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Driving Inertial Confinement Fusion With Strong Pulsed Magnetic Field

Zhisheng Wang[1] & Kexin Yao[1]

[1] Jinan New Technology Research Institute, Jinan, China

Correspondence: Kexin Yao, Jinan New Technology Research Institute, Jinan, Shandong, China. E-mail: yayydwpq@163.com

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Abstract

Regarding inertial confinement fusion (ICF), the current proposed driving energy sources are mainly laser beams or high-energy particle beams. This paper proposes a new method: Adopting a strong pulsed magnetic field as the driving energy source, explored its action principle, derived all relevant formulas, calculated an example and compared it with existing driving methods.The conclusion drawn is that this method can achieve high energy gain, the required equipment is relatively simple and without the disadvantages of other methods, making it a feasible method.

Keywords: strong pulsed magnetic field, inertial confinement, nuclear fusion

0. Preface

There are currently two methods being explored to achieve controlled nuclear fusion: magnetic confinement and inertial confinement.The former is the Tokamak device, and the Chinese have set a world record of 1056 sec and 158 million deg F plasma operation on 12/30/2021. The challenge faced by magnetic confinement is the need to maintain the fusion plasma at temperatures of over a billion degrees for as long as possible, requiring the vessel to withstand extremely high temperatures, requiring the vessel to withstand extremely high temperatures. To address this, a strong magnetic field is used to confine the high-temperature plasma and isolate it from the vessel walls. The Tokamak device must find ways to further extend the presence of the high-temperature plasma to be of practical use.

The principle of inertial confinement is completely different from the above.Taking laser direct drive device as an example: a hollow spherical shell, known as a "target pellet", with a diameter of a few millimeters is made using a polymer.euterium and tritium (DT) fusion fuel are loaded into the sphere, using a uniformly distributed lasers to emit strong laser pulses around the target pellet, irradiating its surface, This causes the shell to instantaneously vaporize and expand, while the resulting reaction force drives the DT fuel to rapidly implode towards the center of the sphere, leading to a sudden increase in temperature and pressure, resulting in fusion , which is equivalent to detonating a miniature hydrogen bomb. This method utilizes the inertial force generated by the implosion of the fuel to constrain the fuel itself, thus enabling the fuel to sufficiently undergo fusion; The difficulty faced by inertial constraint driven by laser or particle beams is that the radiation energy beam cannot fully and uniformly illuminate the surface of the spherical shell. This leads to fluid instability caused by asymmetric implosion, i.e. the rupture of the pellet shell or inner layer during implosion, thereby disrupting the centripetal compression of DT. Additionally, the coronal region formed by the vaporization of the shell has limited transparency to radiation energy; this reduces the efficiency of radiation energy as the driving force input.

The paper proposes using a strong pulsed magnetic field as the driving energy source, and employing a ring target instead of a target pellet. The structure and working principle are illustrated in Figure 1: a hollow ring made of metal coated with DT ice (solid DT) layer on its inner surface, and the DT gas filled inside the DT ice cavity. The ring target is placed in a uniformly distributed driving magnetic field $B_{dr}(t)$ at the initial moment, and when $B_{dr}(t)$ undergoes a instant step change, a great induced current J is generated inside the metal shell, causing the shell to rapidly heat up to the point of breakdown, forming a plasma. The magnetic field B_J generated by J has a pinching effect on J, leading to the shell plasma to be pinched centripetally, compressing the DT to its critical point and initiating fusion reactions.

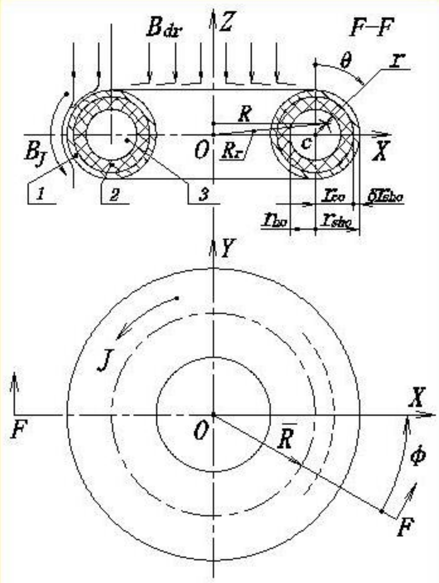


Fig 1. 1.shell 2.DTice 3.DTgas

As the shell is symmetrically acted upon by $B_{dr}(t)$ on all cross-sections simultaneously, the implosion motion of the shell and the DT is symmetrical, preventing fluid instability due to asymmetry, and there are no transparency issues. The entire fusion process of the ring target consists of the following three parts:

Firstly, the explosion induced by discharge～electric explosion, and pinching effect of the ring target shell; Secondly, DT undergoes implosion due to the pinch effect of the ring target shell;

Thirdly, as the implosion approaches the center, the velocity of the DT suddenly decreases to zero, leading to a sharp increase in internal energy. This is an energy conversion caused by stagnation, abbreviated as "stagnate". During stagnation, due to the higher entropy of the central DT gas, it heats up rapidly. At the end of stagnation, the central DT gas first reaches the fusion threshold, forming a"hot spot", which is the"ignition". The fusion energy within the hot spot propagates outward in the form of waves to the surrounding DT ice, resulting in wide range fusion. This method of first forming a hot spot in a small range and then diffusing and igniting requires less driving energy. If the ignition were to occur simultaneously throughout the entire region, a larger driving energy would be needed. Therefore, the structure of "ice wraps gas", as shown in Figure 1, is employed.

The aim of this paper is to derive the appropriate structural dimensions of the ring target, the amount of DT fuel loading, and the suitable pulse waveform of the driving magnetic field, in order to achieve a higher energy gain with lower driving energy input.

1. Fusion Average Reaction Rate $\overline{v\sigma}_{sc}$, as Well as the Power Density W_a Related to a Particles

1.1 Average Fusion Reaction Rate $\overline{v\sigma}_{sc}$

The DT fusion reaction equation involved in this paper is

$$ {}_1^2D+{}_1^3T \rightarrow {}_2^4H_e(3.52MeV)+{}_0^1n(14.06MeV) \tag{1,1-1}$$

There is an interaction potential energy between atomic nuclei, and when the distance between atomic nuclei is greater than a certain value r_V, this potential energy is basically Coulombic potential energy; In order to achieve fusion,two positively charged D and T nuclei must have sufficient kinetic energy of mutual motion to overcome Coulomb potential energy and collide with each other. When $r<r_V$, a pair of DT nuclei will be attracted to each other by nuclear forces, ultimately leading to fusion.

According to the above, if a pair of DT moves relative and a particle hits a circle centered on another particle with a radius of r_V, then the pair of DT may undergo fusion, $\sigma_{sc}=\pi r_V^2$ being the fusion reaction cross-section.

Let the particle flow of particle 2 with a reaction cross-section of σ_{sc} bombard a single particle 1. If the quantity areal density of particle flow 2 is n_s, the probability of particle 1 being hit, i.e. fusion, is $P_{sc}=n_s\sigma_{sc}$; The physical meaning of σ_{sc} can also be: the probability that a particle undergoes fusion under the bombardment of a particle stream with a unit flux density per unit time.

If v is the velocity of particle 2 relative to particle 1 and the quantity volume density of particle 2 is n_{V2}, then $n_{V2}v$ is the quantity flux density of particle 2; If the quantity volume density of particle 1 being bombarded is n_{V1}, the probability of fusion reaction for multiple particles 1 being bombarded by multiple particles 2 within a unit time and volume is $n_{V1}n_{V2}v\,\sigma_{sc}$.

If particle 1 and particle 2 collide and can undergo complete fusion, $n_{V1}=n_{V2}=n_V$ is required, otherwise the excess $\Delta n=\left|n_{V1}-n_{V2}\right|$ particles will be useless due to the absence of particles that collide with them; This means that particles 1 and 2 should take equimolar values, so that D and T nuclei form a one-to-one relationship, known as the "DT pair"; The probability of such a "DT pair" fully undergoing fusion reaction per unit time and volume is $n_V^{\,2}v\,\sigma_{sc}$.

If $n_V=1$, then $v\sigma_{sc}$ becomes: within a unit time,the probability of fusion reactions occurring when two particle streams with the same quantity volume density of 1 move relative to each other at velocity v; $v\sigma_{sc}$ is called fusion reaction rate, and its dimension is $[v\sigma_{sc}]=cm^3/s$.

Note that if n_V is the quantity volume density of the "DT pair", then the quantity volume density of D or T nucleus respectively is also n_V. Therefore, when $n_V=1$, $v\sigma_{sc}$ is the fusion reaction rate of the "DT pair".

In the calculation, the average value $\overline{v}\overline{\sigma}=\int_0^{\infty}v\sigma(v)f(v)dv$ of $v\sigma_{sc}$ must be used, and the probability density $f_i(v)$ ～ $i=1,2$, can be obtained from the Maxwell velocity distribution, thus obtaining $\overline{v}\overline{\sigma}=\iint\left|v_1-v_2\right|\sigma(\left|v_1-v_2\right|)f_1f_2dv_1dv_2$. For this equation, literature (Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008) provides several fitting algorithms obtained through numerical integration, among which the following two approximate formulas are recommended for DT fusion

$$\overline{v}\overline{\sigma}(T)=k_1\exp(-k_2\left|\ln(T/k_3)\right|^{2.13})\ [cm^3s^{-1}] \tag{1.1-2)1}$$

where $k_1=9.10\times10^{-16}[cm^3s^{-1}]$, $k_2=0.572$, $k_3=64.2[KeV]$, and

$$\overline{v}\overline{\sigma}(T)=k_4T^2[cm^3s^{-1}] \tag{1.1-2)2}$$

where $k_4=1.1\times10^{-18}[cm^3s^{-1}KeV^{-2}]$.

In the above two formulas the temperature dimension is $[T]=KeV$; The accuracy of the former is 10% at temperature (3-100) KeV and 20% at (0.3-3) KeV; The accuracy of the latter at (8-25) KeV is 15%.

1.2 Power Density W_a Related to Particle α

The formula for W_a is

$$W_a=A_a\rho_{mh}^{\ 2}\overline{v}\overline{\sigma}(T)\ [erg/(s\cdot cm^3)] \tag{1.2-1)1}$$

$$A_a=8.064\times10^{40}[erg/g^2] \tag{1.2-1)2}$$

Where ρ_{mh} ～mass density of equimolar DT gas.

Argument the above:

For a "DT pair"with a quantity volume density of n_V, the probability of fusion reaction occurring per unit time at temperature T is $n_V{}^2\overline{v\sigma}(T)$; According to equation (1,1-1), the energy carried by AF particles generated after each "DT pair" fusionis is $Q_{DT}=3.52MeV$. Therefore, the power density generated by the α particles participating in fusion is WA=; So, the power density generated by the AF α particles participating in fusion is $W_\alpha = Q_{DT}n_V{}^2\overline{v\sigma}(T)$; In this equation If the average mass of equimolar DT ions is $\overline{m}_{DT}$, then

$$W_\alpha = Q_{DT}(\rho_{mh}/2\overline{m}_{DT})^2\overline{v\sigma}(T) \tag{1.2-2}$$

In the above formula, the total mass of a "DT pair" is $m_{DT}=5m_p$,where the neutron mass is approximated as the proton mass m_p, and the electron mass is ignored.So, due to taking $\overline{m}_{DT}=m_{DT}/2$, there is

$$\overline{m}_{DT}=2.5m_p \tag{1.2-3}$$

Substituting the above formula, proton mass, and Q_{DT} into formula (1.2-2) and converting eV to erg, obtain formulas (1.2-1)1 and (1.2-1)2.

2. The Implosion of DT

2.1 Foreword

DT in implosion can be regarded as an ideal gas because:according to literature (Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008), if a plasma with a mass density of ρ_m satisfies the discriminant $(e^2/k_BT)^3\,2\rho_m/\overline{m}_{DT}<1$, it is an ideal gas, where e is the electron charge and k_B is the Boltzmann constant; At the beginning, an electric explosion occurred on the metal shell outside the DT ice layer.According to literature (Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008), the temperature of the plasma generated by the electric explosion can reach $k_BT=10^7K$ or above. Here, estimating the DT ice layer $k_BT=10^6K=1.38\ \times10^{-10}erg$, and the initial mass density of the DT ice is $\rho_m=\rho_{mio}=0.215g\cdot cm^{-3}$. The above data is substituted into the discriminant formulato obtain $(e^2/k_BT)^3\,2\rho_{mco}/\overline{m}_{DT}=4.986\times10^{-4}<<1$; From this, at the beginning of the electric explosion DTice can be regarded as an ideal gas; At the beginning, DT ice is the coldest and thickest state of DT during the entire implosion process. Since DT ice at this time is an ideal gas, DT should also be an ideal gas in implosion.

DT implosion, as a type of flow, should be expressed using an equation of fluid dynamics, but this equation involves the first law of thermodynamics, which means that the ring target reaches thermal equilibrium internally. Therefore, it is required that the diameter of the hot spot and implosion time be sufficiently large compared to the a mean free path and collision time of particles in the plasma; According to literature (Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008), the typical hot spot diameter of the ICF pellet is $120\times10^{-4}cm$, and the typical implosion time is $10^{-9}s$. When the temperature in the later stage of implosion is $10keV$, the mean free path of the particles is $10^{-4}cm$, and the collision time is $10^{-12}s$; By comparison, it is known that the first two are indeed much larger than the latter two, so it is reasonable to use equation of fluid dynamics to describe implosion.

Implosion is a radial, centripetal, and centrosymmetric motion with no tangential relative motion between streamlines, thus there is no tangential gradient of velocity; However, the viscous force of a fluid is related to the tangential gradient of velocity, so the viscous force term in the equation of fluid dynamics can be ignored. In addition, due to the short duration of the implosion, the heat energy exchange inside and outside the ring target is little and can be ignored.

Therefore, implosion can be considered an isentropic process.

In summary, the equation of isentropic ideal fluid dynamics can be used to describe implosion.

2.2 Equation of Isentropic Ideal Fluid Dynamics

2.2.1 Establishing a Ring Coordinate System

As shown in Figure 1, establish a Ring Coordinate System on the ring target with r, θ, ϕ as coordinate variables and O as the origin, the radius of the circle where the center c of the ring target cross-section is located is $\overline{R}=const$; In this coordinate system, the vector $\mathbf{A}$ is represented as $\mathbf{A}=A_r\mathbf{e}_r+A_\theta\mathbf{e}_\theta+A_\phi\mathbf{e}_\phi$, where $\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_\phi$ are the unit vectors corresponding to r, θ, ϕ.

The gradient of the ring coordinate system is

$$\nabla = \frac{\partial}{\partial r}\mathbf{e}_r + \frac{\partial}{\partial L_\theta}\mathbf{e}_\theta + \frac{\partial}{\partial L_\phi}\mathbf{e}_\phi \tag{2.2-1)1}$$

where $L_\theta = r_k\theta$ and $L_\phi = R\phi$ arc lengths in the $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ directions, respectively.
For implosion

$$\nabla = \frac{\partial}{\partial r}\mathbf{e}_r \tag{2.2-1)2}$$

From the previous equation, it can be inferred that there exists the following formula

$$d(\nabla a)/dt = \nabla(da/dt) \tag{2.2-1)3}$$

Argument the above:

For orthogonal curvilinear coordinates, the gradient formula is $\nabla q = \frac{\partial a}{h_r \partial r}\mathbf{e}_r + \frac{\partial a}{h_\theta \partial \theta}\mathbf{e}_\theta + \frac{\partial a}{h_\phi \partial \phi}\mathbf{e}_\phi$. Where $h_r = \left|\partial \mathrm{R}_r/\partial r\right|$, $h_\theta = \left|\partial \mathrm{R}_r/\partial \theta\right|$ and $h_\phi = \left|\partial \mathrm{R}_r/\partial \phi\right|$ are scale factors, the position vector R_r of the point on the ring target is represented as $\mathrm{R}_r = \overline{\mathrm{R}} + \mathbf{r} = \mathbf{i}(\overline{R} + r\sin\theta)\cos\phi + \mathbf{j}(\overline{R} + r\sin\theta)\sin\phi + \mathbf{k}r\cos\theta$, in the Cartesian Coordinate $X\text{–}Y\text{–}Z$ in Figure 1, resulting in $h_r = 1$, $h_\theta = r$ and $h_\phi = R$, where $R = \overline{R} + r\sin\theta$. Therefore, $\nabla q = (\partial a/\partial r)\mathbf{e}_r + [\partial a/\partial(r\theta)]\mathbf{e}_\theta + [\partial a/\partial(R\phi)]\mathbf{e}_\phi$ is obtained, which is $\nabla q = (\partial a/\partial r)\mathbf{e}_r + (\partial a/\partial L_\theta)\mathbf{e}_\theta + (\partial a/\partial L_\phi)\mathbf{e}_\phi$.
Due to the central symmetry of the implosion motion, the equipotential surface of a should not be related to θ and ϕ. Therefore, $\nabla q = (\partial a/\partial r)\mathbf{e}_r$ can be deduced from the above formula.

