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Statistical Analysis of the Magnetic and Velocity Field Fluctuations for the Solar Wind Turbulence

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Abstract

In this paper, the scaling properties of magnetohydrodynamic (MHD) turbulence fluctuations are studied for different data sets. The Data is collected in two different setups, namely in situ satellite observations of the solar wind and the direct numerical simulation of incompressible MHD equations. It is found that the probability density functions (PDFs) of inertial range energy fluctuations exhibit self-similarity and monoscaling in agreement with solar-wind measurements. Furthermore, the energy PDFs exhibit similarity over all scales of the turbulent system showing no substantial qualitative change of shape as the scale of the fluctuations varies. On the contrary, our results also exhibit that magnetic, velocity and Elsässer field fluctuations have an intermittent structure so that the associated PDFs change from Gaussian at large scales to leptokurtic (fat-tailed) at small scales.

Keywords: probability density function, magnetohydrodynamics, solar wind, self similar, Elsässer field

1. Introduction

Turbulence in electrically conducting magnetofluids is, apart from its importance for laboratory plasmas, see for example (Ortolani & Schnack, 1993), a key ingredient in the dynamics of, e.g., the Earths liquid core and the solar wind, (Biskamp, 2003). A simple description of such plasmas is the frame work of incompressible magnetohydrodynamics (MHD), a fluid approximation neglecting kinetic processes occurring on microscopic scales. This approach is appropriate if the main interest is focused on the nonlinear dynamics and the inherent statistical properties of fluid turbulence. The solar wind provides a unique laboratory for the study of such magnetofluids with a magnetic Reynolds number estimated to exceed 10^5 (Bruno & Carbone, 2005; Goldstein, Roberts, & Matthaeus, 1995). At high frequencies, kinetic effects are dominant, but at frequencies lower than the ion cyclotron frequency, the evolution of plasma can be modeled using the MHD approximation. The satellite observations offer us an almost unique possibility to gain information on the turbulent MHD state of solar wind (Tu & Marsch, 1995; Goldstein, 2001). The concept of self-similarity in turbulence is one of the key hypotheses of Kolmogorov theory (Kolmogorov, 1991), which leads to the famous -5/3 exponent for the power spectrum decay in the inertial range (the intermediate range of scales between the large scales where energy is injected towards the small ones where it is dissipated). The analysis of the probability distribution functions (PDF) of velocity fluctuations at a given scale shows a departure from Gaussianity in the PDF's tails. The same behavior is usually evidenced by looking at the scaling exponents of higher order moments, which appear to be nonlinear functions of the order index. The anomalous scaling in the inertial range and the non-Gaussian PDF of velocity fluctuations are attributed to the intermittency in turbulent flows (Kolmogorov, 1962). However, experimental observations and numerical computational due to varying energy transfer in the inertial range have shown nonlinear behavior of such scaling exponents. The presence of intermittency in MHD flows is simply confirmed by the departure from the Gaussian distribution and suggests a multifractal model to approximate the shapes of the PDFs (Hnat, Chapman, & Rowlands, 2003).

On the other hand, magnetofluids are more complex than hydrodynamic turbulence due to the coupling vector fields, velocity $\boldsymbol{V}$ and magnetic B (Biskamp, 1993). The turbulence in the solar wind as a magnetized plasma has been studied for decades in the past as the possible state that leads to cascade of energy from the observed large scale fluctuations down to dissipation scales that can heat the solar wind plasma.

Observations of Alfvénic fluctuations in the solar wind by Helios and Ulysses spacecraft show that the turbulent energy carried by these fluctuations is distributed in frequency according to a power law, at high frequencies going as $f^{-\frac{5}{3}}$, a Kolmogorov-like spectrum, while at lower frequencies the spectrum close to f^{-1}. Alfvénic turbulence is predominant in fast wind streams while in slow solar wind the turbulence is of more complex nature with low Alfvénicity.

