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Conservation of Energy and Lorentz Covariant Gravitation

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Abstract

We show a class of Lorentz covariant theories of gravitation that includes general relativity do not satisfy conservation of energy.

Keywords: Lorentz transformation, energy, momentum, photon velocity

1. Introduction

General relativity is covariant under general coordinate transformations (Einstein, 1916). Consequently it is covariant under Lorentz transformations. Within a theory of gravitation we will be considering independent systems S_v where $-1 < v < 1$. Each S_v will be examined with respect to two frames of reference F_v and F_v' related by a Lorentz transformation. For each S_v there will be a point mass and a photon moving towards the mass along a line. The gravitational field of the mass will cause energy, momentum, and speed of the photon to change as it moves towards the mass. The energies and momenta of the mass and photon will be described by Lorentz covariant four-vectors. We will restrict to the class of Lorentz covariant theories of gravitation that have only c and G as constants with dimension. This class includes general relativity. Units are chosen so that $c = G = 1$.

2. Frames of Reference

Let F_v be a frame of reference with coordinates t, x, y, z and F_v' a frame of reference with coordinates t', x', y', z'. The coordinates of the frames being related by the Lorentz transformation

$$t = \frac{t'+vx'}{\sqrt{1-v^2}} \quad x = \frac{x'+vt'}{\sqrt{1-v^2}} \quad y = y' \quad z = z' \tag{1}$$

With respect to F_v' let there be a photon γ moving from positive x' infinity towards the origin along the x' axis. Also let there be a point mass A such that when γ is at infinity A is at rest at the origin. When γ is at infinity let M_A' be the energy of A, E_γ' be the energy of γ, $P_A'^{\mu}$ be the components of the energy-momentum four-vector of A and $P_\gamma'^{\mu}$ be the components of the energy-momentum four-vector of γ. With respect to F_v' when γ is at infinity

$$P_A'^0 = M_A' \quad P_A'^1 = P_A'^2 = P_A'^3 = 0$$
$$P_\gamma'^0 = E_\gamma' \quad P_\gamma'^1 = -E_\gamma' \quad P_\gamma'^2 = P_\gamma'^3 = 0 \tag{2}$$

Let (De Paepe, 2010)

$$M_A' = \sqrt{1-v^2}m_A \quad E_\gamma' = \sqrt{\frac{1+v}{1-v}}E_\gamma \tag{3}$$

where m_A and E_γ are positive constants. We will be considering independent systems S_v. For each $-1 < v < 1$ there is a system. We still have a photon moving towards a point mass along the x' axis but for each of the different S_v we are starting with different masses of A and different energies of γ with respect to F_v'. For S_v with respect to F_v' when γ is at infinity the energies of A and γ are given by (3). For S_v with respect to F_v when γ is at infinity the energy of A is using (2) and (3) and the formula for transformation of energy

$$P_A^0 = \frac{P_A'^0+vP_A'^1}{\sqrt{1-v^2}} = \frac{M_A'}{\sqrt{1-v^2}} = \frac{\sqrt{1-v^2}m_A}{\sqrt{1-v^2}} = m_A \tag{4}$$

and the energy of γ is

$$P_\gamma^0 = \frac{P_\gamma'^0 + vP_\gamma'^1}{\sqrt{1-v^2}} = \frac{E_\gamma' + (-E_\gamma')}{\sqrt{1-v^2}} = \sqrt{\frac{1-v}{1+v}}E_\gamma' = \sqrt{\frac{1-v}{1+v}}\sqrt{\frac{1+v}{1-v}}E_\gamma = E_\gamma \tag{5}$$

With respect to F_v the total energy of S_v is then $m_A + E_\gamma$.

