Applied Physics Research; Vol. 7, No. 4; 2015
ISSN 1916-9639 E-ISSN 1916-9647
Published by Canadian Center of Science and Education

Particle limiting velocities from the bicubic equation derived from Einstein's kinematics: PeV electron case

Josip Śoln[1]

[1] JZS Phys-Tech, Vienna, Virginia 22182, USA

Correspondence: Josip Śoln, JZS Phys-Tech, Vienna, Virginia 22182, USA. E-mail: jurasoln@yahoo.com

Received: June 27, 2015 Accepted: July 7, 2015 Online Published: July 14, 2015

doi:10.5539/apr.v7n4p37 URL: http://dx.doi.org/10.5539/apr.v7n4p37

Abstract

Here one reviews the particle limiting velocities derived from the Einstein's kinematic generated bicubic equation: c_1, the primary limiting velocity, c_2, the obscure limiting velocity and c_3, the normal limiting velocity. While c_1 and c_3 are real c_2 is imaginary. Each of these limiting velocities depend on particle physics parameters of energy, E, mass, m, and ordinary velocity, v in such a way that c_1^2, c_2^2 andc_3^2 are all related to each other by simple transforms, leaving invariant the zero sum rule, $c_1^2 + c_2^2 + c_3^2 = 0$. As such, they form the bicubic particle kinematics. Now for the problem at hand, the limiting velocities are calculated specifically for the 0.5 MeV mass electron in the PeV energy region from the 2010 Crab Nebula Flare. Of the three solutions, c_1, c_2 and c_3 one finds c_1 to be very large and likely unphysical, similarly imaginary c_2 with very large absolute value also likely unphysical and both of them Lorentz violating (LV), while the calculated normal limiting velocity c_3 has acceptable values in this high energy case. With the electron energy in the PeV region, the electron mass has very little influence on c_3. Even so one calculates miniscule subluminal and superluminal Lorentz violations, when respectively $c_3 \precsim c$ and $c_3 \succsim c$, and the Lorentz invariance (LI) when the evaluation yields $c_3 = c$. Qualitatively, because of miniscule masses, the calculated electron limiting velocity due to the Crab Nebula Flare PeV events shows great deal of similarities with the calculated neutrino limiting velocity from the OPERA neutrino velocity experiments. To get bigger mass effects on limiting velocities, one needs to go from the energy region $E \gg mc^2$ to the energy region $E \succsim mc^2$. With this, one would see whether in the lower energy region one has also $c_3 \simeq O(c)$ with LI and LV small portions. Also lower energy electron velocities, may even provide physical reasons for the existance or non-existance of c_1 and c_2.

Keywords: Bicubic equation, Limiting velocities, Lorentz violation

1. Introduction

The discussions about the Lorentz invariance (LI) and possible Lorentz violation (LV) go in hand with experimental interests to study the neutrino and electron ordinary as well as limiting velocities. For neutrinos one has the well known 17 GeV OPERA experiment (Strauss, 2014), while in the PeV energy regions, one can study the neutrino and electron velocities from the 2010 Crab Nebula Flare (see Stecker, 2014) and the Ice Cube PeV events (Aartsen et al., 2013; Borriello, Chakraborty, Mirizzi, & Serpico, 2013).

In what follows, the emphasis will be on the electron limiting velocity, which from the practical point of view, complies the best with the solution for the normal limiting velocity c_3, as elaborated in Soln (2014). This will be done in the same manner as it was already done for the neutrino limiting velocity in Soln (2014). However, although in both cases one talks about very high energy particles the difference comes from the fact that neutrino originated from the well defined OPERA laboratory source (Strauss, 2014), while electrons have extragalactic origin (Aartsen et al., 2013; Borriello, Chakraborty, Mirizzi, & Serpico, 2013; Stecker, 2014;). The major difference between the neutrino and the electron cases comes from the fact that the mass of the electron is well known at $m_e c^2 = 0.51 MeV$, while for the flavor neutrinos Dirac masses were chosen through the νSM, the minimal extension of the SM (Standard Model) by introducing the right-handed neutrino field ν_R which happens after spontaneous breaking of the electroweek gauge symmetry (Soln, 2014; Castelo-Branco, & Emmanuel-Costa, 201; Gupta, Joshipura, & Patel, 2013). The freedom in the strength of the spontaneous symmetry breaking allows the values of neutrino flavor masses $m_\nu(\alpha)c^2$, $(\alpha = e, \mu, \tau)$ be consistent with the mass-state neutrino masses $m_\nu(i)c^2, (i = 1, 2, 3)$, (Soln, 2014; Castelo-Branco, & Emmanuel-Costa, 201; Gupta, Joshipura, & Patel, 2013). With tri-bimaxial neutrino matrix from Harrison, Perkins, and Scott (2002) for $\alpha = \mu$, one formulates $m_\nu^2(\alpha)$, as average value of $m_\nu^2(i)$ over fixed flavor $\alpha = \mu$ to obtain $m_\nu(\mu)c^2 = 0.076eV$. This is miniscule when compared to the neutrino energy of $17GeV$. Similarly one sees that also in the case of electron $m_e c^2$ is miniscule when compared to the PeV electron energy, making these two cases similar.

