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Tensor Satisfying Binary Law for the Equation Including the Trigonometric Function 2

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Abstract

This article is revised edition of "Tensor satisfying Binary Law for the equation including the trigonometric function". The revision is classified roughly into two of the next. (a) The revision of the conclusion in the proposition. (b) Revision of the proof method in the proposition. Furthermore, I report that scalar, vector are all Tensor satisfying Binary Law in this article. In other words, I report that scalar, the vector satisfy Binary Law by oneself.

Keywords: tensor, covariant derivative

1. Introduction

I have already reported establishment of $A^{\mu} = Sin(x^{\nu})$. (Ichidayama, 2017, Property of …) $A^{\mu} = Sin(x^{\nu})$ is an equation including the trigonometric function here. However, it isn't investigated Tensor satisfying Binary Law for the equation including the trigonometric function. I investigate Tensor satisfying Binary Law for the equation including the trigonometric function newly and has been reported this result in "Tensor satisfying Binary Law for the equation including the trigonometric function".(Ichidayama, 2024) This article is revised edition of "Tensor satisfying Binary Law for the equation including the trigonometric function". The revision is classified roughly into two of the next. (a) The revision of the conclusion in the proposition. (b) Revision of the proof method in the proposition. Proposition according to (a) is Proposition9,Proposition10,Proposition12 in this article. Proposition according to (b) is Proposition2,Proposition6,Proposition7,Proposition8,Proposition12 in this article. Furthermore, I report that scalar, vector are all Tensor satisfying Binary Law in this article in Proposition1. In other words, I report that scalar, the vector satisfy Binary Law by oneself in Proposition1.

2. Definition

Definition1. $\overline{x^{\mu}} \neq x^{\mu}, \overline{x^{\nu}} \neq x^{\nu}, \overline{x^{\mu}} = x^{\nu}, \overline{x^{\nu}} = x^{\mu}$ is established.(Ichidayama, 2017, Introduction of …)

I named $\overline{x^{\mu}} \neq x^{\mu}, \overline{x^{\nu}} \neq x^{\nu}, \overline{x^{\mu}} = x^{\nu}, \overline{x^{\nu}} = x^{\mu}$ "Binary Law".(Ichidayama, 2017, Introduction of …)

Definition2. If $\overline{x^{\mu}} \neq x^{\mu}, \overline{x^{\nu}} \neq x^{\nu}, \overline{x^{\mu}} = x^{\nu}, \overline{x^{\nu}} = x^{\mu}$ is established, $x_{\mu} = x^{\nu}$ is established.(Ichidayama, 2017, Introduction of …)

Definition3. If $\overline{x^{\mu}} \neq x^{\mu}, \overline{x^{\nu}} \neq x^{\nu}, \overline{x^{\mu}} = x^{\nu}, \overline{x^{\nu}} = x^{\mu}$ is established, $x_{\nu} = -x_{\mu}$ is established.(Ichidayama, 201, Introduction of …7)

Definition4. If $\overline{x^{\mu}} \neq x^{\mu}, \overline{x^{\nu}} \neq x^{\nu}, \overline{x^{\mu}} = x^{\nu}, \overline{x^{\nu}} = x^{\mu}$ is established, $x^{\nu} = -x^{\mu}$ is established.(Ichidayama, 2017, Introduction of …)

Definition5. If $\overline{x^{\mu}} \neq x^{\mu}, \overline{x^{\nu}} \neq x^{\nu}, \overline{x^{\mu}} = x^{\nu}, \overline{x^{\nu}} = x^{\mu}$ is established, $\frac{\partial^3 M^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} = M$ is established.(Ichidayama, 2023)

Definition6. $x^{\mu} = \frac{\partial x^{\mu}}{\partial x^{\nu}} x^{\nu}$ is established.

Definition7. $SinA + SinB = 2Sin\frac{(A+B)}{2}Cos\frac{(A-B)}{2}$ is established. (Spiegel, 1968)

Definition8. $\mathrm{Sin}(n\pi) = 0$ is established. n expresses natural number here.

Definition9. $\int \frac{dX}{\sqrt{A^2-X^2}} = \arcsin\left(\frac{X}{A}\right)$ is established. (Spiegel, 1968)

Definition10. $\mathrm{W}(A \to B) = -\mathrm{U} = \int_A^B \vec{F} \cdot d\vec{r}$ is established. (Kittel, Knight, Ruderman, 1975)

W expresses Work, U expresses Potential Energy, $\vec{F}$ expresses External force vector, and $\vec{r}$ expresses Displacement vector.

Definition11. $E = mc^2$ is established. (Taylor, 1975)

E expresses Energy, m expresses Mass, and c expresses Speed of light.

Definition12. The force that the nucleus attracts electron is expressed in $F = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2} = -\frac{k}{r^2}$.

r expresses distance between nucleus and the electron, ϵ_0 expresses dielectric constant, k expresses constant, Q expresses nuclear charge, q expresses electronic charge.

The force that binary proton repels is expressed in $F = \frac{1}{4\pi\epsilon_0}\frac{QQ}{r^2} = \frac{k'}{r^2}$.

r expresses distance between proton each, k' expresses constant.

Definition13. $y[x] = C[1]Cos[x] + C[2]Sin[x]$ is established as a solution of the equations of $y''[x] + y[x] = 0$. y is function $y = f(x)$ which assumes x an independent variable. I obtained this calculation result by Wolfrem Mathematica 11.3.

Definition14. Even if X and Y are any tensor, $(XY)_{;\sigma} = X_{;\sigma}Y + XY_{;\sigma}$ is managed generally. (Dirac, 1975)

Definition15. If Y is the tensor of the zeroth rank, $Y_{;\sigma} = Y_{,\sigma} = \frac{\partial Y}{\partial x^\sigma}$ is established. (Dirac, 1975)

Definition16. $M^\mu = Mx^\mu, M^\mu_{;\nu} = Mx^\mu_{;\nu}$ is established. M^μ, x^μ are the tensor of the first rank, and M is the tensor of the zeroth rank here. $M^\mu_{;\nu}, x^\mu_{;\nu}$ are the tensor of the second rank.

Notation ";" before the index with the bottom expresses covariant differentiation here.

Definition17. $M^\mu_{;\nu} = \frac{\partial M^\mu}{\partial x^\nu} + M^a\Gamma^\mu_{a\nu}$ is established in the covariant differentiation for the contravariant vector. (Fleisch, 2012)

Hypothesis1. $M \propto m, M = \epsilon m$ is established.

M expresses $\frac{\partial^3 \mathrm{M}^\mu}{\partial \mathrm{x}^\nu \, \partial \mathrm{x}^\nu \, \partial \mathrm{x}^\nu} = \mathrm{M}$, ϵ expresses Proportional constant, and m expresses Mass.

3. Property of Tensor Satisfying Binary Law

Proposition1. The tensor of the first rank is equivalent to the tensor satisfying Binary Law of the first rank.

The tensor of the zeroth rank is equivalent to the tensor satisfying Binary Law of the zeroth rank.

Proof.

$$\vec{M} = M^\mu \overrightarrow{e_\mu} \tag{1}$$

is established. When all coordinate system satisfies Binary Law for component

$$M^\mu \tag{2}$$

of (1), I get

$$M^\mu. \tag{3}$$

As (3) accords in (2), (2) is tensor satisfying Binary Law of the first rank. In other words, the tensor of the first rank is equivalent to tensor satisfying Binary Law of the first rank.

$$M^2 = \vec{M} \cdot \vec{M} = M^\mu \overrightarrow{e_\mu} \cdot M_\mu \overrightarrow{e^\mu} = M^\mu M_\mu \tag{4}$$

is established. $\overrightarrow{e_\mu} \cdot \overrightarrow{e^\mu} = 1$ is established here. When all coordinate system satisfies Binary Law for component

$$M^\mu M_\mu \tag{5}$$

of (4), I get

$$M^\mu M_\mu. \tag{6}$$

As (6) accords in (5), (5) is tensor satisfying Binary Law of the zeroth rank. In other words, the tensor of the zeroth rank is equivalent to tensor satisfying Binary Law of the zeroth rank.

$$M^\mu_{;\nu}\overrightarrow{e_\mu}\overrightarrow{e^\nu} = \left(\frac{\partial M^\mu}{\partial x^\nu} + M^a\,\Gamma^{\;\mu}_{a\nu}\right)\overrightarrow{e_\mu}\overrightarrow{e^\nu} \tag{7}$$

in consideration of Definision17. When all coordinate system satisfies Binary Law for component

$$\frac{\partial M^\mu}{\partial x^\nu} + M^a\,\Gamma^{\;\mu}_{a\nu} \tag{8}$$

of (7), I get

$$\frac{\partial M^\mu}{\partial x^\nu} + M^\nu\,\Gamma^{\;\mu}_{\nu\nu}. \tag{9}$$

As (9) does not accord in (8), (8) is not tensor satisfying Binary Law of the second rank. As vector, scalar are tensor satisfying Binary Law, I think that tensor $\frac{\partial M^\mu}{\partial x^\nu} + M^a\,\Gamma^{\;\mu}_{a\nu}$ may be tensor satisfying Binary Law like these.

