A Paradoxical Deviation from Lavoisier's Postulate Using the Classic Newtonian Gravity

Valeriu Dragan (Corresponding author)
Faculty of Aerospace Engineering, "Politehnica"University of Bucharest
Str. Gheorghe Polizu, nr.1, Sector 1 Bucharest, 011061, Romania
E-mail: drvaleriu@gmail.com

Received: July 12, 2011 Accepted: July 26, 2011 Published: November 1, 2011
doi:10.5539/apr.v3n2p203 URL: http://dx.doi.org/10.5539/apr.v3n2p203

Abstract

The conservation of mass and energy concept is unanimously considered to be a law of nature since its postulation in the 18th century by French chemist Antoine Lavoisier. However, the postulate did not take into consideration the 20th century Theory of Relativity. Based on the observation that the gravitational acceleration is independent of the motion mass of the falling object, we derive a paradoxical thought experiment that deviates from Lavoisier's postulate. We use the mathematical models of the Special Theory of Relativity and Newtonian gravitation to demonstrate that the model permits generation of mass and energy in a manner that is fundamentally different than the one used by Barber's 1982 Self Creation Cosmology. A sample calculation is also provided before the concluding remarks.

Keywords: Newtonian universal gravitation, Special Theory of Relativity, Mass conservation principle

1. Introduction

Mass and energy conservation is universally thought to be valid; however this paper will try to generate a paradox based on the equations of the Special Theory of Relativity and the classical gravitational model. There is a long standing interest in finding a set of equations for describing Gravity, various models have been proposed and tested. We investigate the simpler, textbook variation of the gravitational force, which in conjuncture with the Theory of Relativity will provide a deviation from Lavoisier's conservation principle. Newton's law of universal gravitational attraction is proportional to the mass of the bodies in question, following the mathematical expression of Eq.(1).

$$F = G\frac{m \cdot m_T}{h^2} \tag{1}$$

This leads to the fact that the mass of a falling object does not influence the gravitational acceleration, g , that it is subjected to:

$$g = G\frac{m_T}{h^2} \tag{2}$$

In 1961 the Jordan-Brans-Dicke theory of gravity was formulated, Ref(Brans, C. H.,et al. 1961) and introduced the idea that the universal constant G should be replaced by a variable scalar field φ. The model was further modified by Barber (1982) Ref. (Barber, G.A., 1982) into what is called the Self Creation Cosmology (SCC), a model that allows mass and energy to be created. In Ref. (Brans, C.H., et al. 1987), Barber shows that the SCC is internally consistent. Another important new framework for gravity is the MOND model, Ref.(Milgrom, M., 1983) which predicts the observations of uniform velocity of rotation of galaxies.

Milgrom introduces the a_0 universal constant which is approximated at 10-10 m/s2 and formulates a new equation for the gravitational force:

$$F = m \cdot \mu\left(\frac{a_0}{a}\right) \cdot a \tag{3}$$

Where:

$$\mu\left(\frac{a_0}{a}\right) = 1 \Leftrightarrow a >> 1 \tag{4}$$

$$\mu\left(\frac{a_0}{a}\right) = \frac{a_0}{a} \Leftrightarrow a << 1 \tag{5}$$

A more modern model is Beckenstein's TeVeS in Ref. (Jacob D. Bekenstein, 2004) which, in addition to solving the galaxy rotation problem is meant to predict other cosmological phenomena as well. Although the above cosmological models provide theoretical support for various modern observations that could not be otherwise explained they are so far incomplete and require further study.

In Ref. (V. Alan Kostelecký, et al. 2009), Kostelecký and Tasson present a class of violations of relativity in Matter- Gravity Couplings. Further comments regarding the Theory of Relativity and it's latest improvements are discussed in Ref. (Orfeu Bertolami, et al. 2008). Reference (Pedro G. Ferreira, et al. 2008) provides insight on the necessity of new gravitational models in continuation of Newton's and Einstein's theories. It is also important to point out that although Barber's SCC does allow energy and mass to be generated, the mechanisms by which that are achieved are fundamentally different than those described in this paper. As stated before, the current paper

2. General Considerations

A thought experiment is presented hereby describing a paradoxical outcome derived from applying the relativistic equations to a body falling in a gravitational field.

