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Simplified Calculation of Magnetic Fields Induced by Special Relativity Effects Shows Magnetism Due Largely to Other Causes

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Abstract

According to theory advanced by Albert Einstein, the magnetic force observed from a stationary reference frame is that of the electrostatic force viewed from a moving reference frame in consideration of Special Relativity effects. This paper provides a simplified approach for calculating magnetic forces around electrical conductors resulting from Special Relativity effects. A finding of this paper is that Special Relativity effects contribute to the magnetic field around common electrical conductors, but the effect is negligible. Einstein's explanation of the magnetic field does not account for the full strength of the magnetic field observed around common electrical conductors. Further research is recommended for understanding the cause of magnetism in common electrical conductors, and development of new magnetic theory.

Keywords: magnetism, special relativity, Einstein

1. Introduction

The difference in velocity between the electrons and protons in an electrical conductor is said to produce a magnetic force due to Special Relativity effects. According to theory advanced by Albert Einstein, the magnetic force observed from a stationary reference frame is that of the electrostatic force viewed from a moving reference frame in consideration of Special Relativity effects (Einstein, A., 1905, June 30).

The traditional mathematical approach for calculation of the Special Relativity effects includes calculation of the attraction between two parallel wires with electrical current flowing equally through them in the same direction (Feynman, R., 2010; Purcell, E., & Morin, D., 2013; Griffiths, D., 2017; Schwartz, M., 1972). The calculation is performed from a moving reference frame for consideration of the Special Relativity effects.

This paper presents a more direct calculation of the electrostatic field arising from Special Relativity effects. With the approach presented in this paper, only a single wire is needed for the calculation. The present approach is less laborious and provides a clearer understanding of the underlying physics.

A finding of this paper is that a variance in the electrostatic field of electrical conductors is caused by Special Relativity effects, but that the effect is negligible due to the relatively slow electron drift velocity relative to the speed of light. Einstein's explanation of the magnetic field does not account for the full strength of the magnetic field observed around common electrical conductors. Further research is recommended for understanding the cause of magnetism in common electrical conductors, and development of new magnetic theory.

This paper begins with review of the magnetic field equation for electrical conductors, and review of electrostatic charges in copper wire conductors.

2. Magnetic Field Around an Electrical Conductor

According to Ampere's Law, the magnetic field density (B) around an electrical conductor is equal to,

$$B = \mu_o I/2\pi R \tag{1}$$

Where μ_o is the permeability of free space, I is the electrical current, and R is the radial distance from the electrical conductor.

3. Copper Wire Electrical Conductors

Copper atoms have an equal number of electrons and protons. When viewed from a stationary reference frame, the negative static electric charge from the electrons is exactly canceled out by the positive static electric charge of the protons, leaving the copper atoms with a net static electric charge of zero.

The protons are secured in place in the atom's nucleus, and not free to leave the copper atom. By contrast, one of the outer valance electrons is free to leave the atom, provided that a second electron is added to the atom to take its place. This free motion of the outer electrons enables the flow of electron current through copper wire. So long as the departing electron is replaced by an arriving electron, the static electric charge of the copper remains equal to zero. Electrical current flowing through copper wire does not produce a static electric charge when viewed from a stationary reference frame.

4. Static Electric Linear Charge Density

The negative flux from the electrons is exactly cancelled out by the positive flux from the protons. Let's calculate the flux density from the electrons alone in the copper wire to solve for how large these balanced push-and-pull forces are.

The static electric charge from the free electrons in a length of copper wire is equal to,

$$Q_e = q_e n_c A_b L_S \qquad \text{Units: } (C/e)(e/m^3)\ m^2\ m = C \tag{2}$$

$$q_e = -1.602{,}177 \times 10^{-19}\ C \qquad \text{Units: C} \tag{3}$$

$$n_c = 8.49 \times 10^{28}\ \text{Copper} \qquad \text{Units: } e/m^3 \tag{4}$$

Where q_e is the static electric charge of an individual electron, n_c is the number of free electrons per cubic meter of copper (Walker, J., 2011), A_b is the bare copper wire cross sectional area in meters squared, and L_S is the length of the wire in meters, where the subscript "s" denotes a stationary reference frame.

Static electric flux density E is equal to the charge Q_e divided by the surface area that the charge radiates through and the permittivity constant ε_o. The area A of a cylinder of radius R surrounding the wire is equal to,

$$A = 2\pi R L_S \qquad \text{Units: } m^2 \tag{5}$$

The static electric flux density from the electrons (E_{Se}) at distance R from the copper wire is then,

$$E_{Se} = Q_e/A\varepsilon_o \qquad \text{Units: } C/m^2 \tag{6}$$

$$= q_e n_c A_b L_S/2\pi R L_S \varepsilon_o$$

$$E_{Se} = q_e n_c A_b/2\pi R\varepsilon_o \tag{7}$$

The subscript "$_{Se}$" for flux density E_{Se} denotes a stationary reference frame for the electrons.