From the previous equation there is $\frac{d(\nabla a)}{dt} = \frac{d}{dt}(\frac{\partial a}{\partial r}\mathbf{e}_r + \frac{\partial a}{\partial L_\theta}\mathbf{e}_\theta + \frac{\partial a}{\partial L_\phi}\mathbf{e}_\phi) = \frac{\partial}{\partial r}(\frac{da}{dt})\mathbf{e}_r + \frac{\partial}{\partial L_\theta}(\frac{da}{dt})\mathbf{e}_\theta + \frac{\partial}{\partial L_\phi}(\frac{da}{dt})\mathbf{e}_\phi$, thus, $d(\nabla a)/dt = \nabla(da/dt)$ is obtained.

The divergence of ring coordinate system is

$$\nabla \cdot \mathrm{A} = \partial(rRA_r)/(rR\partial r) + \partial(RA_\theta)/(R\partial L_\theta) + \partial A_\phi/\partial L_\phi \tag{2.2-2)1}$$

where A～vector field.
Argument the above:
For orthogonal curvilinear coordinates, the divergence formula is $\nabla \cdot \mathrm{A} = \frac{1}{h_r h_\theta h_\phi}[\frac{\partial(h_\theta h_\phi A_r)}{\partial r} + \frac{\partial(h_\phi h_r A_\theta)}{\partial \theta} + \frac{\partial(h_r h_\theta A_\phi)}{\partial \phi}]$, resulting in $\nabla \cdot \mathrm{A} = \frac{\partial(rRA_r)}{rR\partial r} + \frac{\partial(RA_\theta)}{R\partial L_\theta} + \frac{\partial A_\phi}{\partial L_\phi}$ for ring coordinate.
The Curl of the ring coordinate system is

$$\nabla \times \mathrm{A} = \frac{1}{rR}\begin{vmatrix} \mathbf{e}_r & r\mathbf{e}_\theta & R\mathbf{e}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_k & rA_\theta & RA_\phi \end{vmatrix} \tag{2.2-2)2}$$

For vector field $\mathrm{A} = A_r\mathbf{e}_r$ that are not related to θ and ϕ, there is

$$\nabla \times \mathrm{A} = 0 \tag{2.2-2)3}$$

Argument the above:

For orthogonal curvilinear coordinates, the curl formula is $\nabla \times \mathrm{A} = \frac{1}{h_r h_\theta h_\phi}\begin{vmatrix} h_r\mathbf{e}_r & h_\theta\mathbf{e}_\theta & h_\phi\mathbf{e}_\phi \\ \frac{\partial}{\partial r_k} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ h_r A_k & h_\theta A_\theta & h_\phi A_\phi \end{vmatrix}$, so for Ring

Coordinate, there is $\nabla\times\mathrm{A}=\frac{1}{rR}\begin{vmatrix}\mathrm{e_r} & r\mathrm{e_\theta} & R\mathrm{e_\phi}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ A_k & rA_\theta & RA_\phi\end{vmatrix}$. Therefore, for vector field $\mathrm{A}=A_r\mathrm{e_r}$ that are not related to θ and ϕ, the expression for curl is $\nabla\times\mathrm{A}=0$.

The Laplace operator in ring coordinate system is

$$\nabla^2=\frac{\partial}{rR\partial r}(rR\frac{\partial}{\partial r})+\frac{\partial}{R\partial L_\theta}(R\frac{\partial}{\partial L_\theta})+\frac{\partial^2}{\partial {L_\phi}^2} \tag{2.2-3}$$

Argument the above:

Substituting formulas (2.2-1)1 and (2.2-2)1, into $\nabla^2 q=\nabla\cdot(\nabla q)$ obtains formulas (2.2-3).

The Euler operators in ring coordinate system

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\mathrm{u}\cdot\nabla \tag{2.2-4)1}$$

where u is the flow velocity in the rest frame, and for implosion $\mathrm{u}=u\mathrm{e_r}$.

For implosion

$$\frac{d}{dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial r} \tag{2.2-4)2}$$

Argument the above:

For fluid parameter $q(\mathrm{R_r},t)$ related to position vector $\mathrm{R_r}=r\mathrm{e_r}+L_\theta\mathrm{e_\theta}+L_\phi\mathrm{e_\phi}$, there is $\frac{dq}{dt}=\frac{\partial q}{\partial t}+\frac{\partial q}{\partial r}\frac{dr}{dt}+\frac{\partial q}{\partial L_\theta}\frac{dL_\theta}{dt}+\frac{\partial q}{\partial L_\phi}\frac{dL_\phi}{dt}$.

From $\mathrm{u}=\frac{d\mathrm{R_r}}{dt}$ and formula (2.2-1)1, it can be inferred that $\frac{d}{dt}=\frac{\partial}{\partial t}+\mathrm{u}\cdot\nabla$; Applying formula (2.2-1)2 to this formula obtains $\frac{d}{dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}$.

If the cross-section radius of a ring is r, and the radius of the circle where the center c of its cross-section is located is $\overline{R}$, then its volume and surface area are respectively

$$V=2\pi^2r^2\overline{R} \quad \text{and} \quad S=4\pi^2r\overline{R} \tag{2.2-5)1,2}$$

2.2.2 Write the Equation System

The equation system of isentropic ideal fluid dynamics consists of three equations: continuity equation, momentum equation, and isentropic energy equation. The following will write these three equations in a ring coordinate system.

The continuity equation is:

$$\partial\rho_m/\partial t+\partial(r\rho_m u)/(r\partial r)=0 \quad \text{or} \quad d\rho_m/dt+\rho_m[\partial(ru)/(r\partial r)]=0 \tag{2.2-6}$$

where ρ_m ～mass density.

Argument the above:

Take a shell like volume element in ring target, with its cross-section center being the center c of the ring target cross-section. Its volume is V, the internal surface area is S, and the shell thickness is δr. Let u be the velocity of the fluid flowing into S, and ρ_m be the mass density of the fluid; The total mass flow rate of the fluid penetrating the inner surface S and exiting the outer surface $S+\Delta S$ is

$\Delta Q=(\rho_m+\Delta\rho_m)\ (u+\Delta u)\ (S+\Delta S)-\rho_m uS$, omitting second order and above small quantities to obtain $\Delta Q=\rho_m S\Delta u+\rho_m u\Delta S+Su\Delta\rho_m$.

On the other hand, the total mass of the fluid in volume $V=\delta rS$ is $\rho_m V=\rho_m\delta rS$, then the change in total mass per unit time is $\Delta Q'=(-\partial\rho_m/\partial t)\delta rS$. Since $\Delta Q=\Delta Q'$ is required, there must be $\rho_m S\Delta u+\rho_m u\Delta S+Su\Delta\rho_m=(-\partial\rho_m/\partial t)\delta rS$. Based on this equation and using the surface areaformula (2.2-5)2, $\frac{\delta\rho_m}{\partial t}+\frac{\partial(r\rho_m u)}{r\partial r}=0$ can be derived; By using the Euler operator(2.2-4)2, $\frac{d\rho_m}{dt}+\rho_m\frac{\partial(ru)}{r\partial r}=0$ can be derived.
The momentum equation is

$$\partial u/\partial t+u(\partial u/\partial r)+\rho_m{}^{-1}(\partial p/\partial r)-f=0 \quad \text{or} \quad du/dt+\rho_m{}^{-1}(\partial p/\partial r)-f=0 \tag{2.2-7)1}$$

where p ～pressure, f ～body force; For implosion, f, p, and u are all radially oriented. For implosion, gravity can be ignored and there is no other form of body force, so $f=0$, resulting in

$$\partial u/\partial t+u(\partial u/\partial r)+\rho_m{}^{-1}(\partial p/\partial r)=0 \quad \text{or} \quad du/dt+\rho_m{}^{-1}(\partial p/\partial r)=0 \tag{2.2-7)2}$$

Argument the above:

According to Xu & Jin, *et al.*, (1981), the general form of the momentum equation is $\rho_m\frac{du}{dt}+\nabla p-\rho_m f=0$. For this equation, use gradient (2.2-1)2 and Euler operators (2.2-4)2 to derive equation (2.2-7)1.
The isentropic energy equation is

$$[d(c_s{}^2/\rho_m{}^{2/3})]/dt=0 \tag{2.2-8}$$

Where c_s ～sound speed.

Argument the above:

According to literature (Jialuan Xu, Shang Xian, 1981) there is $\frac{d}{dt}(c_s{}^2/\rho_m{}^{2/i})=0$, where i ～degree of freedomof particle thermal motion; Treating plasma as a single particle system, thus $i=3$ should be taken, so that equations (2.2-8) obtaining.

2.3 Parameter Discontinuity Interface in Fluids

2.3.1 Existence of Discontinuity Interface

There is an interval Δr distributed along the flow direction. If Δr is small enough, it can be approximated as the the streamline shape within Δr does not change over time, meaning that the flow is stable.Therefore, the fluid parameters, should not be an explicitfunction of t, and within Δr, it can be considered as $r\approx const$.

Thus, within Δr, equations (2.2-6) and (2.2-7) 2 become $\partial(r\rho_m u)/(r\partial r)=0$ and $u\,\partial u/\partial r+\rho_m{}^{-1}\,\partial p/\partial r=0$, and using $r\approx const$ and the former, it can be inferred that $\rho_m u=const$; Based on this and the latter, it can be inferred that $\rho_m u^2+p=const$.

The previous discussion suggests that there can be r_1 and r_2 that meet $r_1-r_2<\Delta r$, and their corresponding $u_1(r_1)$, $\rho_{m1}(r_1)$, $p_1(r_1)$, and $u_2(r_2)$, $\rho_{m2}(r_2)$, and $p_2(r_2)$, making $\rho_{m1}u_1=\rho_{m2}u_2$ and $\rho_{m1}u_1{}^2+p_1=\rho_{m2}u_2{}^2+p_2$; There can be r_s that satisfies $r_2<r_s<r_1$. When $r_1\to r_s^-$ and $r_2\to r_s^+$, although the above two still hold, there can be $u_1\neq u_2$, $\rho_{m1}\neq\rho_{m2}$, and $p_1\neq p_2$; This indicates that there are discontinuities in the fluid parameters ρ_m, u and p on both sides of r_s and r_s is the discontinuous interface.

The fluids on both sides of r_s are flowing in the same direction, and there are two possible directions for r_s movement: One is the same as the flow direction, the second is opposite to the flow direction. Due to the flow of the implosion fluid from the high-energy region to the low-energy region, the upstream side of r_s is the high-energy region and the downstream side is the low-energy region. r_s moving downstream indicates that the high-energy region is expanding, which is in line with the physical meaning of implosion, while r_s moving upstream indicates that the low-energy region is expanding, which is not in line with the physical meaning of implosion. Therefore, the movement direction of r_s should be in moving downstream.

The fluid must flow in from one side of r_s and out from the other side, and the inflow side is called "front" and the outflow side is called "back". Mark "front" and "back" with "1" and "2" respectively, let the propagation speed of r_s is u_s, and the direction of u_s is set to be positive. Therefore, there must be $u_s \geq u_1 > 0$ and $u_s \geq u_2 > 0$, that is, the propagation speed of the discontinuity must be greater than the fluid speed, This forms a trend of continuous expansion of the high-energy region.

For implosion, there exists a parameter discontinuity at the boundary between DT ice and gas at the initial time; After the implosion occurs, the discontinuity will propagate ahead of the DT ice -gas interface, forming a disturbance wave ahead; The forward disturbance wave immediately emitted a reflected shock wave upon reaching the center.

Establish an accompanying reference frame on the discontinuity r_s, denote the velocity in it as v, where $\rho_m u = const$ and $\rho_m u^2 + p = const$ are represented as

$$\rho_m v = J_1 \quad \text{and} \quad \rho_m v^2 + p = J_2 \qquad (2.3\text{-}1)1,2$$

In the equation, J_1 and J_2 ～constants.

2.3.2 Formula for Calculating Discontinuity

In a rest frame the propagation velocity u_s of the discontinuity is

$$u_s = u_1 - J_1/\rho_{m1} = u_2 - J_1/\rho_{m2} \qquad (2.3\text{-}2)$$

Argument the above:

In the accompanying reference frame, $v_1 = u_1 - u_s$, Substituting this into formula (2.3-1)1 obtains $u_s = u_1 - J_1/\rho_{m1}$, and similarly, it can be inferred $u_s = u_2 - J_1/\rho_{m2}$.

J_1 can be represented as

$$J_1 = -[(p_2 - p_1)/(1/\rho_{m1} - 1/\rho_{m2})]^{1/2} \qquad (2.3\text{-}3)$$

Argument the above:

Equations(2.3-1)1,2 can be written as $\rho_{m1} v_1 = \rho_{m2} v_2 = J_1$ and $\rho_{m1} v_1^2 + p_1 = \rho_{m2} v_2^2 + p_2$, resulting in $p_2 - p_1 = J_1 v_1 - J_1 v_2$ and $J_1 = \pm[(p_2 - p_1)/(1/\rho_{m1} - 1/\rho_{m2})]^{1/2}$; But according to formulas $v_1 = u_1 - u_s$, $u_s \geq u_1 > 0$ and (2.3-1)1, there is $J_1 < 0$, so there should be $J_1 = -[(p_2 - p_1)/(1/\rho_{m1} - 1/\rho_{m2})]^{1/2}$.

Regarding $u_1 - u_2$, there are

$$u_1 - u_2 = [(p_1/\rho_{m1})(p_2/p_1 - 1)\ (1 - \rho_{m1}/\rho_{m2})]^{1/2} \qquad (2.3\text{-}4)$$

Argument the above:

According to equation (2.3-2), there is $u_1 - u_2 = J_1(1/\rho_{m1} - 1/\rho_{m2})$, substituting formula (2.3-3) into this formula obtains $u_1 - u_2 = [(p_1/\rho_{m1})(p_2/p_1 - 1)\ (1 - \rho_{m1}/\rho_{m2})]^{1/2}$.

Due to the right side of formula (2.3-4) is greater than zero, there is $u_1 > u_2$: Because the square root on the right side of the formula must also be greater than zero, there must be $p_2 > p_1$, $\rho_{m2} > \rho_{m1}$, or= $p_2 < p_1$, $\rho_{m2} < \rho_{m1}$; However, due to the fact that the "2" side of the discontinuity r_s is in the high-energy region compared to the "1" side, it is necessary to take $p_2 > p_1$ and $\rho_{m2} > \rho_{m1}$ to comply with the physical meaning; So can infer:

In implosion, as the fluid passes through the shock wave surface, its velocity decreases while both pressure and density increase.

If the following formula holds

$$\rho_{m2} >> \rho_{m1},\ p_2 >> p_1 \tag{2.3-5,6}$$

It is called a strong shock wave; If $u_2 \approx 0$, the following formula can be derived by using the above formula and (2.3-4)

$$|u_1| \approx [R_g T_2\, \rho_{m2}/\rho_{m1}]^{1/2} \tag{2.3-7}$$

Similarly, if $u_1 \approx 0$, there exists the following formula

$$|u_2| \approx [R_g T_2\, \rho_{m2}/\rho_{m1}]^{1/2} \tag{2.3-7)'$$

2.4 Solving the Equation of Isentropic Ideal Fluid Dynamics

This article refers to literature (Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008) and explores the derivation of formulas in the ring coordinate.

2.4.1 Variable Substitution of Equation System

The dimensional analysis method was used to perform dimensional analysis on the equation system, introducing dimensionless variables as follows

$$\xi = (r/r_o)|t_o/t|^{\alpha} \tag{2.4-1)1$$

And introduce the dimensionless parameters $U(\xi)$, $C(\xi)$, $G(\xi)$ corresponding to u, c_s, ρ_m to express the fluid state as follows

$$u = (\alpha r/t)U(\xi),\quad c_s = (\alpha r/t)C(\xi),\quad \rho_m = \rho_{mo}(r/r_o)^{\kappa} G(\xi) \tag{2.4-1)2,3,4$$

where r_o and t_o ～the position vector and time corresponding to the reference point, α and k ～ undetermined constant.

Determine the reference vector r_o and calibrate the reference time t_o for streamline $r{=}r(a,\ t)$, since its Lagrangian coordinate a satisfies $a{=}r(a,\ t{=}0)$, so that for the streamline with $a{=}0$, $r(0,\ t{=}0){=}0$ holds; Which means that streamline $r = r(0,\ t)$ reaches the center of the implosion $r{=}0$ at the time $t = 0$, so $(r{=}0,\ t{=}0)$ corresponds to the endpoint of implosion; As the reference time,it must be set at a moment before reaching the center of the implosion, therefore, the reference time must be $t_0{<}0$.

For the reference point (r_o, t_o), according to formula (2.4-1)1, $\xi = 1$; It can be proven that the average value of the disturbance velocity $dr/dt = u_\zeta$ propagating along the curve $\xi = 1$ in the interval $(t_o \le t \le 0)$ is $\bar{u}_\zeta = \frac{1}{0 - t_0}\int_{t_0}^{0} u_\zeta dt = \frac{r_0}{t_0}$.

According to formula (2.4-1)3, there is $c_{so} = (\alpha r_o/t_o)C(\xi = 1)$ at reference point (r_o, t_o), and if the function $C = C(\xi)$ satisfies the condition

$$C(\xi = 1) = 1/\alpha \tag{2.4-2)3$$

Then there is $c_{so} = \frac{r_o}{t_o}$, thus $\bar{u}_\zeta = c_{so}$, that is, the average value of the disturbance velocity propagating along the curve $\xi = 1$ is the sound velocity $c_{so} = r_o/t_o$ originating from the reference point (r_o, t_o).