One aspect of the Theory of Relativity is that, for low velocities, the equations tend to approximate the classical mechanical laws of movement. However, at high enough velocities, relativistic effects can be detected. For instance an constant force acting upon a body will not provide a constant acceleration, this is because, as velocity increases, the motion mass of the object increases, diminishing the acceleration which tends to zero as the relative velocity of the body approaches the speed of light. Hence, in relativistic terms, the acceleration of a body depends on its motion mass. However, since the mathematical expression of the gravitational acceleration does not include the motion mass – or rest mass for that matter- of the accelerated object, it can provide a paradoxical reasoning.

2.1 The thought experiment

Imagine a body of rest mass m_0 situated in a gravitational field at the height *h*. The Newtonian potential energy of the body will be:

$$E_P = m_0 \overline{g} h \tag{6}$$

The velocity before impact will be:

$$v = \sqrt{2\overline{g}h} \tag{7}$$

In Eqs. (6) and (7) we preferred the mean gravitational acceleration for simplicity. Because in the case of gravity, the acceleration of the body is independent of its mass *m,* the relativistic equations will yield the same result.

Since the kinetic energy at the moment of impact depends both on the velocity of the object and its motion mass, one could conclude that the kinetic energy predicted by the relativistic calculations is higher than the potential energy of the object.

$$W_k = \frac{m_0 v^2}{2\sqrt{1-\frac{v^2}{c^2}}} \tag{8}$$

Equation (9) shows the mathematical difference between the relativistic kinetic energy of a body falling in a gravitational field before impact and its potential energy at the initial height. In other words, Eq. (9) shows the kinetic energy of the mass gained by relativistic effects while being accelerated in a gravitational field.

$$W_k - W_p = m_0 \bar{g} h \left(\frac{1}{\sqrt{1 - \frac{2\bar{g}h}{c^2}}} - 1 \right) \tag{9}$$

However the kinetic energy is not the only imbalance predicted. Because of the equivalence of mass and energy Eq.(10), the surplus motion mass, i.e. motion mass minus rest mass, will have a total energy calculated by Eq. (11).

$$W_s = c^2 \Delta m = m_0 c^2 \left(\frac{1}{\sqrt{1 - \frac{2\bar{g}h}{c^2}}} - 1 \right) \tag{10}$$

$$\Delta W_g = m_0 \left(\frac{1}{\sqrt{1 - \frac{2\bar{g}h}{c^2}}} - 1 \right) \left(\bar{g} h + c^2 \right) \tag{11}$$

3. A calculation example

Using the equations described in Section 2, we provide a calculation example for better understanding of the principles at work. It is implicit that the bodies considered for this study will have to be disproportionate in size so that the small falling object will not influence in any way the larger celestial body that it is falling towards.

3.1 Problem statement

For the following conditions, calculate

a. The excess mass before impact;

b. The difference between the kinetic energy before impact and the potential energy,

c. The energy equivalent of the excess mass;

d. The total energy difference generated by the gravitational fall

3.2 Conditions

$\bar{g}$ =100 [m/s^2]

h=2000 [m]

m_0 =1000 [kg]

3.3 Solution

a)

$$\Delta m = m_0 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

$$\Delta m = 0.0000000022253001[\text{kg}]$$

b)

$$W_k - W_p = m_0 \bar{g} h \left(\frac{1}{\sqrt{1 - \frac{2 \bar{g} h}{c^2}}} - 1 \right)$$

$$W_k - W_p = 0.00044506002 \, [\mathrm{J}]$$

c)

$$W_s = c^2 \Delta m = m_0 c^2 \left(\frac{1}{\sqrt{1 - \frac{2 \bar{g} h}{c^2}}} - 1 \right)$$

$$W_s = 199999998.9118558167973764 \, [\mathrm{J}]$$

d)

$$\Delta W_g = m_0 \left(\frac{1}{\sqrt{1 - \frac{2 \bar{g} h}{c^2}}} - 1 \right) \left(\bar{g} h + c^2 \right)$$

$$\Delta W_g = 199999998.9123008768173764 \, [\mathrm{J}]$$

4. Further work for experimental validation/invalidation of the paradox

Although the paradoxical results obtained by applying the relativistic equations to a free falling body may appear difficult to both validate and accept, the only true problems associated with an experiment are finding a suitable celestial body and making a precise measuring of the velocity of the falling object.

A possible experimental verification could be made in the form of accurately measuring the velocity of a free falling body in order to determine any difference between the real velocity and the one predicted by both Newtonian and Relativistic theories.