Let's define a new term, **linear charge density**, denoted "λ", where the linear charge density for the free electrons alone (λ_{Se}) is equal to,

$$\lambda_{Se} = q_e n_c A_b \qquad \text{Units: } (C/e)(e/m^3)(m^2) = C/m \tag{8}$$

Equation 7 can then be simplified to,

$$E_{Se} = \lambda_{Se}/2\pi R\varepsilon_o \tag{9}$$

The linear charge density for the associated protons (λ_{Sp}) is equal in magnitude and opposite in sign to that of the electrons (subscript "s" denotes a stationary reference frame, and subscript "p" denotes protons). When the proton linear charge density λ_{Sp} is added to the electron linear charge density λ_{Se} the net linear charge density (λ_{SN}) equals zero:

$$\lambda_{SN} = \lambda_{Sp} + \lambda_{Se} = 0 \tag{10}$$

And,

$$\lambda_{Sp} = -\lambda_{Se} \tag{11}$$

The net static electric linear charge density observed from the stationary reference frame (E_{SN}) is then,

$$E_{SN} = \lambda_{SN}/2\pi R\varepsilon_o = 0 \tag{12}$$

As Equation 12 indicates, electrical current flowing through copper wire does not produce a static electric charge when viewed from a stationary reference frame.

5. Electrical Current

The electrical current flowing through copper can be solved for in terms of electron charge q_e, free electron density in copper n_c, wire cross sectional area A_b and drift velocity v_d, where,

$$I = q_e n_c A_b v_d \qquad \text{Units: } (C/e)(e/m^3)(m^2)(m/s) = C/s \quad (13)$$

Drift velocity v_d is in units of m/s. Units of meters squared are used for area A_b, and the number of free electrons in a cubic meter of copper n_c is provided by Equation 4. Substituting in Equation 8 for linear charge density λ_{Se} we have,

$$I = \lambda_{Se} v_d = q_e n_c A_b v_d \qquad \text{Units: C/s} \quad (14)$$

In common electrical conductors, electron drift velocity v_d is limited by resistive heating of the wire. Electron drift velocities v_d rarely exceed 0.005 m/s in copper electrical conductors.

6. Time Dilation According to Special Relativity

Time dilation is the stretching of time for objects in motion. The faster an object moves, the slower the object ages.

Let's describe time dilation by way of an example, where Tom conducts a measurement of time on a moving train. The time measured by Tom while he is **traveling** on the train is denoted "t_T". Sally conducts the same measurement of time but she is standing at the train station. The time measured by Sally from the **stationary** train station is denoted "t_S".

Tom's experiment includes a light source, a reference table, a mirror and a clock. The mirror is mounted on the ceiling above the reference table. With the clock, Tom measures the time t_T for a pulse of light to travel vertically from the reference table, bounce off the mirror, and return vertically to the reference table.

The distance between the reference table and ceiling is denoted D, and the speed of the light is denoted c. Knowing that rate times time equals distance, the up and down distance is equal to,

$$ct_T = 2D \quad (15)$$

And dividing Equation 15 by the speed of light c, Tom's measured time t_T is equal to,

$$t_T = 2D/c \qquad \text{Time measured by Tom on the train} \quad (16)$$

Sally measures the elapsed time as well, but she uses two synchronized clocks located on the stationary train station platform. One clock is located adjacent to the train where the light starts, and the other clock is located adjacent to the train where the light ends. From Sally's perspective the light travels vertically distance D twice (up then down), plus the horizontal distance the train traveled down the train track (h). The horizontal distance traveled down the train track h is equal to the velocity of the train (v_S) observed by Sally, times the time measured by Sally t_S, where,

$$h = v_S t_S \quad (17)$$

From Sally's perspective the light travels diagonally up then down due to the vertical motion between the reference table and ceiling of the train, and the horizontal motion due to the train moving down the track. Let's denote each of the diagonal lengths "L". The diagonal distance L as seen by Sally is found using the Pythagorean Theorem, where,

$$L = [(h/2)^2 + D^2]^{0.5} \equiv [(v_S t_S/2)^2 + D^2]^{0.5} \quad (18)$$

From Equation 15, distance D equals,

$$ct_T/2 = D$$

Substituting this value for distance D into Equation 18 we have,

$$L = [(v_S t_S/2)^2 + (ct_T/2)^2]^{0.5} \quad (19)$$

Alternatively, Sally could use her measurement of time t_S to calculate the distance traveled by the light using the simple relationship of rate times time equals distance. For her measured time of t_S and the speed of light c we have the total distance traveled 2L equal to,

$$2L = ct_S \quad (20)$$

Or rearranging terms to solve for the time measured by Sally from her stationary train platform:

$$t_S = 2L/c \qquad \text{Time measured by Sally from the train station} \quad (21)$$