In this Letter we quantify the scaling seen in turbulence fluctuations for different data sets, namely satellite observations of the solar wind and the direct numerical simulation (DNS) of incompressible MHD equations. The scaling exponents extracted from the structure functions (SFs) sensitively distinguish between self-similarity and multifractality in time series. This will allow us to understand the heating process of solar wind, the transport of solar energetic particles and galactic cosmic rays within the heliosphere. Briefly we present the result of this letter as: (i) the power density spectrum (PSD) of this system exhibits a Kolmogorov-like scaling law with the spectral exponent very close to *-5/3* in the inertial range. (ii) magnetic and velocity field fluctuations are multifractal, i.e., the associated PDFs change from Gaussian at large scales to leptokurtic (fat-tailed) at small scales. The scaling exponents show a nonlinear function of the order index as a signature of intermittency. (iii) In contrast the PDFs of magnetic energy, B^2, kinetic energy, V^2, and or total (magnetic + kinetic) energy $(z^{\pm})^2$ (where $z^{\pm} = V \pm B$ are the Elsässer fields) are self- similar over all observed scales, exhibit monoscaling, i.e. a global scale-invariance, and closely resemble gamma distributions.

2. The Datasets

2.1 Numerical Simulation

The dimensionless equations of incompressible MHD, formulated in Elsässer variables $z^{\pm}$ with the fluid velocity $\boldsymbol{V}$ and the magnetic field B which is given in Alfvén speed units, read (Momeni & Müller, 2008)

$$\nabla . z^{\pm} = 0, \tag{1}$$

$$\partial_t z^{\pm} = -z^{\mp} . \nabla z^{\pm} - \nabla P + \eta_{+} \nabla z^{\pm} + \eta_{-} \nabla z^{\mp}, \tag{2}$$

with the total pressure $P = p + \frac{1}{2} B^2$. The dimensionless kinematic viscosity μ and magnetic diffusivity η appear in $\eta_{\pm} = 1/2(\mu \pm \eta)$. We like to remind that z^{+} and z^{-} represent Alfvén modes propagating in opposite directions along the ambient magnetic field. The data used in this work stems from pseudospectral high-resolution DNS based on a set of equations equivalent to Eqs 1 and 2. It describes homogeneous fully developed turbulent MHD flows in a cubic box of linear size 2π with periodic boundary conditions. The initial conditions for the decaying simulation run consist of random fluctuations with total energy equal to unity. In the MHD cases total kinetic and magnetic energy are approximately equal. In the MHD setups magnetic and cross helicity are small so that $z^{+} = z^{-}$. The driven turbulence simulations were run toward quasistationary states whose energetic and helicity characteristics as mentioned above are roughly equal to the decaying run. The MHD magnetic Prandtl number $P_{rm} = \frac{\mu}{\eta}$ and the Reynolds number are unity and of order 10^3 respectively. The simulation of the above- described setup is conducted with a resolution of 1024^2 collocation points. The angle-integrated energy spectrum of this system exhibits a Kolmogorov-like scaling law in the inertial range, i.e., $E_k = k^{-5/3}$ (Müller & Grappin, 2005).

2.2 Solar Wind Observations

The satellite observations of both velocity and magnetic field in the interplanetary space, offer us an almost unique possibility to gain information on the turbulent MHD state in a very large scale range, say from1 AU (astronomical unit, $= 1AU = 1.5 \times 10^8 km$) down to $10^3 km$. Here we limit to analysis only plasma measurements of the bulk velocity $V(t)$ and magnetic field intensity $B(t)$. We based our analysis on plasma measurements as recorded by the ACE spacecraft. The data sets are in geocentric ecliptic (GSE) coordinates. The data sets, that we consider here are then a) magnetic field with 1 second averaged for the interval 01/03/2009-31/08/2009, this consists of $\sim 1.6 \times 10^8$ samples and b) velocity field with 64 second averaged for the interval

01/01/2009-31/01/2010, this consists of $\sim 5\times10^{6}$ samples. The speeds are below 300 km/sec. All of these time series can be downloaded from CDA web http://cdaweb.gsfc.nasa.gov/.

