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Ehrenfest's Paradox and General Relativity

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Abstract

Ehrenfest's paradox is examined within the framework of combined relativity; it means there would be a move from a 2D problem to a 3D problem. By combining special with general relativity, a mathematical solution involving 3D seems to exist. The paradox would disappear: the radius causing the paradox would curve in the third dimension to correspond to the contracted 2D circumference. But the mathematical solution seems incomplete as it includes only one contraction; until now, combined relativity involves two contractions. If the special relativistic part of combined relativity is abandoned, the mathematical solution would be a possibility and general relativity alone would explain the contraction expected with a rotating disk.

Keywords: Ehrenfest's paradox, general relativity, equivalence principle, simultaneity, spacetime

1. Introduction

1.1 Ehrenfest's Paradox

Ehrenfest's paradox was published in (1909) and involves a rotating disk. Normally, the circumference should be $2\pi r$. However, with a high speed, special relativity must be taken into account. The radius remains the same length for the rotating disk, whether at rest or while rotating; in contrast, the circumference contracts when it rotates and would no longer be $2\pi r$.

There is an excellent review by Øyvind Grøn (2004), in which we discover that the paradox has been interpreted in various ways, even being regarded as an illusion. Professor Grøn concluded that because of the problem of clock synchronisation, a solution to Ehrenfest's paradox was unlikely to be found (2007).

The paradox was known to Einstein, who stated: "The treatment of the uniformly rotating rigid body seems to me to be of great importance on account of an extension of the relativity principle to uniformly rotating systems…" (1909).

In his book *Relativity* (1916), Einstein wrote: "in a state of *uniform rectilinear and non-rotary motion*… In this sense, we speak of *special* principle of relativity..." (his emphasis, not mine). Consequently, we cannot use special relativity alone for a "rotary" system as it is currently the case for all attempts to solve the paradox. Einstein presented the reasoning for the solution to the paradox in Section 10 of his book, which can be summarised with the following quote: "Because of the Lorentz contraction in a reference frame that rotates relative to an inertial frame, the laws that govern rigid bodies do not correspond to the rules of Euclidean geometry". It gave Einstein the idea that space is not flat, which is a cornerstone of general relativity.

By using non-Euclidean geometry, Rizzi and Ruggiero (2002) claimed to have resolved the paradox. However, most physicists accept Grøn's kinematic considerations (1975) as the correct approach to Ehrenfest's paradox.

Those kinematic considerations will be completely ignored because the paradox is based on special relativity for which the principle of relativity should be correct: observing a rotating disk should lead to the same result as a rotating observer observing a fixed disk. The rotating camera, section 7 in (Danis, 2024), shows that no contraction is observed. So special relativity may not be correct for a rotating disk, rendering previous considerations incorrect.

1.2 Combined Relativity

Combined relativity is a new concept that has been presented in section 5 of (Danis, 2024). Present readers would benefit from reading the previous paper. Following a problem with the distance between two events, section 5 is exploring the length contraction of the clock with the photon travelling in a direction perpendicular to the motion. This solves the problem of distance and to keep the two clocks synchronised (the photon of the second clock travelling in the direction of the motion), two contractions were needed, one in all directions and the second solely in the direction of motion. This is what I called combined relativity.

One consequence of combined relativity is that clocks of two different frames of reference (one at rest, one moving) are synchronised. The twin paradox may have disappeared. It raises an issue with the definition of the constancy of the speed of light, and the new definition found in section 11 of (Danis, 2024) will be used later.

With combined relativity there is a speed-force which is the equivalent to gravity: with the equivalence principle an acceleration due to a mass creates gravity and an acceleration due to a change of speed creates that speed-force. The linear speed of each point on the disk changes with radius. This, in turn, generates a speed-force. Hence, a speed gradient leads to the deformation of space. The 2D disk could become deformed and the paradox would become a 3D problem. It is this approach that we will explore in this paper.

