076.003 | 978077.558 |
| 42 | 8.3526 | 5.3145 | 978093.177 | 978076.144 | 978077.699 |
| 43 | 8.3363 | 5.3213 | 978093.289 | 978076.257 | 978077.812 |
| 44 | 8.3403 | 5.3206 | 978093.278 | 978076.246 | 978077.800 |
| 45 | 8.3336 | 5.3443 | 978093.672 | 978076.641 | 978078.196 |

| | | | | | |
|---|---|---|---|---|---|
| 46 | 8.2750 | 5.3385 | 978093.575 | 978076.544 | 978078.099 |
| 47 | 8.3200 | 5.3438 | 978093.664 | 978076.633 | 978078.187 |
| 48 | 8.3150 | 5.3348 | 978093.514 | 978076.482 | 978078.037 |
| 49 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 50 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 51 | 8.3453 | 5.1560 | 978090.588 | 978073.549 | 978075.103 |
| 52 | 8.3483 | 5.1533 | 978090.544 | 978073.505 | 978075.060 |
| 53 | 8.3300 | 5.1511 | 978090.509 | 978073.470 | 978075.024 |
| 54 | 8.3266 | 5.1690 | 978090.797 | 978073.759 | 978075.313 |
| 55 | 8.3075 | 5.1750 | 978090.894 | 978073.856 | 978075.410 |
| 56 | 8.3086 | 5.1793 | 978090.964 | 978073.925 | 978075.480 |
| 57 | 8.3005 | 5.1825 | 978091.015 | 978073.977 | 978075.532 |
| 58 | 8.2950 | 5.1921 | 978091.171 | 978074.133 | 978075.688 |
| 59 | 8.2870 | 5.1940 | 978091.201 | 978074.164 | 978075.718 |
| 60 | 8.2781 | 5.1958 | 978091.231 | 978074.193 | 978075.748 |
| 61 | 8.2701 | 5.1926 | 978091.179 | 978074.141 | 978075.696 |
| 62 | 8.2601 | 5.1868 | 978091.085 | 978074.047 | 978075.602 |
| 63 | 8.2528 | 5.1843 | 978091.044 | 978074.006 | 978075.561 |
| 64 | 8.2455 | 5.1808 | 978090.988 | 978073.950 | 978075.504 |
| 65 | 8.2361 | 5.1775 | 978090.934 | 978073.896 | 978075.451 |
| 66 | 8.2193 | 5.1801 | 978090.976 | 978073.938 | 978075.493 |
| 67 | 8.2198 | 5.1825 | 978091.015 | 978073.977 | 978075.532 |
| 68 | 8.2115 | 5.1871 | 978091.090 | 978074.052 | 978075.606 |
| 69 | 8.2000 | 5.1883 | 978091.109 | 978074.071 | 978075.626 |
| 70 | 8.1955 | 5.1836 | 978091.033 | 978073.995 | 978075.550 |
| 71 | 8.1905 | 5.1668 | 978090.762 | 978073.723 | 978075.278 |
| 72 | 8.1831 | 5.1713 | 978090.834 | 978073.796 | 978075.350 |
| 73 | 8.1758 | 5.1676 | 978090.775 | 978073.736 | 978075.291 |
| 74 | 8.1653 | 5.1680 | 978090.781 | 978073.742 | 978075.297 |
| 75 | 8.1570 | 5.1666 | 978090.759 | 978073.720 | 978075.274 |
| 76 | 8.1481 | 5.1668 | 978090.762 | 978073.723 | 978075.278 |
| 77 | 8.1838 | 5.1688 | 978090.794 | 978073.755 | 978075.310 |
| 78 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 79 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 80 | 8.5283 | 5.1755 | 978090.902 | 978073.864 | 978075.418 |
| 81 | 8.5540 | 5.2321 | 978091.821 | 978074.785 | 978076.340 |
| 82 | 8.5675 | 5.2730 | 978092.491 | 978075.457 | 978077.012 |
| 83 | 8.5425 | 5.2721 | 978092.477 | 978075.442 | 978076.997 |
| 84 | 8.5270 | 5.1493 | 978090.480 | 978073.441 | 978074.995 |
| 85 | 8.5273 | 5.1030 | 978089.739 | 978072.698 | 978074.253 |
| 86 | 8.4931 | 5.0851 | 978089.455 | 978072.413 | 978073.967 |
| 87 | 8.5230 | 5.0688 | 978089.197 | 978072.154 | 978073.708 |
| 88 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 89 | 8.5000 | 5.0781 | 978089.344 | 978072.301 | 978073.856 |
| 90 | 8.3386 | 5.0790 | 978089.358 | 978072.316 | 978073.870 |
| 91 | 8.3806 | 5.0741 | 978089.280 | 978072.238 | 978073.792 |
| 92 | 8.4798 | 5.0681 | 978089.186 | 978072.143 | 978073.697 |