3. Results and Discussion

Kolmogorov's K41 turbulence theory was based on the hypothesis that energy is transferred in the spectral domain at a constant rate through local interaction within the inertial range. This energy cascade is self-similar due to the lack of any characteristic spatial scale within the inertial range itself. The best way to indicate this cascade is provided by examining the PSD of solar wind fluctuations, which is equivalent to studying a second order statistic. The PSD in the inertial range reflects the similarity properties of high-Reynolds number turbulence. Figure 1 shows the PSD of magnetic field components of the satellite observations, calculated using the Welch method (Welch, 1967). Qualitatively similar behavior in PSD has also been observed in DNS studies (Müller & Grappin, 2005). One can clearly recognize in PSD two power laws: At low frequencies, the spectrum is f^{-1} which contains the energy that eventually is transferred to smaller scales via turbulence cascade, and at higher frequency range (inertial range) the spectrum is $f^{-5/3}$ as predicted by Kolmogorov (Kolmogorov, 1991). To test for intermittency feature of the fluctuations, we now turn to higher-order statistics. We focus on the statistics of two point increments of a turbulent field component, $\delta p(t,\tau;x,\ell)=p(t+\tau;x+\ell)-p(t;x)$, for time τ or space ℓ lag. The importance of these increments is that they yield a comprehensive and scale-dependent characterization of the statistical properties of turbulent fluctuations via the associated probability density function (PDF). We focus on the absolute moments of the increments, known as structure functions, (SF)

$$S^{(m)}(\tau;\ell)=\frac{1}{N}\sum_{j=1}^{N}|\,\delta p(t_j,\tau;x_j,\ell)\,|^{m}, \tag{3}$$

where m and N are the moment order and sample size, respectively. Let us now consider the scaling as defined by the SFs, $S^{(m)}(\tau,\ell)\propto(\tau^{\xi(m)};\ell^{\xi(m)})$. The linear function $\xi(m)=\alpha m$ reflects the statistical self-similarity, in the monoscaling case. This scaling with $\alpha=1/3$ is characteristic of Kolmogorov's theory turbulence (Kolmogorov, 1991). Monoscaling of SFs of energy fluctuations was shown in Refs. (Momeni & Müller, 2008) (Hnat, Chapman, & Rowlands, 2003). We plot in Figure 2 and Figure 3 the PDFs of energy fluctuations for each data set. It has been shown that the magnetic and kinetic energy density fluctuations, exhibit monoscaling over all observed scales, and closely resemble gamma distributions with $\alpha \sim 0.29\pm0.025$ (Momeni & Müller, 2008). The non-Gaussian nature of the PDFs over all scales is evident. The PDFs are almost symmetrical and become increasingly broader with growing τ or (ℓ) reflecting the increase of turbulent energy toward the largest scales. Interestingly, the PDFs at all scales have the same leptokurtic shape resembling Levy laws (Momeni & Müller, 2008; Momeni & Mahdizadeh, 2008). It can be explained by the presence of the self-similar energy cascade over this range of scales. On the contrary, in some cases, one may observe a multifractal scaling, in the sense that a nonlinear dependence is observed on m where $\xi(m)=m\alpha(m)$ is a convex function of m and $\xi(m+1)>\xi(m)\ \forall m$. This deviation from strict self-similarity over all time scales τ or (ℓ), which termed multifractal scaling, is caused by the intermittent structure of turbulence. The structure functions of velocity and magnetic field fluctuations with order m = 1- 8 for the data interval studied here are shown in Figure 4. On this *log-log* plot the gradients as shown give estimates of the scaling exponents $\xi(m)$ in the inertial range. A linear least-squares fit is carried out to obtain the gradients. Because of increasing statistical errors at the higher orders, such a fitting becomes rather arbitrary. In Figure 5 we estimate the scaling behavior of $\xi(m)$ for both data sets. The inertial range shows multi-exponent scaling as evidenced by a nonlinear function $\xi(m)$. From the point of view of a stochastic process, a given scaling behavior is correspond to a particular process, and thus by extension the stochastic signature of a particular physical mechanism. The degree of intermittency is measured through the distance between the curve $\xi(m)$ and the linear scaling $m/3$. Looking at Figure 5, it can be seen that magnetic field is more intermittent than velocity field. Note that, in numerical simulation, intermittency for Elsässer variables is also a little stronger than for the magnetic field of solar wind. In the incompressible case (the assumption used in simulation setup), we cannot distinguish between scaling laws for velocity and magnetic variables. But the solar wind plasma is compressible and inhomogeneous mixture of mutually interacting regions with different physical characteristics and dynamically important kinetic processes (Goldstein, Roberts, & Matthaeus, 1995; Tu & Marsch, 1995). Thus it is not clear if the above-mentioned solar-wind observations are caused by turbulence or some other physical phenomenon.