Proposition2. $\frac{\partial M}{\partial x^\nu} = 0$ is established in the tensor satisfying Binary Law. "M" expresses $\frac{\partial^3 M^\mu}{\partial x^\nu\,\partial x^\nu\,\partial x^\nu} = M.$

Proof. I get

$$M^\mu = Mx^\mu \tag{10}$$

$$M^\mu_{;\nu} = Mx^\mu_{;\nu} \tag{11}$$

from Definision16. I get

$$M^\mu_{;\nu} = M_{;\nu}x^\mu + Mx^\mu_{;\nu} \tag{12}$$

as covariant differentiation of (10) in consideration of Definision14. I rewrite (12) by (11),Definision15 and get

$$M^\mu_{;\nu} = \frac{\partial M}{\partial x^\nu}x^\mu + M^\mu_{;\nu},$$

$$\frac{\partial M}{\partial x^\nu}x^\mu = 0. \tag{13}$$

As x^μ is any tensor, I get

$$\frac{\partial M}{\partial x^\nu} = 0 \tag{14}$$

from (13).

Proposition3. When all coordinate systems satisfies Binary Law, $\frac{\partial M}{\partial \varphi^\nu} = 0, \frac{\partial M}{\partial \varphi^\mu} = 0$ is established. "M" expresses $\frac{\partial^3 M^\mu}{\partial x^\nu\,\partial x^\nu\,\partial x^\nu} = M.$

Proof. I assume that

$$\frac{\partial M}{\partial \varphi^\nu} = 0 \quad (false) \tag{15}$$

is established.

I rewrite μ,ν-inversion form of (10) and get

$$\varphi^{\nu} = \varphi x^{\nu}. \tag{16}$$

I rewrite (15) using (16) and get

$$\frac{\partial M}{\partial \varphi x^{\nu}} = 0 \quad (false). \tag{17}$$

I multiply the both sides of (17) by φ and get

$$\frac{\varphi\, \partial M}{\partial \varphi x^{\nu}} = \frac{\partial M}{\partial x^{\nu}} = 0 \quad (false). \tag{18}$$

As (18) is not established for (14) ,

$$\frac{\partial M}{\partial \varphi^{\nu}} = 0 \tag{19}$$

is established. I get

$$\frac{\partial M}{\partial \varphi^{\mu}} = 0 \tag{20}$$

as μ,ν-inversion form of (19).

4. Property $\frac{\partial^3 M^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}} = M: (M > 0)$

Proposition4. When all coordinate systems satisfies Binary Law, $\frac{\partial^2 M^1}{\partial \dot{x}^1\, \partial \dot{x}^1} = -M^1, \frac{\partial^2 M^2}{\partial \dot{x}^1\, \partial \dot{x}^1} = -M^2, \dot{x}^1 = \dot{x}^2$ is established for $\frac{\partial^3 M^{\mu}}{\partial x^{\nu}\, \partial x^{\nu}\, \partial x^{\nu}} = M{:}\, M > 0$ if the number of the dimensions is 2.

Proof. I get

$$\frac{\partial^2 M^{\mu}}{\partial x^{\nu}\, \partial x^{\nu}} = \mathrm{M} \int \partial x^{\nu} = \mathrm{M} x^{\nu} \tag{21}$$

in consideration of Proposition2 for Definision5. Two next

$$\frac{\partial^2 M^{\mu}}{\partial x^{\nu}\, \partial x^{\nu}} = -M x^{\mu} = -M^{\mu}, \tag{22}$$

$$\frac{\partial^2 M^{\mu}}{\partial x^{\nu}\, \partial x^{\nu}} = M x_{\mu} = \frac{M}{x^{\mu}} = \frac{(M)^2}{M^{\mu}} \tag{23}$$

can rewrite (21) each using Definision2,Definision4,(10). I get (23) as $x_{\mu} = \frac{1}{x^{\mu}}$ here. I get

$$\frac{\partial^2 M^1}{\partial \dot{x}^1\, \partial \dot{x}^1} = -M^1, \frac{\partial^2 M^1}{\partial \dot{x}^2\, \partial \dot{x}^2} = -M^1,$$
$$\frac{\partial^2 M^2}{\partial \dot{x}^1\, \partial \dot{x}^1} = -M^2, \frac{\partial^2 M^2}{\partial \dot{x}^2\, \partial \dot{x}^2} = -M^2, \tag{24}$$

$$\frac{\partial^2 M^1}{\partial \dot{x}^1\, \partial \dot{x}^1} = \frac{(M)^2}{M^1}, \frac{\partial^2 M^1}{\partial \dot{x}^2\, \partial \dot{x}^2} = \frac{(M)^2}{M^1},$$
$$\frac{\partial^2 M^2}{\partial \dot{x}^1\, \partial \dot{x}^1} = \frac{(M)^2}{M^2}, \frac{\partial^2 M^2}{\partial \dot{x}^2\, \partial \dot{x}^2} = \frac{(M)^2}{M^2} \tag{25}$$

from (22),(23) if I assume a dimensional number 2. I get

$$\frac{\partial^2 M^1}{\partial \dot{x^1}\,\partial \dot{x^1}} = \frac{\partial^2 M^1}{\partial \dot{x^2}\,\partial \dot{x^2}}, \frac{\partial^2 M^2}{\partial \dot{x^1}\,\partial \dot{x^1}} = \frac{\partial^2 M^2}{\partial \dot{x^2}\,\partial \dot{x^2}} \tag{26}$$

from (24),(25). I get

$$\frac{\partial^2 M^1}{\partial \dot{x^1}\,\partial \dot{x^1}} = \frac{\partial^2 M^1}{\partial \dot{x^1}\,\partial \dot{x^1}}\ (false), \frac{\partial^2 M^2}{\partial \dot{x^1}\,\partial \dot{x^1}} = \frac{\partial^2 M^2}{\partial \dot{x^1}\,\partial \dot{x^1}}\ (false) \tag{27}$$

from (26) if I assume establishment of $\dot{x^1} = \dot{x^2}\ (false)$. Because (27) isn't established,

$$\dot{x^1} = \dot{x^2} \tag{28}$$

is established. I get

$$\frac{\partial^2 M^1}{\partial \dot{x^1}\,\partial \dot{x^1}} = -M^1, \frac{\partial^2 M^2}{\partial \dot{x^1}\,\partial \dot{x^1}} = -M^2, \tag{29}$$

$$\frac{\partial^2 M^1}{\partial \dot{x^1}\,\partial \dot{x^1}} = \frac{(M)^2}{M^1}, \frac{\partial^2 M^2}{\partial \dot{x^1}\,\partial \dot{x^1}} = \frac{(M)^2}{M^2} \tag{30}$$

in consideration of (28) for (24),(25).

Proposition.5 When all coordinate systems satisfies Binary Law, $M^1 = MSin(\dot{x^1}), M^2 = MSin(\dot{x^1})$ is established for $\frac{\partial^3 M^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu} = M : M > 0$ if the number of the dimensions is 2.

Proof. When $M > 0$ is established, I get

$$M^1 = C[1]Cos(\dot{x^1}) + C[2]Sin(\dot{x^1}),$$
$$M^2 = C[1]Cos(\dot{x^1}) + C[2]Sin(\dot{x^1}) \tag{31}$$

as a solution of the equations of (29) in consideration of Definision13. In addition, I do not deal in this article about (30). I get

$$M^1 = C[2]Sin(\dot{x^1}), M^2 = C[2]Sin(\dot{x^1}) \tag{32}$$

as $C[1] = 0$ for (31). I assume that

$$x^1 = Sin(\dot{x^1})\ (false), x^2 = Sin(\dot{x^1})\ (false) \tag{33}$$

is established. I rewrite (33) using (10) and get

$$M^1 = MSin(\dot{x^1})\ (false), M^2 = MSin(\dot{x^1})\ (false). \tag{34}$$

I get

$$\frac{d^2 M^1}{d\dot{x^1} d\dot{x^1}} = -MSin(\dot{x^1}) = -M^1\ (false),$$

$$\frac{d^2 M^2}{d\dot{x^1} d\dot{x^1}} = -MSin(\dot{x^1}) = -M^2\ (false) \tag{35}$$

from (34). I get the conclusion that

$$x^1 = Sin(\dot{x^1}), x^2 = Sin(\dot{x^1}) \tag{36}$$

is established as (35) is not established from (29). I get

$$M^1 = C[2]x^1, M^2 = C[2]x^2 \tag{37}$$

from (32),(36). I get

$$C[2] = \frac{Mx^1}{x^1} = \frac{Mx^2}{x^2} = M \tag{38}$$

as (10) in (37). I get

$$M^1 = MSin(\dot{x^1}), M^2 = MSin(\dot{x^1}) \tag{39}$$

from (32),(38). I get

$$M^\mu = MSin(x^\nu) \tag{40}$$

from (39) in consideration of (28). Similarly, I get

$$M^1 = C[1]Cos(\dot{x^1}), M^2 = C[1]Cos(\dot{x^1}) \quad (41)$$

as $C[2] = 0$ for (31). I assume that

$$x^1 = Cos(\dot{x^1})\ (false), x^2 = Cos(\dot{x^1})\ (false) \quad (42)$$

is established. I rewrite (42) using (10) and get

$$M^1 = MCos(\dot{x^1})\ (false), M^2 = MCos(\dot{x^1})\ (false). \quad (43)$$

I get

$$\frac{d^2M^1}{d\dot{x^1}d\dot{x^1}} = -MCos(\dot{x^1}) = -M^1\ (false),$$

$$\frac{d^2M^2}{d\dot{x^1}d\dot{x^1}} = -MCos(\dot{x^1}) = -M^2\ (false) \quad (44)$$

from (43). I get the conclusion that

$$x^1 = Cos(\dot{x^1}), x^2 = Cos(\dot{x^1}) \quad (45)$$

is established as (44) is not established from (29). I get

$$M^1 = C[1]x^1, M^2 = C[1]x^2 \quad (46)$$

from (41),(45). I get

$$C[1] = \frac{Mx^1}{x^1} = \frac{Mx^2}{x^2} = M \quad (47)$$

as (10) in (46). I get

$$M^1 = MCos(\dot{x^1}), M^2 = MCos(\dot{x^1}) \quad (48)$$

from (41),(47). I get

$$M^\mu = MCos(x^\nu) \quad (49)$$

from (48) in consideration of (28).

