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Comprehensive Bicubic Formalism of Ordinary and Novel (dm) Particles With Substituted Virtual Novel Electron Describing the Bohr Atom

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Abstract

The particle limiting velocity solutions from the bicubic limiting velocity equation (Šoln, J., 2021.1.2, 2022) have been very useful in carrying out parallel studies of ordinary and novel particles. This study is facilitated with the help of evolutionary congruent parameters, ordinary$z_1 \leq 1$, novel $z_2 = 1/z_1 \geq 1$ and congruent phase-angle $\alpha \leq \pi/2$. At smooth matching point $z_1 = z_2 = 1$ and $\alpha = \pi/2$, there is no physical difference between an ordinary and novel particle as they have the same limiting velocities. At $\alpha = \pi/3 < \pi/2$, they have already different limiting velocities with different other physical characteristics. Ordinary photon γ and novel doubly photon γ_N are indistinguishable at $\alpha = \pi/2$, while at $\alpha = \pi/3 < \pi/2$, they are already physically different. That applies to any other particle, ordinary proton p versus novel proton p_N, etc. Through trial and error, one finds that c_3 the ordinary particle limiting velocity solution has ratio value of 1 at the comprehensive congruent phase-angle $\alpha_O = \pi/5.1 = 0.616$ which an ordinary particle of velocity v can satisfy at $v \simeq c_3$, in fact, defining the value of c_3. The quantum jump from the ordinary particle comprehensive congruent phase-angle α_O to the novel particle comprehensive congruent phase-angle $\alpha_N = 2\alpha_O = 2\pi/5.1 = 1.232$ yields novel particle limiting velocity solution *doubly* $Rc_{1,3}$ with ratio value of 1 which a novel particle velocity v_N can satisfy with *doubly*$Rc_{1,3} \simeq v_N$, simply defining the value of *doubly* $Rc_{1,3}$. This comprehensive quantum jump with fixed comprehensive congruent phase-angles α_O and α_N apply to any free and interacting ordinary particle (γ, e, ν,etc.) when connecting to the corresponding free and interacting novel particle (γ_N, e_N, ν_N, etc.). Concerning the Bohr's atom, we wish to address the question of negative energy electron e emitting positive energy radiation. To this end we simply substitute e with the virtual novel electron e_N with the same radius r and velocity v. This virtual e_N, due to the Coulomb potential energy V from the sitting proton p plus the centrifugal force, with the quantized orbital virtual novel electron angular momentum yields the positive virtual novel electron energy in the bicubic formalism. The frequencies of emitted radiation from virtual novel electron is practically the same as from the negative energy electron e.

Keywords: limiting velocity, energy, congruent parameter, novel, dark matter particle, quantum jump

1. Introduction

The astrophysical observations indicate that the Universe contains substantial fraction of dark mater or novel particles. A large portion of this d.m. particles is in sub-GeV energy region (Adari, P. et al., 2023). Other gravitational studies with empirical like methods further point to the existence of dark matter (Clowe, D. et al., 2006). The creation of d.m. novel particles is believed to be caused also by creation of black holes (Curd, B. et al., 2024). In (Abe, S. et al., 2023), one pursues a WIMP charged d.m. particle that would form a bound state with a nucleus with insignificant results. Hence, it would appear that dark matter has only gravitational interactions even between themselves (the WIMP d.m.weakly interacting massive particle contains neutral χ^0 and excited charged χ^-states). Other attempts to incorporate novel (dm) particles into the Standard Model usually also yield nebulous results.For example, In the search for Light Dark Photon (Yeong, G-K., et al., 2023) extend the Standard Model with the additional U(1) gauge field which with another U(1) gauge field, after proper redefinitions yield the massless electromagnetic photon gauge field A_μ and massive novel photon gauge field $A_{N\mu}$ which as yet, it has not been obsereved. If this novel photon $A_{N\mu}$ was available it would manifest itself through the Feynman diagrams in the calculations of reaction $\gamma\gamma \rightarrow e^+e^-$ (Xu, I. et al., 2022), whose predictions have not been yet observed. As long as one stays just with the novel (dm) particles, the gravitational studies and empirical like methods can give the proof of the existence of dark matter (Clowe, D. et al. (2006). It appears that novel or d.m. paticles simply cannot get adjusted partially or completely to the S.M. methodology when describing their possible interactions. As we know the S.M. basically can describe the interactions with what we call the ordinary particles. The bicubic formalism developed over some time, (Šoln, J., 2014, 16, 17), (Šoln, J., 2018.1.2), (Šoln, J., 2019, 20), (Šoln, J., 2021.1.2), (Šoln, J., 2022.1.2) and (Šoln, J., 2023), carries out parallel studies of ordinary and novel particles through solutions of the bicubic equation for particle limiting velocities. Through these solutions we are looking for the simplest kinematical relationships between ordinary and novel (dm) particles, reegardless whether they are free or interacting. In this endeavor the most useful solutions are the ones from (Šoln, J., 2021.1.2; 2022.1.2; 2023). The description of relationships between ordinary and novel (dm) particles,is carried out with the help of evolutionary congruent parameters, congruent phase-angle $\alpha \leq \pi/2$ plus respectively ordinary and novel congruent parameters $z_1 \leq 1$ and $z_2 = 1/z_1 \geq 1$. At smooth matching point $z_1 = z_2 = 1$ and $\alpha = \pi/2$, there is no physical difference between an ordinary and novel particle as they have the same limiting velocities.With the specific congruent phase-angle, say $\alpha(e)$ in (Šoln,J., 2022.1.2) is described the individual quantum jump from an ordinary electron e to a novel matched electron e_N , $e \longrightarrow e_N$. Similarly for other ordinary particles, such as ν, γ, p,etc. to matched novel particle ν_N, γ_N, p_N,etc.. Here, we generalize this to comprehensive quantum jump, valid simultaneously for all ordinary and novel matched particles: Ordinary particles with comprehensive congruent phase-angle $\alpha_O = \pi/5.1$ quantum jump to novel particles with comprehensive congruent phase-angle $\alpha_N = 2\alpha_O = 2\pi/5.1 = 1.232$. Already at this level, being able evaluating a novel particle velocity only from the point of view of an ordinary particle, we see that in order to see effects of novel particles we have to take into account similarities, differences with substitutions causing interrelationships between ordinary and novel particles. For example in Bohr atom (see Eisberg, R. M., 1966, p.115)) we substituted the ordinary electron e with the novel virtual electron e_N in order to get positive electron energy at the end. At least theoretically, this suggest that novel (dm) particle could interact not just among themselves but also with ordinary particles. In other words, the interaction involvement of novel particles is facilitated by the presence of ordinary particles.