Substituting distance L from Equation 19 into Equation 21 to solve for the time observed by Sally t_S from the stationary train station platform we have,

$$t_S = 2[(v_S t_S/2)^2 + (ct_T/2)^2]^{0.5}/c \tag{22}$$

Simplifying Equation 22 we have,

$$ct_S/2 = [(v_S t_S/2)^2 + (ct_T/2)^2]^{0.5}$$

$$(ct_S/2)^2 = (v_S t_S/2)^2 + (ct_T/2)^2$$

$$(ct_S/2)^2 – (v_S t_S/2)^2 = (ct_T/2)^2$$

$$(ct_S)^2 – (v_S t_S)^2 = (ct_T)^2$$

$$c^2 t_S^2 – v_S^2 t_S^2 = c^2 t_T^2$$

$$t_S^2 – (v_S^2/c^2)t_S^2 = t_T^2$$

$$t_S^2(1 – (v_S^2/c^2)) = t_T^2$$

$$t_S^2 = t_T^2/(1 – (v_S^2/c^2))$$

$$t_S = t_T/[1 - (v_S/c)^2]^{0.5} \qquad \text{Time dilation} \tag{23}$$

To reduce the number of terms that need to be typed in Equation 23, Hendrik Lorentz defined the Lorentz Factor (γ):

$$\gamma = 1/[1 - (v_S/c)^2]^{0.5} \qquad \text{Lorentz factor} \tag{24}$$

Substituting Equation 24 into Equation 23, the time observed by Sally from the stationary train station platform t_S is then equal to,

$$t_S = t_T\gamma \qquad \text{Time dilation} \tag{25}$$

γ is non-dimensional, as the units for v_S/c cancel out. γ can be thought of as a scalar magnitude. γ is also always equal to or greater than 1, and when the velocity v_s equals zero the Lorentz Factor equals 1:

$$\gamma \geq 1 \tag{26a}$$

$$\text{When } v_s = 0: \quad \gamma = 1 \tag{26b}$$

7. Length Contraction

A related effect of time dilation is length contraction. We will use a moving train and Sally and Tom's reference frames again to describe length contraction. This time, let's assume Sally has a ruler of length L_S extending lengthwise in the direction of the train track.

Tom passes by Sally at velocity v_S. In Sally's reference frame the time for Tom to travel the length of her ruler L_S is,

$$t_S = L_S/v_S \qquad \text{Time measured by Sally} \tag{27}$$

Tom's clock runs slower due to time dilation. Therefore, the time measured by Tom equals,

$$t_T = t_S/\gamma \tag{28}$$

Looking out the window, Tom measures the time to pass the ruler, where,

$$t_T = L_T/v_S \qquad \text{Time measured by Sall} \tag{29}$$

Replacing Tom's time t_T with Equation 28 we have,

$$t_S/\gamma = L_T/v_S$$

$$t_S = \gamma L_T/v_S \tag{30}$$

Replacing Sally's time t_S with Equation 27 we have,

$$L_S/v_S = \gamma L_T/v_S$$

$$L_S = \gamma L_T \tag{31}$$

Because the Lorentz Factor γ is greater than 1, the length measured by Tom is shorter than the length measured by Sally.

8. Sally's View of the Wire

Let's now assume instead of a train Sally is observing a copper wire having an electron current. When Sally views the copper wire she observes the electron current flowing to the right with drift velocity v_d. When Sally measures the static electric field next to her she records a value of zero, because there is an equal number of electrons and protons in a given length of wire.

9. Tom's View of the Wire

Next, let's imagine Tom on a train moving at drift velocity v_d to the right. From his perspective the electrons are stationary and the protons are moving to the left at velocity v_d.

Because Tom is moving, his clock is also ticking slower. His second takes longer than Sally's second. The length of Tom's second is found by time dilation Equation 25. Because of his longer second, Tom observes more protons passing by than a person would without time dilation. Tom observes a proton linear charge density (λ_{Tp}) equal to,

$$\lambda_{Tp} = \lambda_{Sp}\gamma \qquad (32)$$

Where the Lorentz factor γ is calculated using Tom's drift velocity v_d. Subscript "$_T$" denotes a traveling reference frame.