This article selects the starting radius r_{ho} of implosion on the inner surface of DT ice as the reference vector, That is

$$r_{ho} = r_o \tag{2.4-2)1$$

In this way, the reference time t_o can be calibrated as

$$t_o = r_o / c_{so} \tag{2.4-2)2}$$

The following will describe: when $\xi = 1$ is taken, $\xi = (r/r_o)|t_o/t|^a$ draw a curve $r/r_0 \sim t/t_0$ in the coordinate plane; For any streamline $r = r(a,t)$, if the point (r_{ao}, t_{ao}) on it and the reference point (r_o, t_o) are on the same curve $\xi = 1$, then point (r_{ao}, t_{ao}) must be the starting point of implosion for streamline $r = r(a,t)$.
Attention: since the reference point is the starting point of the implosion, and at this starting point, the fluid is in the same state, therefore, c_{so} is the same everywhere.

In addition, at the reference point (r_o, t_o), according to formula (2.4-1)4, there is $\rho_{mo} = \rho_{mo}(r_o/r_o)^{\kappa} G(1)$, thus deriving

$$G(1)=1 \tag{2.4-2)4}$$

The following text intends to substitute formulas (2.4-1)1,2,3,4 into the equation system for variable substitution.

Equations (2.2-6), (2.2-7)1 and (2.2-8) by variable substitution it becomes:

$$dU/d\ln\xi + (U-1)\ (d\ln G/d\ln\xi) + U(k+2) = 0 \tag{2.4-3)1}$$

$$(U-1)\ (dU/d\ln\xi) + [C^2/(5/3)]\ (d\ln GC^2/d\ln\xi) + U(U - 1/\alpha) + C^2\ (k+2)/(5/3) = 0 \tag{2.4-3)2}$$

$$d\ln C^3/d\ln\xi - d\ln G/d\ln\xi + [U(3-k) - 3/\alpha]/(U-1) = 0 \tag{2.4-3)3}$$

Argument the above:

Firstly, the following calculations are made based on formulas (2.4-1)2,3,4

$$\frac{\partial \xi}{\partial t} = \frac{-a}{t}\xi,\quad \frac{\partial \xi}{\partial r} = \frac{1}{r}\xi,\quad \frac{\partial \rho_m}{\partial t} = \frac{-\alpha\xi\rho_m}{t}\frac{d\ln G}{d\xi},\quad \frac{\partial \rho_m}{\partial r} = \frac{\kappa\rho_m}{r} + \frac{\rho_m\xi}{r}\frac{d\ln G}{d\xi},\quad \frac{\partial u}{\partial t} = \frac{-u}{t} + \frac{-\alpha\xi u}{t}\frac{dU}{Ud\xi},$$

$$\frac{\partial u}{\partial r} = \frac{u}{r} + \frac{\xi u}{r}\frac{d\ln U}{d\xi},\quad \frac{\partial c_s}{\partial t} = \frac{-c_s}{t} - \frac{\alpha c_s\xi}{t}\frac{d\ln C}{d\xi},\quad \frac{\partial c_s}{\partial r} = \frac{c_s}{r} + \frac{\xi c_s}{r}\frac{d\ln C}{d\xi} \tag{2.4-3)4,5,6,7,8,9,10,11}$$

Secondly, according to continuity equation (2.2-6) $\frac{\partial \rho_m}{\partial t} + \frac{\partial(r\rho_m u)}{r\partial r} = 0$, there is $\frac{\partial \rho_m}{\partial t} + u\frac{\partial \rho_m}{\partial r} + \rho_m\frac{\partial u}{\partial r} + \frac{\rho_m u}{r} = 0$. Substitute equations (2.4-3)6,7,9 into it to obtain $(1 - \alpha r/ut)\, d\ln G/d\xi + (k+2u)/\xi + dU/Ud\xi = 0$, substituting (2.4-1)2 $u = \alpha r U/t$ into this equations obtains $dU/d\ln\xi + (U-1)\, d\ln G/d\ln\xi + U(k+2) = 0$.

Thirdly, according to momentum equation (2.2-7)2, this equation can be transformed into $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{2c_s}{5/3}\frac{\partial c_s}{\partial r} + \frac{{c_s}^2}{(5/3)\rho_m}\cdot\frac{\partial \rho_m}{\partial r} = 0$ using the single particle gas sound velocity formula ${c_s}^2 = \frac{(5/3)p}{\rho_m}$. Substituting formulas (2.4-3)8,9,11,4 into this formula obtains $(\frac{u^2}{r} - \frac{a\,u}{t})\frac{r}{U}\frac{dU}{d\ln\xi} + \frac{{c_s}^2}{5/3}\frac{d\ln GC^2}{d\ln\xi} + r[\frac{u^2}{r} - \frac{u}{t} + \frac{{c_s}^2}{(5/3)r}(k+2)] = 0$, and then applying formula(2.4-1)2 to obtain $(U-1)\frac{dU}{d\ln\xi} + \frac{C^2}{5/3}\frac{d\ln GC^2}{d\ln\xi} + U(U - \frac{1}{a}) + C^2\frac{k+2}{5/3} = 0$.

Fourthly, according to the energy equation (2.2-8), the Euler operator(2.2-4)2 is used to transform it into ${c_s}^2(-2/3){\rho_m}^{-2/3-1}(\partial\rho_m/\partial t + u\,\partial\rho_m/\partial r) + (2c_s/{\rho_m}^{2/3})\ (\partial c_s/\partial t + u\,\partial c_s/\partial r) = 0$, and formulas(2.4-3)6,7,10,11 are substituted into these formulas to obtain $-(\frac{u}{r} - \frac{a}{t})\frac{d\ln G}{d\ln\xi} + \frac{-uk}{r} + 3(\frac{u}{r} - \frac{1}{t}) + 3\frac{d\ln C}{d\ln\xi}(\frac{u}{r} - \frac{a}{t}) = 0$. Based on these, formula(2.4-1)2 is used to deduce $(U-1)\, d[\ln(C^3/G)]/d\ln\xi + U(3-k) + (-3/\alpha) = 0$.

2.4.2 Further Evolution of the Equations System

The equations (2.4-3)1,2,3 can be further evolved into the following equations

$$dU/dC = \Delta_1(U,C)/\Delta_2(U,C) \tag{2.4-4)1}$$

$$d\ln\xi/dC = \Delta[U(C),C]/\Delta_2[U(C),C] \tag{2.4-4)2}$$

In the above equation:

$$\Delta_1(U,C)=U(U-1)\ (U-1/\alpha)-C^2[2U+(k-2\lambda)/(5/3)] \quad (2.4\text{-}4)3$$

$$\Delta_2(U,C)=C[\ (U-1)\ (U-1/\alpha)+U\ [\ (U-1)+\lambda]/3-C^2+(3\lambda+k)C^2/[5(U-1)\ ]] \quad (2.4\text{-}4)4$$

$$\Delta(U,C)=C^2-(U-1)^2 \quad (2.4\text{-}4)5$$

Where $\lambda=1/\alpha-1$.

Argument the above:

Firstly, according to (2.4-3), there is $d\ln G/d\ln\xi=d\ln C^3/d\ln\xi+[U(3-k)-3/\alpha]/(U-1)$, substituting this equation into (2.4-3)1 to obtain $(C/3)dU+(U-1)dC+C[5U/3-1/\alpha]d\ln\xi=0$. In this equation let $a_1=C/3$, $b_1=U-1$, $d_1=C[5U/3-1/\alpha]$, then $a_1dU+b_1dC+d_1d\ln\xi=0$ is obtained.

Secondly, according to equation (2.4-3)2 there is $(U-1)\frac{dU}{d\ln\xi}+\frac{C^2}{5/3}\cdot\frac{d\ln G}{d\ln\xi}+\frac{C^2}{\gamma}\cdot\frac{d\ln C^2}{d\ln\xi}+U(U-\frac{1}{\alpha})+\frac{C^2}{5/3}(k+2)=0$. Substituting equation(2.4-3)3 into this equation obtains $(U-1)dU+3CdC+\{U(U-\frac{1}{\alpha})+C^2[3+\frac{-3(1/\alpha-1)-k}{(5/3)\ (U-1)}]\}d\ln\xi=0$, and let $\lambda=1/\alpha-1$, $a_2=U-1$, $b_2=3C$, $d_2=U(U-1/\alpha)+C^2[3-[3\lambda+k]/[(5/3)\ (U-1)]]$, then $a_2dU+b_2dC+d_2d\ln\xi=0$ is obtained.

Thirdly, Calculate using the two formulas derived above:

$a_1dU+b_1dC+d_1d\ln\xi=0\times d_2$ - $a_2dU+b_2dC+d_2d\ln\xi=0\times d_1$, from this obtain $dU/dC=(b_1d_2-b_2d_1)/(a_2d_1-a_1d_2)$; In this equation let $\Delta_1(U,C)=b_1d_2-b_2d_1$, $\Delta_2(U,C)=a_2d_1-a_1d_2$, thus, $dU/dC=\Delta_1(U,C)/\Delta_2(U,C)$ is obtained.

Regarding $\Delta_1=b_1\cdot d_2-b_2d_1$, substitute the previously setting $b_1=U-1$, $b_2=3C$, $d_1=C[5U/3-1/\alpha]$, $d_2=U(U-\frac{1}{\alpha})+C^2[3-\frac{3\lambda+k}{(5/3)\ (U-1)}]$ into this equation, and take $\lambda=1/\alpha-1$ to derive $\Delta_1(U,C)=U(U-1)\ (U-1/\alpha)-C^2[2U+(k-2\lambda)/(5/3)]$.

Regarding $\Delta_2=a_2d_1-a_1d_2$, substitute the previously setting $a_1=\frac{C}{3}$, $a_2=U-1$, and $d_1=C[\frac{5U}{3}+\frac{-1}{\alpha}]$, $d_2=U(U-\frac{1}{\alpha})+C^2[3-\frac{3\lambda+k}{(5/3)\ (U-1)}]$ into this equation, derive $\Delta_2(U,C)=C[\ (U-1)\ (U-1/\alpha)+U\ [\ (U-1)+\lambda]/3-C^2+(3\lambda+k)C^2/[5(U-1)\ ]]$.

Fourthly, calculate using the two formulas derived above: $a_1dU+b_1dC+d_1d\ln\xi=0\times a_2$ - $a_2dU+b_2dC+d_2d\ln\xi=0\times a_1$, from this obtain $d\ln\xi/dC=(a_1b_2-a_2b_1)/(a_2d_1-a_1d_2)$, let $\Delta(U,C)=a_1b_2-a_2b_1$ and $\Delta_2(U,C)=a_2d_1-a_1d_2$ has been setted earlier, thus, the original formula becomes $d\ln\xi/dC=\Delta[U(C),C]/\Delta_2[U(C),C]$.

Regarding $\Delta=a_1b_2-a_2b_1$, substituting the previously setting $a_1=C/3$, $a_2=U-1$, and $b_1=U-1$, $b_2=3C$ into this equation, derive $\Delta(U,C)=C^2-(U-1)^2$.

Equations (2.4-4)1,2 must be solved numerically to obtain the solution function

$$U=U[C,\mathrm{C}^{(1)}] \quad (2.4\text{-}5)1$$

and

$$\mathcal{Z}=\mathcal{Z}[C,\mathrm{C}^{(2)}],\quad \mathcal{Z}=\ln\xi \quad (2.4\text{-}5)2$$

where $\mathrm{C}^{(1)}$ and $\mathrm{C}^{(2)}$ ～integral constants.

2.5 The Solution of the Equation System and Its Related Formula

2.5.1 Deriving the Relevant Formula in Implosion

According to fluid mechanics theory, the volume elements in a fluid move along a certain streamline in the flow field, and the position vector of the streamline can be expressed as $r=r(a,\ t)$, where $a=r(a,0)$ is the Lagrangian coordinate of the streamline; Due to the introduction of substitution variable ξ, $r=r(a,\ t)$ can be expressed as the following parametric equation $\begin{cases} r=r(a,\ \xi) \\ t=t(\xi) \end{cases}$. If the function $r=r(a,\ \xi)$ is obtained, the motion law of the implosive fluid is determined; It should be pointed out that according to formula (2.4-1)1, ξ corresponding to $t=0$ is $\xi_\infty=\infty$, so the Lagrangian coordinate $a=r(a,0)$ can also be written as $a=r(a,\xi_\infty)$.

The streamline represented by function $r=r(a,\ \xi)$ is divided into two branches: one starts from the starting point of implosion (r_{ao},t_{ao}) and reaches point $(a,\ \xi_\infty)$, and then the other branch of the streamline starts from point (a,ξ_∞) and intersects with the reflected shock wave.When discussing the $r\sim t$ and $C\sim U$ planes in the following text,it will be pointed out that the streamline originating from the starting point (r_{ao},t_{ao}) is located in the lower half plane of $r\sim t$, corresponding to the upper half plane of $C\sim U$ with $U\geq 0$; The streamline originating from $(a,\ \xi_\infty)$ is located in the upper $r\sim t$ half plane, corresponding to the lower half plane of $C\sim U$ with $U<0$.

The streamline expression originating from the starting point (r_{ao},t_{ao}) of the implosion is

$$r(a,\xi)=a[1-U(\xi)]^{-1/\beta}[\xi^{1/\alpha}C(\xi)]^{-3/\beta} \tag{2.5-1)1}$$

$\beta=2-3\lambda$ in the formula, for the inner surface of DT ice, its starting position vector r_{ho} is the reference vector r_o, and its Lagrangian coordinate is a_{ho}, then the above formula becomes

$$r_h=r(a_{ho},\xi)=a_{ho}[1-U(\xi)]^{-1/\beta}[\xi^{1/\alpha}C(\xi)]^{-3/\beta} \tag{2.5-1)2}$$

where r_h ~inner surface radius of the DT ice.

Formula (2.5-1)1 satisfies the boundary conditions:

$$r\big|_{\xi=1}=r_{ao} \tag{2.5-1)3}$$

To achieve this, the above conditions (2.4-2)3 $C(\xi=1)=1/\alpha$ must be met.

Formula (2.5-1)1 must also meet the boundary conditions

$$r\big|_{\xi=\xi_\infty}=a \tag{2.5-1)5}$$

where $\xi_\infty=\infty$ ~ the ξ value corresponding to $t=0$.

To meet the above two boundary conditions, the following formula must exist

$$a=r_{ao}\{[1-U(1)]/\alpha^3\}^{1/\beta} \text{ and } C(\xi_\infty)=0,\ U(\xi_\infty)=0 \tag{2.5-1)6}$$

and $\xi_\infty{}^{1/\alpha}C(\xi_\infty)=1$, which is the formula (2.5-7)2 that will be derived later.

Argument the above:

Firstly, according to formula $u=dr/dt$ and (2.4-1)2, $\alpha Ud\ln|t|=d\ln r$ can be obtained.

According to formula (2.4-1)1, $d\xi/dt=|t_o/t|^\alpha r_o{}^{-1}(u-r\alpha/t)$ is obtained, substitute formula (2.4-1)2 into it to obtain $d\xi/dt=(\alpha/t)|t_o/t|^\alpha[(U-1)r/r_o]$, then substitute formula (2.4-1)1 into this formula to obtain $d\ln\xi/[\alpha(U-1)]=d\ln|t|$; Substitute the above formula into the previous formula $\alpha Ud\ln|t|=d\ln r$ to obtain $d\ln r/d\ln\xi=U(\xi)/[U(\xi)-1]$.

Secondly, according to equation (2.4-3)1, there is $dU+(U-1)d\ln G+U(\kappa+2)d\ln\xi=0$. Divide the two sides by

$(U-1)$ to obtain $dU/(U-1)+d\ln G+U(U-1)d[(\kappa+2)\ln\xi]=0$. Substitute $d\ln r/d\ln\xi=U/(U-1)$ in "Firstly" into this equation to obtain $d(U-1)/(U-1)+d\ln G+(\kappa+2)d\ln r=0$. U in this equation will be mentioned later: $U\le 1$, and based on this, $(1-U)Gr^{\kappa+2}=C_{ic}^{(3)}$ can be derived, where $C_{ic}^{(3)}$ ∼constant.

Thirdly, the sound velocity formula $c_s{}^2=(5/3)p/\rho_m$ of a single particle gas is substituted into the adiabatic equation $p/\rho_m{}^{5/3}=\mathsf{A}$ to obtain $c_s{}^2\rho_m{}^{-2/3}/\gamma=\mathsf{A}$, substitute formulas(2.4-1)3,4 into this equation and use formula (2.4-1)1 to obtain $[a(\xi/r)^{1/\alpha}rC]^2(r^\kappa G)^{-(\gamma-1)}=[|t_o|/r_o^{1/\alpha}]^2(\rho_{mo}/r_0^k)^{\gamma-1}(5/3)\mathsf{A}$, note that for the adiabatic process $\mathsf{A}=const$ of the same streamline, therefore, the right side of the equation is a constant, set it to $C_{ic}^{(4)}$, and thus $[a(\xi/r)^{1/\alpha}rC]^2(r^\kappa G)^{-2/3}=C_{ic}^{(4)}$ is derived.