Because here we introduced few novel ideas, it is appropriate to show in some details how through solutions of the bicubic equation for particle limiting velocities the simple kinematical relationships between ordinary and novel (dm) particles, which may be free or interacting. Hence, we go directly to upgrading the relativistic kinematics by combining the Einstein's "mass-shell" energy with the particle linear mornentum (Šoln, J. (2014-2023)) to end up with particle limiting velocity c bicubic equation

$$\left(\frac{c^2}{v^2}\right)^3 - \left(\frac{E}{mv^2}\right)^2\left(\frac{c^2}{v^2}\right) + \left(\frac{E}{mv^2}\right)^2 = 0 \tag{1}$$

where c is particle limiting velocity, E is particle energy, m is real particle mass and v^2 particle velocity squared. As in (Šoln, J. (2021.1.2; 2022; 2023)), by relating the energy to real linear congruent parameter z we obtain the Energy and Discriminant for (1) simply as:

$$\begin{aligned} E &= \frac{3\sqrt{3}mv^2}{2z}, z = \frac{3\sqrt{3}mv^2}{2E}; \\ D(z) &= \frac{1}{4}\left(\frac{E}{mv^2}\right)^4\left[1 - \frac{4}{27}\left(\frac{E}{mv^2}\right)^2\right] = \left(\frac{27}{8z^3}\right)^2 (z+1)(z-1) \end{aligned} \tag{2}$$

At this point we designate congruent parameters with kind of particle limiting velocity solutions from (1) (Šoln, J. (2023)):

$$\alpha = 2\tan^{-1}\left(\tan\left(\frac{1}{2}\sin^{-1}\frac{1}{z_2}\right)\right)^{1/3} = 2\tan^{-1}\left(\tan\left(\frac{1}{2}\sin^{-1}z_1\right)\right)^{1/3}, \alpha \leq \frac{\pi}{2} \tag{3}$$

$$z_1(\alpha) = \frac{1}{z_2(\alpha)} = \sin\left[2\tan^{-1}\left(\tan(\frac{\alpha}{2}\right)^3\right], z_1(\alpha) \leq 1, z_2(\alpha) \geq 1 \tag{4}$$

$$z_1(\alpha) = \frac{1}{z_2(\alpha)} = \frac{\sin^3(\alpha)}{4 - 3\sin^2(\alpha)}. \tag{5}$$

The relations (3), (4) and (5) involve real congruent evolution parameters, congruent phase-angle α, linear congruent parameters $z_1(\alpha)$ and $z_2(\alpha)$. Relations (3) and (4) show how they are functionally related to each other, with $z_1(\alpha) \leq 1, z_2(\alpha) \geq 1$, and $\alpha \leq \frac{\pi}{2}$. In (5) $z_1(\alpha)$ is numerically the same as in (4). It was derived by calculating E directly from (1) involving novel limiting velocities and here it is listed for the sake of completeness.

Utilizing (3) and (4), we easily write down the discriminants associated with the ordinary and novel (dm) particles:

$$\begin{aligned} Ordinary\ particles &: D(z_1(\alpha)) \le 0, \alpha \le \frac{\pi}{2}, E(\alpha) = \frac{3\sqrt{3}mv^2}{2z_1(\alpha)} \\ Novel(dm)\ particles &: D(z_2(\alpha)) \ge 0, \alpha \le \frac{\pi}{2}, E(\alpha) = \frac{3\sqrt{3}mv^2}{2z_2(\alpha)} \end{aligned} \tag{6}$$

The relation (6) together with (3) and (4) indicate that the solutions for ordinary and novel particle limiting velocities can be given in terms of one evolutionary parameter, $z_1(\alpha)$, $z_2(\alpha)$ or simply phase-angle α. The energies in (6) are global ordinary and novel particle energies. They are simple in a sense that they give the results if one knows for each of them the particle mas m and particle velocity v which may even come from classical or quantum physics. Numerically the energies are known with insertions of the congruent parameters $z_1(\alpha)$ and $_2(\alpha)$. The more complex energy expressions utilizing explicitly particle limiting velococities (Šoln, J.(2016)) will not be used here.

$$The\ smooth\ matching\ point : D(z_1(\alpha) = z_2(\alpha) = 1) = 0; \alpha = \frac{\pi}{2} \tag{7}$$

At smooth matching point both limiting velocity solutions for ordinary and novel particles smoothly become equal. As $\alpha \le \frac{\pi}{2}$, these velocities diverge in different directions, indicating the difference between ordinary and novel particles. Equally important matching will occur between similar ordinary and novel particles once we relate them with ordinary particle comprehensive congruent phase-angle α_O quantum jump to novel particle comprehensive congruent phase-angle $\alpha_N = 2\alpha_O$. The numerical values of α_O and α_N will be given once the limiting velocity solutions are derived. These comprehensive quantum jumps from α_O to α_N and reverse, between ordinary and novel particles apply to each ordinary and matched novel particle either free or interacting. Mathematical details of smooth matching point are given in Section 3. with the explicit particle limiting velocity solutions from Section 2.

