Applied Physics Research; Vol. 6, No. 1; 2014
ISSN 1916-9639 E-ISSN 1916-9647
Published by Canadian Center of Science and Education

Numerical Simulation to Study Riemann Problem for Euler Equations in Case of Two Supersonic Flows Collide

Layali Yahya Salih Al-Mashhadani[1]

[1] Physics Department, College of Education for Pure Science (Ibn Al-Haitham), Baghdad University, Baghdad, Iraq

Correspondence: Layali Yahya Salih Al-Mashhadani, Physics Department, College of Education for Pure Science (Ibn Al-Haitham), Baghdad University, Baghdad 308, Iraq. E-mail: layaleyahya@gmail.com

Received: November 20, 2013 Accepted: December 13, 2013 Online Published: December 24, 2013
doi:10.5539/apr.v6n1p14 URL: http://dx.doi.org/10.5539/apr.v6n1p14

Abstract

In this work we run numerical simulation of gasdynamic to study Riemann problem for the case of collision between two supersonic flows. We analyzed the profiles of density, velocity, pressure, furthermore we inspect for the late time the Mach number which is represent the shock strengths, the specific internal energy density, the total energy density, and the entropy related quantity.

Our results reveal that the collision between two supersonic flows leads to the formation of two shocks separated by a contact discontinuity, as well as the numerical results for Euler equations and the exact solution are close to each other.

Keywords: astrophysics gas dynamic: Riemann problems, supersonic flow, shock waves

1. Introduction

Riemann problems for the equations of hydrodynamics are defined by the initial conditions (Lowe, 2005)

$$\rho, u, P(x,t) = \begin{cases} \rho_l, u_l, P_l & for\ x < 0 \\ \rho_r, u_r, P_r & for\ x > 0 \end{cases} \quad (1)$$

Values of density, velocity, and pressure (ρ_l, u_l, P_l) are the pre-shock conditions (unshocked gas); the shocked gas (ρ_r, u_r, P_r) are the post-shock conditions, the conditions after the shock.

The general solution depends on the Riemann problem at a hand.

The simplest Riemann problem for Euler equations are that in which $\rho_l = \rho_r, P_l = P_r$, and $u_l = u_r$ and with $u_l > 0$.

In this work we will discuss the case in which two supersonic flows collide. We also assumed that the converging gas that has not yet gone through the shock front is undisturbed.

The problem we have to solve is to find the shock velocity, density, and pressure in the compressed region: u_c, ρ_c, P_c **(**Clarke & Carswell, 2007; Artzi & Falcovitz, 2003).

The Mach number μ of the shock is:

$$\mu = \frac{u_l + u_s}{c_{s,l}} \quad (2)$$

$$c_{s.l} = \sqrt{\gamma \frac{\rho_l}{P_l}} \quad (3)$$

$c_{s,l}$ is the speed of sound for the post-shock gas, and $\gamma = \frac{5}{3}$.

The more common way to define the shock relations is in term of shock's Mach number μ.

The Rankine-Hugoniont conditions can be written as:

$$\frac{\rho_2}{\rho_1} = \frac{u_2}{u_1} = \frac{(\gamma+1)\mu^2}{(\gamma-1)\mu^2+2} \quad (4)$$

$$\frac{P_2}{P_1} = \frac{2\gamma\mu^2-(\gamma-1)}{(\gamma+1)} \tag{5}$$

The internal energy stored in the gas is the work done along the adiabatic path and define by:

$$\varepsilon = \frac{1}{(\gamma-1)}\frac{P}{\rho} \tag{6}$$

The entropy related quantity is

$$\xi = P\rho^{-\gamma} \tag{7}$$

2. Numerical Results and Discussion

The different Riemann problem is when two flows will be collided. Two shocks separated by a contact discontinuity will generate if the flows are supersonic.

We run the simulation with the initial conditions in Table 1, and inspect for a late time the graphs for velocity, density, and pressure, as well as the Mach number, specific internal energy, total energy density, and entropy related quantity.

Table 1. Initial condition of the simulation

Regions	Density(gm cm^{-1} s^{-1})	Velocity(cm s^{-1})	Pressure(gm cm^{-1} s^{-2})
Right	1	2	1
Left	0.1	1	10

The domain size of 1 is divided into 1000 computational cells of length ($\Delta x = 0.001$).

The total time (t = 0.058 s), and the time between outputs ($\Delta t = 5.8 \times 10^{-5}$) is calculated using Equation 8; (Lax & Liu, 1998)

$$\Delta t = N_{CFL}\frac{\Delta x}{max|(u_r-c_s),(u_l-c_s)|} \tag{8}$$

The computation has been performed using CFL number (N_{CFL}) of 0.5.