Fourthly, from the formula $(1-U)\cdot Gr^{\kappa+2}=C_{ic}^{(3)}$ of "Secondly" there is $G(\xi)=C^{(3)}r^{-(\kappa+2)}/(1-U)$, substitute this into the above formula and take $C^{(5)}=[C^{(4)}C^{(3)}]^{1/\beta}$, where $\beta=2-3\lambda$, then the formula for $r=r(a,\xi)$ can be derivedas $r=C^{(5)}[1-U]^{-1/\beta}[\alpha\xi^{1/\alpha}C]^{-3/\beta}$.

Fifthly, determine $C^{(5)}$ in the above formula. The above formula should satisfy the boundary condition (2.5-1)5, so there is $a=C^{(5)}[1-U(\xi_\infty)]^{-1/\beta}[\alpha\xi_\infty{}^{1/\alpha}C(\xi_\infty)]^{-3/\beta}$. Substitute formula(2.5-7)2 $\xi_\infty{}^{1/\alpha}C(\xi_\infty)=1$ derived from the following into this formula, and note that according to formula (2.4-1)1: because there will be no $r=0$ at $t=0$, there must be $\xi_\infty=\infty$; According to formulas (2.4-1)2,3: since u and c_s are finite values at $t=0$, so there must be $C(\xi_\infty)=0$, $U(\xi_\infty)=0$ and thus $a\alpha^{-3/\beta}=C^{(5)}$, substituting this to the original equation obtains $r=a\alpha^{-3/\beta}[1-U]^{-1/\beta}[\alpha\xi^{1/\alpha}C]^{-3/\beta}$.

The above formula should also satisfy the boundary condition(2.5-1)3 formula, so there is $r=a\alpha^{-3/\beta}\cdot[1-U(1)]^{-1/\beta}[\alpha C(1)]^{-3/\beta}$. Substituting (2.4-2)3 formula into this formula obtains $C^{(5)}=r_{ao}[1-U(1)]^{1/\beta}$;

Substituting this into the original formula of r obtains $r=a[1-U(\xi)]^{-1/\beta}[\xi^{1/\alpha}C(\xi)]^{-3/\beta}$.

Combine the obtained $C^{(5)}=r_{ao}[1-U(1)]^{1/\beta}$ and $a\alpha^{-3/\beta}=C^{(5)}$ to obtain $a=r_{ao}\{[1-U(1)]/\alpha^3\}^{1/\beta}$.

The expression for function $\rho_m(\xi)$ is:

$$\rho_m(\xi)=\rho_{mo}[\alpha\xi^{1/\alpha}C(\xi)]^{6/\beta}\{[1-U(\xi)]/[1-U(1)]\}^{3\lambda/\beta} \tag{2.5-2)1}$$

At $t=0$, the above formula becomes

$$\rho_m(\xi_\infty)=\rho_{mo}\,\alpha^{6/\beta}/[1-U(1)]^{3\lambda/\beta} \tag{2.5-2)2}$$

From the above formula: the closer the $U(\xi)$ value is to 1, the greater the $\rho_m(\xi_\infty)$ value, so then the stronger the compression.

Argument the above:

In the process of demonstrating formula (2.5-2)1 there is equation $(1-U)Gr^{\kappa+2}=C_{ic}^{(3)}$, according to this formula $G=C_{ic}^{(3)}r^{-(\kappa+2)}(1-U)^{-1}$ can be obtained,substitute formula (2.5-1)1 into this formula and transform it to obtain $G=(C_{ic}^{(3)}/a^{\kappa+2})\,[\xi^{1/\alpha}C]^{3(\kappa+2)/\beta}[1-U]^{(\kappa+2)/\beta-1}$; From this formula and according to formulas(2.4-2)2,4

$C_{ic}^{(3)} = a^{\kappa+2}\alpha^{3(\kappa+2)/\beta}[1-U(1)]^{1-(\kappa+2)/\beta}$ is obtained, and substitute this into the original formula and transform it to obtain $G = [\alpha\xi^{1/a}C]^{3(\kappa+2)/\beta}\{(1-U)/[1-U(1)]\}^{(\kappa+2)/\beta-1}$; Substitute this formula and formulas(2.5-1)1,6 into formula (2.4-1)4 and transform it ,obtain formula (2.5-2)1.

At $\xi = \xi_\infty$, substituting formula(2.5-1)6, and substituting formula(2.5-7)2 $\xi_\infty{}^{1/\alpha}C(\xi_\infty) = 1$ derived below into the above formula obtains $\rho_m(\xi_\infty) = \rho_{mo}\,\alpha^{6/\beta}/[1-U(1)]^{3\lambda/\beta}$.

The expression for velocity $u = u(a,\xi)$ at $t_o < t < 0$ is

$$u(a,\xi) = -|c_{so}|[r_o/r(a,\xi)]^{1/a-1}[\alpha\xi^{1/a}U(\xi)] \tag{2.5-3)1}$$

The expression for velocity $u(a,\xi_\infty)$ at $t=0$ is

$$u(a,\xi_\infty) = -\alpha\mathbf{M}|c_{so}|(r_o/a)^{1/a-1} \tag{2.5-3)2}$$

Starting time of implosion:

$$u(a,1) = -|c_{so}|[\alpha U(1)] \tag{2.5-3)3}$$

where $\mathbf{M} = U(\xi_\infty)/C(\xi_\infty)$ ～Mach number,will be discussed in formula (2.5-7)1 below.

Argument the above:

Firstly, according to equation (2.4-1)1, there is $|t| = (|t_o|/r_o^{1/a})\,(r/\xi)^{1/a}$. Substituting this into formula (2.4-1)2 obtains $u = -(\alpha r_o/|t_o|)\xi^{1/\alpha}U(\xi)\,(r_{ao}/r)^{1/\alpha-1}$. Substituting formula(2.4-2)2 into this formula obtains equation(2.5-3)1.

Secondly, At $t=0$ i.e. at $\xi = \xi_\infty$, formula(2.5-3)1 becomes $u(a,\xi_\infty) = -|c_{so}|[r_o/r(a,\xi_\infty)]^{1/a-1}[\alpha\xi_\infty{}^{1/a}U(\xi_\infty)]$. Substituting formula(2.5-1)5, and substituting formula (2.5-7)1,(2.5-7)2 which will be discussed below, into this formula obtains $u(a,\xi_\infty) = -\alpha\mathbf{M}|c_{so}|[r_o/a]^{1/a-1}$.

Thirdly, at the starting time $\xi = 1$, so the formula(2.5-3)1 becomes $u(a,1) = -|c_{so}|[r_o/r(a,1)]^{1/a-1}[\alpha U(1)]$. Based on this,can derive $u(a,1) = -|c_{so}|[\alpha U(1)]$ using initial condition(2.5-1)3.

According to formula (2.5-1)1, the following inference can be made:

Inference 1: according to formula (2.5-1)1 and boundary conditions (2.5-1)3, (2.5-1)6:for a given α, if the starting vector r_{ao} of the streamline $r = r(a,\xi)$ is given, then its Lagrangian coordinate a is determined, and thus this streamline is also determined.

Inference 2: There is a relationship between the streamline $r_h = r(a_{ho},\xi)$ with a starting position vector of r_o and the streamline $r = r(a,\xi)$ with a starting position vector of r_{ao}, as follows

$$r(a_{ho},\xi)/r(a,\xi) = r_o/r_{ao} \tag{2.5-4)1}$$

as well as

$$a_{ho}/a = r_o/r_{ao} \tag{2.5-4)2}$$

The above formula indicates that if the streamline $r_h = r(a_{ho},\xi)$ at the inner surface of DT ice is given, then the streamline $r = r(a,\xi)$ of any given starting vector r_{ao} inside DT also is determined.

Argument the above:

For the given α, a_{ho}, a and the same ξ, use the(2.5-1)2 formula÷(2.5-1)1 formula to obtain

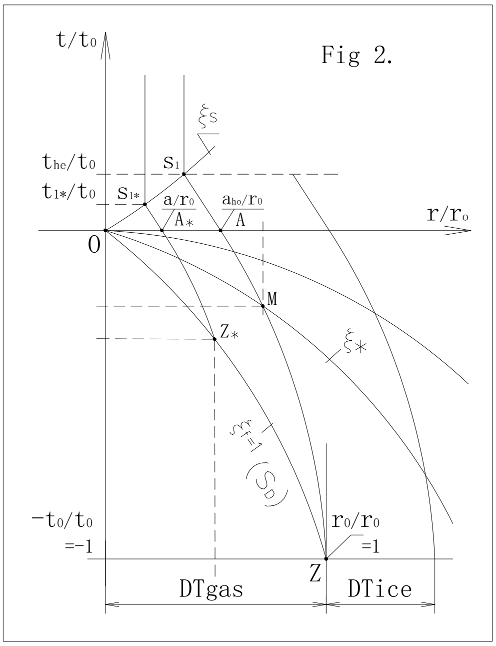
$r(a_{ho}, \xi)/r(a, \xi) = r_o/r_{ao}$, and according to the boundary condition (2.5-1)5 formula, obtain $a_{ho}/a = r_o/r_{ao}$.

2.5.2 Discussion on Content Related to the Function $U\!=\!U[C, C^{(1)}]$ of Solution

The above formulas (2.5-1)2 and (2.5-2) for determining the law and state of implosion flow are all related to the solution function $U\!=\!U[C, C^{(1)}]$, which will be discussed below.

2.5.2.1 Establishing the $r\sim t$ Coordinate Plane

According to the parameter equation $\begin{cases} r = r(a, \xi) \\ t = t(\xi) \end{cases}$: there exists a dimensionless coordinate plane $r/r_0 \sim t/t_0$ as shown in Figure 2, the trace of streamline $r\!=\!r(a,\ t)$ can be drawn in it. Below the $r/r_0 \sim t/t_0$ plane is abbreviated as the $r\sim t$ plane.



Give $\xi = const$ a given value, formula (2.4-1)1 expresses a given curve in the $r\sim t$ plane. Given different values for $const$, then a curve cluster $\xi = (r/r_o)|t_o/t|^{a} = const$ converging at the origin O can be obtained.

There will be intersection points between streamline $r\!=\!r(a,\ t)$ and curve cluster $\xi\!=\!const$. If $r\!=\!r(a,\ t)$ intersects with a certain curve ξ_* in the curve cluster at point M, and draw the vertical line of the r/r_0 axis and t/t_0 axis through point M,then the coordinates at their foot point are the r_*/r_0 coordinate and t_*/t_0 coordinate of streamline $r\!=\!r(a,\ t)$, respectively. Therefore, curve cluster $\xi\!=\!const$ is the parameter coordinate line of parameter equation $\begin{cases} r = r(a, \xi) \\ t = t(\xi) \end{cases}$.

When demonstrating formula (2.5-1)1 it was obtained that $d\ln r/d\ln\xi = U/(U-1)$, for curve cluster $\xi = const$ its left side becomes infinite. In order to this formula to hold true for curve cluster $\xi = const$, the right side must also be infinite,so there must be $U(\xi)\!-\!1\!=\!0$; Thus, $U(\xi)\!-\!1\!=\!0$ corresponds to curve cluster $\xi = const$, but for solution function $U = U[\xi]$, $\xi \neq const$, so there will be no solution function $U[\xi] = 1$.

In addition, due to $1-U(1) \neq 0$ in formula (2.5-2)1, if $C(\xi)\!\to\!\infty$, then $[\alpha\ \xi^{1/a} C(\xi)\ ]^2[1\!-\!U(\xi)\ ]^{\lambda}\}_{\substack{C\to\infty \\ U=1}}$ becomes an indeterminateform, resulting in the uncertainty of $\rho_m(\xi)$ value, which corresponds to the discountinuity of

parameter at the parameter discontinuity interface; But if $C(\xi)$ takes a certain finite value, then $\rho_m(\xi)=0$, indicates that there is a vacuum at that point.
In summary, the curve cluster $\xi = const$ corresponds to $U(\xi)=1$, based on this, the inference is as follows:

Firstly, if $C(\xi)\to\infty$, then corresponds to the parameter discontinuity interface,it will be pointed out in the discussion of singularity in the later:this corresponds to singularity P_6.
Secondly, if $C(\xi)$ takes a finite value, then it corresponds to vacuum.

Regarding the parameter discontinuity interface, the following two will be involved:

Firstly, material interface～the inner surface r_{ho} of DT ice at the starting time of implosion,the density of DT gas at this point is a tiny quantity compared to DT ice, can be approximated as vacuum.This corresponds to a finite value of $C(\xi)$. This is consistent with the previously taken r_{ho} as the reference vector, so $C(\xi=1)=1/a$, Therefore, the parameter discontinuity interface here i.e. material interface and corresponds to $\xi=\xi_f=1$.
Secondly, the reflected shock wave.When the material interface propagates as a parameter discontinuity interface to the center of the implosion, a reflected shock wave is immediately emitted out from the center, this corresponds to $\xi=\xi_s=1$.
Argument the above:

Regarding $\xi_s=1$, referring to reference [4]: for the incident wave (L) and the reflected wave (R), the wave amplitudes are $r_R=|R_{LR}|r_L$ and $u_{\xi R}=R_{LR}u_{\xi L}$ respectively,the radiation power is $P_L=Z_L u_{\xi L}^2$ and $P_R=Z_R u_{\xi R}^2$ respectively; Where r_L and r_R ～fluctuation amplitude, R_{LR} ～reflection coefficient, $u_{\xi L}$ and $u_{\xi R}$ ～amplitude change velocity, Z_L and Z_R ～incident and reflection impedance; If the energy loss during reflection is ignored, then there is $Z_L u_{\xi L}^2 = Z_R u_{\xi R}^2$ due to energy conservation. Substituting $u_{\xi R}=R_{LR}u_{\xi L}$ into this formula obtains $Z_L=Z_R R_{LR}^2$.

Still according to literature (F.S. Crawford, 1981): impedance $Z=f/u_\varphi$, where f ～the damping force, u_φ ～the phase velocity; Due to the incident and reflected waves being in the same medium DT gas, so that there is $Z_L=Z_R$, thus $Z_L=Z_R R_{LR}^2$ becomes $|R_{LR}|=1$. Therefore formula $r_R=|R_{LR}|r_L$ becomes $r_R=r_L$, there is $(r_R/r_o)|t_o/t|^a=(r_L/r_o)|t_o/t|^a$ by this, and this formula becomes $\xi_R=\xi_L$ according to formula (2.4 -1)1; But for implosion,the incident wave surface is the material interface r_{ho}, the reflected wave is the reflected shock wave, and the $\xi_f=1$ corresponding to the material interface is $\xi_L=1$, therefore $\xi_R=1$ is $\xi_s=1$.
For DT ice or gas in the ring target, different layers can be distinguished based on the different position vector r. When the centrally symmetric driving pressure acts on a certain layer, the pressure is transmitted in this way: fluids between different layers move relative to each other due to compression, resulting in compression waves transmitted from the outside to the inside and along the compression direction.

2.5.2.2 In the $r\sim t$ Plane, Representing the Implosion Process

Due to that the DT ice layer is a thin layer,the transmission of compression waves is completed in a very short time, therefore can approximately consider as: all layers within the DT ice will be subjected to the same value, radial driving pressure at the same time.

Regarding the DT ice thickness δr_i, there exists the following formula

$$\delta r_i = \delta r_{io}\, r_c/r_{co} \quad \text{or} \quad \delta r_i = \delta r_{io}\, r_h/r_{ho} \tag{2.5-5)1}$$

Where δr_{io} ～DT ice layer initial thickness; r_{co}, r_c ～DT ice outer surface initial radius, outer surface radius: r_{ho}, r_h ～DT ice inner surface initial radius,inner surface radius.

According to the above formula, regarding the mass density ρ_{mc} of DT ice, approximately consider as: the mass inside the ice layer is uniformly distributed, then the following formula exists

$$\rho_{mc} = \rho_{mco}(\bar{r}_{co}/\bar{r}_c)^2 \tag{2.5-5)2}$$

where ρ_{mc} ～the DT ice density, $(r_c + r_h)/2 = \bar{r}_c$ ～the average radius of the DT ice layer.

Same principle, for the average mass of the center DT gas after the stagnate, and for the mass of the ring target shell, there exists the following formulas

$$\bar{\rho}_{mhe} = \rho_{mho}(r_o/r_{he})^2, \quad \rho_{msh} = \rho_{msho}(\bar{r}_{sho}/\bar{r}_{sh})^2 \tag{2.5-5)3.4}$$

where ρ_{mho}, $\overline{\rho_{mhe}}$ ～center DT gas initial density, end density of stagnate; ρ_{msh}, ρ_{msho}～ring target shell density, shell initial density.