2. The Maths "Solution" to Ehrenfest's Paradox

We saw that *r* in the paradox refers to flat space. Here, it is referred to as Euclidean_*r* with γ being the Lorentz factor that contracts the length in the direction of the motion. The circumference *C* can be determined from the following expression:

$$C = 2\pi\text{Euclidean\_}r \times 1/\gamma, \tag{1}$$

which highlights the problem: *C* is not $2\pi r$ since the circumference has been contracted. Namely, it is now smaller than $2\pi r$ because Euclidean_r/γ is always smaller than *r*.

$$\gamma = 1/\sqrt{(1\text{-}v^2/c^2)} \tag{2}$$

Einstein said that space is not Euclidean for rotating systems. Hence, *r* should be NonEuclidean_*r*. So, the expected result becomes:

$$C = 2\pi\text{NonEuclidean\_}r. \tag{3}$$

With a rotating disk, the speed is known as a function of the radius. Hence, the speed-force can be calculated at each point of the disk. The centre is characterised by zero potential because there is no speed and, thus, no speed-force, which results in a potential value of zero (no deformation). By calculating the work, the deformation of space, the potential and therefore NonEuclidean_*r* can be deduced. How to calculate the work is Einstein's idea; the assessment of the potential (using work) is similar to Appendix III of *Relativity*. However, instead of considering the centrifugal force as Einstein did, the speed-force is implemented.

So we can calculate the potential. But now we need to discover how the potential deforms space to calculate NonEuclidean_*r*. One expression for time dilation using general relativity is as follows:

$$\Delta T/\Delta t = f(V), \tag{4}$$

where ΔT corresponds to one second at the centre of the disk (no time dilation) and Δt relates to one second at the edge of the disk. Moreover, *V* is the difference in potential, which we can calculate with the speed-force (usually, this difference in potential is due to gravity). The quantity $f(V)$ is used here instead of the real expression for the reason given below.

Then, there is time dilation and length contraction from special relativity, which are given by:

$$\Delta t = \gamma\Delta T, \tag{5}$$

$$\Delta x = \Delta X/\gamma, \tag{6}$$

respectively. The meter Δx at the edge is smaller than the meter ΔX at the centre because of the speed. From equations (4), (5) and (6), we find the following:

$$\Delta x/\Delta X = f(V). \tag{7}$$

This is the deformation of space due to the potential of the speed-force. Here, ΔX corresponds to the Euclidean_*r* and Δx relates to the NonEuclidean_*r*. This means that:

$$\text{NonEuclidean\_}r = \text{Euclidean\_}r \times f(V). \tag{8}$$

As we can calculate the potential *V* of the speed-force, we can calculate the NonEuclidean_*r*.

Furthermore, if we combine equations (6) and (7) (or equations (4) and (5)), we find that:

$$1/\gamma = f(V), \tag{9}$$

Therefore, with equation (8), it is determined that:

$$\text{NonEuclidean\_}r = \text{Euclidean\_}r \times 1/\gamma. \quad (10)$$

The right-hand term of equation (10) is in equation (1). If we replace it with the left-hand term, we find the expected result:

$$C = 2\pi\text{NonEuclidean\_}r, \quad (3)$$

QED. But what does that mean?

The first equation where $f(V)$ appears is in equation (4), which should be written as:

$$\Delta T/\Delta t = f(V) = 1 - V/c^2. \quad (11)$$

I have found this equation in Appendix I of "Now. The Physics of Time", a book by Richard Muller (2016). But V/c^2 is not dimensionless while the other side of the equation is dimensionless, which means there seems to be a mistake here. So, I changed "$1 - V/c^2$" with $f(V)$ in the text and I shall leave others to decide whether Dr. Muller's equation is correct or not. By using $f(V)$, the solution does not depend on the correctness of equation (11).

We may realise that equation (7) (using $\Delta x/\Delta X$) is linked to equation (6), so x is in the direction of the circumference (tangential). Then we jump to equation (8) with radius as if equation (7) is radial (using $\Delta y/\Delta Y$).

What I have accomplished is simply applying the fact that the time dilation (equations (4) and (5)) is equal for x, y and z. Equation (6) lies in the direction of the circumference (tangentially), but the resulting "equation (7) transformed" (transformed via time dilation) lies in all directions (tangential and radial) and is:

$$\Delta x/\Delta X = \Delta y/\Delta Y = f(V). \quad (7)$$

The passage between equations (6), (7) and (8) is now justified but is still possibly incorrect because it involves traversing from an infinitesimal dimension to a full radius. To obtain the results and figures that are further down in this paper, I have reinstated the infinitesimal idea.