5. Tensor Satisfying BinaryLaw for the Equation Including the Trigonometric Function

Proposition.6 When all coordinate system satisfies Binary Law, $ASin(x^\nu) = Sin(Ax^\nu)$ is established.
Proof. I get

$$x^\mu = Sin(x^\nu) \quad (50)$$

from (40) in consideration of (10). I rewrite (50) using Definision6,μ, ν-inversion form of Definision6 and get

$$\frac{\partial x^\mu}{\partial x^\nu}x^\nu = Sin\left(\frac{\partial x^\nu}{\partial x^\mu}x^\mu\right). \quad (51)$$

I rewrite (51) by multiplying all x^μ, x^ν by $-A$ and get

$$\frac{\partial -Ax^\mu}{\partial -Ax^\nu}(-Ax^\nu) = \mathrm{Sin}\left(\frac{\partial -Ax^\nu}{\partial -Ax^\mu}(-Ax^\mu)\right),$$

$$\frac{\partial x^\mu}{\partial x^\nu}(-Ax^\nu) = Sin\left(\frac{\partial x^\nu}{\partial x^\mu}(-Ax^\mu)\right). \quad (52)$$

I rewrite (52) using Definision3,Definision4,μ, ν-inversion form of (50) and get

$$\frac{\partial x^\mu}{\partial x^\mu}Ax^\nu = Sin\left(\frac{\partial x^\mu}{\partial x^\mu}Ax^\mu\right),$$

$$ASin(x^\mu) = Sin(Ax^\mu). \quad (53)$$

Proposition.7 When all coordinate system satisfies Binary Law, $ACos(x^\nu) = Cos(Ax^\nu)$ is established.
Proof. I get

$$x^{\mu} = Cos(x^{\nu}) \tag{54}$$

from (49) in consideration of (10). I rewrite (54) using Definision6,μ, ν-inversion form of Definision6 and get

$$\frac{\partial x^{\mu}}{\partial x^{\nu}} x^{\nu} = Cos\left(\frac{\partial x^{\nu}}{\partial x^{\mu}} x^{\mu}\right). \tag{55}$$

I rewrite (55) by multiplying all x^{μ}, x^{ν} by $-A$ and get

$$\frac{\partial - Ax^{\mu}}{\partial - Ax^{\nu}}(-Ax^{\nu}) = Cos\left(\frac{\partial - Ax^{\nu}}{\partial - Ax^{\mu}}(-Ax^{\mu})\right),$$

$$\frac{\partial x^{\mu}}{\partial x^{\nu}}(-Ax^{\nu}) = Cos\left(\frac{\partial x^{\nu}}{\partial x^{\mu}}(-Ax^{\mu})\right). \tag{56}$$

I rewrite (56) using Definision3,Definision4,μ, ν-inversion form of (54) and get

$$\frac{\partial x^{\mu}}{\partial x^{\mu}} Ax^{\nu} = Cos\left(\frac{\partial x^{\mu}}{\partial x^{\mu}} Ax^{\mu}\right),$$

$$ACos(x^{\mu}) = Cos(Ax^{\mu}). \tag{57}$$

Proposition.8 When all coordinate system satisfies Binary Law, $ASin^{-1}(x^{\nu}) = Sin^{-1}(Ax^{\nu})$ is established.

Proof. I get

$$x^{\nu} = Sin^{-1}(x^{\mu}) \tag{58}$$

from (50). I rewrite (58) using Definision6,μ, ν-inversion form of Definision6 and get

$$\frac{\partial x^{\nu}}{\partial x^{\mu}} x^{\mu} = Sin^{-1}\left(\frac{\partial x^{\mu}}{\partial x^{\nu}} x^{\nu}\right). \tag{59}$$

I rewrite (59) by multiplying all x^{μ}, x^{ν} by $-A$ and get

$$\frac{\partial - Ax^{\nu}}{\partial - Ax^{\mu}}(-Ax^{\mu}) = Sin^{-1}\left(\frac{\partial - Ax^{\mu}}{\partial - Ax^{\nu}}(-Ax^{\nu})\right),$$

$$\frac{\partial x^{\nu}}{\partial x^{\mu}}(-Ax^{\mu}) = Sin^{-1}\left(\frac{\partial x^{\mu}}{\partial x^{\nu}}(-Ax^{\nu})\right). \tag{60}$$

I rewrite (60) using Definision3,Definision4,μ, ν-inversion form of (58) and get

$$\frac{\partial x^{\mu}}{\partial x^{\mu}} Ax^{\mu} = Sin^{-1}\left(\frac{\partial x^{\mu}}{\partial x^{\mu}} Ax^{\nu}\right),$$

$$ASin^{-1}(x^{\nu}) = Sin^{-1}(Ax^{\nu}). \tag{61}$$

Proposition.9 When all coordinate system satisfies Binary Law, $SinA^{\nu} + SinB^{\nu} = (SinA^{\nu} + SinB^{\nu})Cos\frac{(A-B)x^{\nu}}{2}$ is established.

Proof. I get

$$SinA^{1} + SinB^{1} = 2Sin\frac{(A^{1} + B^{1})}{2}Cos\frac{(A^{1} - B^{1})}{2},$$

$$SinA^{2} + SinB^{2} = 2Sin\frac{(A^{2} + B^{2})}{2}Cos\frac{(A^{2} - B^{2})}{2}, \cdots \tag{62}$$

from Definision7. I get

$$SinA^{\nu} + SinB^{\nu} = 2Sin\frac{(A^{\nu} + B^{\nu})}{2}Cos\frac{(A^{\nu} - B^{\nu})}{2},$$

$$SinA^{\nu} + SinB^{\nu} = 2Sin\frac{(A+B)x^{\nu}}{2}Cos\frac{(A-B)x^{\nu}}{2} \tag{63}$$

from (62),μ, ν-inversion form of (10). I get

$$SinA^{\nu} + SinB^{\nu} = (A+B)Sinx^{\nu}Cos\frac{(A-B)x^{\nu}}{2} \quad (64)$$

from (63) using μ, ν-inversion form of (53). I get

$$SinA^{\nu} + SinB^{\nu} = (ASinx^{\nu} + BSinx^{\nu})Cos\frac{(A-B)x^{\nu}}{2}$$

$$= (SinA^{\nu} + SinB^{\nu})Cos\frac{(A-B)x^{\nu}}{2} \quad (65)$$

from (64) using μ, ν-inversion form of (10),μ, ν-inversion form of (53).

6. Force in the Tensor Satisfying Binary Law

Proposition.9 When all coordinate system satisfies Binary Law, $M' = MCos\left(\frac{\varphi^{\nu}}{2}\right), M' = M\frac{-1}{B}Cos\left(\frac{-B\varphi^{\mu}}{2}\right): B = \frac{-1}{\sqrt{1-(\varphi^{\mu})^2}}$ is established for $\frac{\partial^3 M^{\mu}}{\partial x^{\nu}\,\partial x^{\nu}\,\partial x^{\nu}} = M{:}M > 0$.

Proof. I rewrite (40) using (10) and get

$$Mx^{\mu} = MSin(x^{\nu}). \quad (66)$$

I get

$$M(x^{\mu} + \varphi^{\mu}) = MSin(x^{\nu} + \varphi^{\nu}) \quad (67)$$

from (66) as $x^{\mu} \to (x^{\mu} + \varphi^{\mu}), x^{\nu} \to (x^{\nu} + \varphi^{\nu})$. I add (66) to (67) and get

$$M(x^{\mu} + \varphi^{\mu}) + Mx^{\mu} = MSin(x^{\nu} + \varphi^{\nu}) + MSin(x^{\nu}). \quad (68)$$

I rewrite right-hand side of (68) in consideration of (65) and get

$$M\big(Sin(x^{\nu} + \varphi^{\nu}) + Sin(x^{\nu})\big)$$

$$= M\big(Sin(x^{\nu} + \varphi^{\nu}) + Sin(x^{\nu})\big)Cos\frac{\varphi^{\nu}}{2},$$

$$M = MCos\frac{\varphi^{\nu}}{2}. \quad (69)$$

I express (69) in

$$M' = MCos\frac{\varphi^{\nu}}{2} \quad (70)$$

to distinguish M of the both sides of (69). I rewrite left-hand side of (68) in consideration of μ, ν-inversion form of (65) and get

$$M(x^{\mu} + \varphi^{\mu}) + Mx^{\mu} = M\big(SinSin^{-1}(x^{\mu} + \varphi^{\mu}) + SinSin^{-1}(x^{\mu})\big)$$

$$= M\big(SinSin^{-1}(x^{\mu} + \varphi^{\mu}) + SinSin^{-1}(x^{\mu})\big)Cos\frac{Sin^{-1}(x^{\mu}+\varphi^{\mu}) - Sin^{-1}(x^{\mu})}{2},$$

$$M = MCos\frac{Sin^{-1}(x^{\mu}+\varphi^{\mu}) - Sin^{-1}(x^{\mu})}{2}. \quad (71)$$