2. Details of the Bicubic Particle Limiting Velocity Solutions for Ordinary and Novel (dm) Particles

We start with the limiting velocity solutions for ordinary particles, satisfying (6), in the original form (Šoln, J., 2021.a.b; 2022; 2023):

$$\begin{aligned} \frac{c_1^2(\alpha)}{v^2} &= \frac{3}{z_1(\alpha)} \sin\left[\frac{1}{3}(\pi - \sin^{-1} z_1(\alpha)\right] > 0, \\ \frac{c_2^2(\alpha)}{v^2} &= -\frac{3}{z_1(\alpha)} \sin\left[\frac{1}{3}(\pi + \sin^{-1} z_1(\alpha)\right] < 0, \\ \frac{c_3^2(\alpha)}{v^2} &= \frac{3}{z_1(\alpha)} \sin\left[\frac{1}{3} \sin^{-1} z_1(\alpha)\right] > 0. \end{aligned} \tag{8}$$

With the help of relations (3) and (4) we present the limiting velocity solutions for ordinary particles from (8) in form emphasizing the congruent phase-angle α :

$$\begin{aligned} C(\alpha) &= \frac{\sqrt{3}}{2}\cos\left[\frac{1}{3}\sin^{-1} z_1(\alpha)\right] = \frac{\sqrt{3}}{2}\cos\left[\frac{2}{3}\tan^{-1}\left(\tan(\frac{\alpha}{2}\right)^3\right], \\ S(\alpha) &= \frac{1}{2}\sin\left[\frac{1}{3}\sin^{-1} z_1(\alpha)\right] = \frac{1}{2}\sin\left[\frac{2}{3}\tan^{-1}\left(\tan(\frac{\alpha}{2}\right)^3\right], \end{aligned} \tag{9}$$

$$\begin{aligned} \frac{c_1^2(\alpha)}{v^2} &= \frac{3}{z_1(\alpha)}\left[C(\alpha) - S(\alpha)\right] = \frac{3\left[C(\alpha) - S(\alpha)\right]}{\sin\left[2\tan^{-1}\left(\tan(\frac{\alpha}{2}\right)^3\right]}, \\ \frac{c_2^2(\alpha)}{v^2} &= -\frac{3}{z_1(\alpha)}\left[C(\alpha) + S(\alpha)\right] = -\frac{3\left[C(\alpha) + S(\alpha)\right]}{\sin\left[2\tan^{-1}\left(\tan(\frac{\alpha}{2}\right)^3\right]}, \\ \frac{c_3^2(\alpha)}{v^2} &= \frac{3}{z_1(\alpha)} 2S(\alpha) = \frac{6S(\alpha)}{\sin\left[2\tan^{-1}\left(\tan(\frac{\alpha}{2}\right)^3\right]}. \end{aligned} \tag{10}$$

Both solutions satisfy the Cardano's relation $c_1^2(\alpha)$+ $c_2^2(\alpha) + c_3^2(\alpha) = 0$.

The limiting velocity solutions for novel particles, satisfying (6), in the original form are in rather lengty expressions:

$$\begin{aligned}
\frac{c^2_{1,3}(\alpha)}{v^2} &= \frac{3}{2z_2(\alpha)}\csc 2\tan^{-1}\left(\tan\left(\frac{1}{2}\sin^{-1}\left(\frac{1}{z_2(\alpha)}\right)\right)\right)^{1/3}, \\
&\pm i\frac{3\sqrt{3}}{2z_2(\alpha)} ctn 2\tan^{-1}\left(\tan\left(\frac{1}{2}\sin^{-1}\left(\frac{1}{z_2(\alpha)}\right)\right)\right)^{1/3}; \\
\frac{c^2_{2}(\alpha)}{v^2} &= -\frac{3}{z_2(\alpha)}\csc 2\tan^{-1}\left(\tan\left(\frac{1}{2}\sin^{-1}\left(\frac{1}{z_2(\alpha)}\right)\right)\right)^{1/3}.
\end{aligned} \tag{11}$$

The solutions (11) are good examples of consistent self-generating evolutionary system with $z_1(\alpha)$, $z_2(\alpha)$ and α which appears quite naturally in $c^2_{1,3}(\alpha)/v^2$ of (11) (compare with (3)). Utilizing (3) and (4) , changes (11) in to

$$\begin{aligned}
\frac{c^2_{1,3}(\alpha)}{v^2} &= \frac{3\left[1 \pm i\sqrt{3}\cos(\alpha)\right]}{2z_2(\alpha)\sin(\alpha)} = \frac{Rc^2_{1,3}(\alpha)}{v^2} + i\frac{Ic^2_{1,3}(\alpha)}{v^2}, \\
\frac{doubly\ Rc^2_{1,3}(\alpha)}{v^2} &= \frac{3}{2z_2(\alpha)\sin(\alpha)},\ \frac{Ic^2_{1,3}(\alpha)}{v^2} = \pm\frac{3\sqrt{3}\cos(\alpha)}{2z_2(\alpha)\sin(\alpha)}, \\
\frac{c^2_{2}(\alpha)}{v^2} &= -\frac{3}{z_2(\alpha)\sin(\alpha)}; \\
Rc^2_1(\alpha) &= Rc^2_3(\alpha),\ Ic^2_3(\alpha) = -Ic^2_1(\alpha), c^2_2(\alpha) = -2Rc^2_{1,3}(\alpha) \\
Rc_{1,3}(\alpha) &= \sqrt{Rc^2_{1,3}(\alpha)},\ \ Ic_{1,3}(\alpha) = \sqrt{Ic^2_{1,3}(\alpha)}
\end{aligned} \tag{12}$$