The uncertainty of this demonstration originates from the fact that general relativity (equation (7)) and special relativity (equation (6)) are mixed. Is the change from y to x via equation (7) correct? This is tricky because special relativity tells us that length contraction is only along the tangential dimension (along x) and the radial dimension (along y) is unaffected. So, with special relativity, $\Delta y/\Delta Y \neq \Delta x/\Delta X$.

There is a contradiction here, as if special and general relativities are not compatible. This solution has replaced special relativity by general relativity. The previous paper (Danis, 2024) presents the idea that the use of special relativity is possibly invalid for a macroscopic disk, so the change should be correct.

The work should stop here but it is not coherent with (Danis, 2024); there is another difficulty as special relativity should be replaced by combined relativity. Combined relativity is different to general relativity and contains two contractions: one contraction follows general relativity as above, the other one follows special relativity.

To go further I found it useful to give a concrete example.

3. Special Relativity

Let us imagine a disk with a radius of 1000 m rotating at 47713 cycles per second (cps). In this scenario, the circumference would reach the speed of light; hence, 47714 cps or 1001 m is not permissible as the speed would be higher than the speed of light.

We now consider 1000 circumferences, each of which is 1 m from each other, with the final one having a radius of 1000 m. We assume that the Lorentz contraction should be applied in one dimension along each circumference, as special relativity tells us. From these contracted circumferences, we can calculate the corresponding radii.

For a radius of 1000 m, the circumference becomes zero, contracted by the Lorentz factor, which is infinity. This means that "speed" (or the paradox) has returned the circumference towards the centre of the disk. The number of rotations per second is known and, if the circumference is contracted to a small radius, it is clearly not rotating at the speed of light. There is a contradiction.

The problem is due to the disk; as soon as the contraction is applied to the circumference, the circumference becomes smaller. As a result, the speed that has produced the Lorentz factor is reduced, rendering it incorrect.

The speed of rotation is known (in cycles per second). To fix the incorrect Lorentz factor, there is a midpoint (and only one for each non-contracted radius) where the value of the speed with a contracted radius corresponds to the speed for the Lorentz factor. There is a mathematical formula that produces such a contracted radius as a function of the non-contracted radius presented in appendix A. Figure 1 presents these radii, i.e., a graph that results from special relativity but, with the preliminary step based on maths, we can choose the speed corresponding to the midpoint. The strangeness of the outcomes described above then disappears.

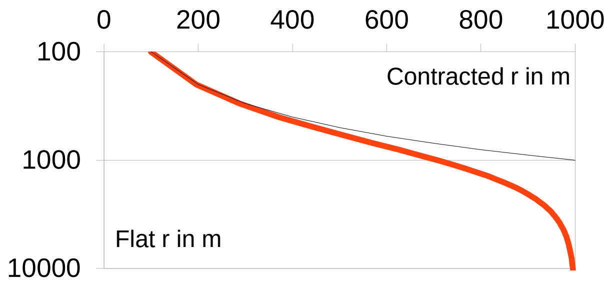

Figure 1. Radius contracted by special relativity

Note. This graph has been obtained using the midpoint (see text for description) for a disk rotating at 47713 cycles per second. The thin line depicts results with no contraction for comparison purposes.

Figure 1 shows that the contracted radius is 707.1 m for a non-contracted radius of 1000 m. Then, whatever the size of the disk, the maximum contracted radius cannot be above 1000 m, so there is a natural process limiting the circumference to the speed of light.

Within Figure 1, there is still something difficult to characterise. The matter representing the disk, which is included between 10,000 and 100,000 m, is reduced to a space of 5 m. If the disk is even larger, the part from 100,000 m to 1,000,000 m would be included in less than 5 cm. If the contraction of the disk is real, it is difficult to imagine how it could work out. As soon as there is a move from 2D to 3D there is also a move from contraction to deformation; at first, the deformation is much easier to visualise as the 900,000m would now be in a direction perpendicular to the disk.