3. Energy and Momentum Functions

Now γ moves from positive x' infinity along the x' axis towards A so with respect to F_v' when γ is at x_γ' let the functions $p_\gamma'^\mu(x_\gamma')$ be the components of the energy-momentum four-vector of γ. The values of M_A', E_γ', and x_γ' completely determines S_v. A component $p_\gamma'^\mu(x_\gamma')$ is then a function of M_A', E_γ', and x_γ' and no other variables. Since we are considering the class of theories of gravitation with only c and G as constants with dimension we have $p_\gamma'^\mu(x_\gamma')/E_\gamma'$ will be a dimensionless function of the dimensionless variables M_A'/x_γ' and E_γ'/x_γ'. Note $M_A'/E_\gamma' = (M_A'/x_\gamma')/(E_\gamma'/x_\gamma')$. There is then a dimensionless function C of M_A'/x_γ' and E_γ'/x_γ' such that

$$p_\gamma'^0(x_\gamma') = E_\gamma' + \frac{M_A'E_\gamma'}{x_\gamma'}C\left(\frac{M_A'}{x_\gamma'}, \frac{E_\gamma'}{x_\gamma'}\right) \tag{6}$$

Similarly for the x' component of momentum there is a dimensionless function D such that

$$p_\gamma'^1(x_\gamma') = -E_\gamma' + \frac{M_A'E_\gamma'}{x_\gamma'}D\left(\frac{M_A'}{x_\gamma'}, \frac{E_\gamma'}{x_\gamma'}\right) \tag{7}$$

The functions C and D will depend on the theory of gravitation.

With respect to F_v' if γ is at the point $(t_\gamma', x_\gamma', 0, 0)$ where t_γ' is a function of x_γ' then with respect to F_v it is at the point $(t_\gamma, x_\gamma, 0, 0)$ where

$$t_\gamma' = \frac{t_\gamma - vx_\gamma}{\sqrt{1-v^2}} \qquad x_\gamma' = \frac{x_\gamma - vt_\gamma}{\sqrt{1-v^2}} \tag{8}$$

and x_γ is a function t_γ. With respect to F_v let the functions $p_\gamma^\mu(t_\gamma)$ be the components of the energy-momentum four-vector of γ at time t_γ. The energy of γ at time t_γ is by (3), (6), (7) and (8)

$$p_\gamma^0(t_\gamma) = \frac{p_\gamma'^0(x_\gamma') + vp_\gamma'^1(x_\gamma')}{\sqrt{1-v^2}} = \frac{E_\gamma' + \frac{M_A'E_\gamma'}{x_\gamma'}C\left(\frac{M_A'}{x_\gamma'}, \frac{E_\gamma'}{x_\gamma'}\right) + v\left[-E_\gamma' + \frac{M_A'E_\gamma'}{x_\gamma'}D\left(\frac{M_A'}{x_\gamma'}, \frac{E_\gamma'}{x_\gamma'}\right)\right]}{\sqrt{1-v^2}}$$

$$= \frac{E_\gamma'}{\sqrt{1-v^2}}\left\{1 - v + \frac{M_A'}{x_\gamma'}\left[C\left(\frac{M_A'}{x_\gamma'}, \frac{E_\gamma'}{x_\gamma'}\right) + vD\left(\frac{M_A'}{x_\gamma'}, \frac{E_\gamma'}{x_\gamma'}\right)\right]\right\}$$

$$= E_\gamma + \frac{(1+v)m_AE_\gamma}{x_\gamma - vt_\gamma}\left[C\left(\frac{(1-v^2)m_A}{x_\gamma - vt_\gamma}, \frac{(1+v)E_\gamma}{x_\gamma - vt_\gamma}\right) + vD\left(\frac{(1-v^2)m_A}{x_\gamma - vt_\gamma}, \frac{(1+v)E_\gamma}{x_\gamma - vt_\gamma}\right)\right] \tag{9}$$

The $v \to 1$ limit of (9) is

$$p_\gamma^0(t_\gamma) = E_\gamma + \frac{2m_AE_\gamma}{x_\gamma - t_\gamma}\left[C\left(0, \frac{2E_\gamma}{x_\gamma - t_\gamma}\right) + D\left(0, \frac{2E_\gamma}{x_\gamma - t_\gamma}\right)\right] \tag{10}$$

