

# Combined Effects of a Magnetic Field and a Helical Force on the Onset of a Rotating Rayleigh-Bénard Convection With Free-Free Boundaries

G. Pomalégn<sup>1</sup> & Jean Bio Chabi Orou<sup>2</sup>

<sup>1</sup> Ecole doctorale Science des matériaux, Faculté des sciences et techniques, Université d'Abomey-Calavi, République du BENIN

<sup>2</sup> Faculté des sciences et techniques, département de physique, Université d'Abomey-Calavi, République du BENIN

Correspondence: Jean Bio Chabi Orou, Faculté des sciences et techniques, département de physique, Université d'Abomey-Calavi, République du BENIN. E-mail: jchabi@yahoo.fr

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## Abstract

We investigate the combined effects of rotation, magnetic field and helical force on the onset of stationary and oscillatory convection in a horizontal electrically conducting fluid layer heated from below with free-free boundary conditions. For this investigation the linear stability analysis studied in detail by Chandrasekhar (1961) is used. We obtain the condition for the formation of a single large-scale structure. In (Pomalégn<sup>1</sup> et al., 2014) it was shown the existence of a critical value  $S_{cr}$  of the intensity of the helical force for which the apparition of two cells at marginal stability for the oscillatory convection is obtained. Then, we have shown here how the increasing of parameter Ta influences this critical value of the helical force intensity.

**Keywords:** rotating Rayleigh-Bénard convection, helical force, electrically conducting fluid

## 1. Introduction

The rotating Rayleigh-Bénard convection (Busse & Heikes, 1980; Cox & Matthews, 2000) and the Rayleigh-Bénard convection in the presence of a magnetic field possess control parameters which are the Taylor number characteristic of the rotation and the Chandrasekhar number characteristic of magnetic field. These control parameters make these systems some interesting cases for the study of hydrodynamic and hydromagnetic stability. The turbulent convection motion, in the presence of magnetic field or in rotating systems such as planetary atmospheres, is known to become helical (Moffatt, 1978; Parker, 1979). Under certain conditions, small-scale helical turbulence is capable of intensifying and sustaining large-scale vortex disturbances by means of energy transfer from small to large scales. The generating properties of small-scale helical turbulence were first discovered in magnetohydrodynamic (Steenbeck et al., 1966). This phenomenon is called the alpha-effect. The realizability of the hydrodynamic alpha effect has been justified theoretically for compressible (Moiseev et al., 1983) and incompressible (Moiseev et al., 1988) fluids. The reviews (Rutkevich, 1993; Kurgansky, 1998; Levina et al., 2000) have examined in detail the publications on the hydrodynamic alpha-effect. It was proposed in (Levina et al., 2000) that in a convective system, the special force also called helical force that has the structure of the alpha term provides the excitation of large-scale instability within the frame work of the hydrodynamic  $\alpha$ -effect model.

The action of the helical force within the Rayleigh-Bénard convection may initiate a new type of instability namely the helical – vortex instability. In (Levina et al., 2004), for the three different types of boundary conditions, it was shown that the helical force decreases the threshold of instability. According to (Essoun & Chabi Orou, 2010), for the free-free boundary conditions, it was found that in the presence of rotation, the helical force has the no- monotonic effects on the onset of convection for all values of Taylor number but decreases the corresponding critical wave number for any value of Taylor number. Also Pomalégn<sup>1</sup> et al. (2014) has shown that the helical force has the no- monotonic effects on both the onset of stationary and oscillatory convection in the presence of a magnetic field with free-free boundaries.

We use in this work, the classical linear stability analysis to examine the combined effects of the magnetic field, the rotation and the helical force on the onset of stationary and oscillatory convections with free-free boundaries. The condition for the formation of a single large-scale structure is derived and we have also shown how the

increasing of the parameter  $T_a$  influences the critical value of the intensity of helical force for which we have the apparition of two cells at marginal stability for the oscillatory convection.

## 2. Mathematical Formulation of the Problem

We consider an infinite horizontal layer of an electrically conducting fluid heated from below upon which is impressed a uniform magnetic field in the presence of rotation and helical force. The characteristic basic equations of our system in Boussinesq approximation are:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = -\frac{\vec{v} p}{\rho_0} + \frac{\mu}{4\pi\rho} (\text{curl} \vec{H}) \times \vec{H} + v \nabla^2 \vec{v} - \alpha \Delta T \vec{g} + a \Omega d \vec{f} + \frac{1}{2} \vec{\nabla} (|\vec{\Omega} \times \vec{r}|^2) - 2(\vec{\Omega} \times \vec{v}) \quad (1)$$

$$\frac{\partial \vec{H}}{\partial t} + (\vec{v} \cdot \nabla) \vec{H} = (\vec{H} \cdot \vec{\nabla}) \vec{v} + \eta \nabla^2 \vec{H} \quad (2)$$

$$\frac{\partial T}{\partial t} + (\vec{v} \cdot \nabla) T = \kappa \nabla^2 T \quad (3)$$

$$\vec{\nabla} \cdot \vec{H} = 0 \text{ and } \vec{\nabla} \cdot \vec{v} = 0 \quad (4)$$

$$\vec{f} = \vec{e} (\text{curl}(\vec{v})) z - \frac{\partial (\vec{e} \times \vec{v})}{\partial z}; \vec{e} = (0, 0, 1)$$