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|---|---|---|---|---|---|
| 93 | 8.4636 | 5.0513 | 978088.920 | 978071.877 | 978073.431 |
| 94 | 8.4408 | 5.0343 | 978088.653 | 978071.608 | 978073.163 |
| 95 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 96 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 97 | 8.4453 | 5.0468 | 978088.849 | 978071.805 | 978073.360 |
| 98 | 8.5008 | 5.0333 | 978088.637 | 978071.593 | 978073.147 |
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| 100 | 8.5405 | 5.0255 | 978088.515 | 978071.470 | 978073.024 |
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| 104 | 8.4666 | 5.0616 | 978089.083 | 978072.040 | 978073.594 |
| 105 | 8.3488 | 5.0693 | 978089.205 | 978072.162 | 978073.716 |
| 106 | 8.3378 | 5.0721 | 978089.249 | 978072.206 | 978073.761 |
| 107 | 8.3238 | 5.0638 | 978089.118 | 978072.074 | 978073.629 |
| 108 | 8.3070 | 5.0561 | 978088.996 | 978071.953 | 978073.507 |
| 109 | 8.3488 | 5.0693 | 978089.205 | 978072.162 | 978073.716 |
| 110 | 8.2500 | 5.0000 | 978088.116 | 978071.070 | 978072.624 |
| 111 | 8.2250 | 5.0000 | 978088.116 | 978071.070 | 978072.624 |
| 112 | 8.2000 | 5.0000 | 978088.116 | 978071.070 | 978072.624 |
| 113 | 8.1500 | 5.0000 | 978088.116 | 978071.070 | 978072.624 |
| 114 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 115 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 116 | 8.3606 | 4.9718 | 978087.677 | 978070.630 | 978072.184 |
| 117 | 8.3655 | 4.9290 | 978087.015 | 978069.967 | 978071.521 |
| 118 | 8.3776 | 5.1965 | 978091.242 | 978074.204 | 978075.759 |
| 119 | 8.3938 | 4.9606 | 978087.503 | 978070.456 | 978072.010 |
| 120 | 8.3883 | 4.9525 | 978087.378 | 978070.330 | 978071.884 |
| 121 | 8.3953 | 4.9468 | 978087.290 | 978070.242 | 978071.796 |
| 122 | 8.4051 | 4.9391 | 978087.171 | 978070.122 | 978071.677 |
| 123 | 8.5683 | 4.9366 | 978087.132 | 978070.084 | 978071.638 |
| 124 | 8.4223 | 4.9368 | 978087.135 | 978070.087 | 978071.641 |
| 125 | 8.4215 | 4.9530 | 978087.385 | 978070.338 | 978071.892 |
| 126 | 8.4386 | 4.9335 | 978087.085 | 978070.036 | 978071.590 |
| 127 | 8.4473 | 4.9480 | 978087.308 | 978070.260 | 978071.815 |
| 128 | 8.4551 | 4.9233 | 978086.928 | 978069.879 | 978071.433 |
| 129 | 8.4611 | 4.0855 | 978075.137 | 978058.057 | 978059.611 |
| 130 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 131 | 8.4641 | 4.9116 | 978086.748 | 978069.699 | 978071.253 |
| 132 | 8.5230 | 4.7930 | 978084.951 | 978067.897 | 978069.451 |
| 133 | 8.5188 | 4.7971 | 978085.013 | 978067.958 | 978069.513 |
| 134 | 8.5120 | 4.8020 | 978085.086 | 978068.032 | 978069.587 |
| 135 | 5.5096 | 4.8111 | 978085.223 | 978068.169 | 978069.723 |
| 136 | 8.5105 | 4.8196 | 978085.350 | 978068.297 | 978069.852 |
| 137 | 8.5075 | 4.8266 | 978085.456 | 978068.403 | 978069.957 |
| 138 | 8.5046 | 4.8460 | 978085.749 | 978068.697 | 978070.251 |
| 139 | 8.5026 | 4.8652 | 978086.040 | 978068.988 | 978070.543 |