I get

$$M = MCos\frac{Sin^{-1}(1+\varphi)x^{\mu} - Sin^{-1}(x^{\mu})}{2}$$

$$= MCos\frac{(1+\varphi)Sin^{-1}(x^{\mu}) - Sin^{-1}(x^{\mu})}{2}$$

$$= MCos\frac{Sin^{-1}(x^{\mu}) + \varphi Sin^{-1}(x^{\mu}) - Sin^{-1}(x^{\mu})}{2}$$

$$= MCos\frac{Sin^{-1}(\varphi x^{\mu})}{2} = MCos\frac{Sin^{-1}(\varphi^{\mu})}{2} \quad (72)$$

using μ,ν-inversion form of (61),(10) from (71). I express (72) in

$$M' = MCos\frac{Sin^{-1}(\varphi^{\mu})}{2} \tag{73}$$

to distinguish M of the both sides of (72). I get

$$\frac{\partial Sin^{-1}(\varphi^{\mu})}{\partial\varphi^{\mu}} = \frac{1}{\sqrt{1-(\varphi^{\mu})^2}} \tag{74}$$

as $A = 1, X \to \varphi^{\mu}$ for Definision9. I get

$$\frac{\partial\varphi^{\mu}}{\partial\varphi^{\mu}} = \frac{\partial Sin^{-1}(\varphi^{\nu})}{\partial\varphi^{\mu}} = \frac{-\partial Sin^{-1}(-\varphi^{\nu})}{\partial\varphi^{\mu}}$$

$$= \frac{\partial Sin^{-1}-(\varphi^{\mu})}{\partial\varphi^{\mu}} = \frac{-1}{\sqrt{1-(\varphi^{\mu})^2}} = Scalar \tag{75}$$

from Definision4,(61),(74),μ,ν-inversion form of (58). I decide to express (75) in

$$\frac{\partial Sin^{-1}-(\varphi^{\mu})}{\partial\varphi^{\mu}} = \frac{-1}{\sqrt{1-(\varphi^{\mu})^2}} = B. \tag{76}$$

I get

$$-\mathrm{Sin}^{-1}(\varphi^{\mu}) = \mathrm{B}\int\partial\varphi^{\mu} = \mathrm{B}\varphi^{\mu},$$
$$Sin^{-1}(\varphi^{\mu}) = -B\varphi^{\mu} \tag{77}$$

from (76). I get

$$M' = MCos\left(\frac{-B\varphi^{\mu}}{2}\right) \tag{78}$$

from (73),(77). I get

$$M' = MCos\frac{\varphi^{\mu}}{2} \tag{79}$$

as μ,ν-inversion form of (70). I get

$$MCos\frac{\varphi^{\mu}}{2} = MCos\left(\frac{-B\varphi^{\mu}}{2}\right) \tag{80}$$

from (78),(79). The establishment of (80) is impossible here. Therefore, I rewrite (78) as $MCos\left(\frac{-B\varphi^{\mu}}{2}\right) \to M\frac{-1}{B}Cos\left(\frac{-B\varphi^{\mu}}{2}\right)$ in consideration of (57) and get

$$M' = M\frac{-1}{B}Cos\left(\frac{-B\varphi^{\mu}}{2}\right). \tag{81}$$

Proposition.10 When all coordinate system satisfies Binary Law, $F_{\nu} = -\frac{\partial M}{\partial x^{\nu}}$ is established.

Proof.

$$\vec{F} = \overrightarrow{e^{\nu}}F_{\nu}, d\vec{r} = \overrightarrow{e_{\nu}}dx^{\nu} \tag{82}$$

is established. I rewrite Definision10 using (82) and get

$$\mathrm{U} = -\int\overrightarrow{\mathrm{e}^{\nu}}\mathrm{F}_{\nu}\cdot\overrightarrow{\mathrm{e}_{\nu}}\mathrm{d}x^{\nu} = -\int\left(\overrightarrow{e^{\nu}}\cdot\overrightarrow{e_{\nu}}\right)F_{\nu}\,dx^{\nu} = -\int F_{\nu}dx^{\nu}. \tag{83}$$

I get (83) as $\overrightarrow{e^{\nu}}\cdot\overrightarrow{e_{\nu}} = 1$ here. I get

$$F_{\nu} = -\frac{\partial U}{\partial x^{\nu}} \tag{84}$$

from (83). I get

$$U = \frac{c^2}{\epsilon}M \tag{85}$$

as E→U from Definision11,Hypothesis1. I get

$$U = M \tag{86}$$

as $\frac{c^2}{\epsilon} = 1$ for (85). In addition, I rewrite Hypothesis1 using $\frac{c^2}{\epsilon} = 1$ and get

$$M = mc^2. \tag{87}$$

I get

$$F_\nu = -\frac{\partial M}{\partial x^\nu} \tag{88}$$

from (84), (86).

Proposition.12 When all coordinate system satisfies Binary Law, $F_\nu{}' = \frac{M}{2}Sin\left(\frac{\varphi^\nu}{2}\right), F_\mu{}' = \frac{M}{2}\frac{-1}{B}Sin\left(\frac{-B\varphi^\mu}{2}\right): B = \frac{-1}{\sqrt{1-(\varphi^\mu)^2}}$ is established for $\frac{\partial^3 M^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu} = M: M > 0$.

Proof. I get

$$F_\nu = -\frac{\partial M}{\partial \varphi^\nu} \tag{89}$$

as $x^\nu \to \varphi x^\nu = \varphi^\nu$ from (88). I get

$$F_\nu{}' = -\frac{\partial M'}{\partial \varphi^\nu} = -\frac{\partial M Cos\left(\frac{\varphi^\nu}{2}\right)}{\partial \varphi^\nu}$$

$$= -\frac{\partial M}{\partial \varphi^\nu}Cos\left(\frac{\varphi^\nu}{2}\right) - \frac{M\,\partial Cos\left(\frac{\varphi^\nu}{2}\right)}{\partial \varphi^\nu} = \frac{M}{2}Sin\left(\frac{\varphi^\nu}{2}\right). \tag{90}$$

from (70),(89),Proposition3. If $F_\nu{}' = 0$ is established,

$$\frac{\varphi^\nu}{2} = \pi n \tag{91}$$

is established in consideration of (90),Definision8. I get

$$\dot{F_1}' = \frac{M}{2}Sin\left(\frac{\dot{\varphi^1}}{2}\right), \tag{92}$$

$$\dot{\varphi^1} = 2\pi n \tag{93}$$

from (90),(91) if I assume dimensionality 1. I show figure of (92) in Figure 1. The value of $\dot{\varphi^1}$ satisfying $\dot{F_1}' = 0$ exists innumerably according to (93). I show this in

$$\dot{\varphi^1} = 0, 2\pi, 4\pi, 6\pi, 8\pi, 10\pi, \cdots. \tag{94}$$

In Fig.1, Positive divergence point in the field of force exists in (94). I show this in

$$\dot{\varphi^1} = 0, 4\pi, 8\pi, \cdots. \tag{95}$$

Negative divergence point in the field of force exists in (94). I show this in

$$\dot{\varphi^1} = 2\pi, 6\pi, 10\pi, \cdots. \tag{96}$$

I think about wave moving in circle length of radius r. Circle length of radius r as $2\pi r$. Wave length of the wave as λ. The phase length of the wave as φ.

$$2\pi r - n\lambda = \varphi \tag{97}$$

is established here.

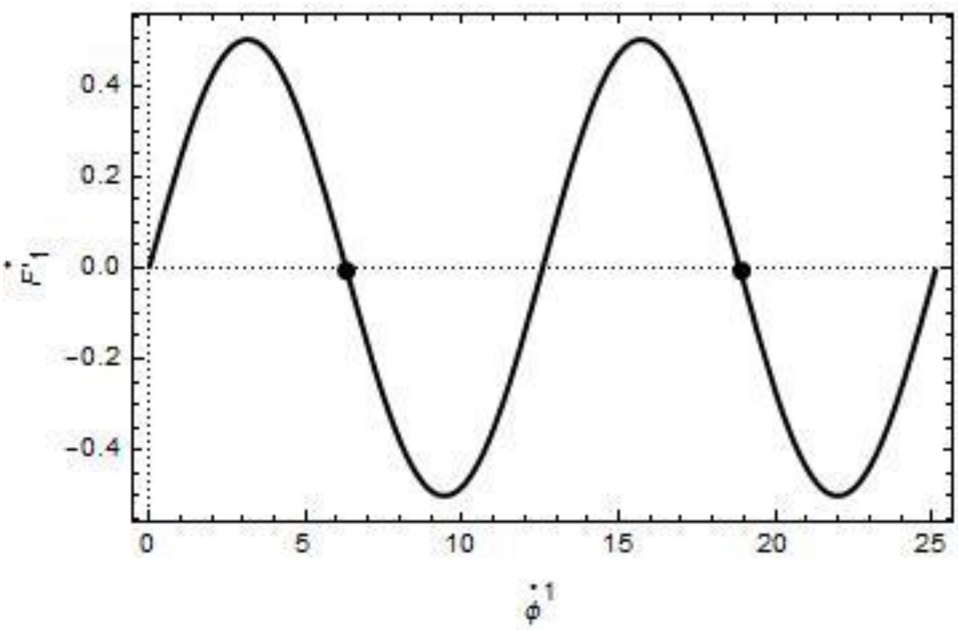

Figure 1. Plot for variable $\dot{\varphi}^1$ of $\dot{F_1}' = \frac{M}{2} Sin\left(\frac{\dot{\varphi}^1}{2}\right) : (M = 1)$