Argument the above:

For δr_i : regarding $\delta r_{io}/\delta r_i = (r_{co} - r_{ho})/(r_c - r_h) = (r_{co}/r_c)(1 - r_{ho}/r_{co})/(1 - r_h/r_c)$ or $\delta r_{io}/\delta r_i = r_{ho}(r_{co}/r_{ho} - 1)/r_h(r_c/r_h - 1)$, discuss r_{co}/r_{ho} in formula: use equation (2.5-4)1 here, where $r(a_{ho}, \xi) = r_h$ and $r_o = r_{ho}$, taken $r(a, \xi) = r_c(a_c, \xi)$, then there is $r_{ao} = r_{co}$, hence there is $r_h/r_c = r_{ho}/r_{co}$, thus deriving $\delta r_i = \delta r_{io}\, r_c/r_{co}$ or $\delta r_i = \delta r_{io}\, r_h/r_{ho}$ from the original formula.

For ρ_{mc}, if approximately consider as: within the DT ice there is no parameter discontinuity interface,and the mass is uniformly distributed, then according to the law of mass conservation there is $\rho_{mc}V_c = \rho_{mco}V_{co}$, where V_{co}, V_c～DT ice layer volume and initial volume; According to formula (2.2-5)1, for DT ice layer this formula holds: $V_c = 2\pi^2\bar{R}(r_c^{\,2} - r_h^{\,2}) = 2\pi^2\bar{R}(r_c + r_h)\,(r_c - r_h) = 4\pi^2\bar{R}[(r_c + r_h)/2]\delta r_i$, note that $(r_c + r_h)/2 = \bar{r}_c$ is the average radius of the DT ice layer, so $V_c = 4\pi^2\bar{R}\bar{r}_c\delta r_i$, hence the original formula becomes $\rho_{mc}\bar{r}_c\delta r_i = \rho_{mco}\bar{r}_{co}\delta r_{io}$; Substituting formula (2.5-5)1 into this formula obtains $\rho_{mc} = \rho_{mco}(\bar{r}_{co}/\bar{r}_c)^2$.

For ρ_{mho} and ρ_{msh}, due to there is no parameter discontinuity interface within the hot spot after stagnate end, and can approximately consider as: there is no parameter discontinuity interface within the ring target shell,so the mass is uniformly distributed.Therefore, according to the law of mass conservation, formulas (2.5-5)3,4 can be derived in the same way as formula (2.5-5)2.

Regarding the central DT gas, as shown in Figure 1, the interface between DT ice and DT gas is a material interface with a position vector r_{ho}, so which is a parameter discontinuity interface; Since the curve cluster $\xi = const$ corresponds to the parameter discontinuity interface, as shown in Figure 2, at the starting time of implosion t_o, the discontinuity interface S_D will propagate as a disturbance wave from the implosion initial point Z to the center point O along the curve $\widetilde{ZO}$, and reach the O point at time $t = 0$; At the same time as the discontinuity interface S_D is emitted,the fluid element at the material interface r_{ho} also starts moving along the streamline $\widetilde{ZA}$ from point Z, and also reaching point A at time $t = 0$.

The discontinuity interface S_D goes to a place, density of the front and rear sides of the place is inconsistent; The S_D is a compressive wave, at the point where the S_D wave surface did not reach, the layer of the DT gas had not been disturbed by compression waves, so remained stationary, only the layer where the S_D wave surface reached began to move. As shown in Figure 2, before the S_D wave surface is reached, the fluid element at point Z_* in DT gas is still at rest; When the S_D wave surface arrives, the fluid element at that point begins to move along the streamline $\widetilde{Z_*A_*}$, and also reaches A_* point at time $t = 0$.

The streamline $\widetilde{ZA}$ and $\widetilde{Z_*A_*}$ are expressed by formulas (2.5-1)2,1 respectively; As mentioned earlier,If the starting position vector r_{ho} is given, then the streamline $\widetilde{ZA}$ is determined; And any streamline $\widetilde{Z_*A_*}$ inside DT is also determined accordingly.

When wave surface S_D propagates to point O, a reflected shock wave ξ_s is immediately emitted from the point O, at this time the stagnate begins: shock wave ξ_s will meet various streamline such as $\widetilde{ZA}$ and $\widetilde{Z_*A_*}$ in sequence; Due to that the shock wave ξ_s is a parameter discontinuity interface, the implosive fluid passes through the shock wave surface and rushes towards the center point O, causing a sharp decrease in velocity to zero, thereby sequentially entering the stagnate; When the reflected shock wave ξ_s reaches the DT ice -gas interface,the velocity inside the center DT gas is all reduced to zero, and the stagnate ends at this time.

2.5.2.3 Establishing the $C\sim U$ Coordinate Plane

As a solution to equation(2.4-4)1, function $U=U[C,C^{(1)}]$ represents a solution curve in the $C\sim U$ coordinate plane.

The range of values for C and U

Firstly, according to formula(2.4-1)1: $\xi \geq 0$, and since C appears in the $[\alpha\xi^{1/\alpha}C(\xi)]^{-3/\beta}$ term of formula (2.5-1)1, therefore there must be $C \geq 0$.

Secondly, due to $u \leq 0$, thus according to formula(2.4-1)2: there must be $U \geq 0$ in the $t_0 \leq t \leq 0$ range, and $U < 0$ in the $t > 0$ range.

Thirdly, there is $U < 1$ in the $t_0 \leq t \leq 0$ range.

Fourthly, in summary, as shown in Figure 3, the range of values for C and U are located in the common area to the right of the U axis and below the $U = 1$ line.

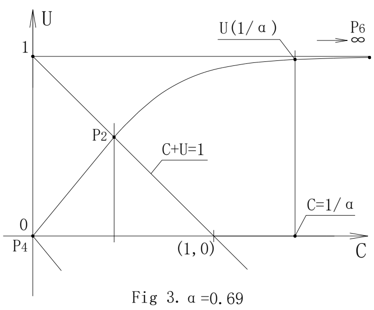


Fig 3. α=0.69

Argument the above:

Regarding $U < 1$, according to formula (2.4-1)1, there is $\xi=0$ at $r = 0$; when changing from $(r = 0, t < 0)$ to $(r = a, t = 0)$ changes from $\xi=0$ to $\xi=\infty$; So, before time $t=0$, ξ is an increasing function; Hence $d\xi>0$, that is $d\ln\xi > 0$, and for implosion $dr < 0$,that is $d\ln r < 0$; Thus, the left side of $\frac{d\ln r}{d\ln\xi}=\frac{U(\xi)}{U(\xi)-1}$ obtained in the proving formula(2.5-1)1 must be less than zero; Due to $U\geq 0$ in the $t_0 \leq t \leq 0$ range, hence $U < 1$ is sure.

2.5.2.4 The Singularities of Equation (2.4-4)1 $dU/dC = \Delta_1(U, C)/\Delta_2(U, C)$ in the $C\sim U$ Plane

If $\frac{\Delta_1(U, C)}{\Delta_2(U, C)}$ is the unique deterministic function at point $P_n(C,U)$ in the $C\sim U$ plane, then the equation has a unique deterministic solution at that point; If the value of $\frac{\Delta_1}{\Delta_2}$ at point $P_n(C,U)$ is uncertain, then point P_n is a singularity, and the equation will have many solution curve passing through this singularity.

Equation $dU/dC = \Delta_1(U, C)/\Delta_2(U, C)$ has seven singularity, and this article involves the following three: P_6, P_2, and P_4.

At point $P_6[C = \infty, U = const]$, it can be proven that $\Delta_1/\Delta_2 = \infty$, so this point is a singularity; If $U = 1$ then the P_6 singularity corresponds to the parameter discontinuity interface.

Argument the above:

If there is $U=1$ at the singularity P_6, then formula (2.5-2)1 becomes an indeterminate form, this makes the ρ_m value uncertain,that corresponds exactly to the parameter discontinuity interface, and this is also consistent with curve cluster $\xi = const$ that corresponds to parameter discontinuity interface; Therefore, the parameter discontinuity interface corresponds to the singularity P_6.

There is a singularity P_2 in the upper half plane of $C \sim U$, which has the following characteristics:

if a solution curve must pass through the straight line $C+U=1$ in the upper half plane, then the solution curve that conforms to the physical meaning must pass at the singularity P_2; When taking $k-2\lambda=0$, the coordinates of P_2 are

$$\begin{cases} C_{P2} = \lambda \\ U_{P2} = 1-\lambda \end{cases} \tag{2.5-6}$$

Where λ must meet $0<\lambda<1$.

Argument the above:

Firstly, according to formula (2.4-4)3 can inferred: there is $\Delta_1/C = U^2 + [\lambda - 1 + (k-2\lambda)/(5/3)]U - (k-2\lambda)/(5/3)$ on the straight line $C+U=1$; In addition, $\Delta_2 = 0$ can also be derived on the straight line $C++U=1$.

Therefore, if equation $U^2 + [\lambda - 1 + (k-2\lambda)/(5/3)]U - (k-2\lambda)/(5/3) = 0$ is satisfied at $C+U=1$, then Δ_1/Δ_2 becomes an indeterminate form, resulting in the existence of a singularity.

When $k-2\lambda=0$ and $1-\lambda>0$ are taken, the above equation has a solution $U=1-\lambda$, and the corresponding point on line $C+U=1$ is $(\lambda, 1-\lambda)$, so the coordinate of P_2 is $(\lambda, 1-\lambda)$.

Secondly, if the solution curve intersects with the straight line $C+U=1$ at point P_*; At this point $\Delta_1 \neq 0$, $\Delta_2 \neq 0$, but at $C=1-U$, according to (2.4-4)5 formula: $\Delta=0$; Thus, according to equations (2.4-4)1,2, $d\xi/dC=0$ and $d\xi/dU=0$ can be derived; From this, it is known that functions $\xi=\xi(C)$ and $\xi=\xi(U)$ have extreme values at point P_* on the $C+U=1$ line.Therefore, $C(\xi)$ and $U(\xi)$ are double valued functions of ξ in the neighbourhood of point P_*. However, the functions $C=C(\xi)$ and $U=U(\xi)$ that conform to physical meaning must be a single valued function of ξ, so the corresponding solution curve will not cross line $C+U=1$ at the points of $\Delta_1 \neq 0$ and $\Delta_2 \neq 0$, but rather at the point of $\Delta_1=0$ and $\Delta_2=0$ $\sim$ the singularity P_2, passes through the straight line $C+U=1$.

At point $P_4(C=0,\ U=0)$, according to formulas (2.4-4)3,4: Δ_1/Δ_2 becomes an indeterminate form, hence P_4 is a singularity. P_4 has the following characteristics:

Firstly, when the solution curve $U=U[C, C^{(1)}]$ enters the lower half plane from the upper half plane of $C \sim U$, it must pass through the singularity P_4, this corresponds to the implosion streamline $r = r(a_{ho}, \xi)$ crossing the r/r_0 axis from the lower half plane of $r \sim t$ into the upper half plane.

Secondly, in the neighbourhood of singularity P_4, there is a linear relationship between U and C

$$U(\xi_\infty) = \pm \mathbf{M}\, C(\xi_\infty) \tag{2.5-7)1}$$

where $\mathbf{M} > \mathbf{0}$ $\sim$constant, i.e. "Mach number"; The above formula takes the "+" sign on the upper half plane and the "-" sign on the lower half plane.

Thirdly, there exists the following formula in the neighbourhood of singularity P_4

$$\xi_\infty^{\ 1/\alpha} C(\xi_\infty) = 1 \tag{2.5-7)2}$$

Argument the above:

Firstly, when discussing the value ranges of C and U ,it has been stated that in the $t_0 \le t \le 0$ ranges: $U \ge 0$, and in the $t > 0$ ranges: $U < 0$ .Therefore, when the implosion streamline $r = r(a_{ho}, \xi)$ crosses the r/r_0 axis from the lower half plane of $r \sim t$ and enters its upper half plane, the solution curve $U = U[C, C^{(1)}]$ corresponding to this process passes through the C axis from the upper half plane of $C \sim U$ and enters its lower half plane; At the intersection of the solution curve and C axis, there should be $U = 0$; But as stated in the proof (2.5-1)1, this corresponds to $\xi = \infty$, $U(\xi_\infty) = 0$, $C(\xi_\infty) = 0$; Therefore, when the solution curve enters the lower half plane from the upper half plane of $C \sim U$, it must pass through point $[C(\xi_\infty) = 0,\ U(\xi_\infty) = 0]$.

Secondly, in the neighbourhood of singularity $P_4(C = 0,\ U = 0)$, formulas(2.4-4)3,4 become $[\Delta_1]_{\substack{C=0\\U=0}} = U/\alpha$ and $[\Delta_2]_{\substack{C=0\\U=0}} = C/\alpha$ after omitting second-order and third-order small quantities, respectively, resulting in $[dU/dC]_{\substack{C=0\\U=0}} = U/C$. Integrating this formula in the neighbourhood of point $P_4(C = 0,\ U = 0)$ obtains $\ln U = \ln C^{(8)} C$, with the constant of integration $C^{(8)} = \pm \mathbf{M}$, resulting in $U = \pm \mathbf{M}\, C$; So, in the first quadrant of $C \sim U$ the formula should take the sign "+", in the second quadrant should take the sign "-".

Thirdly, in the neighbourhood of singularity $P_4(C = 0,\ U = 0)$, formulas (2.4-4)5,4 become $[\Delta(U, C)]_{\substack{C=0\\U=0}} = -1$ and $[\Delta_2]_{\substack{C=0\\U=0}} = C/\alpha$, respectively, resulting in equations (2.4-4)2 becoming $[d \ln \xi / dC]_{\substack{C=0\\U=0}} = -\alpha/C$. By integrating both sides and taking the constant of integration as 1, $\xi_\infty^{\ 1/\alpha} C(\xi_\infty) = 1$ can be derived.

2.5.2.5 The Solution Function $U = U[C, C^{(1)}]$ and the Solution Curve

By using the solution function $U = U[C, C^{(1)}]$, a solution curve can be drawn on the $C \sim U$ plane. According to formula(2.5-1)1, this solution curve corresponds to a cluster of implosive streamline $r = r(a_{ho}, \xi)$;

Since the implosion streamline starts at the material boundary, which is a parameter discontinuity interface, but the parameter discontinuity interface corresponds to P_6, therefore, the solution curve corresponding to the implosion streamline should start at the singularity P_6.

From the implosion initiation to before the stagnate, the streamline $r = r(a_{ho}, \xi)$ reaches the r/r_0 axis within the $t_0 \le t \le 0$ range, so the corresponding solution curve should first pass through the singularity P_2 in the upper half plane of $C \sim U$ and then reach the singularity P_4; In Figure 3, the two solution curves mentioned above are drawed: $P_6 \tilde{P}_2$ and $\overline{P_2 P_4}$.

Regarding the solution curve $\overline{P_2 P_4}$, if $k - 2\lambda = 0$, then it is a straight line $U = \mathbf{M}\, C$ from singularity P_2 to singularity P_4, where

$$\mathbf{M} = (1-\lambda)/\lambda \tag{2.5-9}$$

Argument the above:

Substitute the coordinates $C_{P2} = \lambda$ and $U_{P2} = 1 - \lambda$ of the singularity P_2 at $k - 2\lambda = 0$ into the right side of equation(2.4-4)1 to obtain $dU/dC = U/C$, substitute $C_{P2} = \lambda$ and $U_{P2} = 1 - \lambda$ also into the right of this equation to obtain $\frac{U}{C} = \frac{1-\lambda}{\lambda}$, thereby resulting in $\frac{dU}{dC} = \frac{1-\lambda}{\lambda}$. Integrate to obtain $U = \frac{1-\lambda}{\lambda} C + C^{(9)}$, which is a solution straight line with a slope of $\mathbf{M} = (1-\lambda)/\lambda$ originating from the singularity P_2; If $C^{(9)} = 0$ istaken, then $U = C\,(1-\lambda)/\lambda$ passes through P_4 point, so the straight line $U = \mathbf{M}\, C$ is the solution curve.

2.5.2.6 Summary

Discuss until now, the solution functions $U = U[C]$ corresponding to two solution curves $P_6 \tilde{P}_2$ and $\overline{P_2 P_4}$

have been obtained respectively. Thereby the parameters $r = r(a_{ho}, \xi)$, $\rho = \rho_m(\xi)$, and other parameters of the implosion fluid can be obtained using such as formulas(2.5-1)2 and(2.5-2)1,2, etc, therefore describing the physical process of implosion.

3. The Stagnate, Self-heating, and Ignition of DT

3.1 Foreword

The occurrence of stagnate,as shown in Figures 2 and 3, when the implosion proceeds to $t>0$, the streamline enters the upper half plane of $r \sim t$; The streamline then passes through the reflected shock wave and rushes towards the center, causing $r \to 0$ due to the implosion velocity $u<0$. Thereby according to equation (2.4-1)2, this causing $u \to 0$, so results in the stagnate of DT.

At the moment $t=0$ when the reflected shock wave ξ_s is emitted, the stagnate begins; As the reflected shock wave advances, the velocity of any streamline that meets it rapidly decreases to zero after passing through the shock wave, until the fluid element at the inner surface of the DT ice layer along the streamline $A\tilde{S}_1$ meets the reflected shock wave. Since then, the flow velocity of all DT gases has decreased to zero, thus achieving complete stagnate. The corresponding moment is $t = t_{he}$, the center DT gas with zero kinetic energy forms at the moment, its energy will be converted into internal energy.