Where  $\vec{r}$  is the radius vector of a fluid particle from the rotation axis,  $\vec{v}$  is the fluid particle velocity vector,  $p$  is pressure,  $T$  is temperature,  $\vec{f}$  is helical force,  $\vec{e}$  is the unit vector along the vertical axis,  $\vec{H}$  is magnetic field,  $\mu$  is magnetic permeability,  $\rho$  is density of fluid particle,  $\rho_0$  is density of fluid particle at temperature  $T_0$ ,  $v$  is fluid's kinematic viscosity,  $\eta$  is resistivity,  $\kappa$  is thermal diffusivity of the fluid particle,  $\vec{\Omega}$  is angular rotation vector,  $d$  is layer height,  $\kappa$  is heat conductivity of the fluid particle,  $C_v$  is specific heat capacity of the fluid

particle.  $\kappa = \frac{\kappa}{\rho_0 C_v}$ ,  $a \Omega d$  is the amplitude of model force,  $\alpha$  is coefficient of volume expansion,  $\Delta T$  is difference temperature between the boundaries of the layer,  $\vec{g}$  is gravity acceleration.

From these equations, one could notice that in the absence of the magnetic field we obtain the system studied in (Essoun & Chabi Orou, 2010) and in (Essoun & Chabi Orou, 2012). Furthermore, when only rotation is absent, the system studied in (Pomalégni et al., 2014) is obtained. So, the joint effects of the helical force, rotation and magnetic force is described by this above set of equations.

## 3. Linear Stability Analysis

We use the linear stability analysis studied in detail by Chandrasekhar (1961) for the study of our system. The required variables by the analysis of hydrodynamic equations describing the system are the  $z$ -component of the velocity field  $w$ , the  $z$ -component of vorticity  $\zeta$ , the  $z$ -component of the current density  $\xi$  and  $\theta$  the fluctuations of the temperature.

The linear perturbation equations in terms of  $w$ ,  $\zeta$ ,  $\xi$  and  $\theta$  obtained from the basic equations stand as:

$$\frac{\partial(\nabla^2 w)}{\partial t} = \alpha g \nabla_h^2 \theta + v \nabla^4 w + \frac{\mu H}{4\pi\rho} D \nabla^2 h_z - 2\Omega D \zeta + a \Omega d (\nabla_h^2 - D^2) \zeta \quad (5)$$

$$\frac{\partial \zeta}{\partial t} = v \nabla^2 \zeta + a \Omega d D^2 w + \frac{\mu H}{4\pi\rho} D \xi + 2\Omega D w \quad (6)$$

$$\frac{\partial h_z}{\partial t} = \eta \nabla^2 h_z + H D w \quad (7)$$

$$\frac{\partial \xi}{\partial t} = \eta \nabla^2 \xi + H D \zeta \quad (8)$$

$$\frac{\partial \theta}{\partial t} = \kappa \nabla^2 \theta + \beta w \quad (9)$$

where

$$D = \frac{\partial}{\partial z}, \quad \nabla_h^2 = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \text{ and } \nabla^2 = \nabla_h^2 + D^2.$$

With the analysis into normal modes (Chandrasekhar, 1961), the linearized governing equations (5), (6), (7), (8), and (9) become:

$$(D^2 - k^2)(D^2 - k^2 - \sigma)W = \frac{g\alpha d^2 k^2 \theta}{\nu} - \frac{\mu H d}{4\pi\rho\nu} D(D^2 - k^2)K + \frac{2\Omega d^3}{\nu} DZ + \frac{a\Omega d^3}{\nu} (D^2 + k^2)Z \quad (10)$$

$$(D^2 - k^2 - \sigma)Z = -\frac{a\Omega d}{\nu} D^2 W - \frac{\mu H d}{4\pi\rho\nu} DX - \left(\frac{2\Omega d}{\nu}\right) DW \quad (11)$$

$$(D^2 - k^2 - P_m \sigma)K = -\frac{H d}{\eta} DW \quad (12)$$

$$(D^2 - k^2 - \sigma P_m)X = -\frac{H d}{\eta} DZ \quad (13)$$

$$(D^2 - k^2 - P_r \sigma)\theta = -\frac{\beta d^2}{\kappa} W \quad (14)$$

where  $W$ ,  $Z$ ,  $K$ ,  $X$  and  $\theta$  are the amplitude of vertical velocity, vertical vorticity, vertical magnetic field, vertical current density and temperature respectively,  $Pr = \frac{\nu}{\kappa}$  is Prandtl number,  $P_m = \frac{\nu}{\eta}$  is magnetic Prandtl number,  $k$  is the dimensionless wave number,  $\sigma$  is the stability parameter which in general is a complex variable denoted by  $\sigma = \gamma + i\omega$ . In this expression of  $\sigma$ ,  $\gamma$  is the growth rate of the instability and  $\omega$  is the frequency. The disturbance grows and the system becomes unstable when  $\gamma > 0$  and the disturbance decays and the system becomes stable when  $\gamma < 0$ . Whereas, if  $\gamma = 0$  we have the marginal state characterized by  $\sigma = i\omega$ . When  $\omega = 0$  the marginal state sets in as stationary motions and if  $\omega \neq 0$  the marginal state sets in as oscillatory motions.