The black dot expresses $\dot{\varphi}^1$ satisfying $\dot{F}_1{}' = 0$, and this is negative divergence point in the field of force more. When $\varphi \neq 0$ is established, φ of the wave is only out of φ every time for each $2\pi r$. As overlap between the wave which φ is different each occurs, I get

$$Sin(x) + Sin(x + \varphi). \tag{98}$$

(98) accords with right-hand side of (68). I get

$$2\pi \dot{r}^1 - n\lambda = \dot{\varphi}^1, 2\pi \dot{r}^2 - n\lambda = \dot{\varphi}^2, 2\pi \dot{r}^3 - n\lambda = \dot{\varphi}^3, \cdots. \tag{99}$$

from (97). I get

$$2\pi r^\nu - n\lambda = \varphi^\nu \tag{100}$$

from (99). I get

$$2\pi \dot{r}^1 - n\lambda = \dot{\varphi}^1 \tag{101}$$

from (100) if I assume dimensionality 1. I express domain of $\dot{\varphi}^1$ in

$$0 \leq \dot{\varphi}^1 < \lambda. \tag{102}$$

I get

$$\frac{n\lambda}{2\pi} \leq \dot{r}^1 < \frac{(n+1)\lambda}{2\pi} \tag{103}$$

as domain of $\dot{r}^1$ from (101),(102). I get

$$\dot{F_1}' = \frac{M}{2} Sin\left(\frac{2\pi \dot{r}^1 - n\lambda}{2}\right) = \frac{M}{2} Sin\left(\pi \dot{r}^1 - \frac{n\lambda}{2}\right) \tag{104}$$

from (92) as (101). In the case of $n = 1$, I get

$$\dot{F_1}' = \frac{M}{2} Sin\left(\pi \dot{r}^1 - \frac{\lambda}{2}\right),$$

$$\frac{\lambda}{2\pi} \leq \dot{r}^1 < \frac{2\lambda}{2\pi} \tag{105}$$

from (103),(104) here. Furthermore, in the case of $n = 2$, I get

$$F_1{}^{\dot{}}{}' = \frac{M}{2} Sin\left(\pi \dot{r^1} - \frac{2\lambda}{2}\right),$$

$$\frac{2\lambda}{2\pi} \le \dot{r^1} < \frac{3\lambda}{2\pi} \quad (106)$$

from (103),(104). In the case of $n = 3$, I get

$$F_1{}^{\dot{}}{}' = \frac{M}{2} Sin\left(\pi \dot{r^1} - \frac{3\lambda}{2}\right),$$

$$\frac{3\lambda}{2\pi} \le \dot{r^1} < \frac{4\lambda}{2\pi} \quad (107)$$

from (103),(104). As described above, I can obtain it even if I arrive in the case of $1 \le n < \infty$. I show figure of (105) in Figure 2.

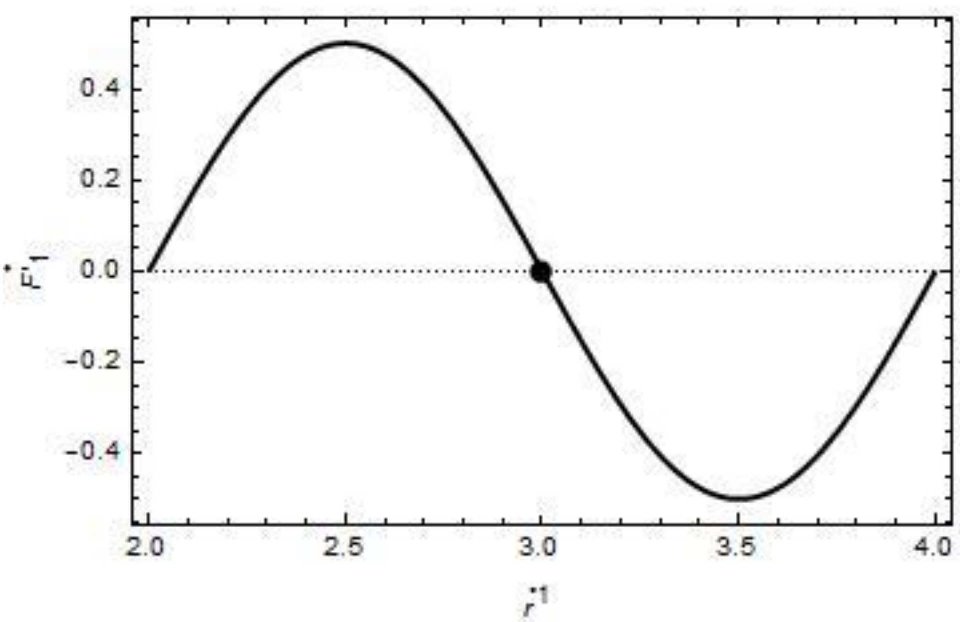

Figure 2. Plot for variable $\dot{r^1}$ of $\dot{F_1}' = \frac{M}{2} Sin\left(\pi \dot{r^1} - \frac{\lambda}{2}\right), \left(\frac{\lambda}{2\pi} \le \dot{r^1} < \frac{2\lambda}{2\pi}\right) : (M = 1, \lambda = 4\pi)$

The black dot expresses $\dot{r^1}$ satisfying $\dot{F_1}' = 0$, and this is negative divergence point in the field of force more.

When binary particle with the opposite charge is located each as distance r. Force F' which one particle receives is obtained as

$$F' = -\frac{k}{(r)^2} \quad (108)$$

in consideration of Definision12. I get

$$\dot{F_1}' = -\frac{k}{\left(\dot{r^1}\right)^2} = -k(\dot{r_1})^2, \dot{F_2}' = -\frac{k}{\left(\dot{r^2}\right)^2}, \dot{F_3}' = -\frac{k}{\left(\dot{r^3}\right)^2}, \cdots \quad (109)$$

from (108). I get

$$F_\nu' = -\frac{k}{(r^\nu)^2} \quad (110)$$

from (109). I get

$$\dot{F_1}' = -\frac{k}{(\dot{r^1})^2} \tag{111}$$

from (110) if I assume dimensionality 1. I show figure of (111) in Figure 3. I add (111) to (105) and get

$$\dot{F_1}' = \frac{M}{2} Sin\left(\pi \dot{r^1} - \frac{\lambda}{2}\right) - \frac{k}{(\dot{r^1})^2},$$

$$\frac{\lambda}{2\pi} \leq \dot{r^1} < \frac{2\lambda}{2\pi}. \tag{112}$$

I show figure of (112) in Figure 4. Similarly, I add (111) to (106) and get

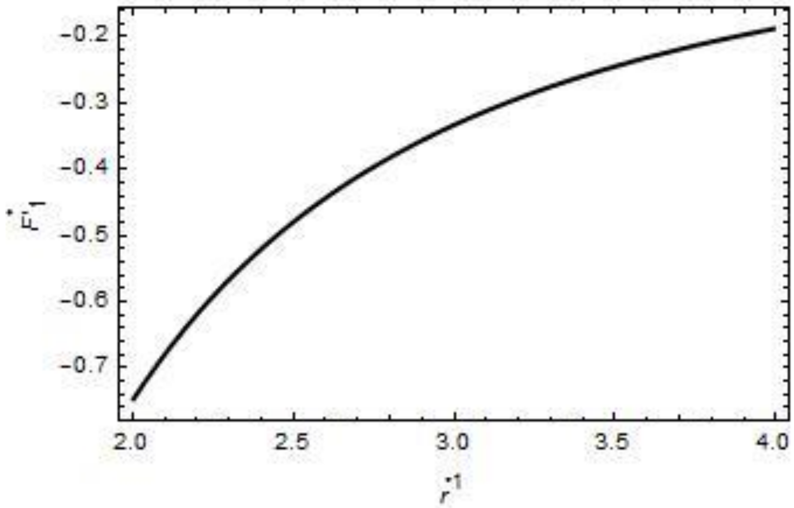

Figure 3. Plot for variable $\dot{r^1}$ of $\dot{F_1}' = -\frac{k}{(\dot{r^1})^2} : (k = 3)$

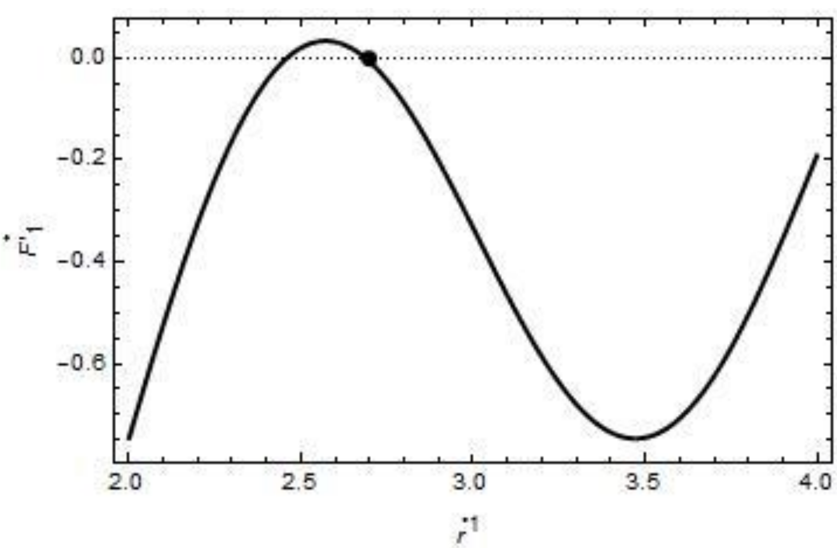

Figure 4. Plot for variable $\dot{r^1}$ of $\dot{F_1}' = \frac{M}{2} Sin\left(\pi \dot{r^1} - \frac{\lambda}{2}\right) - \frac{k}{(\dot{r^1})^2}, \left(\frac{\lambda}{2\pi} \leq \dot{r^1} < \frac{2\lambda}{2\pi}\right) : (k = 3, M = 1, \lambda = 4\pi)$