4.1 The experimental layout

The experiment would have two vehicles, one orbiter and a free falling impactor. The orbiter will hold a steady geostationary orbit around the celestial body and launch the impactor towards it. Since the only force compensating for the gravitational pull is the centrifugal force, all that needs to be done is slow the tangential velocity of the impactor to zero, hence leaving the body to fall freely towards the surface of the celestial body.

Although the velocities of the impactor will be quite small in comparison to the velocity of light, we must account for each relativistic effect it will encounter and factor it in while measuring the falling velocity. One way to achieve this is to measure the redshift of a signal emitted by the impactor and calculate the impactor's falling velocity. The redshift will be affected by three effects:

1. Gravitational redshift due to the fact that it will be closer to the center of the celestial body towards which it is falling

$$z = \sqrt{1 - \frac{2GM_{star}}{r \cdot c^2}} - 1 \tag{12}$$

Where:

G is the gravitational constant

M_{star} is the mass of the body

c is the velocity of light

r is the distance from the body

2. Relativistic Doppler effect which will factor in the observer's orbital velocity. For an oblique beam of light the Doppler redshift, compensated for the relativistic aberration is:

$$f_{obs} = \sqrt{1 - \frac{v^2}{c^2}} \left(1 - \frac{v \cdot \cos(\theta_{source})}{c} \right) \cdot f_{source} \tag{13}$$

3. Hubble's cosmological redshift, which due to the size of the experiment is negligibly small.

The total redshift measured will be the summation of all of the above. By knowing the altitude of the impactor and the orbiter velocity we should be able to deduce the falling velocity.

4.2 The precision required

In the past section we stated that in order to test the hypothesis presented in this paper we must only measure the velocity of a free falling object accurately enough. The question that remains is just how accurately does the falling velocity need to be measured.

Logically, the paradoxical situation presented will be solved if the terminal velocity of the impactor were smaller than derived in this paper. In order to negate any surplus in kinetic energy, the velocity needs to be contracted by the square root of the Lorentz factor:

$$v_i = v_R \sqrt[4]{1 - \frac{v_R^2}{c^2}} \tag{14}$$

Where:

v_i is the velocity needed in order to invalidate the paradox

v_R is the velocity predicted by the theory Which if introduced into Eq.(8) would yield:

$$W_k = \frac{1}{2} \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \left(v_R \sqrt[4]{1 - \frac{v_R^2}{c^2}} \right)^2 \tag{15}$$

Hence providing exactly the same kinetic energy as the potential energy, thus disproving the paradox.

Because of the expression in Eq. (14), the difference between the predicted velocity and the velocity needed to invalidate the hypothesis is very small at low terminal velocities. In Figs. 1 and 2 portray those differences by comparison and by subtraction respectively.

In the case of a calculated terminal velocity of 2000 m/sec, the accuracy of the measurements will have to be higher than 10^-9 percent, which is unreasonably high. The margin of error increases however as the velocity increases, as shown in Fig.3, at approximately 20 000 m/sec the margin of error is of about 0.1% which is acceptable.

In order to obtain such high velocities simply by freefall, the celestial body onto which the drop is made will have to be massive in order to have a gravitational field intense enough to achieve high accelerations over large distances. The only possible candidate within our solar system will have to be the Sun itself. This will complicate matters even more due to solar winds and radiation which may adversely impact the accuracy of the measuring instruments. Further problems might be posed by the lack of homogeneity in the gravitational field of the celestial body and also by its rotation.

The method described above is not the only one imaginable for the test of the claims of this paper, others may be imagined that could be more precise or easier to put into practice, the purpose of this example is to show the level of precision needed for such measurements.

5. Conclusions

A though experiment is presented which, based on the equations of the Theory of Relativity provides a paradoxical counter example to Lavoisier's postulate that mass and energy must be conservative. The key observation is that, in the theory, gravitational acceleration does not change its mathematical expression in the relativistic case, retaining its Newtonian form. Based on this observation, a logical reasoning is made that lead to the following conclusions:

The kinetic energy of a free falling impactor does not equal its potential energy, being always greater as a result of the motion mass being greater that its rest mass. Equations have been derived in order to quantify the surplus kinetic energy of the impactor.

Also it is possible to calculate the total energy imbalance by using Eq.(8). An example of such a calculation is presented in subsection 2.3. showing that if correct, the energetic imbalance is within reasonable measurable limits.