Technically, it will be squares (c_i^2, $i = 1, 2, 3$) of the three electron limiting velocities, primary, c_1, obscure, c_2, and normal, c_3, that will be derived in exact forms from the bicubic equation as shown in Soln (2014). To better exemplify the high energy behaviors from exact solutions, the Taylor series expansions are done in terms of $(mv^2/E) < (2/3\sqrt{3})$ with m, v and E being particle mass, ordinary velocity and energy, respectively. Actually, the parameters m, v and E for both the muon neutrino OPERA experiment (Strauss, 2014; Soln, 2014) and the electron 2010 Crab Nebula Flare data from Stecker (2014) obey much stronger condition $(mv^2/E) \ll 1$. This condition assures that one of the limiting velocities,the normal limiting velocity, c_3, will yield $c_3 \simeq O(v)$. As a consequence, any time v is measured to be close to c one has the approximate LI. Otherwise when and if $v < c$ or $v > c$ one will have either subluminal or superluminal LV. As to the primary $c_1$and obscure, c_2 limiting velocities for the electron 2010 Crab Nebula Flare data they are likely unphysical to be observable.

In Section 2, the general outline of the bicubic kinematics through the bicubic equation, with three exact solutions for squares of theoretical particle limiting velocities, is given. Following Soln (2014) these squares of theoretical particle limiting velocities are exhibited as functions of particle ordinary velocity v, particle mass m, and particle energy E. Here, one introduces also the expressions for the squares of the ordinary particle velocity v, by inverting the solutions for the squares of the particle limiting velocities, c_1^2, c_2^2 and c_3^2 to be used as consistency checks for the limiting velocity solutions. Here also three limiting velocity sum rules are introduced of which one being the exact zero sum rule $c_1^2 + c_2^2 + c_3^2 = 0$ indicating that one, here c_2, is imaginary.

Theoretical particle limiting velocity solutions are analyzed in Section 3, where one finds out that the solution c_3, the normal limiting velocity, fits best the astrophysical electron velocity data from the 2010 Crab Nebula Flare or qualitatively the Ice Cube PeV events from Stecker (2014). The theoretical normal electron limiting velocity solution c_3 will be qualitatively compared to the normal neutrino limiting velocity solution c_3 in Soln (2014) for the OPERA experiments from Strauss (2014).

Section 4 is devoted to final remarks and conclusion. Comparison with other approaches in the literature treating the possibility of Lorentz invariance violation from changes in Dirac equation to changes in the relativistic kinematics will be made.

2. Recapitulations and Bicubic Kinematics from the Bicubic Equation Particle Limiting Velocity Solutions

Here, in order to establish the particle bicubic kinematics, one briefly recapitulates the derivation of particle limiting velocity expressions of the primary c_1,the obscure c_2, and the normal one c_3, in terms of particle parameters m, v and E as done in Soln (2014). To complete the loop, here one also expresses the particle velocity v in terms of each of limiting velocities c_1, c_2, and c_3, which facilitates judging their relative to each other values without detailed calculations.