We have eliminated  $K$ ,  $Z$ ,  $X$  and  $\theta$  from these equations to obtain the dimensionless governing equation that stands as:

$$\frac{(D^2 - k^2 - Pr\sigma)}{Ta^2 SD(2D^2 + k^2)} \left[ (D^2 - k^2)[(D^2 - k^2 - \sigma)(D^2 - k^2 - P_m\sigma) - QD^2]^2 + (D^2 - k^2 - P_m\sigma)^2 [S^2 D^2 (D^2 + k^2) + TaD^2 + \right. \\ \left. Rak^2 [(D^2 - k^2 - \sigma)(D^2 - k^2 - P_m\sigma) - QD^2](D^2 - k^2 - P_m\sigma)W] \right] \quad (15)$$

In this equation,  $S = \frac{a\Omega d^2}{\nu}$  is a dimensionless parameter that characterizes the intensity of force  $\vec{f}$ ,  $Ra = \frac{g\alpha\beta d^4}{\nu\kappa}$

is dimensionless Rayleigh number,  $Ta = \frac{4\Omega^2 d^4}{\nu^2}$  is the Taylor number,  $Q = \frac{\mu H^2 d^2}{4\pi\rho\nu\eta}$  is a Chandrasekhar number.

We consider in this work, the free-free boundary conditions characterized by the fact that  $W$  and all its even derivatives vanish at  $z = 0$  and  $z = 1$ . Hence, the solution of equation (15) takes the following form  $W = \sin(\pi z)$ .

Substituting  $W = \sin(\pi z)$  and  $\sigma = i\omega$  in Eqn (15), we get:

$$Ra = \frac{(\pi^2 + k^2 + i\omega Pr) \left\{ (\pi^2 + k^2)[(\pi^2 + k^2 + i\omega)(\pi^2 + k^2 + i\omega P_m) + Q\pi^2]^2 \right. \\ \left. + (\pi^2 + k^2 + i\omega P_m)^2 [Ta\pi^2 + S^2\pi^2(k^2 - \pi^2)] \right\}}{k^2(\pi^2 + k^2 + i\omega P_m)[(\pi^2 + k^2 + i\omega)(\pi^2 + k^2 + i\omega P_m) + Q\pi^2]} \quad (16)$$

For the stationary convection characterized by  $\omega = 0$ , we have got the following Rayleigh number:

$$Ra_s = \frac{(\pi^2 + k_s^2) \left\{ (\pi^2 + k_s^2)^2 + Q\pi^2 \right\}^2 + (\pi^2 + k_s^2)[Ta\pi^2 + S^2\pi^2(k_s^2 - \pi^2)]}{k_s^2[(\pi^2 + k_s^2)^2 + Q\pi^2]} \quad (17)$$

We recover the results obtained by Chandrasekhar (1961) when  $S=0$  in equation (17), while  $Ta=0$  and  $Q=0$  corresponds to those obtained by Levina et al. (2004). Also when  $Q=0$  in equation (17), we recover the results obtained by Essoun and Chabi Orou (2010).  $Ta=0$  corresponds to those obtained by Pomalégné et al. (2014).

The critical value of  $Ra_s$  is obtained for  $k_s = k_{sc}$  where  $k_{sc}$  is the positive root of the following equation.

$$\left(-4 + \frac{4S^2 - 8Q}{\pi^2} - \frac{4Q^2 + 4Ta}{\pi^4}\right) \left(\frac{k_{sc}}{\pi}\right)^2 + \left(-3 + \frac{6S^2 - 2Q}{\pi^2} + \frac{QS^2 + Q^2 - 6Ta}{\pi^4} + \frac{QTa}{\pi^6}\right) \left(\frac{k_{sc}}{\pi}\right)^4 + \left(10 + \frac{12Q + 4S^2}{\pi^2} + \frac{2Q^2(1+S^2) - 4Ta}{\pi^4}\right) \left(\frac{k_{sc}}{\pi}\right)^6 + \\ \left(25 + \frac{13Q + S^2}{\pi^2} - \frac{Ta}{\pi^2}\right) \left(\frac{k_{sc}}{\pi}\right)^8 + \left(24 + \frac{4Q}{\pi^2}\right) \left(\frac{k_{sc}}{\pi}\right)^{10} + 11 \left(\frac{k_{sc}}{\pi}\right)^{12} + 2 \left(\frac{k_{sc}}{\pi}\right)^{14} = 1 + \frac{3Q - S^2}{\pi^2} + \frac{Ta + 3Q^2 - QS^2}{\pi^4} + \frac{Q^3 + QTa}{\pi^6} \quad (18)$$