Self heating, if at the end of stagnate, the internal energy shows an increasing trend, i.e:

$$dE_{he}/dt > 0 \tag{3.1-1}$$

where E_{he} ∼internal energy density of the central gas. This causes the temperature of the central gas to continuously increase, leading to nuclear fusion .The process described in the above formula is called "self heating".

Ignition, If at the end of the stagnate, the central gas occurs sufficient strong nuclear fusion due to self heating, the gas mass formed in this way is called a "hot spot"; If the hot spot continuously heats up,and the fusion energy inside it can continuously increase, causing the fusion energy to be transmitted externally, leading to complete nuclear fusion of the outer DT ice layer, then this process is called "ignition".

3.2 The Center DT Gas Energy Equation and Self-heating Conditions

3.2.1 Central DT Gas Energy Equation

The energy equation for the center DT gas during stagnate is

$$dE_{he}/dt = W_{dep} - W_e - W_r - W_m \tag{3.2-1}$$

where $dE_{he}/dt = W_\varepsilon$ ∼internal energy power density, W_{dep} ∼the power density of α particle fusion deposited in the central DT gas, "deposition" refers to: when fusion occurs, α particles with greater kinetic energy escape and enter the surrounding DT ice layer, leaving behind α particles, which are "deposited" α particles; W_e ∼thermal conduction power density, W_r ∼radiation power density, W_m ∼pressure energy power density.

According to reference [1], the formula for W_{dep} is

$$W_{dep}(T_{he}) = W_a(T_{he}) f_a [erg/cm^3 s] \tag{3.2-2)1}$$

In the above formula, according to (1.2-1)1,2formula $W_a = A_a \rho_{mh}{}^2 \overline{v\sigma}(T)[erg/cm^3 s]$ and $A_a = 8.064 \times 10^{40} [erg/g^2]$, $f_a \sim \alpha$ particle deposition ratio coefficient.

The formula for f_a is

$$f_a = \begin{cases} 3\tau_a/2 - 4\tau_a{}^2/5\,,\ if\ \tau_a \le 1/2 \\ 1 - 1/4\tau_a + 1/160\tau_a{}^3\,,\ if\ \tau_a \ge 1/2 \end{cases} \tag{3.2-2)2}$$

where τ_a ∼the "light thickness" of α particles, i.e. the penetrability of α particles. The formula for τ_a is

$$\tau_a(T_{he}) = 2.035\times10^2\,\rho_{mhe}\;r_{he}/T_{he}^{\;3/2} \tag{3.2-2)3}$$

where ρ_{mhe} ～mass density of DT gas at the end of stagnate, $T_{he}[KeV]$ ～the center DT gas temperature at the end of stagnate.

Argument of formula (3.2-2)3: $\partial\ln\Lambda/\partial n_e = \partial\{\ln[(K_B T_{he})^3/4\pi n_e e^6]^{1/2}\}/\partial n_e = 1/2n_e$

Using formula $\tau_a = r_{he}/L_a = 9(\rho_{mhe}\;r_{he}/T_{he}^{\;3/2})\ln\Lambda$ from literature (Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008), the Coulomb logarithm $\ln\Lambda$ in the formula comes from formula $\Lambda = 4\pi n_e \lambda_D^{\;3}$ in Xu & Jin, *et al.*, (1981), where $\lambda_D = (K_B T_{he}/4\pi n_e e^2)^{1/2}$ ～the Debye length, n_e ～ the electron number density; From this, it can be derived that $\ln\Lambda = \ln[(K_B T_{h0})^3/4\pi n_e e^6]^{1/2}$; Calculate $\partial\ln\Lambda/\partial n_e = 1/2n_e$, due to $n_e >> 1$, this formula causes $\partial\ln\Lambda/\partial n_e \approx 0$, so there is $\ln\Lambda \approx const$ regarding n_e. Therefore, $n_e = 2.69\times10^{19}[cm^{-3}]$ under standard conditions can be used, note that $K_B T_{he}$ is the energy carried by α particles during fusion $3.52[MeV]$ [see formula (1,1-1)], thereby $\ln\Lambda = 2.261\times10$ is obtained, substituting this formula into the formula for τ_a obtains $\tau_a = 2.035\times10^2\,\rho_{mhe}\;r_{he}/T_{he}^{\;3/2}$.

The formula for W_e is

$$|W_e(T_{he})| = A_e T_{he}^{\;7/2}/r_{he}^{\;2}\,[erg/cm^3 s] \tag{3.2-3)1}$$

$$A_e = 4.282\times10^{18}[erg/cm\cdot s\cdot KeV^{7/2}] \tag{3.2-3)2}$$

where the dimension of T_{he} in the formula is KeV.

Argument the above:

According to heat transfer theory, the heat flux density of the thermal conductor is $\mathrm{q} = -x_e\nabla T$, where x_e ～ coefficient of thermal conductivity, due to comparing with electron, the very small coefficient of thermal conductivity of ions,it can be ignored. Therefore, the coefficient of thermal conductivity in the following text refers to the electronic coefficient of thermal conductivity x_e. In the above formula, according to the gradient formula (2.2-1)2 of the ring coordinate, $\nabla T = (\partial T/\partial r)\mathrm{e_r}$ is hold, thereby, there is $q = -x_e\,\partial T/\partial r$.

According to the above formula, it can be concluded that the heat conduction power density of the center DT gas is $W_e = \{[q(S+\Delta S) - qS]/\Delta V\}_{\Delta S\to0} = -x_e(\partial T/\partial r)\,(dS/dV)$, where ΔS and ΔV are the change in surface area and corresponding volume change. Using equation (2.2-5)1,2 and approximating $T(r)$ as a linear relationship $T = C^{(10)}r$, $W_e = -(x_e/r_{he})\,(T_{he}/r_{he})$ can be derived.

In literature (Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008), the Fokker Planker equation of plasma dynamics was used to introduce the formula for x_e. For equimolar DT there is $x_e = [(8/\pi)^{3/2}/4.3][K_B(K_B T_{he})^{5/2}/e^4 m_e^{\;1/2}\ln\Lambda]$, Substituting this and all relevant data into the previous formula obtains $W_e = -4.282\times10^{18}T_{he}^{\;7/2}/r_{he}^{\;2}[erg/cm^3 s]$.

The formula for W_r is

$$|W_r(T_{he})| = A_r\rho_{mhe}^{\;2}T_{he}^{\;1/2}[erg/cm^3 s] \tag{3.2-4)1}$$

$$A_r = 3.058\times10^{23}[erg\cdot cm^3/g^2\cdot s\cdot KeV^{1/2}] \tag{3.2-4)2}$$

where the dimension of T_{he} is $[KeV]$.

Argument the above:

According to literature (Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008), the radiation loss of DT fusion plasma is mainly bremsstrahlung, with a power density of $\left|W_r\right| = (64/3\sqrt{2\pi})\ [e^6(K_BT)^{1/2}/c^3\hbar m_e{}^{3/2}]n_e{}^2$. Substituting the electron number density n_e of equimolar DT and other data into it, $\left|W_r\right| = 3.058\times10^{23}\rho_{mhe}{}^2T_{he}{}^{1/2}$ can be obtained.

The formula for W_m is

$$\left|W_m(T_{he})\right| = A_m(\rho_{mho}/\rho_{mco})^{1/2}[\rho_{mhe}T_{he}{}^{3/2}/r_{he}]\ [erg/cm^3s] \tag{3.2-5)1}$$

In the formula

$$A_m = 4.240\times10^{22}[erg/(g\cdot KeV)]^{3/2} \tag{3.2-5)2}$$

the dimension of T_{he} is $[KeV]$.

Argument the above:

W_m is the work done by the central DT gas pressure to the external, During the period $[0,\ t_{he}]$ of stagnate, the DT ice layer on the outside contracts inward, while the reflected shock wave on the inside expands outward; There is a layer of DT gas between the former and the latter; But because the reflected shock wave is a strong shock wave, there is $p_2 >> p_1$, therefore, it can be approximated that the DT gas pressure p_2 after the reflected shock wave directly acts on the inner surface of the DT ice layer, causing resistance to its inward contraction.

The work done by pressure p_2 is $p_2dV = p_2Sdr_h$, where V and S ～respectively represent the volume and surface area of the center DT gas, and the corresponding power density is $W_m = p_2Sdr_h/Vdt = p_2u_h\,S/V$.
Substituting formulas(2.2-5)1,2 into this formula obtains $\left|W_m\right| = 2\left|p_2u_h\right|/r$.

In the above formula, u_h is the flow velocity in front of the reflected shock wave during stagnate, which can be calculated using formula (2.3-7).The original formula can be writte as

$\left|W_m\right| = 2R_g{}^{3/2}(\rho_{mho}/\rho_{mco})^{1/2}\ \rho_{mhe}T_{he}{}^{3/2}/r_{he}$ using the ideal gas law,substituting the gas constant of equimolar DT into this formula, thereby $\left|W_m\right| = A_m(\dfrac{\rho_{mho}}{\rho_{mco}})^{1/2}\dfrac{\rho_{mhe}T_{he}{}^{3/2}}{r_{he}}$ and $A_m = 4.240\times10^{22}[erg/(g\cdot KeV)]^{3/2}$ are obtained.

3.2.2 Deriving Self-heating Conditions

Substituting equation (3.2-1) into formula (3.1-1), and then using formulas (3.2-2)1, (3.2-3)1, (3.2-4)1 and (3.2-5)1, the following inequality can be derived,this is the self heating condition:

$$A(T_{he})(\rho_{mhe}r_{he})^2 + B(T_{he})\ (\rho_{mhe}r_{he}) - C(T_{he}) > 0 \tag{3.2-6)1}$$

where

$$A(T_{he}) = A_\alpha\overline{v\ \sigma}(T_{he})f_\alpha - A_rT_{he}{}^{1/2},\quad B(T_{he}) = A_m(\rho_{mho}/\rho_{mco})^{1/2}T_{he}{}^{3/2},\quad C(T_{he}) = A_eT_{he}{}^{7/2} \qquad (3.2\text{-}6)2,3,4$$

3.3 Ignition and Ignition Criterion

3.3.1 Physical Process and Ignition Conditions of Ignition

If the center DT gas has reached the self heating condition at the end of the stagnate, keeping its temperature continuously rising, causing sufficient fusion energy W_α to be generated inside, forming a hot spot. And through the outer surface r_{he} of the hot spot, the energy is transferred to the DT ice layer in the neighbourhood dr, causing the internal energy to rise and undergo fusion, thereby forming the outer propagation surface $r_{h1} = r_{he} + dr$ of the fusion, while the hot spot shrinks inward.

The fusion energy inside the wave surface $r_{h1} = r_{he} + dr$ also transfers some of the energy to the neighbourhood

dr, forming new wavefronts $r_{h2} = r_{h1} + dr$ ... $r_{hi} = r_{hi-1} + dr$ ..., If it continues like this, then a continuously spreading fusion wavefront is formed.

To continue the above process, sufficient fusion energy W_a must be generated at the end of the stagnate, and the fusion energy W_a should be trend of increased, the fusion energy of outward transmission should be also increasing; This is the "ignition" process, the ignition conditions can be expressed as

$$[dW_a(T)/dt]_{T=T_{fus}} > 0 \tag{3.3-1}$$

where T_{fus} ~the temperature required for reaching fusion.

When the fusion energy is transmitted to the neighbourhood dr, the substances outside the original wave surface r_{hi} enters dr and becomes the substances inside the new wave surface $r_{hi+1} = r_{hi} + dr$, thereby increasing the mass inside the new wave surface $r_{hi+1} = r_{hi} + dr$.

3.3.2 Time Related to Fusion and Conditions for Completing Fusion

Inertial constraint time t_{hfe}, in the process of the fusion wave surface layer by layer spreading outward, the hot spot always provides fusion energy; With the continuous external transmission of fusion energy, the colder and denser substances outside the hot spot will enter the hot spot layer by layer, forming an inward contraction wave surface that is transmitted layer by layer, causing the hot spot to gradually decrease.The fusion wave propagates outward from the end time t_{he} of the stagnate, the heat spot disappears at t_{hf}; The existence time $t_{hfe} = t_{hf} - t_{he}$ of the hot spot is referred to as the "inertial constraint time ". According to literature (Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008), the inertial constraint time is

$$t_{hfe} = r_{he}/c_{she} \tag{3.3-2}$$

where $c_{she} = [(5/3)T_{he}R_g]^{1/2}$ ~the sound velocity within the hot spot.

The complete fusion reaction time t_{fus} refers to the time during which all DT within a certain volume can participate and complete fusion, and its expression is

$$t_{fus} = 25m_p{}^2/\rho_{mhe}M_{ho}\overline{v}\overline{\sigma}(T_{he}) \tag{3.3-3}$$

where M_{ho} ~ the initial mass of the center DT gas.

Argument the above:

At a certain time neighborhood t at the end of stagnate, the average probability of equimolar DT occurring fusion reaction in volume V is $n_{DT}{}^2\overline{v}\overline{\sigma}(T_{he})tV/4$; If all DT in V must participate in completing the fusion reaction, then $n_{DT}{}^2\overline{v}\overline{\sigma}(T_{he})tV/4 = 1$ is required, so it can be derived that $t \geq 25m_p{}^2/[\rho_{mhe}M_{ho}\overline{v}\overline{\sigma}(T_{he})]$ is required, hence $t_{fus} = 25m_p{}^2/[\rho_{mhe}M_{ho}\overline{v}\overline{\sigma}(T_{he})]$ should be taken.

The condition for all DT to participate in and complete fusion within the inertial constraint time is

$$r_{he}\rho_{mhe}T_{he}(\rho_{mce}/\rho_{mhe})^{1/2} \geq [2.26\times10^{-21}/M_{ho}T_{he}{}^{1/2}]\,(\rho_{mce}/\rho_{mhe})^{1/2} \tag{3.3-4}$$

where ρ_{mce} ~the mass density of DT ice at the end of stagnate, the T_{he} dimension is KeV.

Argument the above:

In order for all DT to participate in and complete fusion within time t_{hfe}, there must be $t_{hfe}/t_{fus} \geq 1$; Substitute formulas(3.3-2,3), sound velocity $c_{she} = [(5/3)T_{he}R_g]^{1/2}$, and (1.1-2)2 into this formula to obtain $r_{he}\rho_{mhe}T_{he}(\rho_{mce}/\rho_{mhe})^{1/2} \geq \{25[(5/3)R_g]^{1/2}m_p{}^2\}/(m_{ho}k_4T_{he}{}^{1/2})(\rho_{mce}/\rho_{mhe})^{1/2}$, and then substitute all relevant data into this formula to obtain formula(3.3-4).

3.3.3 Ignition Energy Equation

Derive ignition energy equation

In the propagation of fusion wave surface r_{hi}, wave surface r_{hi} outputs energy to its forward neighborhood dr; Developing from r_{hi} to $r_{hi+1} = r_{hi} + dr$, the energy equation within the wave surface r_{hi+1} is

$$u_{hi}\, d(\varepsilon_{hi} m_{hi}) / dr_{hi} = (W_a - |W_r|)V_{h1} - p_{hi} S_{hi} u_{hi} \tag{3.3-5)1}$$

The energy equation corresponding to the increase in mass in dr is

$$\varepsilon_{hi} u_{hi}\, dm_{hi} / dr_{hi} = [W_a(1 - f_a) + W_e]V_{hi} \tag{3.3-5)2}$$

u_{hi} in the above formula～the velocity behind the wave surface, its expression is

$$u_{hi} = (R_g T_{hi}\, \rho_{mhi} / \rho_{mhj})^{1/2} \tag{3.3-5)3}$$

in the above equations, ε_{hi}～ pecific internal energy within the r_{hi} wave surface, m_{hi}～mass of the DT within r_{hi} wave surface, V_{hi} and S_{hi}～the volume and surface area enclosed by the r_{hi} wave surface, p_{hi}～pressure on the r_{hi} wave surface, ρ_{mhi} and T_{hi}～DT density and temperature within the r_{hi} wave surface, ρ_{mhj}～the density of DT ice in front of the r_{hi} wave surface.

Argument the above:

Firstly, wave surface r_{hi} developing into wave surface $r_{hi+1} = r_{hi} + dr$, the energy equation within wave surface r_{hi+1} is $W_\varepsilon = W_{dep} + W_q - W_m$ according to the first law of thermodynamics.

Regarding W_ε: the internal energy increment within the r_{hi+1} wave surface is $d(\varepsilon_{hi} m_{hi})$, and thus the corresponding internal energy power density is $W_\varepsilon = \dfrac{d(\varepsilon_{hi} m_{hi})}{V_{h1} dt}$, but $dt = \dfrac{dr_{hi}}{u_{hi}}$ so there is $W_\varepsilon = u_{hi} \dfrac{d(\varepsilon_{hi} m_{hi})}{V_{h1} dr_{hi}}$.

Regarding W_{dep}: the dr obtains fusion energy that is introduced from r_{hi}, and the fusion power density in $r_{hi+1} = r_{hi} + dr$ is W_a, while in r_{hi+1} the fusion energy will continue to be transmitted outward.

If the fusion energy shows an increasing trend, within r_{hi+1} can still maintain the W_a value after energy transfer.

Regarding $W_q = W_e + W_r$: due to that heat conduction should occur at the interface that is relatively stationary with the thermal conductive medium, but the wave surface r_{hi+1} is rapidly propagating forward, so the thermal conductivity power density at this location should be disregarded, i.e. $W_e = 0$; But there is outward radiation, hence $W_r < 0$, resulting in $W_q = -W_r$.