We can rewrite relations (12) to better emphasize the dependence on the congruent phase-angle α :

$$\begin{aligned}
\frac{c^2_{1,3}(\alpha)}{v^2} &= \frac{Rc^2_{1,3}(\alpha)}{v^2} + i\frac{Ic^2_{1,3}(\alpha)}{v^2}, \\
\frac{doubly\ Rc^2_{1,3}(\alpha)}{v^2} &= \frac{3\sin\left[2\tan^{-1}\left(\tan(\frac{\alpha}{2}\right)^3\right]}{2\sin(\alpha)}, \\
\frac{Ic^2_{1,3}(\alpha)}{v^2} &= \pm\frac{\sqrt{3}3\sin\left[2\tan^{-1}\left(\tan(\frac{\alpha}{2}\right)^3\right]\cos(\alpha)}{2\sin(\alpha)} = \pm\sqrt{3}\frac{Rc^2_{1,3}(\alpha)}{v^2}\cos(\alpha), \\
\frac{c^2_{1,3}(\alpha)}{v^2} &= \frac{Rc^2_{1,3}(\alpha)}{v^2}(1 \pm i\sqrt{3}\cos(\alpha)) \\
\frac{c^2_{2}(\alpha)}{v^2} &= -\frac{3\sin\left[2\tan^{-1}\left(\tan(\frac{\alpha}{2}\right)^3\right]}{\sin(\alpha)} = -2\frac{doubly\ Rc^2_{1,3}(\alpha)}{v^2}
\end{aligned} \tag{13}$$

At this point we wish to epmphasize the number of Cardano's relations which follow from (11), (12) and (13)

$$\begin{aligned}
\frac{Rc^2_3(\alpha)}{v^2} + \frac{Rc^2_1(\alpha)}{v^2} + \frac{c^2_2(\alpha)}{v^2} &= 0, \\
\frac{Ic^2_{,3}(\alpha)}{v^2} + \frac{Ic^2_{,1}(\alpha)}{v^2} &= 0, \\
\frac{c^2_3(\alpha)}{v^2} + \frac{c^2_1(\alpha)}{v^2} + \frac{c^2_2(\alpha)}{v^2} &= 0.
\end{aligned} \tag{14}$$

The importance of these relations for (8,10,11,12) comes from the fact that in each group the elements are complement physically to each other and, as such, can contribute to enclosed physical picture. For instance, $c^2_2(\alpha)/v^2$ complements physically *doubly* $Rc^2_{3,1}(\alpha)/v^2$, by giving maximal real novel particle velocities. Similarly $c^2_{3,1}(\alpha)/v^2$, $c^2_2(\alpha)/v^2$ and $Ic^2_{,3,1}(\alpha)/v$ together give unphysical fudge novel particle velocities that will be discussed later.

3. Smooth Matching Point at the Congruent Phase-angle $\alpha = \pi/2$

Next, using relations (3) and (4) together with ordinary and novel particle limiting velocity solutions (10), (11) and (12) we wish to verify the smooth matching point between the ordinary and novel particles at $\alpha = \pi/2$. The meaning of the smooth matching point is very simple: There are no differences between the ordinary and novel particle limiting velocities as particles have the same mass m and same velocity v at $\alpha = \pi/2$. Physically they are the same. Beyond $\alpha = \pi/2$ things change dramatically, ordinary and novel particles disengage completely. Thus, using relations (5), (6) an (8), we wish to verify the smooth matching point between the ordinary and novel particle limiting velocity squares at $\alpha = \pi/2$, where one should notice that there are no imaginary contributions for novel particle limiting velocity squares. Next, in Tables 1 through 5, we give simple mathematical examples that as soon as the congruent phase-angle α moved from $\alpha = \pi/2$ to $\alpha = \pi/3$, the limiting velocities of ordinary and novel particles move from their equalities to the distinct inequalities.

Table 1. Ordinary particle limiting velocity squares at smooth matching point of $z_1 = 1, \alpha = \pi/2$

$$\begin{pmatrix} c_1^2(\alpha)/v^2 & c_2^2(\alpha)/v^2 & c_3^2(\alpha)/v^2 \\ 1.5 & -3 & 1.5 \end{pmatrix}$$

Table 2. Novel particle limiting velocity squares at smooth matching point of $z_2 = 1, \alpha = \pi/2$

$$\begin{pmatrix} \begin{matrix} c_1^2(\alpha)/v^2 \\ = Rc_1^2(\alpha)/v^2 + i0 \end{matrix} & c_2^2(\alpha)/v^2 & \begin{matrix} c_3^2(\alpha)/v^2 \\ = Rc_3^2(\alpha)/v^2 + i0 \end{matrix} \\ 1.5 & -3 & 1.5 \end{pmatrix}$$

Clearly, at the smooth matching point of $\alpha = \pi/2$, limiting velocity squares of ordinary and novel particles have the same values. To show how fast these values diverge away from $\alpha/2$, we now take $\alpha = \pi/3$:

Table 3. Ordinary particle limiting velocity squares at $z_1 = 0.371, \alpha = \pi/3$

$$\begin{pmatrix} c_1^2(\alpha)/v^2 & c_2^2(\alpha)/v^2 & c_3^2(\alpha)/v^2 \\ 6.436 & -7.458 & 1.022 \end{pmatrix}$$

Table 4. Novel (dm) particle limiting velocity squares at $z_2 = 1/z_1 = 2.695, \alpha = \pi/3$

$$\begin{pmatrix} \begin{matrix} c_1^2(\alpha)/v^2 \\ = Rc_1^2(\alpha)/v^2 + iIc_1^2(\alpha)/v^2 \end{matrix} & c_2^2(\alpha)/v^2 & \begin{matrix} c_3^2(\alpha)/v^2 \\ = Rc_3^2(\alpha)/v^2 + iIc_3^2(\alpha)/v^2 \end{matrix} \\ 0.643 + i0.557 & -1.286 & 0.643 - i0.557 \end{pmatrix}$$

It is immediately evident to the large differences in limiting velocities between the matched ordinary and novel particles as we move away from $\alpha = \pi/2$ to $\alpha = \pi/3$. One should notice that at $\alpha = \pi/3$ while for ordinary particles the limiting velocities c_1, c_2 and c_3 are all larger than the particle velocity v, the opposite is true for the novel particle limiting velocities $doublyRc_{1,3}^2$ which are smaller than v^2 with exception for c_2^2 which is larger tan v^2. Next, in Table 5, we wish show how the change in congruent parameters affect the Global "pseudo-relativistic" kinetic energies from (6).