The oscillatory convection ( $\omega \neq 0$ ) is characterized by a Rayleigh number  $Ra$  which from equation (16) will be complex while the physical meaning of  $Ra$  requires it to be real. Hence, the condition for that  $Ra$  is real implies that the imaginary part of equation (16) is zero, i.e:

$$Ra_o = \frac{\pi^2+k^2}{k^2} \left\{ \begin{array}{l} [(\pi^2+k^2)^2 - Pr\omega^2] + \frac{Q\pi^2[(\pi^2+k^2)^2+PrP_m\omega^2]}{(\pi^2+k^2)^2+\omega^2P_m^2} \\ + \frac{S^2\pi^2(k^2-\pi^2)+Ta\pi^2}{k^2+\pi^2} \frac{[(\pi^2+k^2)^2-P_m\omega^2+Q\pi^2][(\pi^2+k^2)^2-PrP_m\omega^2]+\omega^2(\pi^2+k^2)^2(P_m+Pr)(P_m+1)}{[(\pi^2+k^2)^2-P_m\omega^2+Q\pi^2]^2+\omega^2(\pi^2+k^2)^2(P_m+1)^2} \end{array} \right\} \quad (19)$$

and

$$\frac{S^2\pi^2(k^2-\pi^2)+Ta\pi^2}{k^2+\pi^2} \frac{(\pi^2+k^2)^2(Pr-1)+Q\pi^2(P_m+Pr)+\omega^2P_m^2(Pr-1)}{[(\pi^2+k^2)^2-P_m\omega^2+Q\pi^2]^2+\omega^2(\pi^2+k^2)^2(P_m+1)^2} + \frac{Q\pi^2(Pr-P_m)}{(\pi^2+k^2)^2+\omega^2P_m^2} + 1 + Pr = 0 \quad (20)$$

When  $S=0$  in equations (19) and (20), we recover the results obtained by Chandrasekhar (1961) while  $Ta=0$  and  $Q=0$  correspond to those obtained by Levina et al. (2004). Also when  $Q=0$  in equation (19), we recover the results obtained by Essoun et al. (2010).  $Ta=0$  corresponds to those obtained by Pomalégni et al. (2014).

#### 4. Results and Discussion

From equation (18), when  $S > \pi + \frac{Q}{\pi}$  and  $(\pi^2 + Q)^2 - S^2\pi^2 + Ta = 0$ , the critical wave number for the onset of stationary convection tends to zero. In other words, the critical wave number  $k_{sc}$  tends to zero while the critical Rayleigh number  $Ra_{sc}$  tends to a limiting value  $Ra_s^* = 4\pi^4 + \frac{Ta_{cr}\pi^2}{\pi^2+Q_{cr}}$  when  $S$  approaches some critical value  $S_{cr} = -\sqrt{Ta + (\pi^2 + Q)^2}$ , or  $Ta$  approaches  $Ta_{cr} = S^2\pi^2 - (\pi^2 + Q)^2$  and or  $Q$  approaches  $Q_{cr} = -\pi^2 + \sqrt{S^2\pi^2 - Ta}$ .

We have shown from figures 1, 2 and 3 the respective influence of the parameters  $S$  (characteristic of helical force),  $Q$  (characteristic of magnetic field) and  $Ta$  (characteristic of rotation) on the onset of the stationary convection. Then we have made to vary the value of the parameter of which we want to study the influence on the onset of the stationary convection and we have kept fixed the values of other parameters.

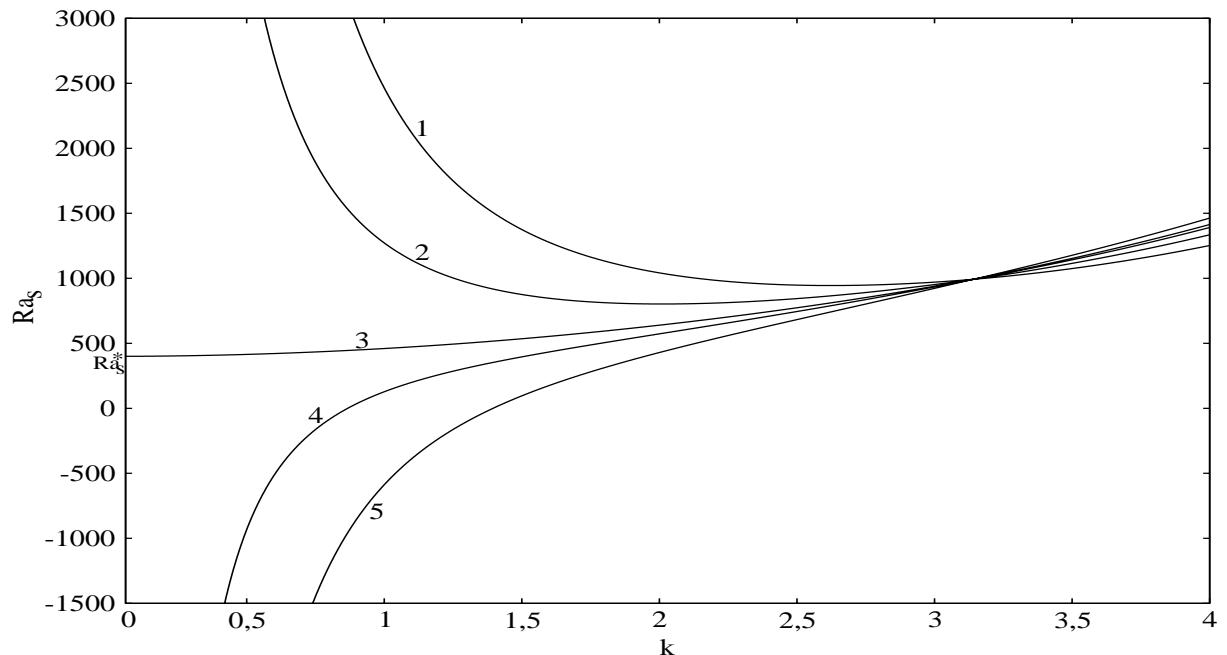


Figure 1. Neutral stability curves for the onset of stationary convection in the case  $Ta=20$ ,  $Q=10$  and for several values of  $S$ : (1)  $S=0$ ; (2)  $S=5$ ; (3)  $S=6.48291$ ; (4)  $S=7$ ; (5)  $S=8$

From Figure 1 we observe that the helical force decreases the threshold of stationary convection when  $Ta = 20$  and  $Q = 10$ .