To detail the particle bicubic kinematics, as shown inSoln (2014), one starts with the Einstein's kinematics of the particle mass shell condition $\vec{p}^{\,2}c^2 - E^2 + m^2c^4 = 0$ into which one substitutes the momentum $\vec{p} = E\,\vec{v}c^{-2}$ and then identifying c, the velocity of light, as actually the particle limiting velocity,to end up for c with the bicubic equation

$$m^2c^6 - E^2c^2 + E^2v^2 = 0 \qquad (1.1)$$

where, m, v and E , as already mentioned, being particle mass, velocity and energy. From (1.1) one is to calculate the particle limiting velocities. As shown in Soln (2014), the solutions of (1.1) are,

$$
\begin{aligned}
z &= \frac{3\sqrt{3}mv^2}{2E},\ z^2 \le 1;\\
D &= \frac{1}{4}\left(\frac{3\sqrt{3}}{2z}\right)^4\left[1-\frac{4}{27}\left(\frac{3\sqrt{3}}{2z}\right)^2\right] \prec 0;\\
c_1^2 &= \frac{2E}{\sqrt{3}m}\sin\left[\frac{\pi}{3}-\frac{1}{3}\sin^{-1}(z)\right] \succ 0,\\
c_2^2 &= -\frac{2E}{\sqrt{3}m}\cos\left[\frac{1}{3}\sin^{-1}(z)-\frac{\pi}{6}\right] \prec 0,\\
c_3^2 &= \frac{2E}{\sqrt{3}m}\sin\left[\frac{1}{3}\sin^{-1}(z)\right] \succ 0
\end{aligned}
\tag{2.1}
$$

As $c_2^2 \prec 0$ one can consider the obscure c_2 ,being imaginary,primarily as unphysical. However, for the sake of completeness, at least on the formal level, relations (2.1) are taken as the basis for the particle bicubic kinematics as every c_i , $i = 1,2,3$, is related to m, v and E. At this point, the fact that $c_{1,}^2 c_2^2$ and c_3^2 in (2.1), as solutions of bicubic equation (1.1) do not contain explicitly the velocity of light c, allows us to rewrite (1.1) by replacing c with yet to be determined limiting velocities $c_{1,}c_2$ and c_3 ($c \to c_i$). This yields the bicubic equation (1.2) without explicit reference to c but equivalent to(1.1),

$$\left[\left(\frac{mc_i^2}{E}\right)^2 - 1\right]c_i^2 + v^2 = 0, i = 1,2,3 \tag{1.2}$$

Now, in the bicubic solutions (2.1) there are no beforehand reference to the velocity of light c; it can only occur in the solutions (2.1) numerically. Of course, bicubic equations (1.1) and (1.2) yield exactly the same solutions(2.1) where to solutions from (1. 2) at the end one assigns $i = 1,2$ or 3. However, equation (1.2) can be used also directly to verify solutions particularly the ones with approximations.

Simple refinements to the bicubic kinematics come directly from solutions in (2.1) where one explicitly derives three sum rules for squares of limiting velocities of which one is the zero sum rule for the squares of particle limiting velocities,

$$c_1^2 = \frac{E}{m}\cos\left[\frac{1}{3}\sin^{-1}(z)\right] - \frac{1}{2}c_3^2 \succ 0 \tag{2.2}$$

$$c_2^2 = -\frac{E}{m}\cos\left[\frac{1}{3}\sin^{-1}(z)\right] - \frac{1}{2}c_3^2 \prec 0 \tag{2.3}$$

$$c_1^2 + c_2^2 + c_3^2 = 0 \tag{2.4}$$

These relations amplify the interdependence between the primary,c_1, obscure,c_2, and normal,c_3, limiting velocities for the same physical parameters m, v and E. Of course, these relations demand that at least one be imaginary (but no more than two). These interdependences (2. 2, 3, 4) also suggest that one actually has a simple, say, bicubic particle system with fixed physical parameters m, v and E. The individual particles associated with c_1 , c_2 and c_3 limiting velocities, can be called , respectively, primary, obscure and normal particles. Now, one can even entertain the idea that the obscure particle could even be the elusive dark matter particle which here cannot be directly observed because it has imaginary limiting velocity c_2.