Table 5. The values of ordinary and novel Global particle"pseudo-relativistic" energies at $\alpha = \pi/2$ and $\pi/3$ with fixed mass m and velocity v

$$E(z_1(\alpha)) = \frac{3\sqrt{3}mv^2}{2}(1/z_1)\ ,\ E(z_2(\alpha)) = \frac{3\sqrt{3}mv^2}{2}(1/z_2):$$

$$\begin{pmatrix} \alpha & \pi/2 & \pi/3 \\ E(z_1(\alpha)) & \frac{3\sqrt{3}}{2}mv^2 & 2.694\frac{3\sqrt{3}}{2}mv^2 \\ E(z_2(\alpha)) & \frac{3\sqrt{3}}{2}mv^2 & 0.371\frac{3\sqrt{3}}{2}mv^2 \end{pmatrix}$$

These exemplary energy cases demonstrate dramatic change in energy from ordinary to novel particle if $z_2 \gg 1$.

4. Comprehensive Quantum Matching Jump Between Ordinary and Novel Particles

Before getting into details of comprehensive quantum jump let us list in Table 6 a sample of ordinary particles with their masses that one should be able with the comprehensive quantum jump to kinematically connect to the existing corresponding novel particles.

Table 6. A sample of ordinary particles with their masses

$$Photon, \gamma, m(\gamma) = 4.5x10^{-15}eV/c^2;$$
$$Leptons : electron, e, m(e) = 0.511MeV/c^2;$$
$$muon, \mu, m(\mu) = 105.7MeV/c^2;$$
$$neutrino, \nu_\mu, m(,\nu_\mu) = 0.76^{-15}eV/c^2,$$
$$Baryons : proton, p, m(p) = 938.3MeV/c^2;$$
$$neutron, n, m(n) = 939.4MeV/c^2;$$
$$Lambda, \Lambda, m(\Lambda) = 1115.7MeV/c^2,$$
$$eV/c^2 = 4/3x10^{-33}g$$

Ordinary masses are simply denoted with m If the novel mass is different from m we will denote that separately. Concerning the mass of the ordinary photon , $m(\gamma)$ rather than zero, we chose the tiny mass of $6x10^{-48}g = 4.5 \times 10^{-15}eV$as quoted in (Lin, H.-L., Tang, L., Zou, R., 2023) and in (Itzykson, C. and Zuber J.-B., 1980), 138). However, in (Frerick, J., et al., 2023) through Lagrangian formalism, the derived dark matter (novel) photon γ' is even smaller with limits of $4 \times 10^{-19}eV/c^2 \prec m(\gamma') \prec 3 \times 10^{-17}eV/c^2$.

In what follows, to simplify the notation any comprehensive ordinary particle from Table 6 is represented with comprehensive symbol d, while similarly, any corresponding novel particle is denoted with comprehensive symbol d_N. In this notation there is understanding that one can have $d = \gamma, e, n, etc.; d_N = \gamma_N, e_N, n_N, etc.$,where they are distinguished by their attributes. Again there is no prior reason that some or most d_N not to be the same as d. Next, for a comprehensive ordinary particle d we find through trial and error from (8) or (10) that $c_3(\alpha o)$,the ordinary particle limiting velocity solution has ratio value of 1 at the fixed comprehensive congruent phase-angle $\alpha(d) = \alpha_O = \pi/5.1 = 0.616$. This the comprehensive particle d can satisfy with velocity $v(d) \simeq c_3(\alpha o)$, basically defining $c_3(\alpha o)$ numerically for particle d. Of course $c_3(\alpha o) = v(d) = c_3(d)$ will change from particle to particle in Table 6, which is all right as long as $v(d) \leq c$ the velocity of light, which empirically is always accepted. Likely, it is possible that an individual ordinary particle may deviate slightly from the comprehensive congruent phase-angle of the representative ordinary particle d. Similar slight deviations are expected for novel particles, to be yet discussed.

Table 7. The input comprehensive congruent parameters together with the values of limiting velocities for or a comprehensive ordinary particle d

$$\alpha(d) = \alpha_O = \pi/5.1 = 0.616 \quad z_1(\alpha(d)) = z_1(\alpha_O) = 0.064322 \quad z_2(\alpha(d)) = z_2(\alpha_O) = 15.625$$
$$\begin{pmatrix} c_1^2(\alpha o)/v(d)^2 & c_2^2(\alpha o)/v(d)^2 & c_3^2(\alpha o)/v(d)^2 \\ 39.881619 & -40.88228 & 1.0006 \end{pmatrix}$$

One can verify the correctness of solutions by adding them up: $c_1^2((\alpha(d))/v^2) + c_2^2(\alpha(d)/v^2) + c_3^2(\alpha(d)/v^2)$ = -8.6$\times 10^{-6} \approx 0$, satisfying the Cardano's relation (for details, see (Šoln, J., 2014).

Next, we start with novel comprehensive novel particles represented by the symbol d_N Here we utilize the verified quantum jump from the ordinary d to the novel $d_N : \alpha(d) = \alpha_O \to \alpha(d_N) = \alpha_N = 2\alpha_O$ yielding the value $\alpha(d_N) = \alpha_N = 2\pi/5.1 = \pi/2.55 = 1.232$. The evaluation of the comprehensive novel particle d_N limiting velocities utilizes solutions (11), (12) and (13). Again, an individual novel particle may deviate slightly from the comprehensive congruent phase-angle of the representative novel particle d_N.