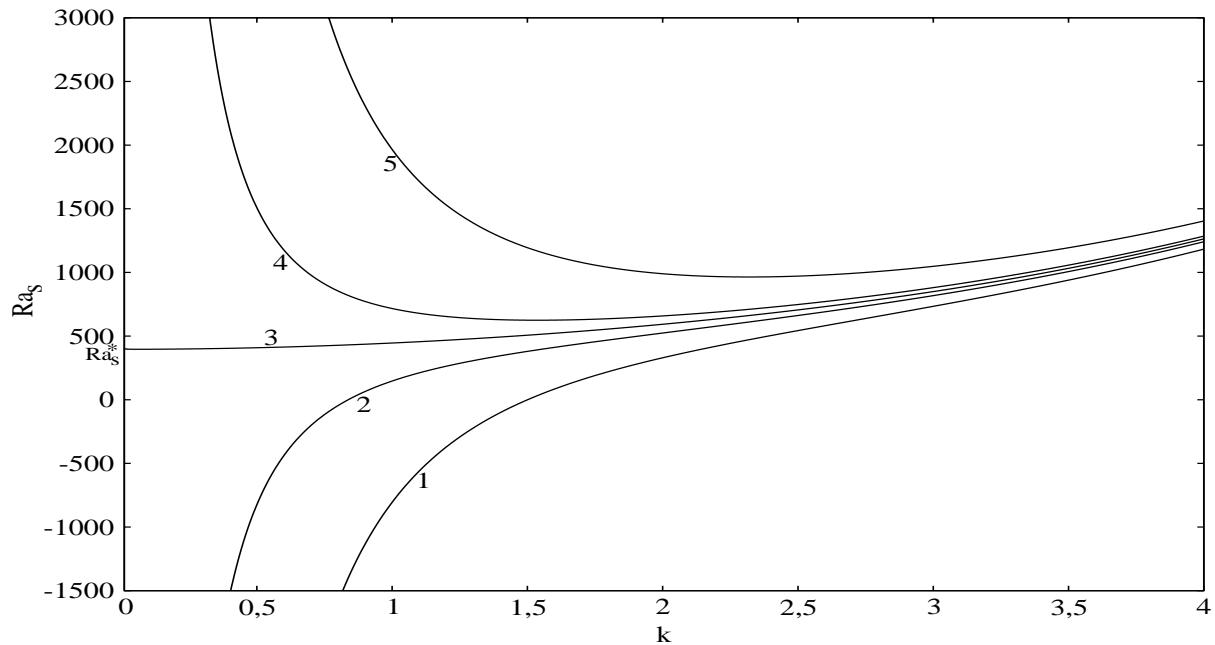


Figure 2. Neutral stability curves for the onset of stationary convection in the case  $Ta=10$ ,  $S=5$  and for several values of  $Q$ : (1)  $Q=0$ ; (2)  $Q=4$ ; (3)  $Q=5.51676$ ; (4)  $Q=7$  and (5)  $Q=15$

Figure 2 shows the stabilizing effects of the magnetic field on the onset of stationary convection when  $S=5$  and  $Ta=10$ .