Regarding W_m: this is the work with a value $W_m = p_{hi} S_{hi} u_{hi} / V_{hi}$ done outward by a fluid with a pressure of p_{hi} and a u_{hi} rate of flow.

In summary, the energy equation in the neighborhood dr ahead becomes $u_{hi}\, d(\varepsilon_{hi} m_{hi}) / dr_{hi} = (W_a - W_r)V_{h1} - p_{hi} S_{hi} u_{hi}$.

Secondly, if the mass increment in the new wave surface r_{hi+1} is Δm_{hi}, the corresponding internal energy increment is $\varepsilon_{hi} \Delta m_{hi}$, make the internal energy power density $W_\varepsilon = \varepsilon_{hi} u_{hi}\, dm_{hi} / V_{hi} dr_{hi}$. Corresponding to Δm_{hi} is the fusion power density $W_a(1 - f_a)$ outputing to the neighboring dr in front through r_{hi}, and W_e input into dr in the form of thermal conduction; Therefore, the energy equation should be $\varepsilon_{hi} u_{hi}\, dm_{hi} / dr_{hi} = [W_a(1 - f_a) + W_e]V_{hi}$.

Thirdly, the DT ice in front of the fusion wave surface r_{hi+1} has a velocity $u_1 \approx 0$ due to the disturbance not reaching; After r_{hi+1} sweeps away, the fluid velocity u_{hi} behind it can be derived as $u_{hi} = (R_g T_{hi}\, \rho_{mhi} / \rho_{mhj})^{1/2}$ using the(2.3-7)’ formula of a strong shock wave.

Further changes in the ignition energy equation

The following ignition energy equation can be further derived from equations (3.3-5)1,2,3

$$(\tau/T_{hi})\,(dT_{hi}/dt)=K_a f_a-K_r-K_e-4/3 \tag{3.3-6)1}$$

$$(\tau/\rho_{mhi})\,(d\rho_{mhi}/dt)=K_a(1-f_a)+K_e-2 \tag{3.3-6)2}$$

$$dW_a/dt=(2W_a/\tau)\,(K_a-K_r-10/3) \tag{3.3-6)3}$$

where dimensionless quantity in the equation

$$K_a=W_a\tau/\rho_{mhi}\varepsilon_{hi},\quad K_r=|W_r|\tau/\rho_{mhi}\varepsilon_{hi},\quad K_e=W_e\tau/\rho_{mhi}\varepsilon_{hi} \tag{3.3-6)4,5,6}$$

Where $\tau=r_{hi}/u_{hi}$ ~characteristic time, its expression is

$$\tau=[r_{hi}/(R_gT_{hi})^{1/2}]\,(\rho_{mic}/\rho_{mhi})^{1/2} \tag{3.3-6)7}$$

Argument the above:

Firstly, $m_{hi}\,d\varepsilon_{hi}/dt=(W_af_a-W_r-W_e)V_{h1}-p_{hi}S_{hi}u_{hi}$ is obtained by changing equations3.3-5)1and (3.3-5)2; The left side of this equation uses $m_{hi}=\rho_{mhi}V_{hi}$ and $\varepsilon_{hi}=1.5R_gT_{hi}$, while the right side uses $p_{hi}=R_g\rho_{mhi}T_{hi}$, $\varepsilon_{hi}=\frac{3}{2}R_gT_{hi}$ and (2.2-5)1,2, can convert it to $(W_af_a-W_r-W_e)V_{h1}-\varepsilon_{hi}\rho_{mhi}\frac{4V_{hi}}{3r_{hi}/u_{hi}}$; Let $\tau=\frac{r_{hi}}{u_{hi}}$ and $K_a=\frac{W_a\tau}{\rho_{mhi}\varepsilon_{hi}}$, $K_r=\frac{|W_r|\tau}{\rho_{mhi}\varepsilon_{hi}}$ and $K_e=\frac{W_e\tau}{\rho_{mhi}\varepsilon_{hi}}$, then $\frac{\tau}{T_{hi}}\frac{dT_{hi}}{dt}=K_\alpha f_\alpha-K_r-K_e-\frac{4}{3}$ can be derived from the original equation.

Secondly, substituting $m_{hi}=\rho_{mhi}V_{hi}$ and $dr_{hi}=u_{hi}dt$ into the left side of equation (3.3-5)2 obtains $\varepsilon_{hi}u_{hi}\frac{dm_{hi}}{dr_{hi}}=\varepsilon_{hi}[\rho_{mhi}\frac{dV_{hi}}{dt}+\frac{d\rho_{mhi}}{dt}V_{hi}]$, and then substituting equation (2.2-5)1 into this equation obtains $\varepsilon_{hi}u_{hi}\frac{dm_{hi}}{dr_{hi}}=\varepsilon_{hi}V_{hi}\frac{\rho_{mhi}}{\tau}(2+\frac{\tau}{\rho_{mhi}}\frac{d\rho_{mhi}}{dt})$, which is then transformed to derive $\frac{\tau}{\rho_{mhi}}\frac{d\rho_{mhi}}{dt}=K_\alpha(1-f_\alpha)+K_e-2$.

Thirdly, by using equations(3.3-6)1+(3.3-6)2, $(\tau/T_{hi}\rho_{mhi})\,[d(T_{hi}\rho_{mhi})/dt]=K_\alpha-K_r-10/3$ is obtained through transformation; Substitute formula(1.1-2)2 into formula(1.2-1)1 to obtain $\rho_{mhi}T_{hi}=(W_\alpha/k_4A_\alpha)^{1/2}$, and then substitute this into the previous formula to obtain $dW_\alpha/dt=(2W_\alpha/\tau)\,(K_\alpha-K_r-10/3)$.

Fourthly, substituting(3.3-5)3 formula $u_{hi}=(R_gT_{hi}\frac{\rho_{mhi}}{\rho_{mhj}})^{1/2}$ into $\tau=\frac{r_{hi}}{u_{hi}}$, so $\tau=\frac{r_{hi}}{(R_gT_{hi})^{1/2}}(\frac{\rho_{mij}}{\rho_{mhi}})^{1/2}$ is obtained.

3.3.4 Deriving Ignition Criterion

The analytical formula for the ignition criterion is

$$\rho_{mhe}r_{he}T_{he}(\rho_{mce}/\rho_{mhe})^{1/2}-1.2T_{he}^{\;2}/[T_{he}^{\;3/2}-3.4][g\cdot KeV/cm^2]>0 \tag{3.3-7)1}$$

The approximate formula for the ignition criterion is

$$\rho_{mhe}r_{he}T_{he}>1.60[g\cdot KeV/cm^2] \tag{3.3-7)2}$$

The dimension of T_{he} in the formula is KeV. The approximate ignition criterion equivalent to the above formula is

$$\mathfrak{F}_{DT}>1.226\times10^2[MJ/cm^2] \tag{3.3-7)3}$$

where $\mathfrak{F}_{DT} \equiv p_{he} r_{he}$ in the formula.

Argument the above:

According to equation (3.3-6)3, in order for the ignition condition(3.3-1) to be valid, it is necessary to satisfy $[K_a - K_r - \frac{10}{3}]_{T=T_{fus}} > 0$. Substitute formulas(3.3-6)4,5 into this formula,and apply formulas (1.2-1)1, (3.2-4)1, (3.3-6)6, (1.1-2)2 and $\varepsilon_{hi} = 3R_g T_{hi} / 2$ to this formula to obtain $\{A_a \rho_{mhi}{}^2 k_4 T_{hi}{}^2 - A_r \rho_{mhi}{}^2 T_{hi}{}^{1/2} - 5\rho_{mhi}[(R_g T_{hi})^{3/2} / r_{hi}]\,(\rho_{mhi} / \rho_{mhj})^{1/2}\}_{T_{hi}=T_{fus}} > 0$. If the required temperature for fusion, i.e. $T_{hi} = T_{he} = T_{fus}$, is reached at the end of the stagnate, thereby $\rho_{mhe} r_{he} T_{he} (\rho_{mce} / \rho_{mhe})^{1/2} > 5R_g{}^{3/2} T_{he}{}^2 / (A_a k_4 T_{he}{}^{3/2} - A_r)$ is obtained by changing the above formulas; Substituting relevant data into this formulas obtains $\rho_{mhe} r_{he} T_{he} (\rho_{mce} / \rho_{mhe})^{1/2} > 1.2 T_{he}{}^2 / (T_{he}{}^{3/2} - 3.4)$.

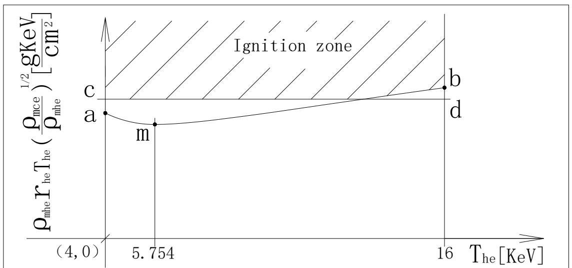


Fig 4.

The graph corresponding to the equation corresponding to the above inequality is plotted in interval $4 \le T_{he} \le 16KeV$ of Figure 4 according to literature (Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008), the temperature required to achieve fusion is $T_{fus} = 5 \sim 15KeV$). $\rho_{mhe} r_{he} T_{he} (\frac{\rho_{mce}}{\rho_{mhe}})^{1/2} = y$ is used as a function of $T_{he} = x$ in the graph,and the $y = y(x)$ graph is drawn ～ curve $\widetilde{amb}$; According to the figure, $\rho_{mhe} r_{he} T_{he} (\frac{\rho_{mce}}{\rho_{mhe}})^{1/2}$ reaches its maximum value at point b;In addition, according to literature (Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008), for fuel with approximately equal pressure due to thermal equilibrium during ignition, there is $\rho_{mhe} / \rho_{mce} \approx 0.1$. Substituting this data and $T_{he} = 16KeV$ into the right side of formula (3.3-7)1 obtains $\rho_{mhe} r_{he} T_{he} > 1.60$.

Substitute the ideal gas law $p_h = R_g T_h \rho_{mh}$ into the definitional formula $\mathfrak{F}_{DT} \equiv p_{he} r_{he}$ to obtain $\mathfrak{F}_{DT} = R_g \cdot \rho_{mhe} r_{he} T_{he}$, and then substitute the ignition criterion(3.3-7)2 and R_g values into this formula to obtain $\mathfrak{F}_{DT} > 1.226 \times 10^2 [MJ / cm^2]$.

3.4 Regarding Temperature T_{he}

The above, whether it is the self heating condition(3.2-6)1, ignition criterion(3.3-7)1, or the condition(3.3-4) that enables all DT to participate in and complete fusion within the inertial constraint time, all depend on the center DT gas stagnate temperature T_{he}; In the following text, there is(6.2-1)b formula $T_{ce} = a_f \mathbf{A}_f \rho_{mce}{}^{2/3} / R_g$ for the DT ice temperature during stagnate, which can be used as an approximate estimate of T_{he}; In fact, for the colder and denser DT ice surrounding the central DT gas, there is $T_{he} > T_{hc}$, so taking $T_{he} \approx T_{hc}$ is a conservative valuation.

3.5 Summary

In order to initiate nuclear fusion after the end of stagnate, must meet self heating conditions, and the conditions for all DT to participate in and complete fusion must also be reached, as well as the ignition criterion must be met; Among them, the self heating condition is formula(3.2-6)1, the condition for completing fusion is formula (3.3-4), and the ignition criterion is formula(3.3-7)2.

The equation corresponding to formula(3.3-4) is $r_{he}\rho_{mhe}T_{he}(\rho_{mce}/\rho_{mhe})^{1/2}=[2.26\times10^{-21}/M_{ho}T_{he}^{1/2}](\rho_{mce}/\rho_{mhe})^{1/2}$. When the values of r_{he}, ρ_{mce}/ρ_{mhe}, and T_{he} have been calculated and M_{ho} is given, the right side of the equation is a fixed value.Therefore, in the coordinate system of Figure 4, the equation is a straight line $\overline{cd}$ parallel to the horizontal axis; In this way, the range of ignition values can be achieved～ignition zone, should be located in the common part above curve $a\tilde{mb}$ and straight line $\overline{cd}$.

4. Explosion Induced by Discharge (EIB) of Ring Target Shell

4.1 Foreword

The driving force of inertial confinement fusion discussed in this article is an external strong pulse magnetic field acting on the ring target. The pulse magnetic field causes a strong induced current in the ring target shell, causing a sharp phase change in the ring target shell and causing an EIB. The generated plasma undergoes a pinch effect due to the Lorentz force.

As earlier in "2. The implosion of DT" mentioned: if the streamline $r=r(a_{ho},\xi)$ at the inner surface of DT ice is determined, then the flow field inside DT is also determined, which determines the energy accumulation during the implosion stagnate, and thus determines whether fusion can be triggered; So the pinch effect of the target ring shell caused by electric explosion should also be constrained by function $r=r(a_{ho},\xi)$.

4.2 The Occurrence and Resistivity of EIB

Under the action of a pulse magnetic field, the Joule heat generated by the induced current J causes the ring target shell to rapidly heat up, resulting in a phase change: solid state→liquid state→gas state→breakdown. At the same time, the resistivity also changes accordingly. As the change in resistivity closely corresponds to the phase change process, therefore, changes in resistivity can be used to reflect the physical process of EIB.

There are several methods for expressing changes in resistivity, and this article intends to use the "Tucker specific action model" among them (refer to literature (Yexun Li, 2002)); This model suggests that due to the completion of EIB in microseconds, the energy losses from heat conduction, convection, and radiation can be ignored, thus can be considered that explosion is only an adiabatic process in which resistance generates Joule heat.

This process can be divided into three categories: unchanged of state, change of state and breakdown process.unchanged of state ,only increases temperature and resistivity; change of state, the temperature remains unchanged and the resistivity continues to increase; When the breakdown occurs, plasma is formed, and the resistivity drops sharply; Below, $j=1,2,3$ is used to represent three states: solid, liquid and gas.

4.2.1 Expression of the Law of Resistivity Change

When the unchanged of state, the j state resistivity is expressed by the following formula:

$$\rho_j(g)=\rho_{jo}A_{jj}^{\,g(t)-g(t_{jo})} \tag{4.2-1)1}$$

In the above formula:

$$A_{jj}=(\rho_{je}/\rho_{jo})^{1/[g(t_{je})-g(t_{jo})]} \tag{4.2-1)a}$$

According to literature (Yexun Li, 2002), $g(t)$ in the above formula is referred to as the specific action, which is defined as

$$g(t)=\int J^2dt \tag{4.2-1)2}$$

Where ρ_{jo}, $\rho_{je}\sim$ the starting and ending resistivity of j state, t_{jo}, $t_{je}\sim$ the starting and ending times of j state, $J(t)\sim$ current density; $g(t)$ should be zero at startup, therefore, there is $g(t_{1o})=g_{1o}=0$.

Argument the above:

The energy equation for the unchanged of state is $[J(t)S_j]^2R_j dt=c_j\rho_{jm}S_j\tilde{L}dT$, but $R_j=\rho_j\tilde{L}/S_j$, which is changed to obtain $dT=(\rho_j/c_j\rho_{jm})J^2dt$, in the equation, $S_j\sim j$ state diversion cross-sectional area, $R_j\sim$ j state resistance, $c_j\sim$specific heat capacity of j state, $\rho_{jm}\sim$mass density of j state ring target shell, $\tilde{L}\sim$ diversion length, $\rho_j\sim$the resistivity of j state metal. Moreover, the resistivity of solid or liquid phase metals generally varies linearly with temperature, i.e. $\rho_j=\rho_{jo}(1+a_jT)$, where $a_j\sim$the temperature coefficient of j state resistivity, hence $d\rho_j=\rho_{jo}a_jdT$; Substitute the previous equation $dT=(\rho_j/c_j\rho_{jm})J^2dt$ into this equation and integrate it to obtain $\ln(\rho_j/\rho_{jo})=(\rho_{jo}a_j/c_j\rho_{jm})\int J^2dt+C^{(11)}$, this equation must satisfy $\rho_{jo}=\rho_j\big|_{t=t_{j0}}$ and $\rho_{je}=\rho_j\big|_{t=t_{je}}$,let $g(t)=\int J^2dt$, $A_{jj}=(\rho_{je}/\rho_{jo})^{1/[g(t_{je})-g(t_{jo})]}$, thereby can be derived $\rho_j(g)=\rho_{jo}A_{jj}{}^{g(t)-g(t_{jo})}$.

Change of state: the resistivity when j state is transformed into k state is expressed by the following formula

$$\rho_{jk}(g)=(\rho_{je}\big/\sqrt{A_{jk}})\big/\sqrt{F_{jk}-g(t)} \tag{4.2-2}$$

in the formula

$$A_{jk}=(\rho_{ko}{}^2-\rho_{je}{}^2)\big/\{\rho_{ko}{}^2[g(t_{ko})-g(t_{je})]\},\quad F_{jk}=1\big/A_{jk}+g(t_{je}) \tag{4.2-2)a,b}$$

where $\rho_{ko}\sim$the starting resistivity of k state, $t_{ko}\sim$the starting time of k state.