The black dot expresses $\dot{r^1}$ satisfying $\dot{F_1}' = 0$, and this is negative divergence point in the field of force more.

$$\dot{F_1}' = \frac{M}{2} Sin\left(\pi \dot{r^1} - \frac{2\lambda}{2}\right) - \frac{k}{\left(\dot{r^1}\right)^2},$$

$$\frac{2\lambda}{2\pi} \leq \dot{r^1} < \frac{3\lambda}{2\pi}. \tag{113}$$

I show figure of (113) in Figure 5. Similarly, I add (111) to (107) and get

$$\dot{F_1}' = \frac{M}{2} Sin\left(\pi \dot{r^1} - \frac{3\lambda}{2}\right) - \frac{k}{\left(\dot{r^1}\right)^2},$$

$$\frac{3\lambda}{2\pi} \leq \dot{r^1} < \frac{4\lambda}{2\pi}. \tag{114}$$

I show figure of (114) in Figure 6.

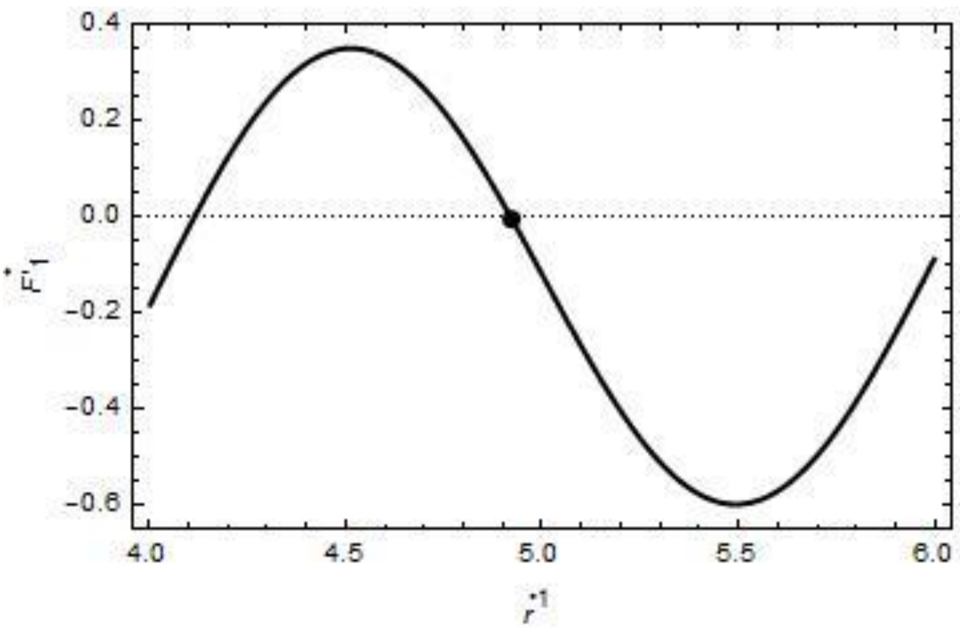

Figure 5. Plot for variable $\dot{r^1}$ of $\dot{F_1}' = \frac{M}{2} Sin\left(\pi \dot{r^1} - \frac{2\lambda}{2}\right) - \frac{k}{\left(\dot{r^1}\right)^2}, \left(\frac{2\lambda}{2\pi} \leq \dot{r^1} < \frac{3\lambda}{2\pi}\right) : (k = 3, M = 1, \lambda = 4\pi)$

The black dot expresses $\dot{r^1}$ satisfying $\dot{F_1}' = 0$, and this is negative divergence point in the field of force more.

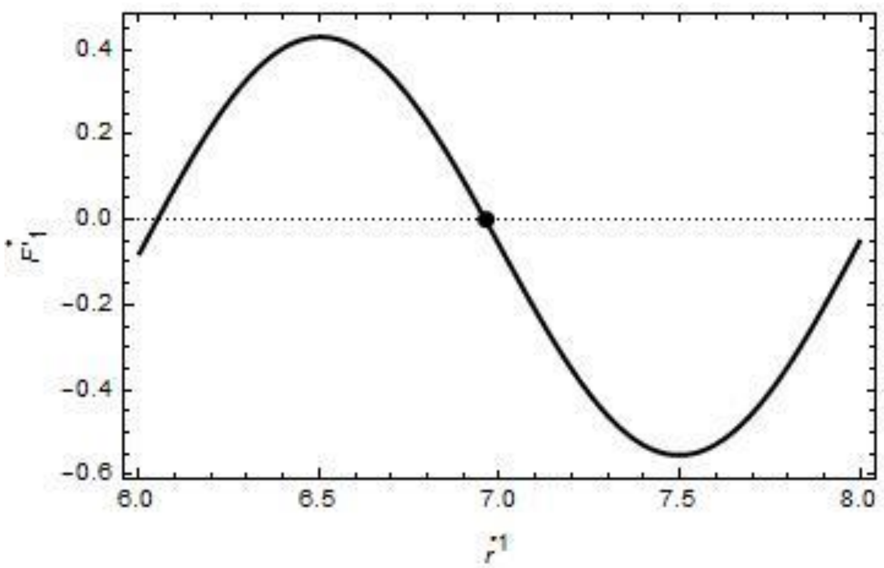

Figure 6. Plot for variable $\dot{r^1}$ of $\dot{F_1}' = \frac{M}{2} Sin = \left(\pi \dot{r^1} - \frac{3\lambda}{2}\right) - \frac{k}{\left(\dot{r^1}\right)^2}, \left(\frac{3\lambda}{2\pi} \leq \dot{r^1} < \frac{4\lambda}{2\pi}\right) : (k = 3, M = 1, \lambda = 4\pi)$

The black dot expresses $\dot{r^1}$ satisfying $\dot{F_1}' = 0$, and this is negative divergence point in the field of force more. I get

$$F_\mu = -\frac{\partial M}{\partial \varphi^\mu} \tag{115}$$

than μ, ν-inversion form of (89). I get

$$F_\mu{}' = -\frac{\partial M'}{\partial \varphi^\mu} = -\frac{\partial M\left(\frac{-1}{B}\right) Cos\left(\frac{-B\varphi^\mu}{2}\right)}{\partial \varphi^\mu} \tag{116}$$

from (81),(115). I rewrite (116) in consideration of (57) and get

$$F_\mu{}' = -\frac{\partial M\left(\frac{-1}{B}\right) Cos\left(\frac{-B\varphi^\mu}{2}\right)}{\partial \varphi^\mu} = -\frac{\partial M\left(\frac{-1}{B}(-B)\right) Cos\left(\frac{\varphi^\mu}{2}\right)}{\partial \varphi^\mu}. \tag{117}$$

I get

$$F_\mu{}' = -\frac{\partial M\left(\frac{-1}{B}(-B)\right) Cos\left(\frac{\varphi^\mu}{2}\right)}{\partial \varphi^\mu} = -\frac{\partial M}{\partial \varphi^\mu}\left(\frac{-1}{B}(-B)\right) Cos\left(\frac{\varphi^\mu}{2}\right)$$

$$-M\frac{\partial \frac{-1}{B}(-B)}{\partial \varphi^\mu} Cos\left(\frac{\varphi^\mu}{2}\right) - M\left(\frac{-1}{B}(-B)\right)\frac{\partial Cos\left(\frac{\varphi^\mu}{2}\right)}{\partial \varphi^\mu}$$

$$= \frac{M}{2}\left(\frac{-1}{B}(-B)\right) Sin\left(\frac{\varphi^\mu}{2}\right). \tag{118}$$

from (117),Proposition3. I rewrite (118) in consideration of (53) and get

$$F_\mu{}' = \frac{M}{2}\frac{-1}{B} Sin\left(\frac{-B\varphi^\mu}{2}\right). \tag{119}$$

If $F_\mu{}' = 0$ is established,

$$\frac{-B\varphi^\mu}{2} = n\pi \tag{120}$$

is established in consideration of (119),Definision8. I get

$$F_1{}' = \frac{M}{2}\sqrt{1-(\varphi^1)^2} Sin\left(\frac{1}{2}\frac{\varphi^1}{\sqrt{1-(\varphi^1)^2}}\right), \tag{121}$$

$$\frac{(\varphi^1)^2}{1-(\varphi^1)^2} = 4\pi n, \varphi^1 = \sqrt{\frac{4\pi n}{1+4\pi n}} \tag{122}$$

from (76),(119),(120) if I assume dimensionality 1. I show figure of (121) in Figure 7, Figure 8, Figure 9.