We finally look scale-by-scale at the individual PDFs of magnetic, velocity and Elsässer field fluctuations over all scales. The PDFs display a different and well-known behavior as can be seen from Figure 6 and Figure 7. The distributions lose their small-scale leptokurtic character as the scales increases. Due to the lacking correlation of distant turbulent fluctuations the associated distributions become approximately Gaussian at large scales. This is a signature of the intermittent small-scale structure of turbulence as we would anticipate from their distinction $\xi(m)$ shown in Figure 5.

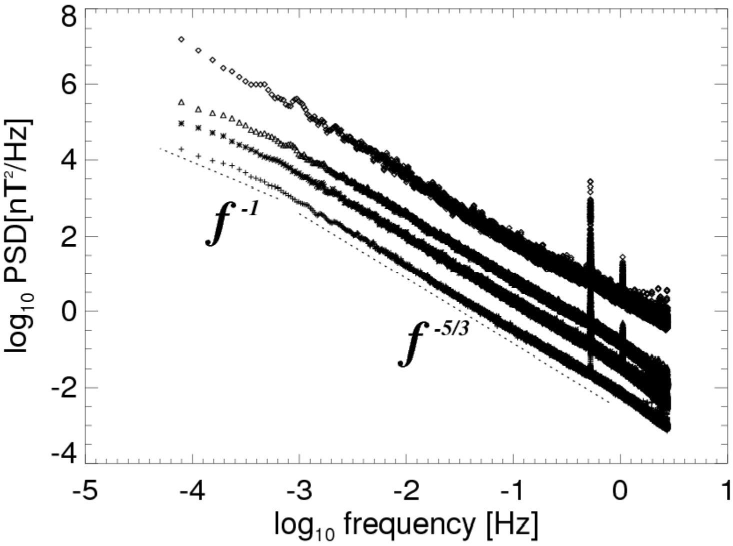


Figure 1. PSD plots of the components of the magnetic field with 1 Hz resolution : (◇) magnitude $|B|$, (+) B_x, (*) B_y and (·) B_z