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|---|---|---|---|---|---|
| 140 | 8.4921 | 4.8778 | 978086.231 | 978069.181 | 978070.735 |
| 141 | 8.4775 | 4.8891 | 978086.404 | 978069.353 | 978070.908 |
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| 143 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 144 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 145 | 8.3613 | 5.1375 | 978090.291 | 978073.251 | 978074.805 |
| 146 | 8.3566 | 5.1370 | 978090.283 | 978073.243 | 978074.797 |
| 147 | 8.3640 | 5.1366 | 978090.276 | 978073.236 | 978074.791 |
| 148 | 8.3670 | 5.1360 | 978090.267 | 978073.227 | 978074.781 |
| 149 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 150 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 151 | 8.1780 | 5.1674 | 978090.771 | 978073.733 | 978075.287 |
| 152 | 8.1806 | 5.0800 | 978089.374 | 978072.332 | 978073.886 |
| 153 | 8.1848 | 5.1571 | 978090.606 | 978073.566 | 978075.121 |
| 154 | 8.1894 | 5.1487 | 978090.470 | 978073.431 | 978074.986 |
| 155 | 8.1943 | 5.1419 | 978090.361 | 978073.321 | 978074.876 |
| 156 | 8.1922 | 5.1339 | 978090.233 | 978073.193 | 978074.747 |
| 157 | 8.1892 | 5.1262 | 978090.110 | 978073.069 | 978074.624 |
| 158 | 8.1877 | 5.1176 | 978089.972 | 978072.931 | 978074.486 |
| 159 | 8.1826 | 5.1090 | 978089.835 | 978072.794 | 978074.348 |
| 160 | 8.1845 | 5.1022 | 978089.727 | 978072.685 | 978074.240 |
| 161 | 8.1903 | 5.1022 | 978089.727 | 978072.685 | 978074.240 |
| 162 | 8.1964 | 5.0899 | 978089.531 | 978072.489 | 978074.044 |
| 163 | 8.2033 | 5.0843 | 978089.442 | 978072.400 | 978073.954 |
| 164 | 8.2110 | 5.0789 | 978089.357 | 978072.314 | 978073.869 |
| 165 | 8.2183 | 5.0724 | 978089.254 | 978072.211 | 978073.765 |
| 166 | 8.2206 | 5.0649 | 978089.135 | 978072.092 | 978073.646 |
| 167 | 8.2246 | 5.0577 | 978089.021 | 978071.978 | 978073.532 |
| 168 | 8.2180 | 5.0511 | 978088.917 | 978071.873 | 978073.428 |
| 169 | 8.2327 | 5.0433 | 978088.794 | 978071.750 | 978073.305 |
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| 171 | 8.2430 | 5.0292 | 978088.573 | 978071.528 | 978073.083 |
| 172 | 8.2503 | 5.0890 | 978089.517 | 978072.475 | 978074.029 |
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| 174 | 8.2617 | 5.0103 | 978088.276 | 978071.231 | 978072.786 |
| 175 | 8.2533 | 5.0031 | 978088.164 | 978071.118 | 978072.673 |
| 176 | 8.2735 | 4.9957 | 978088.048 | 978071.002 | 978072.557 |
| 177 | 8.2764 | 4.9885 | 978087.936 | 978070.890 | 978072.445 |
| 178 | 8.2789 | 4.9840 | 978087.866 | 978070.820 | 978072.374 |
| 179 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 180 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 181 | 8.1743 | 5.1704 | 978090.820 | 978073.781 | 978075.336 |
| 182 | 8.1745 | 5.1776 | 978090.936 | 978073.898 | 978075.452 |
| 183 | 8.1751 | 5.1810 | 978090.991 | 978073.953 | 978075.507 |
| 184 | 8.1713 | 5.1905 | 978091.145 | 978074.107 | 978075.662 |
| 185 | 8.1614 | 5.9626 | 978104.568 | 978087.566 | 978089.120 |
| 186 | 8.1542 | 5.2009 | 978091.313 | 978074.276 | 978075.831 |

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|---|---|---|---|---|---|
| 187 | 8.1471 | 5.2068 | 978091.409 | 978074.372 | 978075.927 |
| 188 | 8.1404 | 5.2124 | 978091.500 | 978074.463 | 978076.018 |
| 189 | 8.1283 | 5.2157 | 978091.554 | 978074.517 | 978076.072 |
| 190 | 8.1165 | 5.2206 | 978091.634 | 978074.597 | 978076.152 |
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| 192 | 8.0899 | 5.2317 | 978091.815 | 978074.779 | 978076.333 |
| 193 | 8.0841 | 5.2363 | 978091.890 | 978074.854 | 978076.409 |
| 194 | 8.0667 | 5.0855 | 978089.461 | 978072.419 | 978073.974 |
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| 197 | 8.2820 | 5.2944 | 978092.844 | 978075.811 | 978077.366 |
| 198 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 199 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 200 | 8.1773 | 5.1027 | 978089.735 | 978072.693 | 978074.248 |
| 201 | 8.1724 | 5.0986 | 978089.669 | 978072.628 | 978074.182 |
| 202 | 8.1673 | 5.0919 | 978089.563 | 978072.521 | 978074.075 |
| 203 | 8.1613 | 5.0845 | 978089.445 | 978072.403 | 978073.958 |
| 204 | 8.1561 | 5.0773 | 978089.331 | 978072.289 | 978073.843 |
| 205 | 8.1536 | 5.0707 | 978089.227 | 978072.184 | 978073.738 |
| 206 | 8.1572 | 5.0638 | 978089.118 | 978072.074 | 978073.629 |
| 207 | 8.3410 | 5.1360 | 978090.267 | 978073.227 | 978074.781 |
| 208 | 8.3410 | 5.1360 | 978090.267 | 978073.227 | 978074.781 |
| 209 | 8.3319 | 5.3391 | 978093.585 | 978076.554 | 978078.109 |
| 210 | 8.3408 | 5.3417 | 978093.629 | 978076.598 | 978078.152 |
| 211 | 8.3473 | 5.3481 | 978093.735 | 978076.705 | 978078.259 |
| 212 | 8.3380 | 5.3493 | 978093.755 | 978076.725 | 978078.279 |
| 213 | 8.3605 | 5.3517 | 978093.795 | 978076.765 | 978078.319 |
| 214 | 8.3664 | 5.3517 | 978093.795 | 978076.765 | 978078.319 |
| 215 | 8.3731 | 5.3495 | 978093.759 | 978076.728 | 978078.283 |
| 216 | 8.3808 | 5.3497 | 978093.762 | 978076.731 | 978078.286 |
| 217 | 8.3886 | 5.3486 | 978093.744 | 978076.713 | 978078.268 |
| 218 | 8.3945 | 5.3502 | 978093.770 | 978076.740 | 978078.294 |
| 219 | 8.4018 | 5.3509 | 978093.782 | 978076.751 | 978078.306 |
| 220 | 8.3855 | 5.3400 | 978093.600 | 978076.569 | 978078.124 |
| 221 | 8.3782 | 5.3342 | 978093.504 | 978076.472 | 978078.027 |
| 222 | 8.3827 | 5.3300 | 978093.434 | 978076.402 | 978077.957 |
| 223 | 8.3902 | 5.3259 | 978093.366 | 978076.334 | 978077.889 |
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| 225 | 8.4012 | 5.3170 | 978093.218 | 978076.186 | 978077.741 |
| 226 | 8.3943 | 5.3197 | 978093.263 | 978076.231 | 978077.785 |
| 227 | 8.3900 | 5.3186 | 978093.245 | 978076.213 | 978077.767 |
| 228 | 8.3842 | 5.3170 | 978093.218 | 978076.186 | 978077.741 |
| 229 | 8.3794 | 5.3133 | 978093.157 | 978076.124 | 978077.679 |
| 230 | 8.3761 | 5.3079 | 978093.067 | 978076.035 | 978077.589 |
| 231 | 8.2217 | 5.3033 | 978092.991 | 978075.958 | 978077.513 |
| 232 | 8.3640 | 5.2993 | 978092.925 | 978075.892 | 978077.447 |
| 233 | 8.3612 | 5.2946 | 978092.847 | 978075.814 | 978077.369 |