The density, velocity, and pressure profiles in Figure 1 shows the collision leads to two discontinuities; a shock in the fast flow (S1), a shock in the low flow (S2), and a contact discontinuity in between the colliding flows.

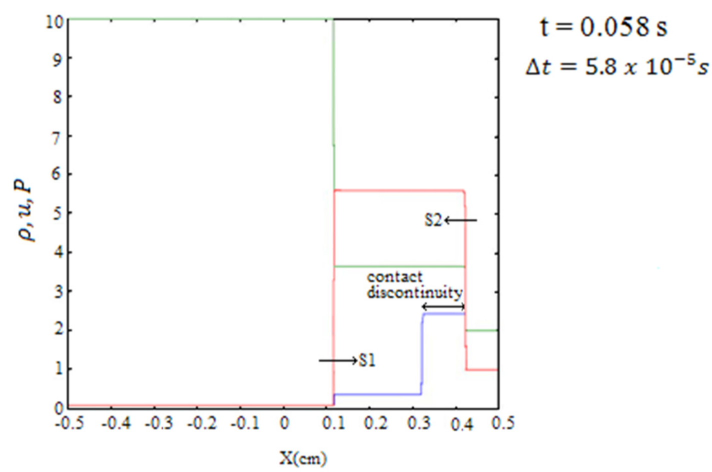

Figure 1. Density, velocity, and pressure as a function of position. The three profiles correspond to: density (the blue line), velocity (the green line), and pressure (the red line)

The gas on the left in Figure 1 is moving with a positive velocity and it is faster than the gas on the right which is move with negative velocity. The two shocks propagating one toward the other, with a sharp jump in the pressure profile. The shock moving to the right sweeps up the materials into a high-density layer.

The shock strength which is characterizes by the Mach numbers in the shocks frame and shock's velocities in the lab frame have been measured from the pre and post shocks conditions and the results illustrates in Table 2.

Table 2. Mach numbers and shock's velocity in the lab frame

Regions	Mach Number	Shock's velocity (cm s^{-1})
Right	2.12	4.67
Left	6.73	1.4

The densities of the exact solution at different times of the simulation have been changed linearly with time (Quartapelle, Castelletti, Guardone, & Quaranta, 2003), as it is shown in Figure 2.

Figure 2 explains the time evolution of the two shocks and the contact discontinuity at four time steps of the simulation.

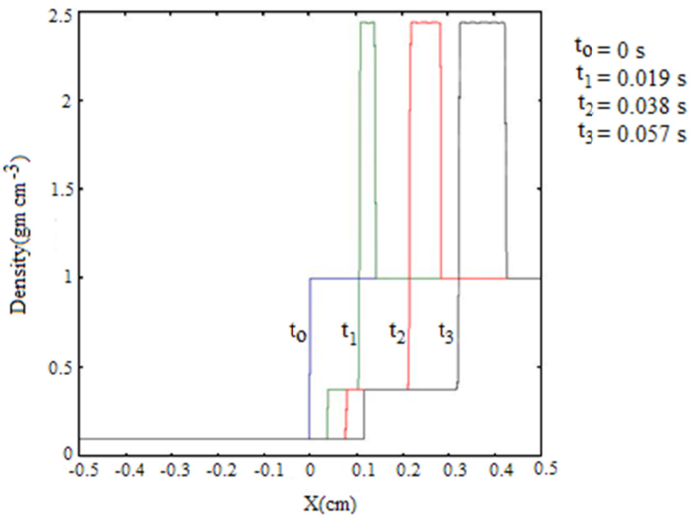

Figure 2. The density profiles as a function of position for four outputs files; at the start of the simulation at time (t = 0 s), in between when the time reaches to 0.019 s, 0.038 s, and at end time of the simulation at time (t = 0.057 s)

Speed of the three discontinuities in Figure 2 can be measured easily by looking at the range of the outputs. therefore, we have speed of the right shock is 4.91 cm s^{-1}, the left shock moving with speed of 1.51 cm s^{-1}, and the contact discontinuity traveling with speed of 3.512 cm s^{-1}.

If we compare with the previous values in Table 2, they are very close to each other. Depending on this result we can conclude that the numerical solution is very close to the exact solution for this Riemann problem.

The computational results of the Mach number in the lab frame and the specific internal energy density are propagating from the left to the right and they are clear in Figure 3.