4. Combined Relativity

4.1 First Deformation

Let us build an identical figure for general relativity, as expressed in the maths “solution”. Thus, we assume that the Lorentz contraction should be applied to all dimensions (i.e., x, y and z). The further from the centre, the faster the disk and the larger the Lorentz factor. Therefore, the shorter the meter rod. Here, we must repeat the trick of Figure 1, i.e., choosing the Lorentz factor of the midpoint where the speed of the contracted radius corresponds to the value of the Lorentz factor. But we also re-instate the infinitesimal idea: that midpoint is calculated for each circumference and the corresponding Lorentz factor is applied to each meter rod, not to the full radius as previously. Each meter rod is added to the previous sum of meter rods.

Figure 2 represents this apparent contraction of the radius of a disk rotating at 47713 cycles per second.

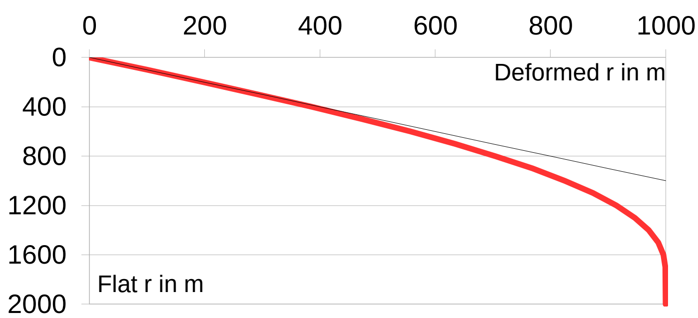

Figure 2. Change of the radius due to the deformation of space

Note. This is for a disk rotating at 47713 cycles per second. The thin line denotes no contraction.

The last section is vertical and, however long the disk is, it would never break the speed of light; it is another natural process to limit the speed under the speed of light. The radius for 1000 m is 839.8 m. Details of how the figure was obtained are presented in appendix A.

The "contraction" of the disk is different. In section 3, the space was flat, so the contraction was strange or an illusion. Here, space is non-flat. We cannot observe that space is non-flat, but we can imagine that the flat disk is deformed into another dimension (such as time), as we can imagine a disk deformed into a bowl in our usual third dimension.

Ehrenfest's paradox gave Einstein the idea that space is deformed, leading to general relativity. With general relativity, all the dimensions are contracted because space is deformed. Possibly, in this case, the word "contracted" is incorrect. In Figure 2, there is no contraction but a deformation of space and the flat disk is deformed following the shape of this deformed space. The length of the curve should be the radius of the flat disk.

Because the deformation uses time and we cannot observe this dimension, it means that the disk will probably look flat to us with a smaller radius than without the rotation.

Stopping here would correspond to the maths solution. But combined relativity requires a second contraction.

4.2 Second Contraction

Our starting point is Figure 2, in which the "contraction" due to general relativity has already been implemented. The rotational speed (in cycles per second) is still the same for all the circles, so we know the linear speed for each circle deformed by general relativity and can also calculate the contraction due to special relativity. The mathematical trick (i.e., the midpoint) used to obtain Figure 1 is again employed.

The original disk has been deformed and looks like a bowl (except to see the bowl shape, we probably need access to the fourth dimension) with a radius of 839.8 m. Due to the additional contraction from special relativity, the disk is further deformed, i.e., the radius is contracted to a value of 643.1 m. The result is a figure similar to Figure 2.

5. The Problem of Combined Relativity

I believe the maths solution corresponds to section 4.1. That second contraction of section 4.2 may imply a further deformation of space, further reducing the apparent radius. It seems that, with that second contraction, we are back to the original paradox as the circumference is contracted but the radius is theoretically untouched.

As combined relativity doesn't solve the paradox; does it imply that it is incorrect? That second contraction is a problem: it is not in the mathematical solution, its justification is the simultaneity of the two photon clocks as shown in section 5 of (Danis, 2024). That simultaneity was reached using the old definition of the constancy of the speed of light as described in section 11 of (Danis, 2024).

With the old definition, there was a problem as the tick and the tock of the two clocks couldn't be simultaneous for an observer in a different frame of reference. Indeed, the photon travelling in the direction of the motion has to catch up with a mirror (resulting in a late tick) then travels toward the incoming mirror (resulting in a rapid tock). The first clock is doing: "tick,tock,,,,,,,,,,tick,tock,,,,,,," the other clock having a regular tick and tock.