Section 3 also tries to help shape a possible experimental research on the matter for validation or invalidation of the theoretical reasoning presented in the paper. The argument is made that for invalidation of the paradox, the free fall velocity must be smaller than the Newtonian and Relativistic prediction and have a more complex expression presented in Eq. (14). Because of that, at low falling velocities, the difference between the Newtonian predicted velocity and the one needed to invalidate the arguments is almost impossible to measure. In order to be able to prove/disprove the paradox, the free fall velocity must be quite high – in the vicinity of 20000 m/sec. This is achievable only if the free fall is towards a celestial body near the size of the Sun.

It is an opinion that such paradoxes may prove relevant in formulating modifications on the theories of gravity which are, for now, incomplete or provide equivalence to theories that permit mass and energy to be created.

References

Barber, G.A., Gen. (1982). On Two Self Creation Cosmologies. *Relativ Gravit.,* 14, 117. http://dx.doi.org/10.1007/BF00756918

Blatt, S. et al. (2008). New limits on coupling of fundamental constants to gravity using 87Sr optical lattice clocks, *Phys. Rev. Lett.,* 100, 140801. http://dx.doi.org/10.1103/PhysRevLett.100.140801

Brans, C. H.; Dicke, R. H. (1961). Mach's Principle and a Relativistic Theory of Gravitation. *Physical Review,* 124 (3): 925–935. Bibcode 1961PhRv. http://dx.doi.org/10.1103/PhysRev.124.925.

Brans, C.H., Gen. (1987). Consistency of field equations in self-creation cosmologies. *Relativ Gravit.*19, 949. http://dx.doi.org/10.1007/BF00759299

Dimopoulos, S., Graham, P. W., Hogan, J. M. & Kasevich, M. A. (2007). Testing general relativity with atom interferometry, *Phys. Rev. Lett.,* 98, 111102. http://dx.doi.org/10.1103/PhysRevLett.98.111102

Jacob D. Bekenstein. (2004) Relativistic gravitation theory for the modified Newtonian dynamics paradigm, *Phys. Rev. D,* 70 (8): 083509. http://dx.doi.org/10.1103/PhysRevD.70.083509

Milgrom, M. (1983). A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. *Astrophysical Journal* 270: 365–370. http://dx.doi.org/10.1086/161130

Orfeu Bertolami, Jorge Páramos and Slava G. Turyshev. (2008). General Theory of Relativity: Will It Survive the Next Decade?, Lasers, Clocks and Drag-Free Control, Explration of Relativistic Gravity in Space, ISBN: 978-3-540-34376-9. http://dx.doi.org/10.1007/978-3-540-34377-6_2

Pedro G. Ferreira,and Glenn D. Starkman. (2009). Einstein's Theory of Gravity and the Problem of Missing Mass, *Science,* ISSN: 0036-8075 (print), 1095-9203 (online) no6: Vol. 326 no. 5954 pp. 812-815 http://dx.doi.org/10.1126/science.1172245.

Peters, A., Chung, K.-Y. & Chu, S. (2001). High-precision gravity measurements using atom interferometry, *Metrologia,* 38, 25–61. http://dx.doi.org/10.1088/0026-1394/38/1/4

V. Alan Kostelecký and Jay D. Tasson. (2009). Prospects for Large Relativity Violations in Matter-Gravity Couplings, *Phys. Rev. Lett.* 102, 010402. http://dx.doi.org/10.1103/PhysRevLett.102.010402

Vitushkin, L. et al. (2002). Results of the sixth international comparison of absolute gravimeters, *Metrologia,* 39, 407–424. http://dx.doi.org/10.1088/0026-1394/39/5/2