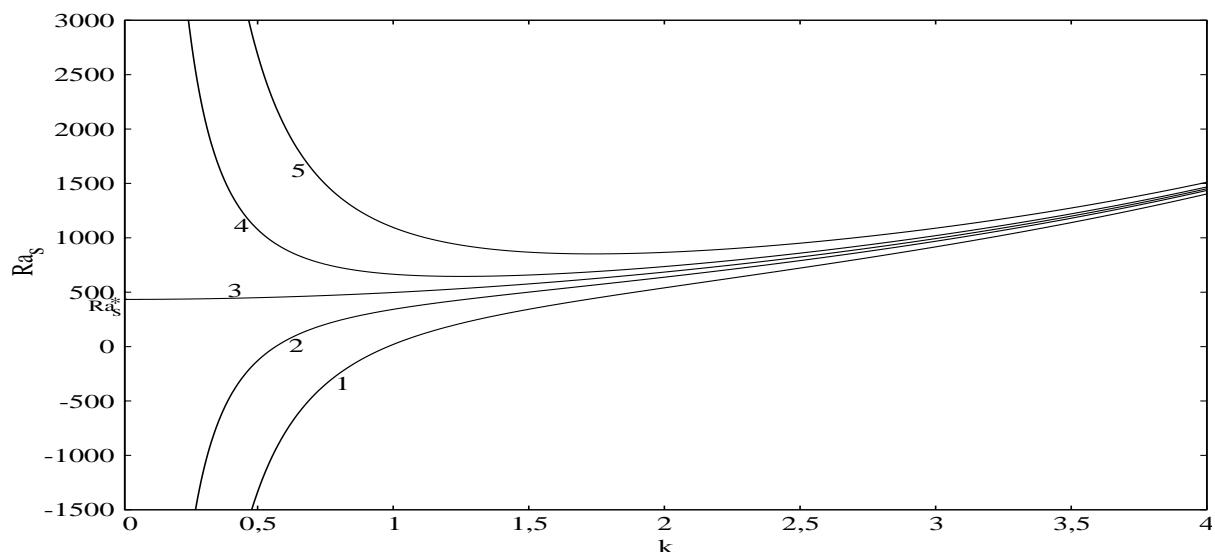


Figure 3. Neutral stability curves for the onset of stationary convection in the case  $Q=10$ ,  $S=7$  and for several values of  $Ta$ : (1) $Ta =0$ ; (2) $Ta=60$ ; (3) $Ta =88.80944$ ; (4)  $Ta = 120$  and (5)  $Ta =200$

We observe through Fig3 the stabilizing effects of rotation on the onset of stationary convection when  $Q= 10$  and  $S=7$ .

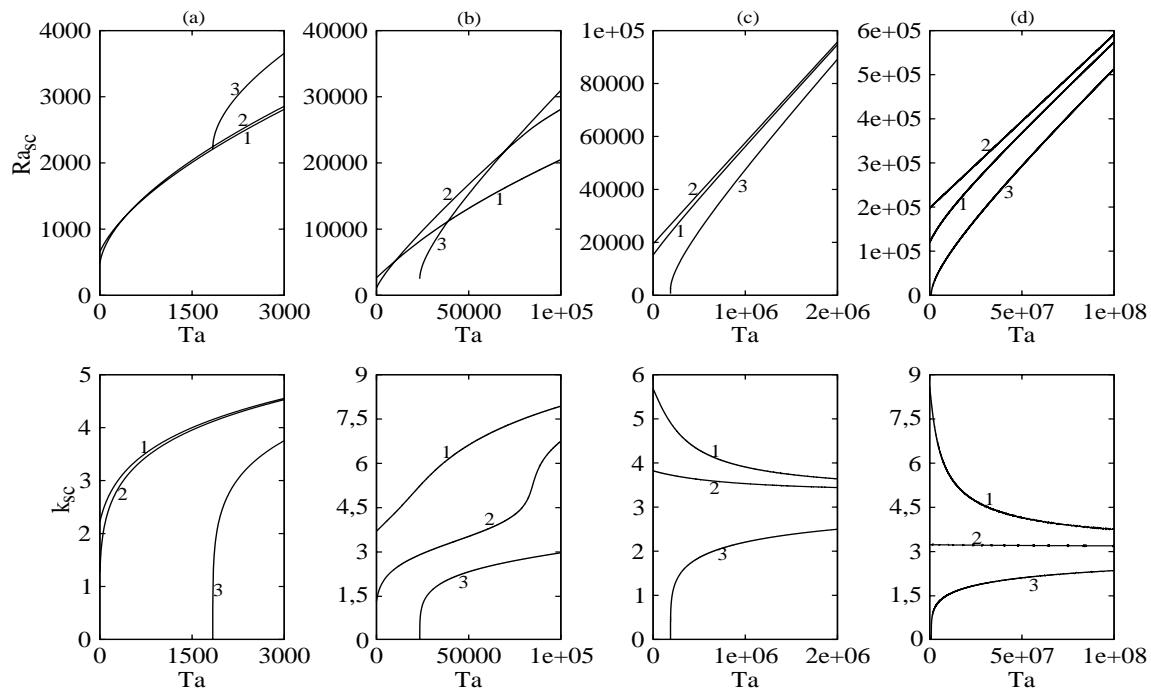


Figure 4. (a) Critical Rayleigh number  $Ra_{sc}$  and critical wave number  $k_{sc}$  as function of  $Ta$  when  $Q=0$  for several values of  $S$ : (1)  $S=0$ ; (2)  $S=3$  and (3)  $S=14$

(b) Critical Rayleigh number  $Ra_{sc}$  and critical wave number  $k_{sc}$  as function of  $Ta$  when  $Q=100$  for several values of  $S$ : (1)  $S=0$ ; (2)  $S=34$  and (3)  $S=60$ .

(c) Critical Rayleigh number  $Ra_{sc}$  and critical wave number  $k_{sc}$  as function of  $Ta$  when  $Q=1000$  for several values of  $S$ : (1)  $S=0$ ; (2)  $S=321$  and (3)  $S=350$ .

(d) Critical Rayleigh number  $Ra_{sc}$  and critical wave number  $k_{sc}$  as function of  $Ta$  when  $Q=10000$  for several values of  $S$ : (1)  $S=0$ ; (2)  $S=1500$  and (3)  $S=3200$ .

In Figure 4a we recover the results obtained by Essoun and al (2010). Through this figure they have shown that for all values of  $Ta$ , the helical force doesn't have the monotonic effect on the onset of stationary convection, but has a monotonic decreasing effect on the corresponding wave number and a monotonic increasing effect on the horizontal scale of the structures merging. With the comparison of Figs 4b, 4c and 4d to Fig 4a we can say that the increasing of parameter  $Q$  reinforces the no-monotonic effects of the helical force on the onset of stationary convection and his monotonic decreasing effect on the corresponding wave number.