| | | | | | |
|---|---|---|---|---|---|
| 234 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 235 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |
| 236 | 8.4691 | 4.9118 | 978086.751 | 978069.702 | 978071.256 |
| 237 | 8.4752 | 4.9151 | 978086.802 | 978069.752 | 978071.307 |
| 238 | 8.4796 | 4.9213 | 978086.897 | 978069.848 | 978071.402 |
| 239 | 8.4801 | 4.9213 | 978086.897 | 978069.848 | 978071.402 |
| 240 | 8.4890 | 4.9315 | 978087.054 | 978070.005 | 978071.560 |
| 241 | 8.4971 | 4.9346 | 978087.101 | 978070.053 | 978071.607 |
| 242 | 8.5042 | 4.9346 | 978087.101 | 978070.053 | 978071.607 |
| 243 | 8.5118 | 4.9412 | 978087.203 | 978070.155 | 978071.709 |
| 244 | 8.5206 | 4.4935 | 978080.607 | 978063.542 | 978065.096 |
| 245 | 8.5294 | 4.9449 | 978087.260 | 978070.212 | 978071.767 |
| 246 | 8.5382 | 4.9474 | 978087.299 | 978070.251 | 978071.805 |
| 247 | 8.5401 | 4.9450 | 978087.262 | 978070.214 | 978071.768 |
| 248 | 8.5454 | 4.9453 | 978087.266 | 978070.218 | 978071.773 |
| 249 | 8.5510 | 5.0100 | 978088.272 | 978071.226 | 978072.781 |
| 250 | 8.5570 | 4.9432 | 978087.234 | 978070.186 | 978071.740 |
| 251 | 8.5631 | 4.9434 | 978087.237 | 978070.189 | 978071.743 |
| 252 | 8.5694 | 4.9449 | 978087.260 | 978070.212 | 978071.767 |
| 253 | 8.5755 | 4.9394 | 978087.175 | 978070.127 | 978071.682 |
| 254 | 8.5827 | 4.9337 | 978087.088 | 978070.039 | 978071.594 |
| 255 | 8.9264 | 4.9304 | 978087.037 | 978069.988 | 978071.543 |
| 256 | 8.6000 | 4.9359 | 978087.121 | 978070.073 | 978071.628 |
| 257 | 8.6091 | 4.9371 | 978087.140 | 978070.092 | 978071.646 |
| 258 | 8.3410 | 5.1306 | 978090.180 | 978073.140 | 978074.694 |

Table 3. Descriptive Statistics of normal gravity field from regional modeling

| Variable | N | Mean | SE Mean | StDev | Minimum | Q1 | Median | Q3 | Maximum |
|---|---|---|---|---|---|---|---|---|---|
| GRS30 | 27 | 978184 | 20.4 | 106 | 978055 | 978088 | 978162 | 978272 | 978394 |
| GRS67 | 27 | 978167 | 20.4 | 106 | 978038 | 978071 | 978145 | 978255 | 978378 |
| WGS84 | 27 | 978169 | 20.4 | 106 | 978040 | 978072 | 978146 | 978256 | 978379 |

Table 4. Descriptive statistics of normal gravity field from local modeling

| Variable | N | Mean | SE Mean | StDev | Minimum | Q1 | Median |
|---|---|---|---|---|---|---|---|
| GRS30mGal | 258 | 978090 | 0.158 | 2.53 | 978075 | 978089 | 978090 |
| GRS67mGal | 258 | 978073 | 0.158 | 2.54 | 978058 | 978072 | 978073 |
| WGS84mGal | 258 | 978074 | 0.158 | 2.54 | 978060 | 978073 | 978075 |
| WGS84-GRS67mGal | 258 | 1.5545 | 0.0000312 | 0.000500 | 1.5540 | 1.5540 | 1.5540 |

| Variable | Q3 | Maximum | Skewness |
|---|---|---|---|
| GRS30mGal | 978091 | 978105 | -0.22 |
| GRS67mGal | 978074 | 978088 | -0.22 |
| WGS84mGal | 978076 | 978089 | -0.22 |
| WGS84-GRS67mGal | 1.5550 | 1.5550 | 0.09 |