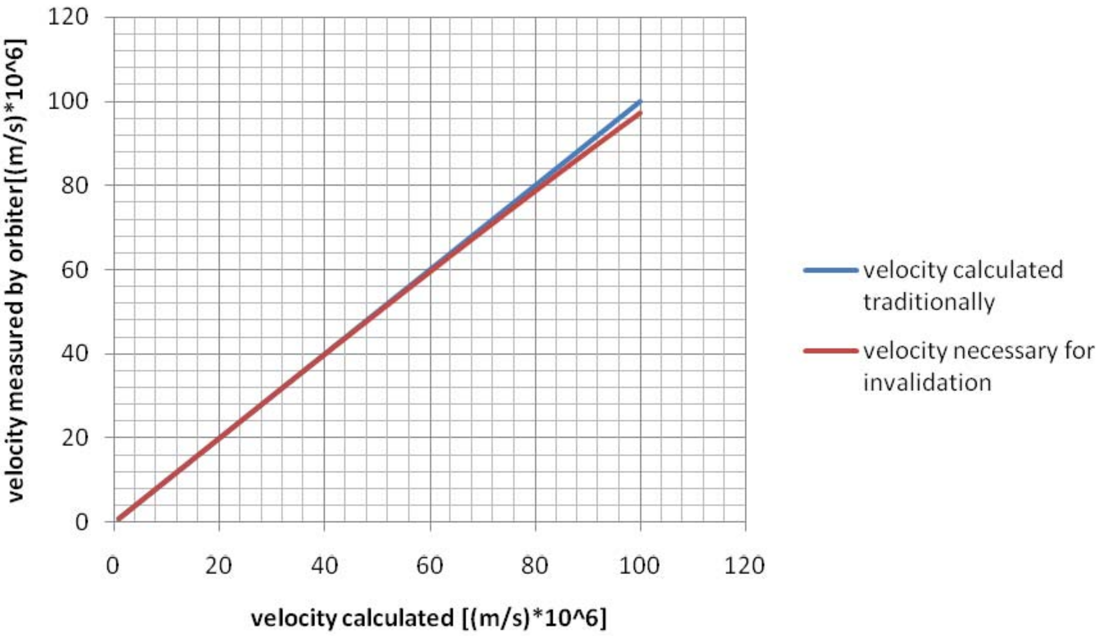

Figure 1.The calculated and invalidation velocities, v_i and v_R versus v_R

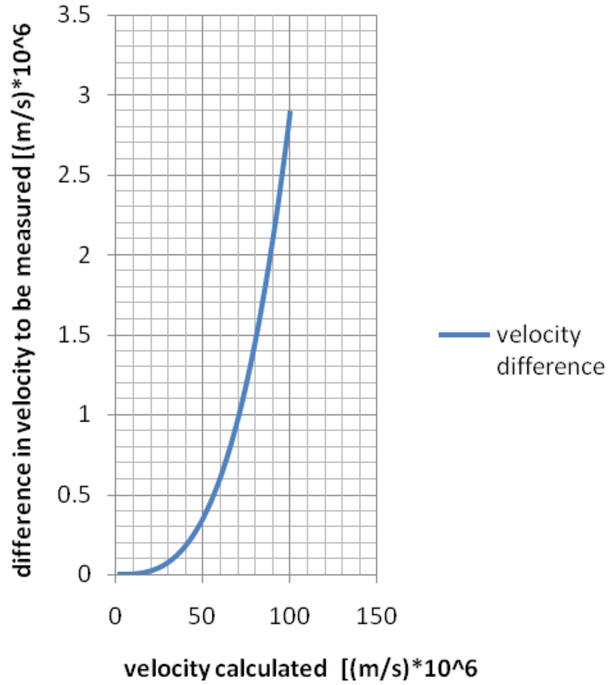

Figure 2. The velocity difference $v_i - v_R$ versus the theoretical velocity, v_R

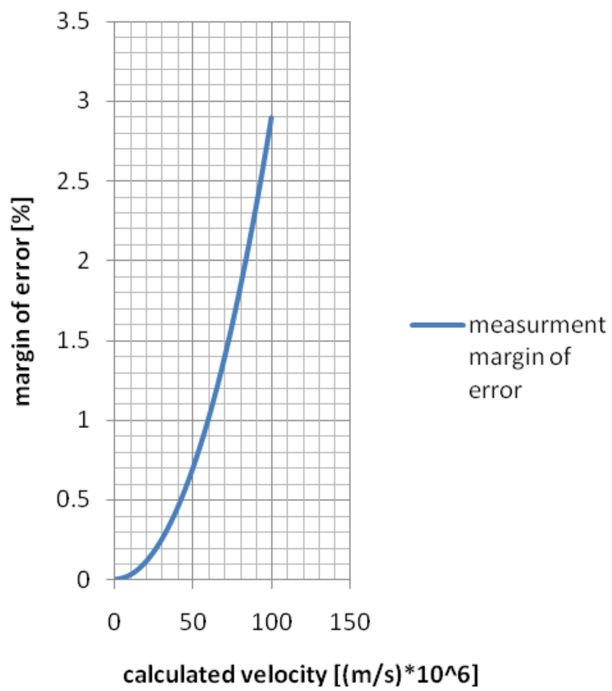

Figure 3. The margin of error versus the theoretical free fall velocity, v_R