Performing the inversions of (2.1) one sees other aspects of bicubic particle kinematics such as complementary to

(2, 2, 3, 4) restrictions on primary,c_1, obscure,c_2, and normal,c_3 limiting velocities.

$$z = \frac{3\sqrt{3}mv^2}{2E}; z^2 \leq 1$$

$$\frac{\sqrt{3}mc_1^2}{2E} \leq 1 : z = \sin\left[\pi - 3\sin^{-1}\left(\frac{\sqrt{3}mc_1^2}{2E}\right)\right], \tag{3.1}$$

$$-\frac{\sqrt{3}mc_2^2}{2E} \leq 1 : z = \sin\left[\frac{\pi}{2} + 3\cos^{-1}\left(-\frac{\sqrt{3}mc_2^2}{2E}\right)\right] \tag{3.2}$$

$$\frac{\sqrt{3}mc_3^2}{2E} \leq 1 : z = \sin\left[3\sin^{-1}\left(\frac{\sqrt{3}mc_3^2}{2E}\right)\right]. \tag{3.3}$$

$$\sin^{-1}\left(\frac{3\sqrt{3}mv^2}{2E}\right) = 3\sin^{-1}\left(\frac{\sqrt{3}mc_3^2}{2E}\right) : v < c_3. \tag{3.4}$$

Numerically z is the same in (3. 1, 2, 3 and 4). As such it demands restrictions on primary, c_1, obscure,c_2, and normal,c_3,limiting velocities on the left hand side of (3. 1, 2, 3 and 4). From inversion of (3.3) one has (3.4) which yields important general information that $v < c_3$.

Refinements to the particle bicubic kinematics, appearing on the left hand sides of (3. 1, 2, 3) come from restrictions on the inverse trigonometric functions $\sin^{-1}$ and $\cos^{-1}$ to their principal values. These restrictions can be emphasized explicitly on primary,c_1, obscure,c_2 and normal,c_3, limiting velocities, respectively, as follows,

$$c_1^2 \leq \frac{2E}{\sqrt{3}m}, \ -c_2^2 \leq \frac{2E}{\sqrt{3}m}, \ c_3^2 \leq \frac{2E}{\sqrt{3}m} \tag{3.5,6,7}$$

The importance of relations (3. 5, 6, 7) is to verify that explicit solutions for c_1^2, c_2^2 and c_3^2 are within allowed values.

By taking $v \longrightarrow 0$ limits in (2.1) one notices the difference in behaviors of bicubic and relativistic particle kinematics. To begin with, the limit of $v \longrightarrow 0$ implies the vanishing of the normal limiting velocity, $c_3 \rightarrow 0$, implying that at the rest normal limiting velocity energy with c_3^2 is not there. But for other two limiting velocities, primary, c_1, and obscure, c_2, one derives that for $v \longrightarrow 0$, $E \rightarrow mc_1^2$ and $E \rightarrow m(-c_2^2)$ each of them becoming the respective rest primary limiting velocity energy and rest obscure limiting velocity energy both of them positive since on general grounds from (2.1) $c_2^2 < 0$. Of course now, the zero sum rule (2. 4) for limiting velocities shrinks simply to $c_1^2 + c_2^2 = 0$. This zero sum rule for the squares of primary,c_1, and obscure,c_2, limiting velocities imply the equivalence of equal mass primary and obscure particle rest energies, $mc_1^2 = m(-c_2^2)$. However, as it is likely that c_2 is unphysical, this equality would imply that c_1 is also likely unphysical.

In high energy region,$E > mv^2$, the Taylor series of (2.1) for squares of primary, obscure and normal limiting velocities become, respectively

$$\begin{aligned}
&z^2 < 1;\\
&c_1^2 = \frac{E}{m} - \frac{v^2}{2} - \frac{3mv^4}{8E} - \frac{m^2v^6}{2E^2} + O\left[v^2\left(\frac{mv^2}{E}\right)^3\right] > 0,\\
&c_2^2 = -\frac{E}{m} - \frac{v^2}{2} + \frac{3mv^4}{8E} - \frac{m^2v^6}{2E^2} + O\left[v^2\left(\frac{mv^2}{E}\right)^3\right] < 0,\\
&c_3^2 = v^2 + \frac{m^2v^6}{E^2} + \frac{69}{32}\frac{m^4v^{10}}{E^4} + O\left[v^2\left(\frac{mv^2}{E}\right)^6\right] > 0
\end{aligned} \tag{4}$$

Of course, the zero sum rule for the squares of particle limiting velocities (2. 4) is also present in (4). Also, the restrictions (3. 5, 6, 7) on the limiting velocities can be easily satisfied by the individual solutions in (4).