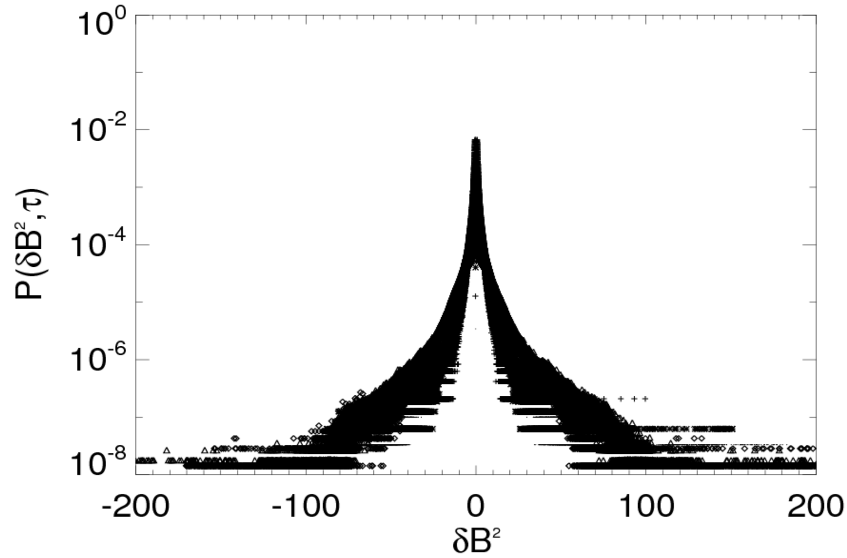


Figure 2. The PDFs of magnetic energy fluctuations of solar wind on five different time scale $\tau = 2 \times 2^k$ with k = 0 (+), k = 6 (*), k = 8 (.), k = 11 (×), and k = 15 (◇)

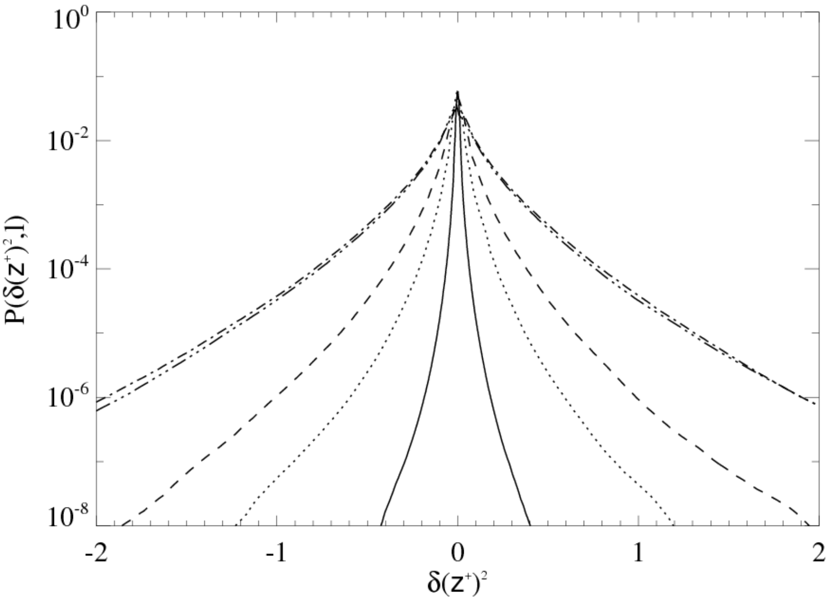
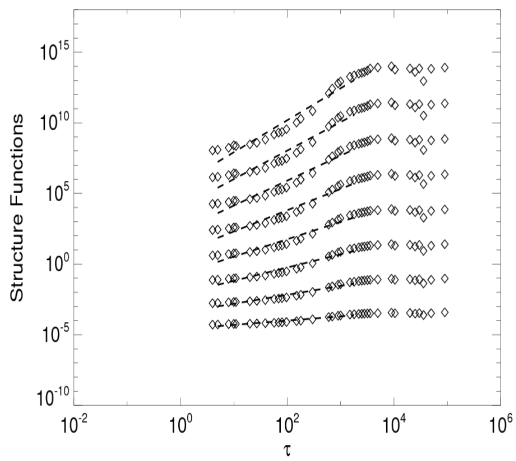


Figure 3. The PDFs of total energy fluctuations calculated by DNS on five different scales $\ell = \pi / k$, with k = 1 (three-dot-dashed), k = 4 (dot-dashed), k = 46 (dashed), k = 130 (dotted), k = 511 (solid) on five different space scale