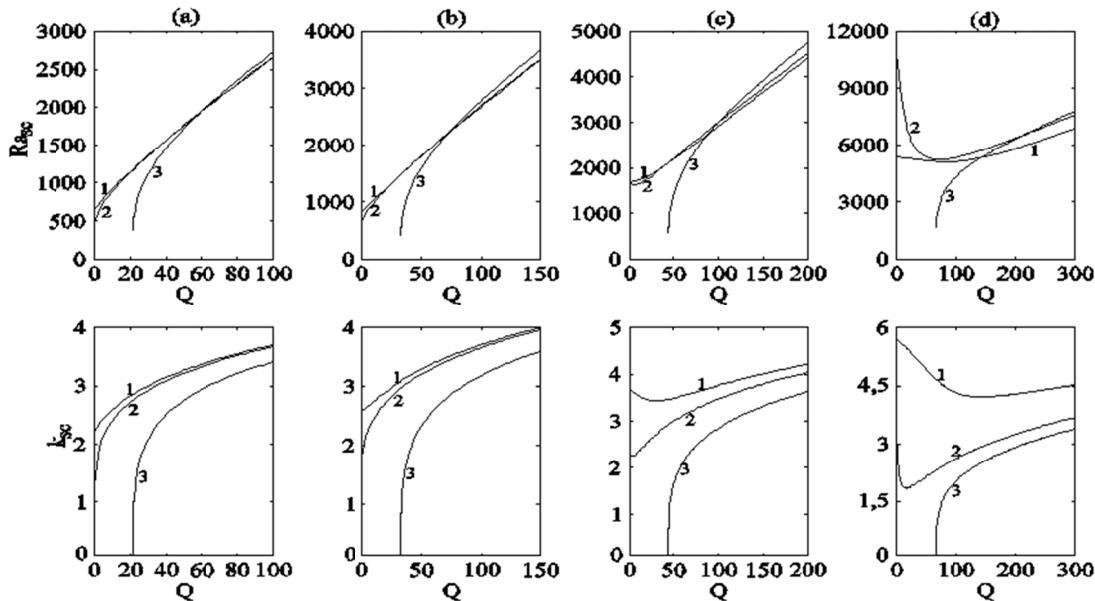


Figure 5. (a) Critical Rayleigh number  $Ra_{sc}$  and critical wave number  $k_{sc}$  as function of  $Q$  when  $Ta=0$  for several values of  $S$ : (1)  $S=0$ ; (2)  $S=3$  and (3)  $S=10$

(b) Critical Rayleigh number  $Ra_{sc}$  and critical wave number  $k_{sc}$  as function of  $Q$  when  $Ta=100$  for several values of  $S$ : (1)  $S=0$ ; (2)  $S=4$  and (3)  $S=14$ .

(c) Critical Rayleigh number  $Ra_{sc}$  and critical wave number  $k_{sc}$  as function of  $Q$  when  $Ta=1000$  for several values of  $S$ : (1)  $S=0$ ; (2)  $S=10$  and (3)  $S=20$ .

(d) Critical Rayleigh number  $Ra_{sc}$  and critical wave number  $k_{sc}$  as function of  $Q$  when  $Ta=10000$  for several values of  $S$ : (1)  $S=0$ ; (2)  $S=31$  and (3)  $S=40$ .

In Figure 5a we recover the results obtained by Pomalégni et al (2014). Through this figure the no-monotonic effects of the helical force on the onset of stationary convection and his monotonic decreasing effect on the corresponding wave number were shown. With the comparison of Fig 5b, 5c and 5d to Fig 5a we can say that the increasing of parameter  $Ta$  reinforces the no-monotonic effect of the helical force on the onset of stationary convection and his monotonic decreasing effect on the corresponding wave number.