Tom also experiences length contraction due to his velocity. Accordingly, Tom's ruler becomes shorter than Sally's ruler. When Tom measures the spacing between the electrons next to him, he observes a larger spacing than Sally does, because his ruler is shorter. The larger spacing between the electrons means that Tom observes a smaller linear electron charge density (λ_{Te}) than what Sally observes. Tom observes an electron linear charge density (λ_{Te}) equal to,

$$\lambda_{Te} = \lambda_{Se}/\gamma \qquad (33)$$

Where the Lorentz factor γ is calculated using Tom's drift velocity v_d. The net linear charge density (λ_{TN}) observed by Tom is the sum of the positive proton linear charge density λ_{Tp} and the negative electron linear charge density λ_{Te}, where,

$$\lambda_{TN} = \lambda_{Tp} + \lambda_{Te} \qquad (34)$$

Replacing linear charge densities λ_{Tp} and λ_{Te} with Equations 32 and 33 we have,

$$\lambda_{TN} = \lambda_{Sp}\gamma + \lambda_{Se}/\gamma \qquad (35)$$

Replacing λ_{Sp} with Equation 11 we have,

$$\lambda_{TN} = -\lambda_{Se}\gamma + \lambda_{Se}/\gamma$$

$$\lambda_{TN} = \lambda_{Se}(-\gamma + 1/\gamma) \qquad (36)$$

The net electrostatic field density observed by Tom (E_{TN}) is then equal to,

$$E_{TN} = \lambda_{TN}/2\pi R\varepsilon_o \qquad (37)$$

And recalling from Equation 11, we have the net electrostatic field density observed by Sally (E_{SN}),

$$E_{SN} = \lambda_{SN}/2\pi R\varepsilon_o = 0 \qquad (12)$$

Equation 37 provides the electrostatic field density observed from a moving reference frame with consideration of Special Relativity effects, that is perceived as a magnetic field density when observed from a stationary reference frame.

10. Conclusion

The difference in velocity between the electrons and protons in an electrical conductor is said to produce a magnetic force due to Special Relativity effects. According to theory advanced by Albert Einstein, the magnetic force observed from a stationary reference frame is that of the electrostatic force viewed from a moving reference frame in consideration of Special Relativity effects (Einstein, A., 1905, June 30). The traditional method of calculating the electrostatic force from a moving reference frame and considering Special Relativity effects is laborious. This paper presents a more direct approach for calculating the electrostatic field that involves the following four short equations:

$\lambda_{Se} = q_e n_c A_b$	Electron linear charge density	(8)
$\gamma = 1/[1 - (v_S/c)^2]^{0.5}$	Lorentz factor	(24)
$\lambda_{TN} = \lambda_{Se}(-\gamma + 1/\gamma)$	Net linear charge density	(36)
$E_{TN} = \lambda_{TN}/2\pi R\varepsilon_o$	Net electrostatic field density	(37)

The net electrostatic field density E_{TN} due to Special Relativity effects is found to be due to the electron drift velocity, where electron drift velocity v_S in Equation 24 is measured from Sally's stationary reference frame.

For common electrical conductors the ratio of v_S/c is negligibly small. Electron drift velocities in common electrical conductors rarely exceed 0.005 m/s, and the speed of light is 299,792,455 m/s. Therefore, from Equation 24 we find that the Lorentz factor γ for common electrical conductors is approximately equal to 1. With $\gamma = 1$, $E_{TN} = 0$ (solved for with Equations 36 and 37). The net electrostatic field density E_{TN} therefore does not account for the magnetic field of Equation 1 for common electrical conductors.

Einstein describes in his 1905 paper that magnetism is a variance in the electrostatic force field that is found by applying Special Relativity. At the time a reason for the non-existents of magnetic charges (monopoles) was needed for a more complete explanation of the Maxwell equations. According to Einstein's logic, magnetism never existed in the first place, it is simply a manifestation of the electrostatic force field from charged particles in motion.

This paper finds a variance in the electrostatic field of electrical conductors is caused by Special Relativity effects, but that the effect is negligible due to the relatively slow electron drift velocity relative to the speed of light. Electron drift velocity is limited by resistive heating of the conductor, and it is therefore only with superconducting materials that the drift velocity may be fast enough for Special Relativity effects to be observed. Einstein's explanation of the magnetic field does not account for the full strength of the magnetic field observed around common electrical conductors. Further research is recommended for understanding the cause of magnetism in common electrical conductors, and development of new magnetic theory.

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