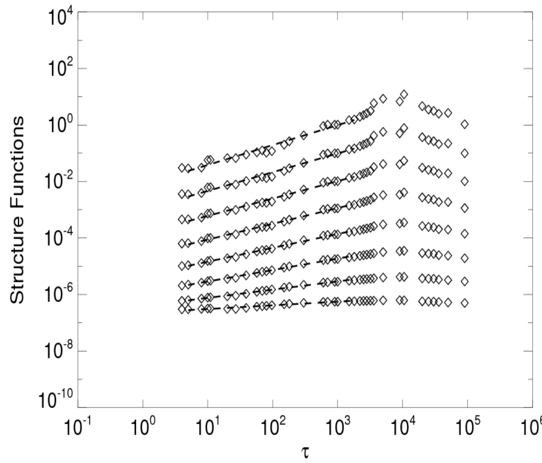


Figure 4. The SFs of the magnetic and velocity field fluctuations of solar wind for m= 1-8 in the upper and lower plots, respectively

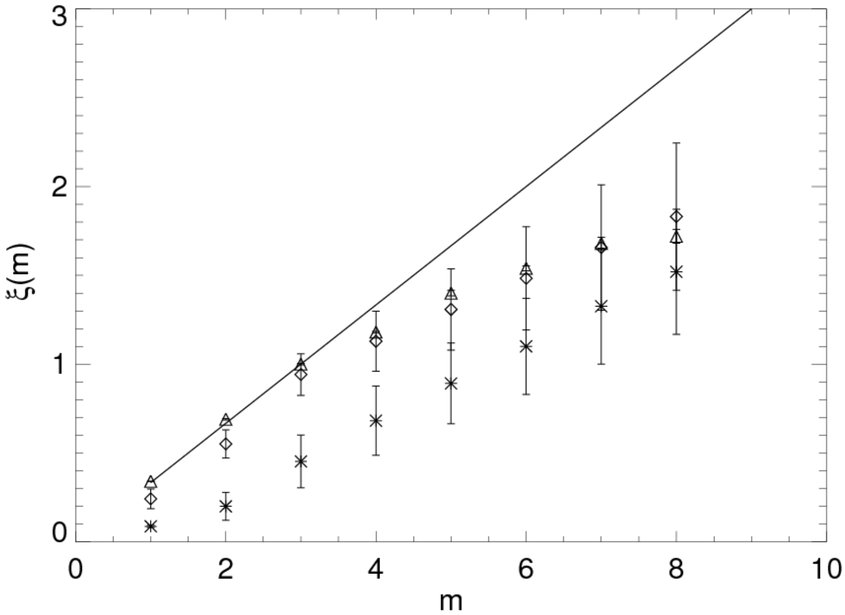


Figure 5. Scaling exponents ξ with order m for magnetic $\triangleright$, velocity* and Elsässer field. A concave shape on this plot indicates multifractal nature of the inertial range. The solid line indicates linear Kolmogorov's theory turbulence m=3