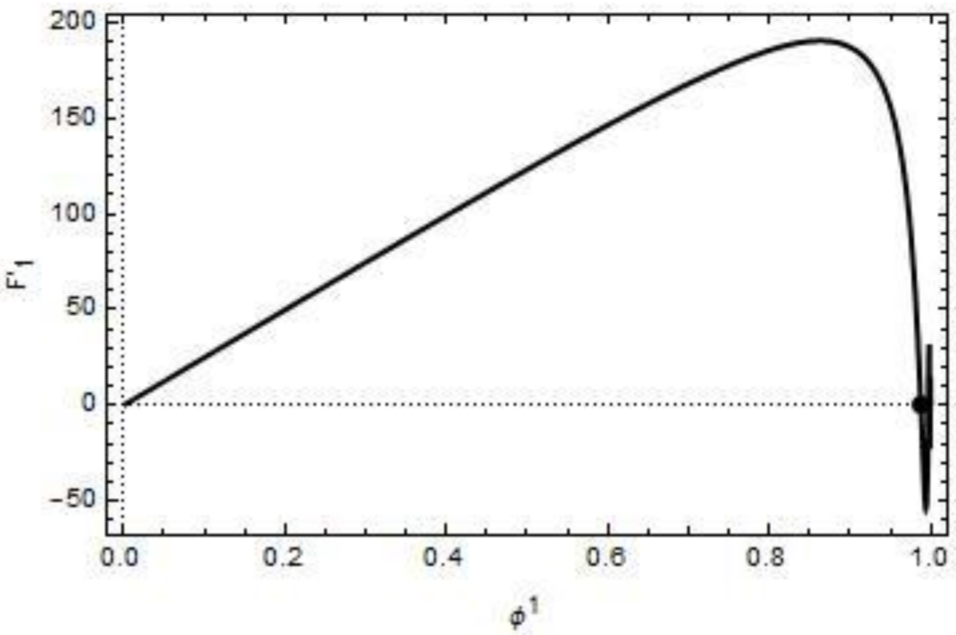

Figure 7. Plot for variable φ^1 of $F_1{}' = \frac{M}{2}\sqrt{1-(\varphi^1)^2}Sin\left(\frac{1}{2}\frac{\varphi^1}{\sqrt{1-(\varphi^1)^2}}\right), (0 \le \varphi^1 < 1): (M = 10^3)$

The black dot expresses φ^1 satisfying $F_1{}' = 0$, and this is negative divergence point in the field of force more.

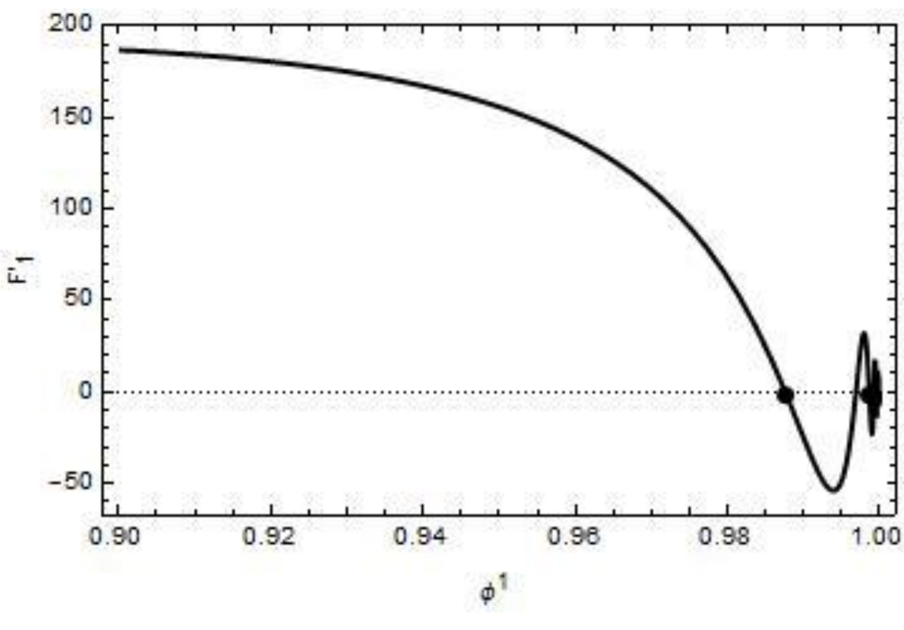

Figure 8. Plot for variable φ^1 of $F_1{}' = \frac{M}{2}\sqrt{1-(\varphi^1)^2}Sin\left(\frac{1}{2}\frac{\varphi^1}{\sqrt{1-(\varphi^1)^2}}\right), (0.9 \le \varphi^1 < 1): (M = 10^3)$

The black dot expresses φ^1 satisfying $F_1{}' = 0$, and this is negative divergence point in the field of force more.

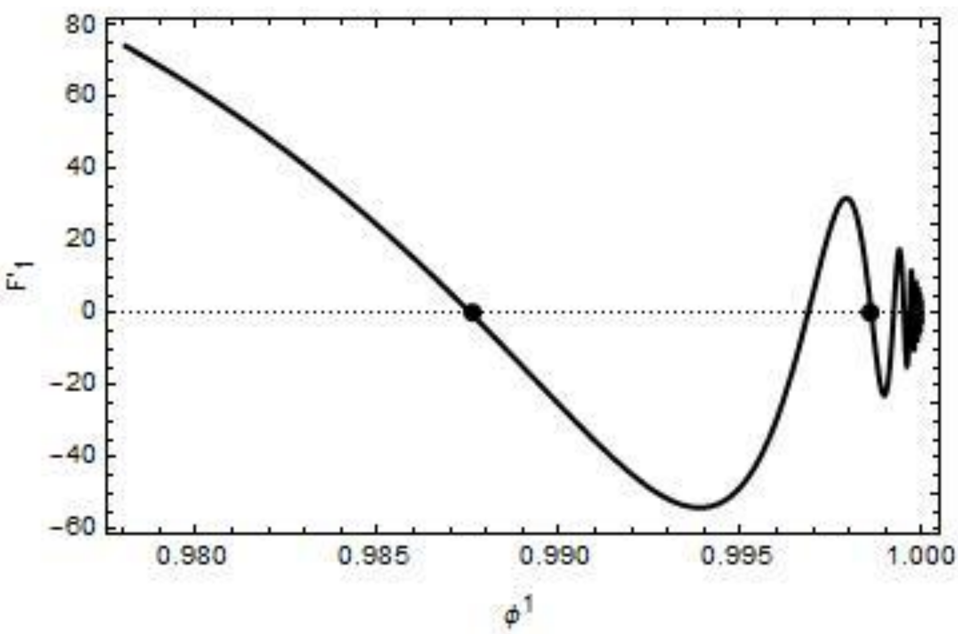

Figure 9. Plot for variable φ^1 of $F_1{}' = \frac{M}{2}\sqrt{1-(\varphi^1)^2}Sin\left(\frac{1}{2}\frac{\varphi^1}{\sqrt{1-(\varphi^1)^2}}\right), (0.978 \le \varphi^1 < 1): (M = 10^3)$

The black dot expresses φ^1 satisfying $F_1{}' = 0$, and this is negative divergence point in the field of force more.

The value of φ^1 satisfying $F_1{}' = 0$ exists innumerably according to (122). I show this in

$$\varphi^1 = 0, \sqrt{\frac{4\pi}{1+4\pi}}, \sqrt{\frac{8\pi}{1+8\pi}}, \sqrt{\frac{12\pi}{1+12\pi}}\sqrt{\frac{16\pi}{1+16\pi}}, \sqrt{\frac{20\pi}{1+20\pi}}, \cdots. \tag{123}$$

In Figure 7,Figure 8,Figure 9, Positive divergence point in the field of force exists in (123). I show this in

$$\varphi^1 = 0, \sqrt{\frac{8\pi}{1+8\pi}}, \sqrt{\frac{16\pi}{1+16\pi}}, \cdots. \tag{124}$$

Negative divergence point in the field of force exists in (123). I show this in

$$\varphi^1 = \sqrt{\frac{4\pi}{1+4\pi}}, \sqrt{\frac{12\pi}{1+12\pi}}, \sqrt{\frac{20\pi}{1+20\pi}}, \cdots. \tag{125}$$

When binary particle with the same charge is located each as distance φ. Force F'which one particle receives is obtained as

$$F' = \frac{k2}{(\varphi)^2} \tag{126}$$

in consideration of Definision12. I get

$$F_1' = \frac{k2}{(\varphi^1)^2} k2 = (\varphi_1)^2, F_2' = \frac{k2}{(\varphi^2)^2}, F_3' = \frac{k2}{(\varphi^3)^2}, \cdots \tag{127}$$

from (126). I get

$$F_\mu' = \frac{k2}{(\varphi^\mu)^2} \tag{128}$$

from (127). I get

$$F_1' = \frac{k2}{(\varphi^1)^2} \tag{129}$$

from (128) if I assume dimensionality 1. I show figure of (129) in Fig.10.