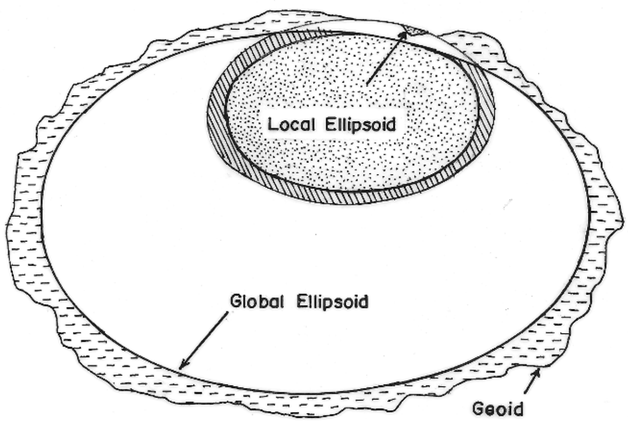


Figure1a. The earth as an ellipsoid

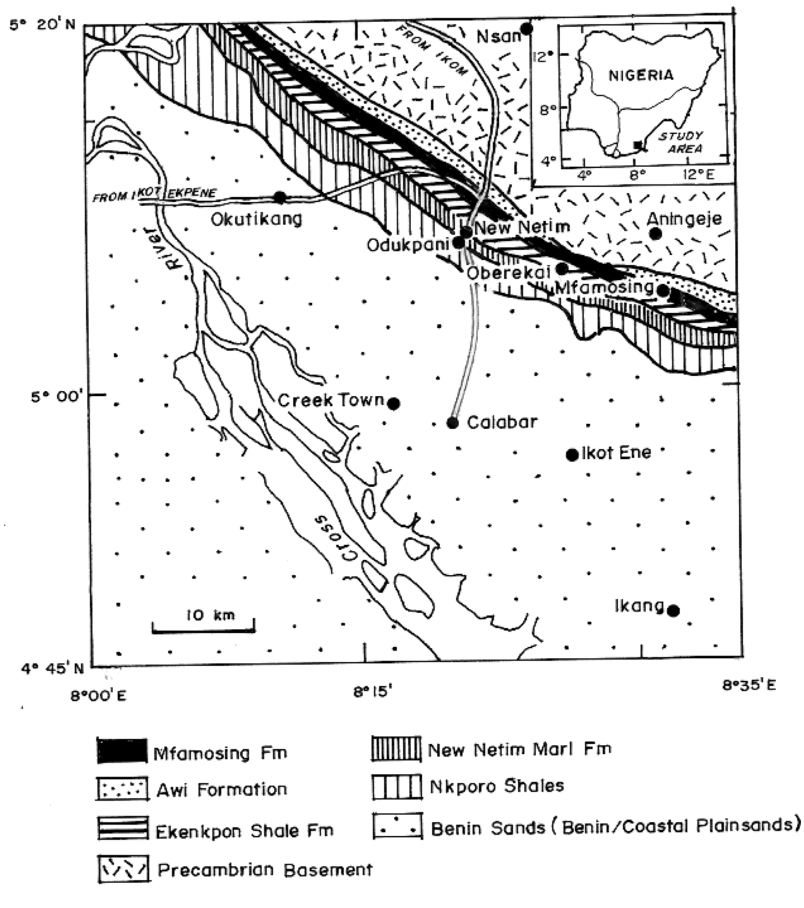


Figure 1b. Geological map of the Calabar Flank, Southeastern Nigeria

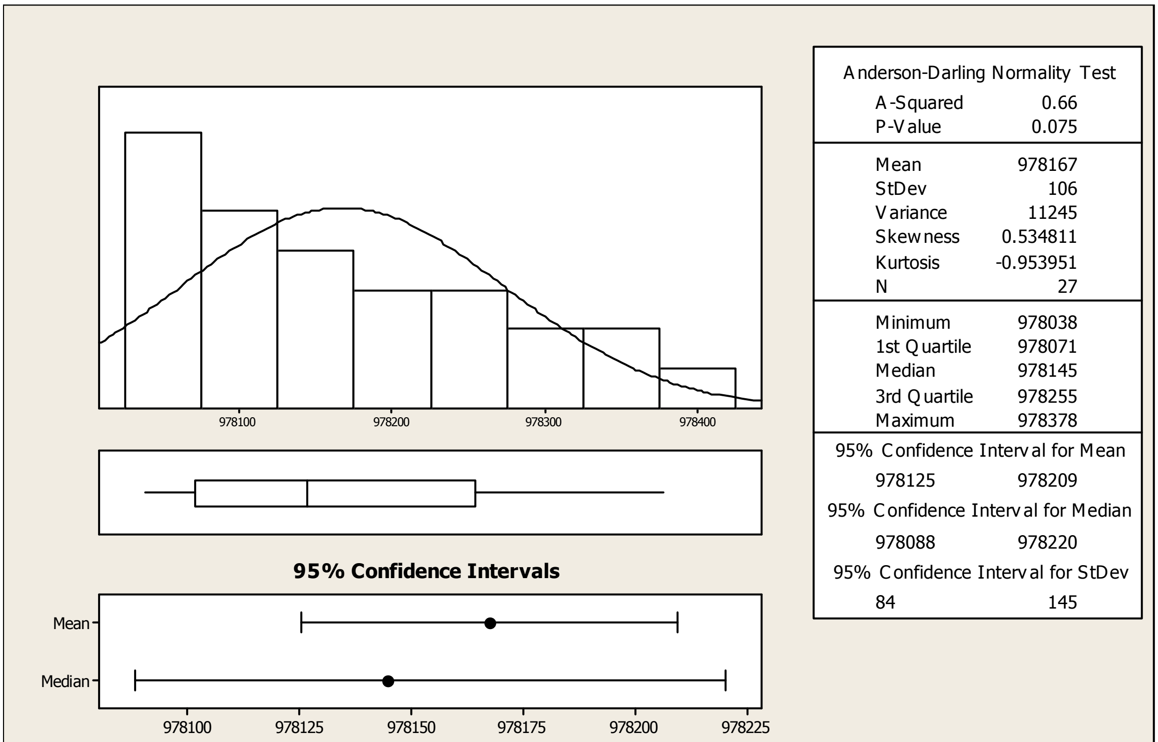