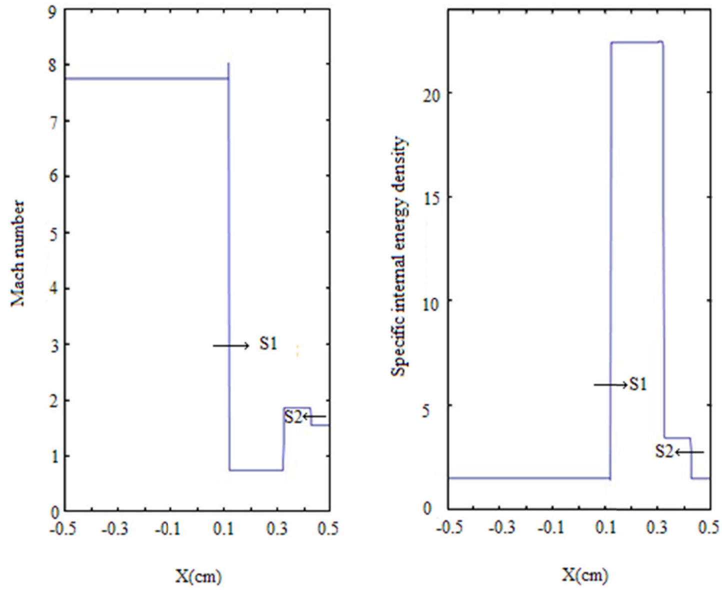

Figure 3. Profiles of the Mach number in the lab frame and the specific internal energy density as a function of position

Again there are two shocks S1, S2, and a contact discontinuity in Figure 3. The first shock wave is propagating from the left to the right, while the second shock wave is spread from the right to the left. The specific internal energy density reaches into the high levels at the shock wave moving to the right.

The total energy density is low across the first shock wave. Because of they are an entropy waves, which is changes the direction of the flow for the shock waves moving to the right. This is illustrated in Figure 4.

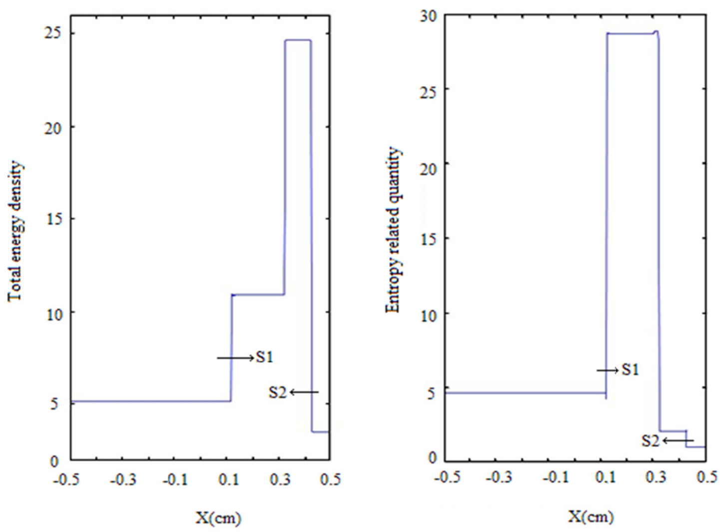

Figure 4. The total energy density and the entropy related quantity as a function of position

3. Conclusions

Conclusions of the present numerical study can be summarized as follows.

1) Collision of two supersonic flows leads to forward Shock and reverse Shock moving toward each other and a contact discontinuity. The forward Shock moving to the right and will sweeping up the materials into a high-density layer.

2) The numerical results for Euler equations are close to the exact solution and the flows are allowed to move

with arbitrary velocities tangent to the initial discontinuity.

3) The Mach number, specific internal energy density is not constant across the contact discontinuity surface.

4) The line dividing between two regions is a contact discontinuity, will be separated two flows of different entropy but the same pressure and velocity.

References

Artzi, M. B., & Falcovitz, J. (2003). *Generalized Riemann Problems in Computational Fluid Dynamics*. New York: Cambridge University Press.

Clarke, C., & Carswell, B. (2007). *Principle of Astrophysical Fluid Dynamics*. New York: Cambridge University Press.

Lax, P., & Liu, X. (1998). Solution of Two-Dimensional Riemann Problems of Gas Dynamic y Positive Schemes. *SIAM Journal on Scientific Computing, 19*(2), 319-340. http://dx.doi.org/10.1137/S1064827595291819

Lowe, C. A. (2005). Two-Phase Shock-Tube Problems and Numerical Methods of Solution. *Journal of Computational Physics, 204*(2), 598-632. http://dx.doi.org/10.1016/j.jcp.2004.10.023

Quartapelle, L., Castelletti, L., Guardone, A., & Quaranta, G. (2003). Solution of the Riemann Problem of Classical Gasdynamics. *Journal of Computational Physics, 190*(1), 118-140. http://dx.doi.org/10.1016/S0021-9991(03)00267-5

Copyrights