Trying to keep the simultaneity of the two clocks is a mistake; it is simply not possible with current proposals. The second contraction of combined relativity, which is here to keep the simultaneity of the two clocks, is a mistake. Therefore, having a special relativistic component in combined relativity is a mistake. With just one contraction the paradox would be solved.

If combined relativity doesn't have the second contraction, combined relativity is equal to general relativity; it would be the other side of the equivalence principle, one side corresponding to the acceleration due to a mass, the other side corresponding to the acceleration due to a change of speed. This idea would satisfy Occam's razor (*entia non sunt multiplicanta absque necessitate*). This paper would be the last about combined relativity as combined relativity is now general relativity. This idea would unify gravity and the fifth force (Danis,2025a) and therefore would also unify Λ (lambda, the cosmological constant) with *G* (the gravitational constant).

In section 11 of (Danis, 2024), there is a new definition of the constancy of the speed of light; it would be constant relative to the surroundings but a final conclusion wasn't reached because the second contraction muddied the water. Now, for Bob observing Alice in a spaceship, a photon in Alice's spaceship would travel at the speed of light compared to the spaceship (that is the new definition). All clocks would be contracted once but the "tick,tock,,,,,,,,,,tick,tock,,,,,,," has disappeared. As the speed of light is now defined within the spaceship, Bob would observe a contraction but the speed of the photon would be reduced by the same factor: Alice's clocks would tick at the same rhythm as Bob's. Please check section 11 of (Danis,2024) for more details.

Combined relativity explained the twin paradox because the clock with the “tick,tock,,,,,” was ignored, then the clock in a frame of reference at rest was synchronised with the other clock in a moving frame of reference. With general relativity we observed the same synchronisation. The synchronisation has been deduced for frames of references that are not accelerating. The time dilation of general relativity is due to the acceleration. There is no contradiction due to the synchronicity of clocks and time dimension could be re-instated as the fourth dimension for the deformation of space.

6. Discussions

If general relativity should be used for a macroscopic disk, it makes sense that Grøn (2007) reached an impasse with special relativity.

The most recent paper of interest is from Rumler et al. (2023). What is interesting is their Fig.14 which shows an “Isometric embedding of the proper 2-dimensional disc space into 3-dimensional Euclidean space…” The result is a bowl, similar to what is expected with our Fig.2. So, if I understood (and that is a big “if”) the idea of a move from 2D to 3D already exists. One main difference is that the bottom of the bowl from our Fig.2 is flat, or with an extremely small deformation compared to their bowl which seems deformed from the centre. The mechanism they have use is indeed different to what is used here so a difference should be expected. I have no doubt that their theoretical study is correct but does it correspond to reality?

Back to this study. The centrifugal force has been ignored. One reason to ignore the centrifugal force is because a natural rotating disk is a galaxy (not an artificial disk on earth). There is a study that would show that there is no centrifugal force in a galaxy as the inertial frame of reference would be rotating because of Mach’s principle (Danis, 2025b).

In the maths demonstration, to pass from x to y, the fact that the time flow would be identical with special and general relativity was assumed. In appendix B we will find that it is not the case. Is that a reason to reject the maths? The maths started from special relativity. If special relativity is wrong, the move from special relativity to general relativity is not necessary, so the equality of time flow wouldn’t be important because $\Delta x/\Delta X = \Delta y/\Delta Y$ with general relativity. That difference in time flow could be an indication that there are two times as suggested in (Danis 2024); one in the macro world and one for quantum mechanics.

If the time-flows of special and general relativity are not the same, it could mean that the general relativity contraction would not be by Lorentz factor. The correct mathematical demonstration should replace Lorentz factor by the correct factor; it should be straightforward. The main point is, that with general relativity, the radius is also contracted.

The title of section 2, “solution” is in quotation marks. One part of the solution would be to have shown that special relativity is not at work with a macro disk. The Ehrenfest paradox should be expressed with general relativity as in section 4.1. Is there still a paradox? Probably yes. The factor for the contraction of the radius changes along the radius. The factor for the contraction of the circumference is the last factor, not an average of all factors. So the radius doesn’t correspond to the circumference. As we understand all steps in the maths, we could use the last meter rod and calculate the radius with the rod of the circumference. Expressed with general relativity, the radius has a reason to change, which was missing with special relativity.