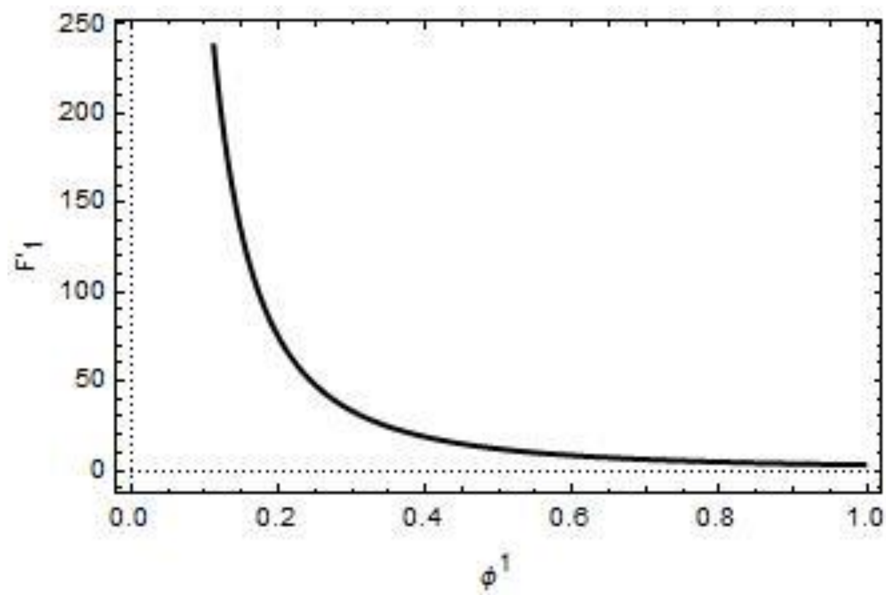

Figure 10. Plot for variable φ^1 of $F_1{}' = -\frac{k2}{(\varphi^1)^2}, (0 \leq \varphi^1 < 1): (k2 = 3)$

I add (129) to (121) and get

$$F_1{}' = \frac{M}{2}\sqrt{1-(\varphi^1)^2}Sin\left(\frac{1}{2}\frac{\varphi^1}{\sqrt{1-(\varphi^1)^2}}\right) + \frac{k2}{(\varphi^1)^2}. \tag{130}$$

I show figure of (130) in Figure 11, Figure 12.

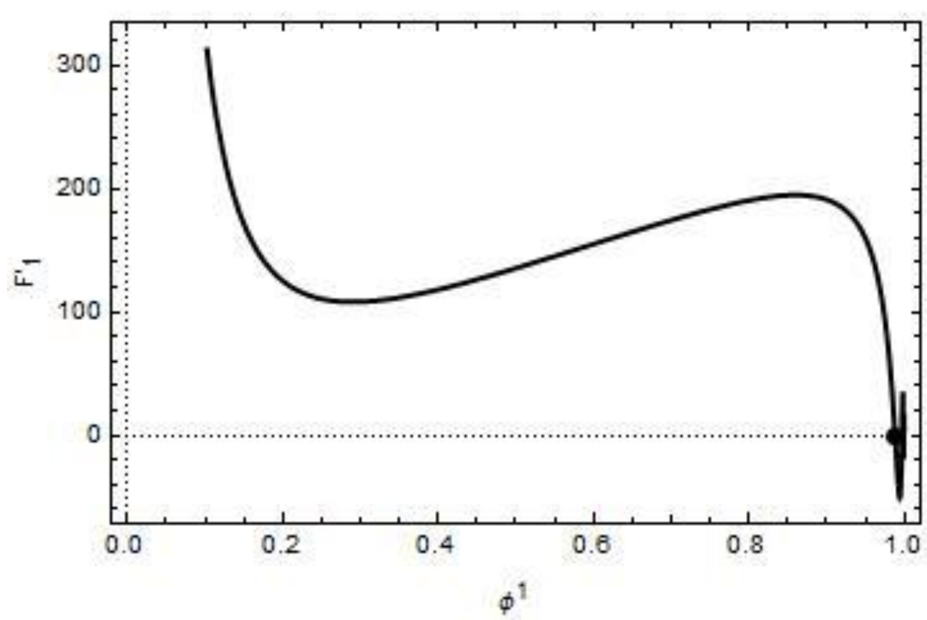

Figure 11. Plot for variable φ^1 of $F_1{}' = \frac{M}{2}\sqrt{1-(\varphi^1)^2}Sin\left(\frac{1}{2}\frac{\varphi^1}{\sqrt{1-(\varphi^1)^2}}\right) + \frac{k2}{(\varphi^1)^2}, (0 < \varphi^1 < 1): (M = 10^3, k2 = 3)$

The black dot expresses φ^1 satisfying $F_1{}' = 0$, and this is negative divergence point in the field of force more.

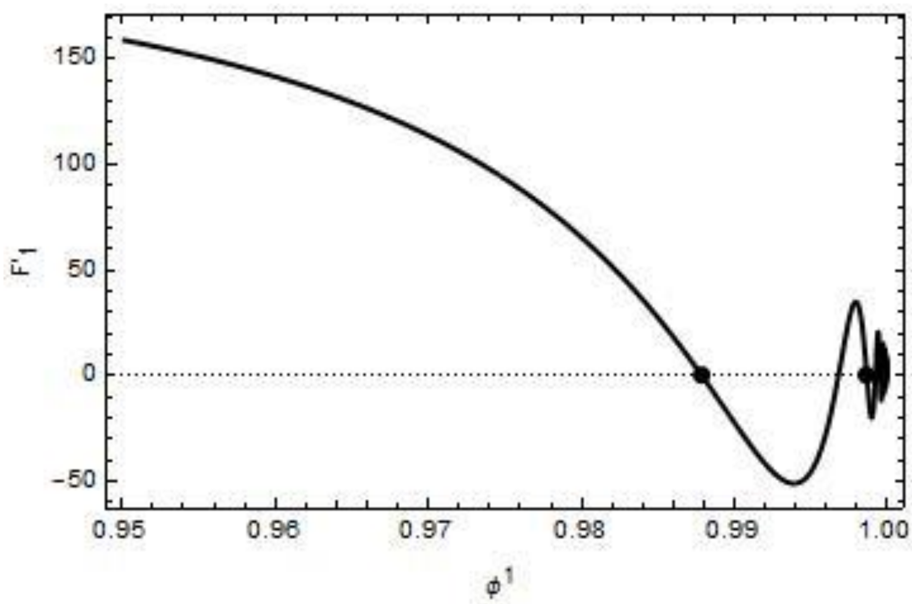

Figure 12. Plot for variable φ^1 of $F_1{}' = \frac{M}{2}\sqrt{1-(\varphi^1)^2}Sin\left(\frac{1}{2}\frac{\varphi^1}{\sqrt{1-(\varphi^1)^2}}\right) + \frac{k2}{(\varphi^1)^2}, (0.95 \le \varphi^1 < 1): (M = 10^3, k2 = 3)$

The black dot expresses φ^1 satisfying $F_1{}' = 0$, and this is negative divergence point in the field of force more.

7. Discussion

About Figure 4, Figure 5, Figure 6

When binary particle with the opposite charge is located each as distance $\dot{r^1}$. It is decided that I assume binary particle with atomic nucleus and electron each. The domain of $\dot{r^1}$ is $\left(\frac{n\lambda}{2\pi} \le \dot{r^1} < \frac{(n+1)\lambda}{2\pi}\right): \left(0 < n \le \infty\right)$ in (104). In other words, the field of force exists to infinity. The black dot expresses $\dot{r^1}$ satisfying $\dot{F_1}{}' = 0$, and this is negative divergence point in the field of force more. And these exist innumerably. Only one points is displayed in Figure 6 here. The electron satisfies $\dot{F_1}{}' = 0$ in $\dot{r^1}$ of the black dot, and the momentum does not change. When the electron satisfies $\dot{F_1}{}' \ne 0$ any place other than $\dot{r^1}$ of the black dot, the momentum changes. If electron moves from black dot point only to $\pm d\dot{r}^1$, the direction of force $\dot{F_1}{}' \ne 0$ which electron receives intends to be pulled back to a black dot. It is thought that the electron continues vibrating in the range of $\pm d\dot{r}^1$ for $\dot{r^1}$ of the black dot. As the electron has charge, an electromagnetic wave is caused by vibration. And the electron will stay in $\dot{r^1}$ of the black dot because energy dissipation occurs.

When it was only (111), the force to act on electron in atom was recognized. I report (105) as force to act on electron in atom in this article other than (111). When electron stays in steady orbit in atom, the force that electron receives must be zero. Force (111) is denied by existence of force (105). Thus, the force that electron receives can become the zero. In other words, it is interpretability that electron stays in steady orbit in atom.

About Figure 11, Figure12

When binary particle with the equal charge is located each as distance φ^1. It is decided that I suppose binary particle to be proton each. The domain of φ^1 is $\{0 < \varphi^1 \le \infty\}$ in (129). In other words, the field of force exists to infinity. In contrast, The domain of φ^1 is $\{0 \le \varphi^1 < 1\}$ in (121). In other words, the outreach of the field of force is limited. The black dot expresses φ^1 satisfying $F_1{}' = 0$, and this is negative divergence point in the field of force more. And these exist innumerably. Only two points are displayed in Figure 12 here. The proton satisfies $F_1{}' = 0$ in φ^1 of the black dot, and the momentum does not change. When the proton satisfies $F_1{}' \ne 0$ any place other than φ^1 of the black dot, the momentum changes. If proton moves from black dot point only to $\pm d\varphi^1$, the direction of force $F_1{}' \ne 0$ which proton receives intends to be pulled back to a black dot. It is thought that the proton continues vibrating in the range of $\pm d\varphi^1$ for φ^1 of the black dot. As the proton has charge, an electromagnetic wave is caused by vibration. And the proton will stay in φ^1 of the black dot because energy dissipation occurs.

Competing interests

The competition does not occur between other researchers by releasing this article.

Informed consent

Obtained.

Ethics approval

The Publication Ethics Committee of the Canadian Center of Science and Education.

The journal and publisher adhere to the Core Practices established by the Committee on Publication Ethics (COPE).

Provenance and peer review

Not commissioned; externally double-blind peer reviewed.

Data availability statement

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

Data sharing statement

No additional data are available.

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