Let us mention that while globally the high energy astrophysical electron velocity data from 2010 Crab Nebula Flare, as collected by Stecker (2014), will be compared and discussed systematically in terms of primary, obscure and normal electrons, each of them with the same physical parameters, m, v and E. Presently, however, it is the normal electron with normal limiting velocity c_3 from (4) that will describe pretty much the data at hand, rather in the same way as it described the neutrino OPERA limiting velocity measurements (Strauss, 2014; Soln, 2014). The similarity comes from the fact that in either case,a particle mass is practically negligible when compared to its energy. As seen from (4), the normal limiting velocity c_3 is then basically determined by the measured velocity v.

3. Limiting velocities of the astrophysical electron

Analytical particle limiting velocity solutions from the bicubic equation in Soln (2014), applied already to the OPERA neutrino velocity experiments (Strauss, 2014), are now expanded and applied to the astrophysical electron velocity data from the 2010 Crab Nebula Flare and qualitatively to the Ice Cube PeV events as presented in Stecker (2014). All the limiting velocities, primary c_1, obscure c_2 and normal c_3 will be analyzed with these real data. In the PeV energy region, this analysis will show very interesting and considerable differences between c_1, c_2 and c_3 although only, at the moment,the normal limiting velocity $c_3$will indicate to be physically measurable.

The electron data of energy, mass and velocity from Stecker (2014) for the Crab Nebula Flare in 2010 that will be analyzed here can be summarized as follows

$$\begin{aligned} E_e &\simeq 5.1PeV, \;\; m_e c^2 \simeq 0.51MeV, \\ v_e &\simeq (1+\delta)\,c, \\ -8\times 10^{-17} &\lesssim \delta \lesssim 5\times 10^{-21} \end{aligned} \tag{5}$$

where the negative, zero and positive δ corresponds respectively to the subluminal, luminal and superluminal electron. Because $m_e c^2/E_e \simeq 10^{-10} \leqslant 1$ it is readily seen that the perturbation solutions (4) need just few first terms to deduce the limiting velocities. Hence, for Crab Nebula Flare electrons in 2010, from relation (4) one calculates analytically the squares of limiting velocities with the data in (6) to obtain,

$$\begin{aligned} c_1^2 &= \frac{E_e}{m_e} - \frac{v_e^2}{2} - O\left(\frac{m_e v_e^4}{E_e}\right) \simeq \left[10^{10} - \frac{1}{2}(1+\delta)^2\right] c^2, \\ c_2^2 &= -\frac{E_e}{m_e} - \frac{v_e^2}{2} + O\left(\frac{m_e v_e^4}{E_e}\right) \simeq \left[-10^{10} - \frac{1}{2}(1+\delta)^2\right] c^2, \\ c_3^2 &= v_e^2 + O\left(\frac{m_e v_e^4}{E_e}\right) \simeq (1+\delta)^2\, c^2. \end{aligned} \tag{6}$$

Restrictions (3. 5, 6, 7) are obeyed in respective solutions of limiting velocities. Here one also notices that the limiting velocity zero squares sum rule (2. 4) is satisfied. Finally from (6) one deduces approximately the linear limiting velocities as,

$$\begin{aligned} c_1 &\simeq \left(\frac{E_e}{m_e}\right)^{\frac{1}{2}} - \frac{1}{4} v_e^2 \left(\frac{m_e}{E_e}\right)^{\frac{1}{2}} \simeq \left[10^5 - \frac{1}{4}(1+\delta)^2\, 10^{-5}\right] c, \\ c_2 &\simeq \pm\, i\left[\left(\frac{E_e}{m_e}\right)^{\frac{1}{2}} + \frac{1}{4} v_e^2 \left(\frac{m_e}{E_e}\right)^{\frac{1}{2}}\right] \simeq \pm i\left[10^5 + \frac{1}{4}(1+\delta)^2\, 10^{-5}\right] c, \\ c_3 &\simeq v_e \simeq (1+\delta)c; \; -8\times 10^{-17} \lesssim \delta \lesssim 5\times 10^{-21}. \end{aligned} \tag{7}$$