Table 8. The input comprehensive congruent parameters together with the values of limiting velocities for a comprehensive novel particle d_N

$$\alpha(d_N) = \alpha_N = 2\pi/5.1 = 1.232 \quad z_1(\alpha(d_N)) = z_1(2\alpha_O) = 0.630 \quad z_2(\alpha(d_N)) = z_2(2\alpha_O) = 1.587$$

$doublyR(\alpha_N)c_{3,1}^2/v^2(d_N)$	$c_2^2(\alpha_N)/v^2(d_N)$	$I(\alpha_N)c_{3,1}^2/v^2(d_N)$	$c_{3,1}^2(\alpha_N)/v^2(d_N)$
1.002	−2.004	∓0.5768	1.002 ∓ i0.5768

Regardless of the $v(d_N)$ value we can make different limiting velocity sums, each of them satisfying Cardano's relation of zero value: (a): $c_3^2(\alpha_N)/v^2(d_N) + c_1^2(\alpha_N)/v^2(d_N) + c_2^2(\alpha_N)/v^2(d_N) \approx 0$,

(b): $I(\alpha_N)c_3^2/v^2(d_N) + I(\alpha_N)c_1^2/v^2(d_N) \approx 0$ and (c): $doublyR(\alpha_N)c_{3,1}^2/v^2(d_N) + c_2^2(\alpha_N)/v^2(d_N) \approx 0$. For example, the importance of relation (a) and (c) is in the fact that these three velocities in them are real and measurable.

As we see from Table 8, for *doubly* $R(\alpha(_N)c^2_{3,1} \simeq v^2(d_N)$ is the maximum real physical squared velocity of the doubly comprehensive novel particle d_N, regardless what one might "measure" eventually for a specific novel particle d_N. What is the value of the comprehensive maximum real physical squared velocity for d_N? Because $\alpha_N = \pi/2.55 < \pi/2$ we may for this endeavor of the novel comprehensive particle look at $doublyR(\alpha_N)c^2_{3,1} \simeq v^2(d_N)$ how it compares relative to $c^2_3(\alpha o) \simeq v^2(d)$. Then from (8), (9) and (10) we evaluate:

$$\begin{aligned} \frac{c^2_3(\alpha o)}{doublyR(\alpha_N)c^2_{3,1}} &= \frac{v^2(d)}{v^2(d_N)} = \frac{6S(\alpha_N)}{z_1(\alpha_N)} = 1.072593, \\ v^2(d_N) &= 0.932320\, v^2(d),\ v(d_N) = 0.965567v(d) \leq c \end{aligned} \tag{15}$$

Special Theory of Relativity is not violated by (15). Here are few examples from Table (6): Novel doubly photon γ_N: $v(\gamma_N) == 0.965567v(\gamma) = 0.965567\ c$; Novel doubly electron e_N: $v(e_N) == 0.965567v(e) \simeq 0.965567\ c$; Novel doubly neutrino ν_N : $v(\nu_N) = 0.965567v(\nu) = 0.965567\ c$; Novel doubly proton p_N : $v(p_N) == 0.965567v(p)$. What $v(p)$ is depends on the ongoing experiment. Quite often $v(p) = 0$. As we see, with certainty one knows the velocities of ordinary photon $v(\gamma) = c$ and ordinary neutrino $v(\upsilon) = c$, while the ordinary electron may achieve $v(e) \simeq c$. This explains why these three doubly novel particle velocities $v(\gamma_N), v(\nu_N)$ and $v(e_N)$ are so close to c. In fact, from last line in (15) one should notice that $v(d_N) \simeq v(d)$. The comprehensive doubly novel particle velocity is practically the same as the comprehensive ordinary particle velocity.

5. The Comprehensive Novel Particle Fudge Velocity

The comprehensive fudge velocity is unique to the novel particles as its imaginary portion is directly connected to the comprehensive novel congruent phase-angle $\alpha_N = \alpha(d_N) = 2\alpha_O = 2\pi/5.1$. It is simply defined as an average of the total $c^2_{1,3}(\alpha_N)$. With taking into account the average value $.< (1 \pm i\sqrt{3}\cos(\alpha_N)) >= (1 + 3\cos^2\alpha_N)^{\frac{1}{2}} = 1.15)$. In Table 9, we show the evolution of $c^2_{1,3}(\alpha_N)$ into the comprehensive fudge velocity $doublyc^2_{1,3,F}\ (d_N)$

Table 9. The evolution of comprehensive novel particle squared fudge velocity

$$\begin{aligned} &(15):\ doublyR(\alpha_N)c^2_{1,3} = v^2(d_N) = 0.93232v^2(d) \\ &(13):\ c^2_{1,3}(\alpha_N) = doublyR(\alpha_N)c^2_{1,3}(1 \pm i\sqrt{3}\ \cos(\alpha_N)), \\ &< doublyc^2_{1,3}(\alpha_N) >= 0.93232v^2(d) < (1 \pm i\sqrt{3}\ \cos(\alpha_N)) >, \\ &v^2_F(d_N) =< doublyc^2_{1,3}(\alpha_N) >= 1.076v^2(d) \end{aligned}$$

The comprehensive novel particle squared fudge velocity $v^2_F\ (d_N)$, although unphysical is numerically practically the same as $v^2(d)$. So it looks like that all novel particle comprehensive velocities are very close to the ordinary particle comprehensive velocity $v(d)$. For novel photon, $v^2_F\ (\gamma_N) = 1.076\ v^2(\gamma) = 1.076\ c^2$ which barely violates Special Theory of Relativity.