**Fig.2 Descriptive statistics of normal gravity field from GRS67**

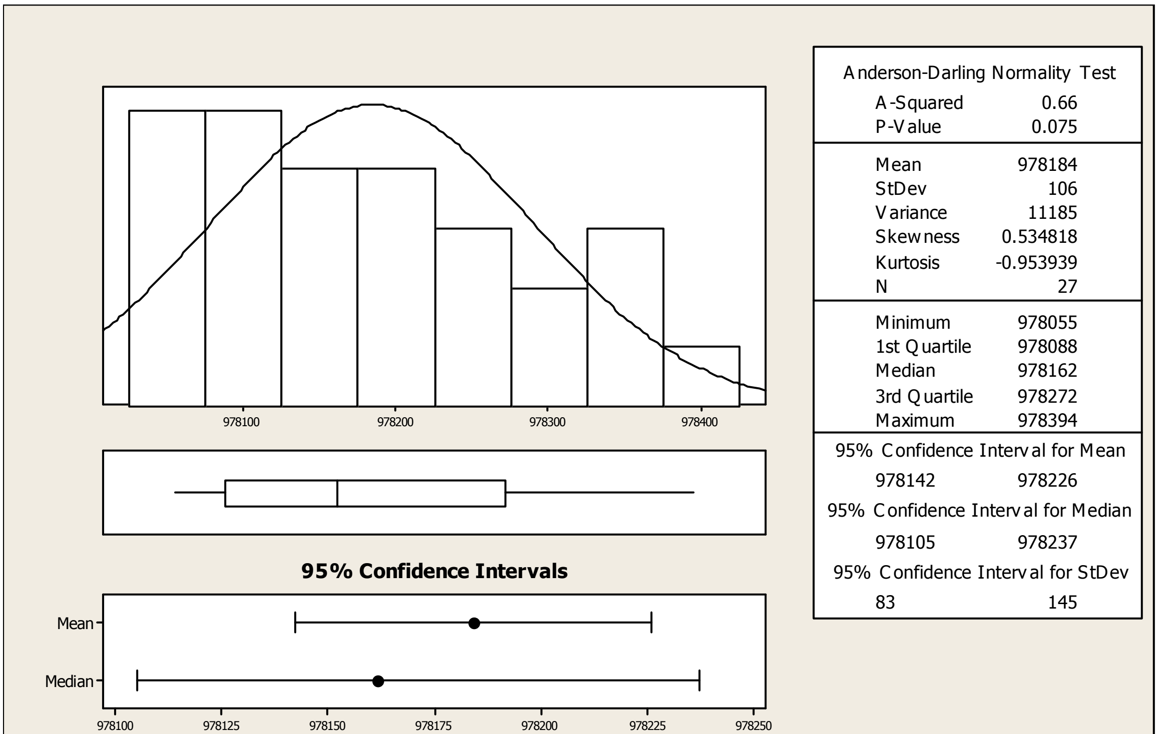


**Fig.3 Descriptive statistics of normal gravity field from GRS30**

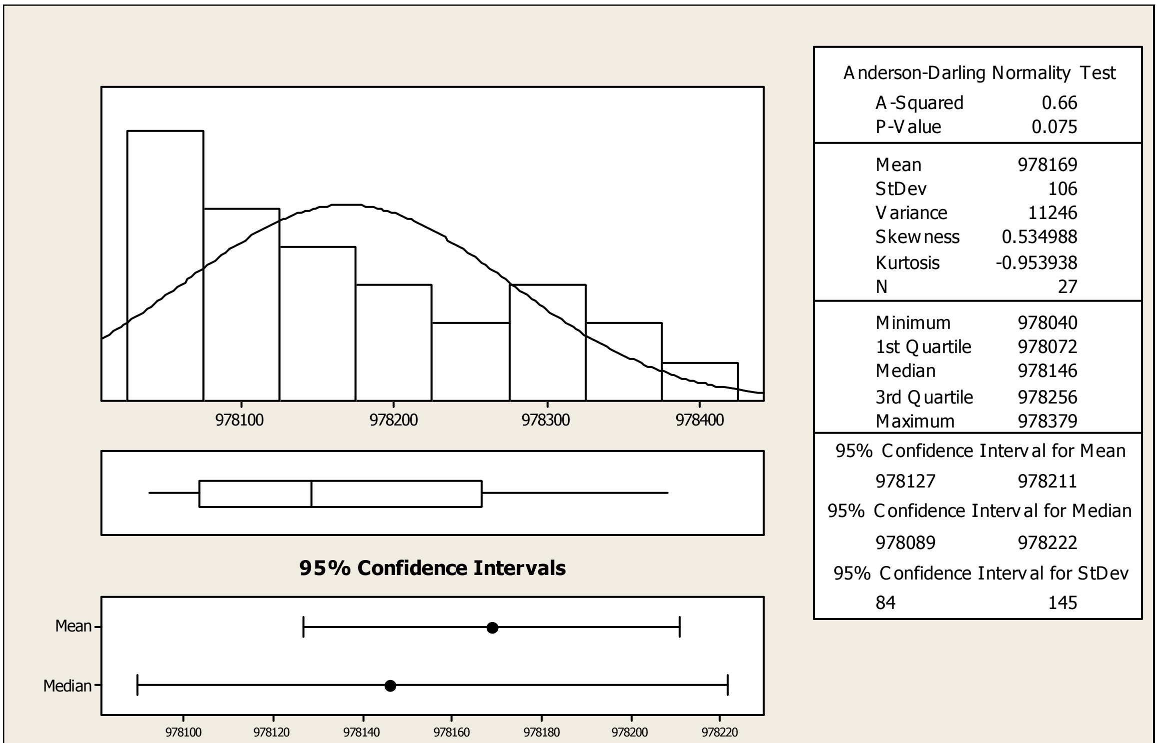