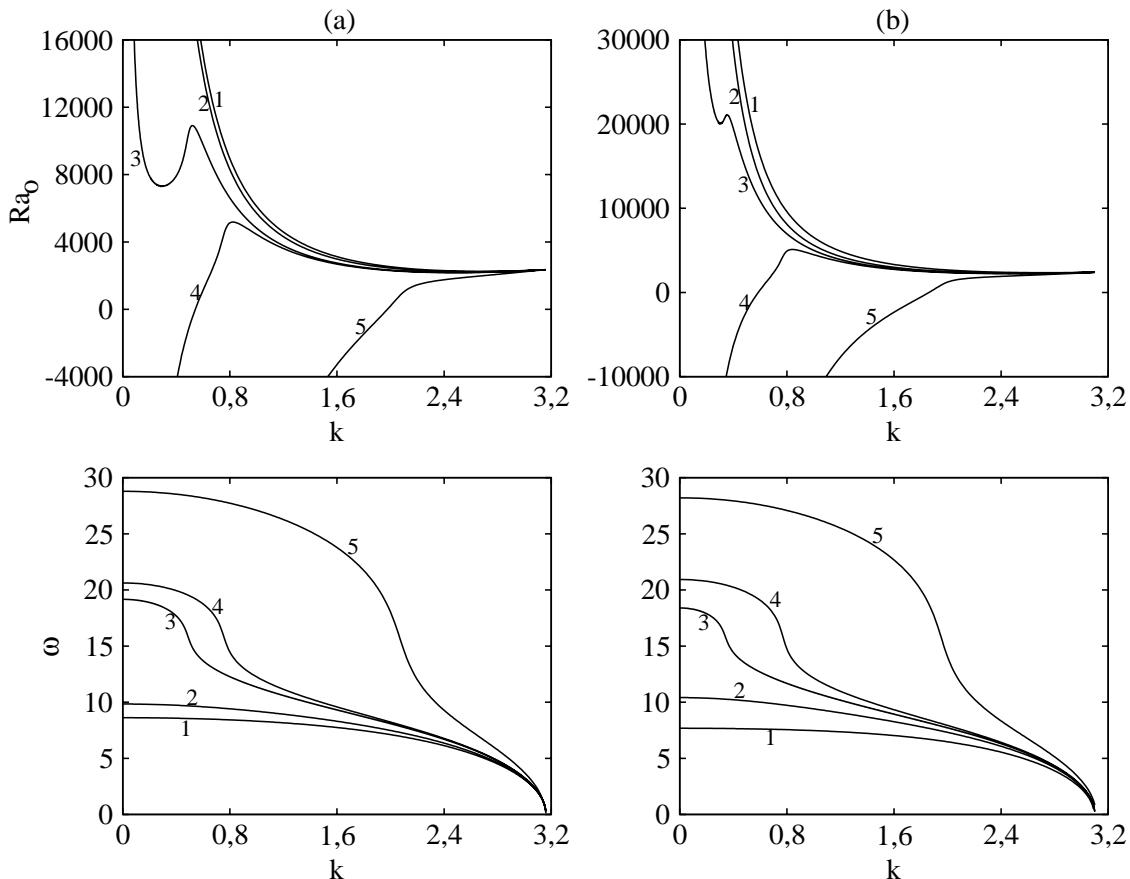


Figure 6. (a) Neutral stability curves for the onset of oscillatory (dashed) convection and frequency  $\omega$  in the case  $Q=80$ ,  $Ta=0$  and  $Pr = 1$ ,  $P_m = 2$  for several values of  $S$ : (1)  $S=0$ ; (2)  $S=3$ ; (3)  $S=4.7$ ; (4)  $S=5$ ; (5)  $S=10$ .

(b) Neutral stability curves for the onset of oscillatory (dashed) convection and frequency  $\omega$  in the case  $Q=80$ ,  $Ta = 100$  and  $Pr = 1$ ,  $P_m = 2$  for several values of  $S$ : (1)  $S=0$ ; (2)  $S=4.7$ ; (3)  $S=5.6$ ; (4)  $S=6$ ; (5)  $S=10$

From Figure 6 we observe that the helical force decreases the threshold of oscillatory convection and increases the frequency when  $Q = 80$ ;  $Ta = 0$  and when  $Q=80$ ;  $Ta = 100$ . Also we note the existence of a critical value of  $S$  i.e. the intensity of helical force for which the curve  $Ra_o(k)$  exhibit two minima at marginal stability. Then we can say that the increasing of the parameter  $Ta$  doesn't influence the effects of helical force on the threshold of oscillatory convection and on the frequency but influence the critical value of  $S$  for which the curve  $Ra_o(k)$  exhibits two minima i.e. the critical value of  $S$  for which we have the apparition of two cells at marginal stability for the oscillatory convection. In other words, the critical value of  $S$  increases when  $Ta$  increases.

## 5. Conclusion

In this work, the combined effects of the magnetic field, rotation and the helical force on the onset of stationary and oscillatory convections in a horizontal electrically conducting fluid layer heated from below with free-free boundary conditions are examined by the application of the linear stability analysis. For all values of Taylor number  $Ta$  or the parameter  $Q$  which characterizes the magnetic field, it was found that the helical force has no monotonic effects on the onset of convection but decreases the corresponding critical wave number. The condition for the formation of a single large-scale structure is obtained, in other words, when the parameter  $S$  which characterizes the intensity of helical force approaching some critical value  $S_{cr} = \frac{1}{\pi} \sqrt{Ta + (\pi^2 + Q)^2}$  or the Chandrasekhar number  $Q$  which characterizes the magnetic field approaches  $Q_{cr} = -\pi^2 + \sqrt{S^2 \pi^2 - Ta}$  and or  $Ta$  approaches  $Ta_{cr} = S^2 \pi^2 - (\pi^2 + Q)^2$ , the critical wave number  $k_{sc}$  tends to zero while the critical Rayleigh number  $Ra_{sc}$  tends to a limiting value  $Ra_s^* = 4\pi^4 + \frac{Ta_{cr}\pi^2}{\pi^2 + Q_{cr}}$ . Formally, this corresponds to an infinite horizontal dimension of a supercritical flow pattern compared with the natural convection: instead of a set of relatively small cells there appears a large-scale structure that occupies the whole available space. We also have shown how the increasing of the Taylor number influence the critical value of the intensity of the helical force for which we have

the apparition of two cells at the marginal stability in the case of oscillatory convection. In other words, when the Taylor number increases this critical value of S also increases.

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