6. Similarities, Differences With Substitutions and Selections Between Free Ordinary and Novel Particles

We start with listing the energy expressions with some substitutions of ordinary and novel particles which demand some interrelationships between them in (16):

$$\begin{aligned} 1 &: \ E(d) = \frac{3\sqrt{3}}{z_1(\alpha_O)}\frac{m(d)v^2(d)}{2}, E(d_N) = \frac{3\sqrt{3}}{z_2(\alpha_N)}\frac{m(d_N)v^2(d_N)}{2} \\ 2 &: \ z_1(\alpha_O) = 0.064322,\ z_2(\alpha_N) = 1.587, v^2(d_N) \simeq v^2(d), \\ 3 &: \ m(d_N) = m(d) : \frac{E(d_N)}{E(d)} = \frac{z_1(\alpha_O)}{z_2(\alpha_N)} \simeq 0.04 = \frac{1}{25} \\ 4 &: \ E(d_N) = E(d) : \frac{m(d_N)}{m(d)} = \frac{z_2(\alpha_N)}{z_1(\alpha_O)} = 24.67 \simeq 25 = \frac{1}{0.04} \end{aligned} \tag{16}$$

To begin with, as shown in Tables 7 and 8, as well as in (15) it is seen that to a good approximation, $v^2(d_N) \simeq v^2(d)$. Even with this, one sees why it is so difficult to observe ordinary and novel particles together if expressions in (16) hold; When particles have the same energies the masses diverge, and the oposite, when the masses are the same energies diverge. The approximate equality $v^2(d_N) \simeq v^2(d)$ one may use to find some inter-relationships with substitutions and selections between ordinary and novel particles at respective comprehensive congruent phase-angles of $\alpha_O = \pi/5.1$ and

$\alpha_N = \alpha(d_N) = 2\alpha_O = 2\pi/5.1$. It is interesting to note that substitutions 3: and selections 4: are inversely symmetrical. With $v^2(d_N) \simeq v^2(d)$, the point 3: in (16) with the assumed substitution $m(d_N) = m(d)$ becomes "natural" to do further explorations. In doing so, one has to be aware that the free energies are not the same any more and one has to make selection choice between $E(d_N)$ and $E(d)$. As there is no physical reason why a novel particle cannot participate in interaction with ordinary particles, one simply has to be aware that to either selected $E(d_N)$ or $E(d)$ one has to add interaction energy. For instance, in the case of Bohr atom, we shall postulate virtual novel electron with substitution mass $m(e_N) = m(e)$, and then to selected $E(e_N)$ adds interaction with the Coulomb field, which is consistent with space-time connections of ordinary and novel particles (Šoln, J., 2022).

The point 4: in (16) with substitution of equal energy $E(d_N) = E(d)$ is not only interesting but also intriguing as it points to changing mass values by means of comprehensive congruent parameters $z_1(\alpha_O)$ and $z_2(\alpha_N)$, when the energies are equal, which is the case for γ and γ_N yielding different masses.

Although comprehensive quantum jump kinematically matches free and interacting ordinary particles to existing free and interacting novel particles, one should not expect to be able straightforwardly to quantum jump a system of interacting ordinary particles to a similar system of interacting novel particles, except in some special cases.

7. Bohr's Atom With Positive Virtual Novel Electron Energy

Here we take the same model of the atom as done originally with,the usual electron (see Eisberg, R. M., 1966, 115) which in the final "state" ended up with negative energy. Since we know in advance that the sign of resulting energy of ordinary electron is wrong, then because the space-time connections of ordinary and novel particles (Šoln, J., 2022), are applicable also for the Bohr atom we use both of comprehensive congruent phase-angles, for ordinary particles $\alpha_O = \pi/5.1$ and novel particles $\alpha_N = \alpha(e_N) = 2\pi/5.1$ in evaluation. Hence, assuming 3: from (16) we substitute ordinary electron e with the virtual novel electron e_N denoted as "Novel" with postulated same mass as ordinary electron, $m(e_N) = m(e)$. It interacts as original electron e; that is, it circulates around the fixed nucleus of mass M at radial distance $r(e_N)$ with perpendicular to its velocity $v(e_N)$. The stability of the orbit is guarantied by the equality of Coulomb force to the Centrifugal force, as shown in (17).

$$\frac{e^2(M)}{r^2(e_N)} = m(e_N)\frac{v^2(e_N)}{r(e_N)} \tag{17}$$

$$m(e_N)v^2(e_N) = \frac{e^2(M)}{r(e_N)} = -V((r(e_N) \tag{18}$$

Relation (18) with the negative Coulomb potential is the consequence of (17). Next in 1: of (16) we select the free energy $E(e_N)$ of the virtual novel electron e_N to which we add the (interacting) orbiting virtual novel electron e_N energy from Coulomb potential $V = -e^2(p)/r(e_N)$ to obtain the total virtual novel electron energy, for simplicity also denoted as $E(e_N)$:

$$E(e_N) = \frac{3\sqrt{3}}{z_2(e_N)}\frac{m(e_N)v^2(e_N)}{2} + V = \frac{3\sqrt{3}}{z_2(e_N)}\frac{m(e_N)v^2(e_N)}{2} - m(e_N)v^2(e_N) \succ 0. \tag{19}$$

$$E(e_N) = \frac{3\sqrt{3}}{z_2(e_N)}\frac{m(e_N)v^2(e_N)}{2} - 2\frac{m(e_N)v^2(e_N)}{2} = \left(\frac{3\sqrt{3}}{z_2(e_N)} - 2\right)\frac{m(e_N)v^2(e_N)}{2} \succ 0. \tag{20}$$

$$E(e_N) = \left(\frac{3\sqrt{3}}{z_2(e_N)} - 2\right)\frac{e^2(M)}{2r(e_N)} = 1.27\frac{m(e_N)v^2(e_N)}{2} = 1.27\frac{e^2(M)}{2r(e_N)} \succ 0. \tag{21}$$

The positive virtual novel electron energy is represented in (19) and explicitly given in (20), and ealuated in (21) in two equivalent expressions with the comprehensive linear congruent parameter $z_2(\alpha(e_N)) = z_2(2\alpha(e)) = 1.587 \prec \sqrt{3}$. If the comprehensive congruent parameter $z_2(\alpha(e_N))$ were $\sqrt{3}$, $E(e_N)$ would simply be $m(e_N)v^2(e_N)/2 = e^2(p)/2r(e_N)$, which is positive with the same absolute value as the negative original result (see Eisberg, R. M., 1966, 115). This example already shows that the notions of ordinary and novel (dm) particles make sense, particularly if they can be interrelated.