**Fig. 4 Descriptive statistics of normal gravity field from WGS84**

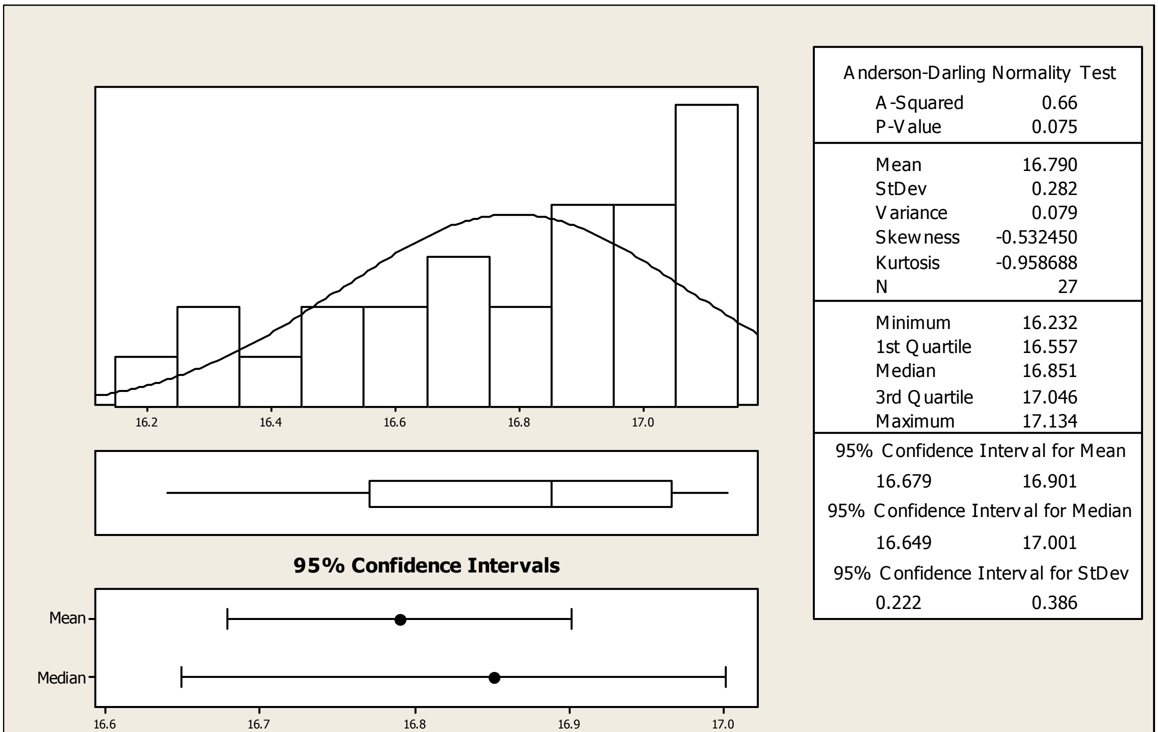


**Fig.5 Descriptive statistics of the difference in values between GRS30 and GRS67**

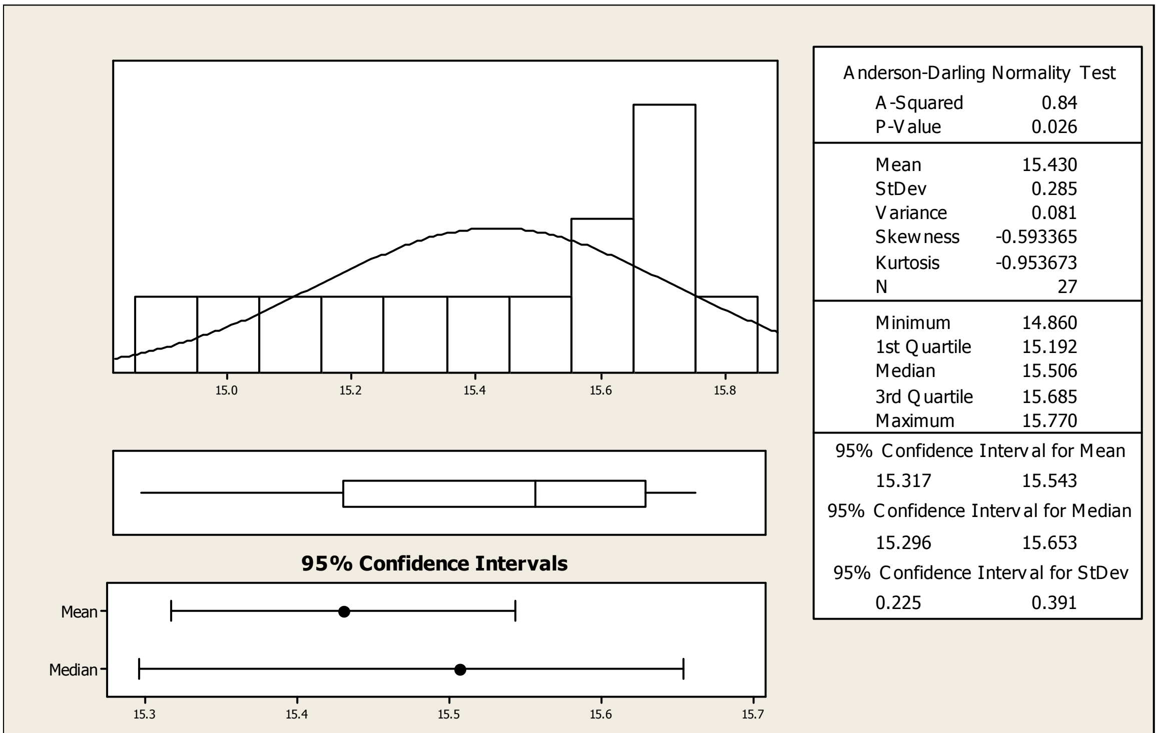