The normal limiting velocity c_3,as seen from the last line in (7), is describing numerically the Crab Nebula Flare in 2010 data from (5) dutifully both in their quadratic and linear forms, in relations (6) and (7), respectively. From relation (7) one sees that the normal limiting velocity c_3 supports LI when $c_3 = c$. When $\delta > 0$ the normal limiting velocity, satisfying $c_3 \gtrsim c$,mostly will support small LV. When $\delta < 0$, because electron mass being negligible, the normal limiting velocity, satisfying $c_3 \lesssim c$,will mostly support small LV. What one also notices is that with Crab Nebula Flare in 2010 data from (6) the primary c_1 and the obscure c_2 limiting velocities are numerically far superior to the normal limiting velocity c_3 as the absolute values of the primary c_1 and the obscure c_2 limiting velocities is about 10^5 times larger and as such support only LV. As such they are likely unphysical. The remote chance for any possible measurable effects by c_1 and c_2 could perhaps occur in experiments with much lower electron energies. This so in particular as imaginary c_2 is a good candidate for describing the obscure "dark" mater particles. The lower electron energy would also allow bigger role for the electron mass in deciding the value of c_3.

4. Final remarks and conclusion

When analyzing the limiting velocity results with solutions (6) and (7) from bicubic equation of Soln (2014) one cannot but notice that the electron energy of $5.1PeV$ makes the electron mass energy of $0.51MeV$ miniscule by comparison. This can be taken as the main reason why the normal limiting velocity c_3 , on one hand can be so close to c, and on the other hand somewhat different from c, the velocity of light. The same situation occurred in the analysis of the OPERA neutrino velocity experiments (Strauss, 2014), with solutions from bicubic equation in Soln (2014) for the normal limiting velocity c_3 also being very close to c. Hence, it would be desirable to have electron velocity experiments where the electron energy would not be too far away from the GeV values.

The humongous value of about 10^5c of the primary limiting velocity c_1 is difficult not to notice in (6) and (7). It almost offers itself for a question: Was this the velocity that was important in the Universe only in the far past ? More study should go into the primary limiting velocity c_1. As it stands, it is likely unphysical.

The imaginary value of the obscure limiting velocity c_2 while being somewhat unusual may have explanation. Namely,it is expressed in terms of real E_e, m_e and v_e and the imaginary value of c_2 allows the obscure or "dark" electron to be only in an enclosed kinematical region. Like for c_3 and $c_{1,}$ better understanding could also be obtained for c_2 if dealing with lower energy particles, $E \ll 5.1PeV$. This may allow to see better weather c_1 and c_2 have a fundamental role in particle physics and also more details about LV caused by c_3. In other words, sticking with the electron, in order to see full effects of v_e, m_e and E_e on c_1, c_2 and particularly on c_3, the electron energy should be lowered to values satisfying $E_e \gtrsim m_ec^2$.

References

Aartsen, M. G., Abbasi, R., Abdou, Y., Ackermann, M., Adams, J., Aguilar, J. A., ... & Christy, B. (2013). First observation of PeV-energy neutrinos with IceCube. *Physical review letters, 111*(2), 021103.

Borriello, E., Chakraborty, S., Mirizzi, A., & Serpico, P. D. (2013). Stringent constraint on neutrino Lorentz invariance violation from the two IceCube PeV neutrinos. *Physical Review D, 87*(11), 116009.

Castelo-Branco, G., & Emmanuel-Costa, D. (2015). Flavour Physics and CP Violation in the Standard Model and Beyond. In *Lectures on Particle Physics, Astrophysics and Cosmology* (pp. 145-186). Springer International Publishing.

Gupta, S., Joshipura, A. S., & Patel, K. M. (2013). How good is - symmetry after results on non-zero 13?. *Journal of High Energy Physics, 2013*(9), 1-17.

Harrison, P. F., Perkins, D. H., & Scott, W. G. (2002). Tri-bimaximal mixing and the neutrino oscillation data. *Physics Letters B, 530*(1), 167-173.

Soln, J. (2014). Theoretical particle limiting velocity from the bicubic equation: Neutrino example. *Physics Essays, 27*(3), 448-453.

Stecker, F. W. (2014). Limiting superluminal electron and neutrino velocities using the 2010 Crab Nebula flare and the IceCube PeV neutrino events. *Astroparticle Physics, 56*, 16-18.

Strauss, T. (2014). Opera Highlights. *arXiv preprint arXiv:1405.6847*.