Now we move to the Bohr's quantization rules. Because the virtual novel electron angular momentum is constant it is chosen for quantization involving the Planck constant h.

$$L(e_N) = m(e_N)\, v(e_N)\, r(e_N) = n\,\hbar, \hbar = h/2\pi \tag{22}$$

In separating dependences for $v(e_N)$ and $r(e_N)$, we relay on equation (18)

$$m(e_N)v(e_N)\,r(e_N) = \frac{e^2(M)}{v(e_N)} = n\hbar \tag{23}$$

$$v(e_N) = \frac{e^2(M)}{n\hbar}, r(e_N) = \frac{n\hbar}{m(e_N)v(e_N)} \tag{24}$$

$$r(e_N) = \frac{n^2\hbar^2}{m(e_N)e^2(M)} \tag{25}$$

The rules (24) and (25) when respectively applied to (20) and (21) yield the same quantized virtual novel,electron expression with nucleus of mass M plus Z ionized atoms (Z=1, 2,3, etc=neutral hydrogen, singly ionized helium atom, doubly ionized helium atom, etc.), reads now for the n energy level:

$$E(e_N)_n = \left(\frac{3\sqrt{3}}{z_2(e_N)} - 2\right)\frac{Z^2\ e^4(M)\ m(e_N)}{2n^2\hbar^2}, n = 1,2,3,.. \tag{26}$$

$$z_2(\alpha(e_N)) = 1.587 :\ E(e_N)_n = 1.27\frac{Z^2\ e4(M)\ m(e_N)}{2n^2\hbar^2} = \frac{E(e_N)_1}{n^2},\ \ n = 1,2,3,.. \tag{27}$$

In Table 10, we make simple comparisons with $Z = 1$ between the hydrogen atom energy levels calculated from (27) and Bohr original negative energies (Eisberg, R. M., 1966, 115):

Table 10. Comparison of hydrogen atom energy levels ($eV = 1.596 \times 10^{-12}\ erg$)

n	1	2	3	4
$E(e_N)_n$	$17.3eV$	$4.32eV$	$1.92eV$	$1.08eV$
$-E_n$	$13.6eV$	$3.4eV$	$1.52eV$	$0.85eV$

We wish to point out that the comprehensive linear congruent parameter $z_2(\alpha(e_N) = 1.587$ is very close to $\sqrt{3} = 1.732$ at which the upper and lower energy levels become equal. Next define the arbitrary energy difference for quantized virtual novel, electron in Z ionized helium atom at some $n \geq 1$:

$$\begin{aligned}\Delta E(e_N; n_i, n_f) &= 1.27\frac{Z^2\ e4(M)\ m(e_N)}{2\hbar^2}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right), n_i \prec n_f \\ &= Z^2 27.6\times 10^{-12} erg\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = Z^2 17.27\ eV\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right),\end{aligned} \tag{28}$$

$$\nu = \frac{\Delta E(e_N; n_i, n_f)}{4\pi\hbar^3},\ k = \frac{\nu}{c} = \frac{1}{\lambda} = 1.27\frac{Z^2\ e4(M)\ m(e_N)}{c4\pi\hbar^3}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$\frac{1}{\lambda} = R(\alpha_N)Z^2\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) \tag{29}$$

$$R(\alpha_N) = \frac{1.27e4(M)\ m(e_N)}{c4\pi\hbar^3} = 139290\ cm^{-1} \simeq 140000\ cm^{-1} \tag{30}$$

The constant quantity $R(\alpha_N)$ goes by the name Rydberg constant for the hydrogen so that one gets from (29) the wave number $k = 1/\lambda = \nu/c$ for evaluation of number of spectral lines (wavelength ranges) for the hydrogen in the range: $n_i \succ n_f$:

$$k = R(\alpha_N)\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right), n_i \prec n_f \tag{31}$$

Relation (31) represents the wavelength ranges (number of spectral lines) for hydrogen series: Lyman: Ultraviolet $n_i = 1, n_f = 2,3,4$; Balmer: Ultraviolet-Visible, $.n_i = 2, n_f = 3,4,5.$; Pashen: Infrared, $n_i = 3, n = 4,5,6.$: Brackett: Infrared $n_i = 4, n = 5,6,7$: Pfund: Infrared $n_i = 5, n = 6,7,8$. The analysis of these data should tell weather the value of the novel particle comprehensive parameter $z_2(\alpha(e_N)) = 1.587$ is correct or weather should be adjusted specifically for the virtual novel electron. The radiated photons by the quantized virtual novel electron energy differences are ordinary photons.

8. Discussion and Conclusion

The relations (15) and(16) indicate to the approximate equalities between comprehensive ordinary particle d and novel particle d_N velocities: $v^2(d_N) \simeq v^2(d)$. These equalities are consistent with the Special Theory of Relativity. The unphysical novel particle fudge velocity which is not present in ordinary particles has a numerical velocity value close to $v(d)$ and violates the Special Theory of Relativity slightly.This approximate velocity equalities made it easy to substitute the novel virtual electron mass with the ordinary electron mass so that the selected virtual novel electron energy in the description of the Bohr atom yields positive electron energy.

In relation (16) we introduced simple two inversely symmertrical substitutions. One can envision more complicated designated substitutuions to deal with other involved physical problems of ordinary and novel particles; examples being: fractional $m(d)$ to $md_N)$ substitution and related likely to fractional $E(d)$ to $Ed_N)$ selection, etc.

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