**Fig.6 Descriptive statistics of the difference in values between GRS30 and GRS84**

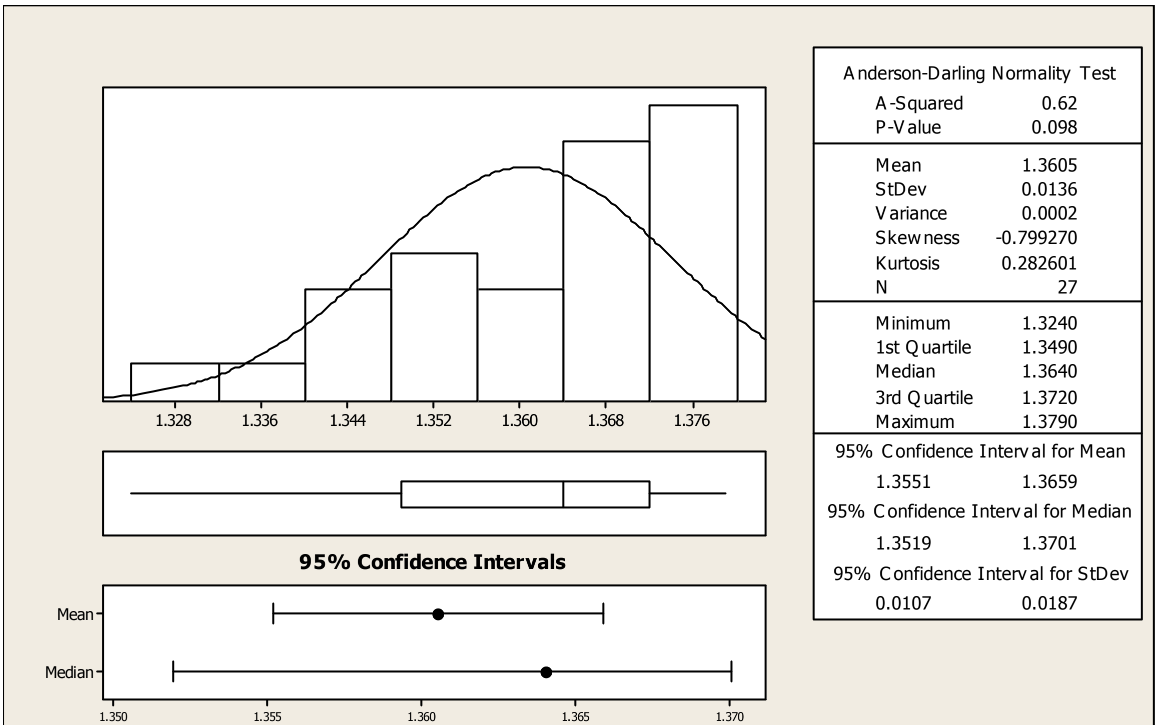


**Fig.7 Descriptive statistics of the difference in values between GRS84 and GRS67**