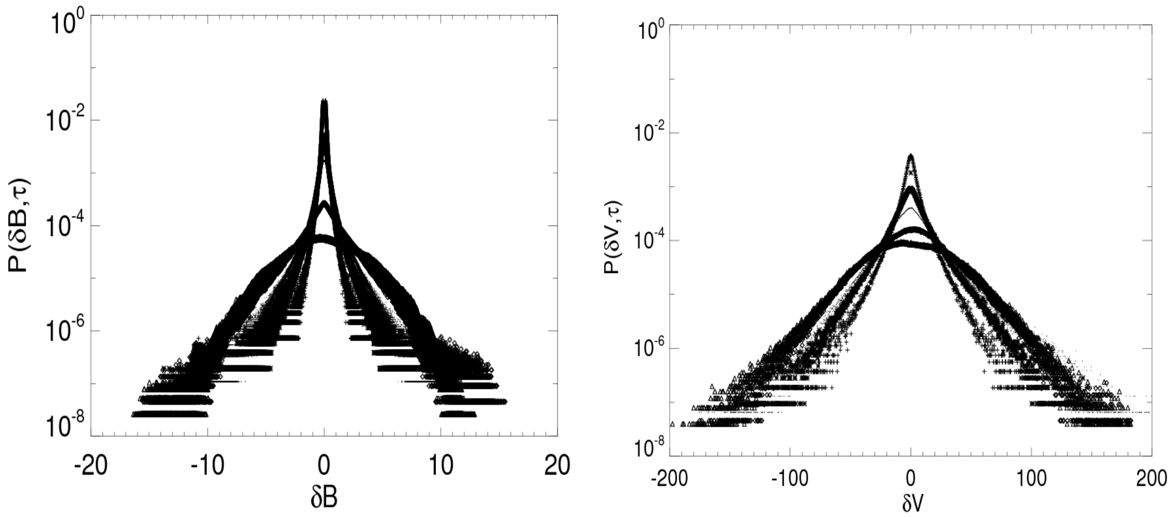


Figure 6. The PDFs of the magnetic and velocity field fluctuations of solar wind, upper and lower respectively, for the same five different time scales as in Figure 2

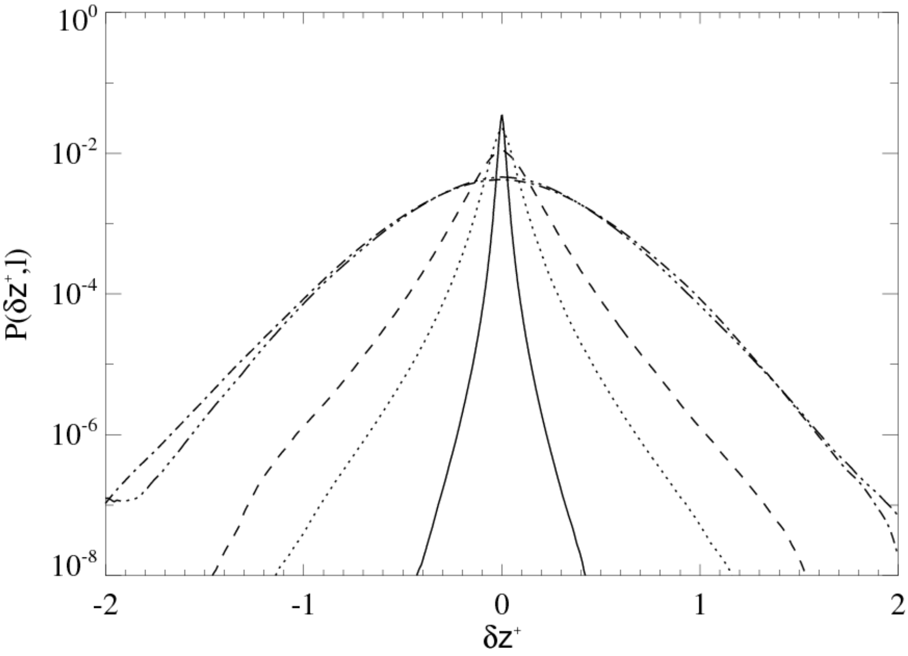


Figure 7. The PDFs of the Elsässer field fluctuations calculated by DNS for the same five different space scales as in Figure 3

In summary it has been shown by satellite data and direct numerical simulations of incompressible turbulent magnetohydrodynamic flows that the monoscaling of energy fluctuation PDFs is the consequence of lacking Galilei invariance of energy increments in combination with self-similar scaling of the underlying turbulent fields. On the contrary, we found that magnetic, velocity and Elsässer field fluctuations show a significant departure from the Kolmogorov linear scaling that is, real scaling exponents are anomalous and show intermittent multifractal scaling in the higher-order statistics.

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