Similarly for the x component of momentum the $v \to 1$ limit is

$$p_\gamma^1(t_\gamma) = -E_\gamma + \frac{2m_AE_\gamma}{x_\gamma - t_\gamma}\left[C\left(0, \frac{2E_\gamma}{x_\gamma - t_\gamma}\right) + D\left(0, \frac{2E_\gamma}{x_\gamma - t_\gamma}\right)\right] \tag{11}$$

Subtracting (10) and (11) gives

$$p_\gamma^0(t_\gamma) - p_\gamma^1(t_\gamma) = 2E_\gamma \tag{12}$$

4. Velocity of Photon

In general relativity the velocity of a photon does not depend on its energy. We will also restrict to the class of theories of gravitation where the velocity of γ does not depend on its energy. With respect to F_v' the velocity dx_γ'/dt_γ' of γ will then be a function M_A' and x_γ' Consequently dx_γ'/dt_γ' will be a dimensionless function of

the dimensionless variable M'_A/x'_γ. There is then a dimensionless function S such that

$$\frac{dx'_\gamma}{dt'_\gamma} = -1 + \frac{M'_A}{x'_\gamma} S\left(\frac{M'_A}{x'_\gamma}\right) \tag{13}$$

Finally we restrict to theories of gravitation where $S(0) > 0$. General relativity (Einstein, 1916) has approximately for small M'_A/x'_γ that

$$\frac{dx'_\gamma}{dt'_\gamma} = -1 + \frac{2M'_A}{x'_\gamma} \tag{14}$$

Consequently for general relativity $S(0) > 0$. With respect to F_v the velocity of γ is using (3), (8), (13) and the velocity addition formula

$$\frac{dx_\gamma}{dt_\gamma} = \frac{\frac{dx'_\gamma}{dt'_\gamma}+v}{1+v\frac{dx'_\gamma}{dt'_\gamma}} = \frac{-1+\frac{M'_A}{x'_\gamma}S\left(\frac{M'_A}{x'_\gamma}\right)+v}{1+v\left[-1+\frac{M'_A}{x'_\gamma}S\left(\frac{M'_A}{x'_\gamma}\right)\right]} = \frac{-1+\frac{(1+v)m_A}{x_\gamma-vt_\gamma}S\left(\frac{(1-v^2)m_A}{x_\gamma-vt_\gamma}\right)}{1+\frac{v(1+v)m_A}{x_\gamma-vt_\gamma}S\left(\frac{(1-v^2)m_A}{x_\gamma-vt_\gamma}\right)} \tag{15}$$

The $v \to 1$ limit of (15) is

$$\frac{dx_\gamma}{dt_\gamma} = \frac{-1+\frac{2m_A}{x_\gamma-t_\gamma}S(0)}{1+\frac{2m_A}{x_\gamma-t_\gamma}S(0)} \tag{16}$$

Solving this differential equation gives

$$x_\gamma - t_\gamma - (x_1 - t_1) + 2m_A S(0)\, ln\left(\frac{x_\gamma-t_\gamma}{x_1-t_1}\right) = 2(t_1 - x_1) \tag{17}$$

where the point $(t_1, x_1, 0, 0)$ with $x_1 - t_1 > 0$ is on the path of γ. There is then no point $(t_2, x_2, 0, 0)$ with $x_2 - t_2 = 0$ on the path of γ hence for all points on the path $x_\gamma - t_\gamma > 0$. From (17) then $x_\gamma - t_\gamma \to 0$ as $t_\gamma \to \infty$.

5. Conclusion

By (12), (16), and the formula $p^1_\gamma(t_\gamma) = (dx_\gamma/dt_\gamma)p^0_\gamma(t_\gamma)$ we have

$$p^0_\gamma(t_\gamma) = \left[1 + \frac{2m_A}{x_\gamma-t_\gamma}S(0)\right]E_\gamma \tag{18}$$

The energy of γ becomes greater than the total energy of the system $m_A + E_\gamma$ as $x_\gamma - t_\gamma \to 0$. We have shown a class of Lorentz covariant theories of gravitation that includes general relativity do not satisfy conservation of energy.

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