At this point, the scientific community should check both papers. The change from special to general relativity is justified only if the previous paper is correct, i.e. special relativity cannot be applied to a macroscopic disk. So the solution to the paradox should be here and it is relatively simple because it involves general relativity; previous scientists were blocked because of special relativity. It is a big claim, is all this correct/coherent/consistent? Is it rigorous enough? Is there any point that I have forgotten to consider in the demonstration or the discussions? Any comment would be most welcome.

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Appendix A: Results of Section 4

The first step is to calculate the middle point where the speed of the contracted radius (Column 3 of Table A1) corresponds to the speed used in Lorentz factor (column 2). The formula I used to calculate this speed v is:

$$v = \sqrt{(\omega^2 r^2/(1+\omega^2 r^2/c^2))} \quad (12)$$

Table A1. Results of combined relativity, section 4

	First deformation		Second contraction	
	Lorentz F	Apparent r	Lorentz F	Apparent r
0	1	0	1	0
100	1.00499	99.50373	1.00494	99.01477
200	1.02010	197.53338	1.01932	193.78886
300	1.04595	293.14041	1.04208	281.30331
400	1.08372	385.41490	1.07170	359.62932
500	1.13533	473.49480	1.10643	427.94726
600	1.20366	556.57490	1.14445	486.32449
700	1.29298	633.91561	1.18399	535.40449
800	1.40972	704.85174	1.22344	576.12263
900	1.56373	768.80138	1.26137	609.49901
1000	1.77072	825.27554	1.29656	636.51184
1100	2.05702	873.88949	1.32803	658.03357
1200	2.46986	914.37768	1.35502	674.80907
1300	3.10185	946.61650	1.37698	687.45989
1400	4.15841	970.66415	1.39362	696.50662
1500	6.18210	986.83989	1.40493	702.41103
Unit: m		m		m

Note. First column is the radius before rotation, second column is the Lorentz factor of the midpoint: where the speed of the circumference equals the speed used for that Lorentz factor. Third column is the resulting radius: only the last 100m is contracted by this Lorentz factor and is added to the previous radius. Third column corresponds to Fig.2. Forth column is the second Lorentz factor of the midpoint applied to the third column. Fifth column is the final apparent radius.

This formula (12) seems correct for the first line (r=100m) but from the second line, I need a small correction in order that the speed of the circumference equals the speed used in Lorentz factor. I cannot calculate a better formula so there is an iteration process to reach the correct Lorentz factor in the third column. The “little correction” is about 1% for r=200m and 26% for r=1000m.

The apparent r of the third column has been built by adding the contracted 100m to the previous apparent r. This is the main difference with the second contraction. That difference is most probably the reason for the little correction. But more than this, I cannot say.

In the main text, the result is different because I used a step of 10m for the main text instead of 100m as in table1. It means that the result in the main text is good but not accurate. The correct formula has to be found to reach the accurate result.

The second contraction, the radius of the previous circumference is not involved because the corresponding circumference is contracted by special relativity which doesn’t consider the previous circumference. So the formula for the speed of the middle point is correct: no little correction.

Appendix B. Time-Flow Dilation

The goal is to check that the time-flow dilation of special relativity equals the time-flow dilation of general relativity.

General relativity and special relativity allow us to calculate a time-flow difference against a reference time-flow. The time-flow difference can be noted as $\Delta\Delta t$, where Δt is the time-flow.

One obvious link between mass (general relativity) and speed (special relativity) is the centrifugal force. The value of the potential V associated with the centrifugal force is given by:

$$V = -\omega^2 r^2/2 = -v^2/2. \quad (13)$$

This formula is not mine; it is from Einstein's Relativity in Appendix III. Here, $\Delta\Delta t$ is due to the potential and, via general relativity, is written as:

$$\Delta\Delta t = 1 - \sqrt{(1 + 2\ V/c^2)} = 1 - \sqrt{(1 - v^2/c^2)}. \quad (14)$$

In turn, $\Delta\Delta t$ due to the speed is given by special relativity as:

$$\Delta\Delta t = [1/\ \sqrt{(1 - v^2/c^2)}] - 1. \quad (15)$$

For a speed below 1000 km/s, equations (14) and (15) are equivalent; but beyond that speed, there is a difference that I cannot resolve.