Argument the above:

The energy equation for change of state is $(JS_{jk})^2R_{jk}dt=H_{jk}dm_k$, in which $S_{jk}\sim$the diversion cross-sectional area when j state and k state coexist, $R_{jk}\sim$the total resistance when j state and k state coexist, $H_{jk}\sim$ the latent heat of state transiting from j state to k state, $m_k\sim$mass of substance of k state; This leads to $J^2dt=(H_{jk}\rho_{km}/\rho_{jk}S_{jk})dS_k$, where $\rho_{km}\sim$the density of k state, $S_k\sim$diversion cross-sectional area of k state, $\rho_{jk}\sim$the total resistivity when j state and k state coexist.

Regarding R_{jk}, the coexistence of two states is equivalent to the parallel connection of two states, thereby resulting in $R_{jk}=R_jR_k/(R_j+R_k)$. Additionally, due to $R_{jk}=\rho_{jk}\tilde{L}/S_{jk}$, $R_j=\rho_j\tilde{L}/S_j$, $R_k=\rho_k\tilde{L}/S_k$ and S_{jk}=S_j+S_k, therefore $\rho_{jk}=\rho_j\rho_k/[\rho_j(1-\varepsilon_{jk})+\rho_k\varepsilon_{jk}]$ can be derived, where $\varepsilon_{jk}=S_j/S_{jk}$ and $1\geq\varepsilon_{jk}\geq0$. Substitute the above equation into $J^2dt=(H_{jk}\rho_{km}/\rho_{jk}S_{jk})dS_k$ and perform the integration for ε_{jk}, and let $(\rho_{ko}{}^2-\rho_{je}{}^2)\big/\{\rho_{ko}{}^2[g(t_{ko})-g(t_{je})]\}=A_{jk}$ and $1/A_{jk}+g(t_{je})=F_{jk}$, can exported $\rho_{je}(1-\varepsilon_{jk})+\rho_{ko}\varepsilon_{jk}=\rho_{ko}\sqrt{A_{jk}}\sqrt{F_{jk}-g(t)}$, take ρ_j=ρ_{je} and ρ_k=ρ_{ko} approximately in the calculation; Substitute this equation back to $\rho_{jk}=\rho_j\rho_k/[\rho_j(1-\varepsilon_{jk})+\rho_k\varepsilon_{jk}]$, thus deriving $\rho_{jk}(g)=\rho_{je}\big/[\sqrt{A_{jk}}\sqrt{F_{jk}-g(t)}]$.

The Tucker model suggests that when the liquid→gas state transition ends, there is no longer low impedance substances that are connected in the conductive channel, the resistivity quickly reaches maximum, and no longer increasing. If the energy is large enough at this time, breakdown will occur, resulting in plasma discharge.

The breakdown stage can be considered as a continuation of the gas state stage. Using formula (4.2-1)1 of the gas state stage ,its resistivity can be written as $\rho_3(g)=\rho_{3o}A_{33}^{\,g(t)-g(t_{3o})}$. Due to $g(t)>g(t_{3o})$ and the sharp decreasing in resistance after breakdown there is $\rho_3<\rho_{3o}$, so the expression for the resistivity of the breakdown stage should be written as

$$\rho_3(g)=\rho_{3o}A_{33}^{\,-[g(t)-g(t_{3o})]} \tag{4.2-3)1}$$

In the formula

$$A_{33}=(\rho_{3o}/\rho_{3e})^{1/(g_{he}-g_{3o})},\quad g_{3e}-g_{3o}\approx[B_{Ju}{}^2-B_3(g_{3o})^2][\ln(\rho_{3o}/\rho_{3e})]/8\pi\rho_{3o} \tag{4.2-3)2,3}$$

where $B_{Ju}{}^2$ ~the given upper bound of magnetic fields $B_{dr}{}^2$ and $B_J{}^2$, ρ_{3e} is the resistivity of the plasma after breakdown, which can be calculated using the following formula

$$\rho_{3e}=2.407\times10^{-9}\ln[5.334\times10^3\sqrt{M_{mo1}/\rho_{1m}Z}\,][\Omega\cdot m] \tag{4.2-3)4}$$

In the formula, M_{mo1}, Z are the molar mass, atomic number of the ring target shell metal material, and ρ_{1m} is its density in the standard state.

The argument for formulas(4.2-3)2,3 will be presented in 4.5 Appendix III.

Argumenting (4.2-3)4 formula:

According to literature (JialuanXu, Shangxian, 1981) the resistivity of the plasma after breakdown:

$\rho_{3e}=e^2m_e{}^{1/2}\ln\Lambda/[32\sqrt{2\pi}\varepsilon_0{}^2(K_BT_{she})^{3/2}][\Omega\cdot m]$, where the Coulomb logarithm $\ln\Lambda=\ln[(4\pi/\sqrt{n_e})(\varepsilon_oK_BT_{she}/e^2)^{3/2}]$;

When proving equation(3.2-2)3 earlier, it is proven that there is $\ln\Lambda\approx const$ regarding n_e; Therefore, can use $n_e=\frac{\rho_{1m}Z}{M_{mo1}m_p}$ in the standard state, substitute it into $\ln\Lambda$, then substitute it into ρ_{3e}, finally, by substituting the values of e, m_e, ε_o and m_p, $\rho_{3e}=\frac{3.906\times10^{-33}}{(K_BT_{she})^{3/2}}\ln[3.287\times10^{27}\sqrt{\frac{M_{mo1}(K_BT_{she})^3}{\rho_{1m}Z}}][\Omega\cdot m]$ can be obtained, where K_BT_{she} ~the temperature of the ring target shell at the end of the stagnate, according to literature ongminZhang, Weibo Yao, 2018), the electron temperature of the plasma generated by EIB of a metal wire is generally $K_BT_{she}=10^7[K]=1.381\times10^{-16}[J]$, substituting this into the original formula obtains formula(4.2-3)4.

4.2.2 Experimental Data

The relevant data ρ_{jo}, ρ_{je}, $g(t_{jo})$ and $g(t_{je})$ in the above formulas must be obtained through experimental measurement.

Table 1 lists Tucker's measured data, which is sourced from literature (Xinggen Gong, 2002-07):

Table 1. Measured data of six types of metal wires in Tucker

Metal	melting beging			melting end			vaporing beging			exploding			Diameter / mm	ρ_b/ρ_0
	ρ / (nΩm)	Q / (Jg^{-1})	g / ($10^{17} A^2 sm^{-4}$)	ρ / (nΩm)	Q / (Jg^{-1})	g / ($10^{17} A^2 sm^{-4}$)	ρ / (nΩm)	Q / (Jg^{-1})	g / ($10^{17} A^2 sm^{-4}$)	ρ / (nΩm)	Q / (Jg^{-1})	g / ($10^{17} A^2 sm^{-4}$)		
Ag	86	245	0.61682	159	356	0.71771	273	710	0.90132	8590	3425	1.12290	0.12	540
Cu	99	459	0.80492	189	663	0.94228	263	1409	1.24008	6200	5909	1.73000	0.10	350
AI	112	623	0.25238	231	1021	0.32055	415	2981	0.48561	3930	9782	0.65776	0.12	139
Au	121	124	0.42816	260	189	0.50180	493	472	0.64950	11240	1897	0.83157	0.12	460
Ni	592	647	0.17233	796	974	0.21156	834	1812	0.30173	6660	5492	0.56007	0.12	85
W	903	995	0.24270	1161	637	0.27831	1236	1042	0.34175	2300	3936	0.75059	0.12	41

This article intends to use the data of Ag wire in the table as the estimated value.

4.3 Using the Ideal Magneto-fluid Mechanics Equation to Express the EIB process

4.3.1 Introduction

The premise for discussing the EIB of the ring target shell in this article is:

Firstly, due to the thin shell of the ring target and the fluid generated by the EIB is constrained by the pinch effect, can considered that the mass density of the shell is uniformly distributed radially along its cross-section.

Secondly, due to the skin effect, it can be considered that the induced current Q1 is concentrated in the thin layer on the surface of the ring target and evenly distribute; Thereby, the pinch force will act simultaneously, equally and radially on all layers within the cross-section of the ring target shell, causing its fluid element to move towards the center along a radial stable streamline, So that is a steady flow, and thus the position vector $r(a,t)$ of the streamline cannot be an explicitfunction of t.

Thirdly, there exists the following formulas from the same principle as formula (2.5-5)1

$$\delta r_{sh} = \delta r_{sho}\, r_{sh} / r_{sho} \quad \text{or} \quad \delta r_{sh} = \delta r_{sho}\, r_c / r_{co} \tag{4.3-1)1}$$

as well as

$$\rho_{msh} = \rho_{msho} (\bar{r}_{sho} / \bar{r}_{sh})^2 \tag{4.3-1)2}$$

Where δr_{sh}, δr_{sho} ~the thickness and initial thickness of the ring target shell, r_{sh}, r_{sho} ~the outer surface radius and initial radius of the ring target shell, r_c, r_{co} ~outer surface radius and initial radius of DT ice, ρ_{msh}, ρ_{msho} ~ring target shell density, initial density, $(r_{sh} + r_c)/2 = \bar{r}_{sh}$ ~the average radius of the ring target shell.

The EIB discussed in this article is a ring target shell flow process driven by a pulse magnetic field $\mathrm{B}_{\mathrm{dr}}(t)$, the expression of this process requires the use of ideal magneto-fluid mechanics equations including equation of ideal fluid dynamics and Maxwell's equations. Regarding the Maxwell's equations used here,according to literature (JialuanXu, Shangxian, 1981), since the fluid in question is a good conductor and its characteristic time of field change is much greater than the particle collision time, displacement current, convection current, etc. can be omitted and referred to as the "quasi static equation". Thus, the form of Maxwell's equations is $\nabla \times \mathrm{E} = -c^{-1} \partial \mathrm{B} / \partial t$ and $\nabla \times \mathrm{B} = 4\pi \mathrm{J}/c$; Additionally, the generalized Ohm's law $\mathrm{J} = (\mathrm{E} + c^{-1} \mathrm{u} \times \mathrm{B}) / \rho(g)$ needs to be used.

4.3.2 Magnetic Field Involved in EIB

Driving magnetic field $\mathrm{B}_{\mathrm{dr}}(t)$. According to the theorem of frozen-in field: the closed circuit of an ideal conductive fluid cannot undergo relative motion perpendicular to the applied magnetic induction Iine; The ring target shell after EIB can be approximated as an ideal conductive fluid, so the pinch velocity u_{sh} of the ring target shell will not cut the magnetic induction line of $\mathrm{B}_{\mathrm{dr}}(t)$, that is, there is $\mathrm{u}_{\mathrm{sh}}=0$ relative to $\mathrm{B}_{\mathrm{dr}}(t)$, so the generalized Ohm's law becomes $\mathrm{E}=\rho(g)\mathrm{J}$; However, due to the presence of opposing electromotive force in the ring target shell, J will not tend to infinity; If the ring target shell is made of a good conductor, that is,its resistivity $\rho(g)\approx 0$, then there must be $\mathrm{E}\approx 0$. Therefore, according to Maxwell's equations $\nabla\times\mathrm{E}=-c^{-1}\partial\mathrm{B}/\partial t$: there is $\partial\mathrm{B}/\partial t\approx 0$, so $\mathrm{B}_{\mathrm{dr}}\approx const$ is obtained. $const$ is a constant regarding t, which should be equal to $\mathrm{B}_{\mathrm{dr}}(0)$ at $t=0$, but in reality, $\mathrm{B}_{\mathrm{dr}}(t)$ value constantly changes with t; This can only be explained as follows: the external magnetic field $\mathrm{B}_{\mathrm{dr}}(t)$ detours outside the area enclosed by the ring target, wile the magnetic field $\mathrm{B}_{\mathrm{dr}}(t)$ within the area enclosed by the ring target is constant with respect to t.

According to another expression of the theorem of frozen-in field: any closed curve moving with an ideal conductive fluid, the conservation of magnetic flux passing through the area enclosed by this curve. Based on this, if the area enclosed by the ring target is S, the magnetic flux passing through S should be $\mathrm{B}_{\mathrm{dr}}S=const$. Since the magnetic field $\mathbf{B}_{\mathrm{dr}}$ within the area enclosed by the ring target is constant with respect to t, so there must be $S=const$, that is, the area enclosed by $\overline{R}$ is constant. Therefore can inferred that the ring target shell is equivalent to a circular current carrying wire with a radius of $\overline{R}$.

Induced magnetic field B_{J}. $\mathrm{B}_{\mathrm{dr}}(t)$ generates an induced electric field E_{dr} in the ring target shell, and the ring target shell generates a self induction electromotive force due to self inductance, which corresponds to the self induced electric field E'; The total electric field acting on the target ring shell should be $\mathrm{E}=\mathrm{E}_{\mathrm{dr}}+\mathrm{E}'$, E generates current J, as shown in Figure 1, J generates an induced magnetic field Q10 surrounding B_{J}.

The Lorentz force $c^{-1}\mathrm{J}\times\mathrm{B}_{\mathrm{J}}$ generated by B_{J} and J is the driving force for ICF discussed in this article; However, due to the skin effect, it can be considered that J is concentrated on the surface of the shell, so only B_{J} at the surface r_{sh} of the ring target shell needs to be considered, therefore this B_{J} must have the same circular shape as r_{sh}.

There is a relationship between the driving magnetic field $\mathrm{B}_{\mathrm{dr}}(t)$ and the induced electric field $\mathrm{E}_{\mathrm{dr}}(t)$ as follows

$$\mathrm{E}_{\mathrm{dr}}(t)=\mathrm{e}_{\Phi}(-\overline{R}/c)\,d[B_{dr}(t)]/dt \tag{4.3-2}$$

Argument the above:

$\mathrm{B}_{\mathrm{dr}}(t)$ and $\mathrm{E}_{\mathrm{dr}}(t)$ can be represented as $\mathrm{B}_{\mathrm{dr}}(t)=B_{dr}(t)\mathrm{e}_{\mathrm{z}}$ and $\mathrm{E}_{\mathrm{dr}}(t)=E_{dr}(t)\mathrm{e}_{\Phi}$ in the cylindrical coordinate system $(R,\ \phi,\ z)$ of Figure 1; $\mathrm{B}_{\mathrm{dr}}(t)$ and $\mathrm{E}_{\mathrm{dr}}(t)$ should satisfy Maxwell's equations $\nabla\times\mathrm{E}_{\mathrm{dr}}=c^{-1}\partial\mathrm{B}_{\mathrm{dr}}/\partial t$,and because the ring target shell is equivalent to a circular wire with a radius of $\overline{\mathrm{R}}$, thereby can derived $\mathrm{E}_{\mathrm{dr}}(t)=\mathrm{e}_{\Phi}(-\overline{R}/c)\,d[B_{dr}(t)]/dt$ in Lagrangian form.

4.3.3 Deriving the equation of magnetic field motion

The induced current J and Induced magnetic field B_{J} are represented as $\mathrm{J}=J\mathrm{e}_{\Phi}$ and $\mathrm{B}_{\mathrm{J}}=B_J\mathrm{e}_{\theta}$ in the ring coordinate system of Figure 1, respectively; The equation of magnetic field motion in Lagrangian form exists on the surface r_{sh} of the ring target shell

$$\partial B_J/[4\pi\rho(g)\partial g]=[\partial(r_{sh}B_J)/(r_{sh}\partial r_{sh})]^{-2}\{\partial[\partial(r_{sh}B_J)/(r_{sh}\partial r_{sh})]/\partial r_{sh}\} \tag{4.3-3}$$

The dimensions of the above equations are in the Gaussian system.

Argument the above:

Using the curl(2.2-2)2 formula of the ring coordinate system, write the Maxwell's equations as $\partial E/\partial r_{sh} = c^{-1}\,\partial B_J/\partial t$ and $r_{sh}^{-1}\,\partial(r_{sh}B_J)/\partial r_{sh} = 4\pi J/c$. Using these equations and the generalized Ohm's law, can obtain $\partial B_J/\partial t = [c^2\rho(g)/4\pi]\,\{\partial[r_{sh}^{-1}\,\partial(r_{sh}B_J)/\partial r_{sh}]/\partial r_{sh}\} - B_J\,\partial u_{sh}/\partial r_{sh} - u_{sh}\,\partial B_J/\partial r_{sh}$.

The continuity equation (2.2-6) can be used to change the above equation to $d(B_J/r_{sh}\rho_{msh})/dt = [c^2\rho(g)/4\pi\ \rho_{msh}]\,\{\partial[\partial(r_{sh}B_J)/(r_{sh}\partial r_{sh})]/r_{sh}\partial r_{sh}\}$; Use formula (4.3-1)2 for ρ_{msh} in this equation and note $\bar{r}_{sho}/\bar{r}_{sh} = r_{sho}/r_{sh}$,can obtain $d(r_{su}B_J)/dt = (c^2\rho(g)/4\pi)\,\{r_{su}\partial[\partial(r_{sh}B_J)/(r_{sh}\partial r_{sh})]/\partial r_{sh}\}$.

Convert r_{sh} in the above equation to the Lagrangian form $r_{sh} = r_{sh}(a_{sh},t)$, then $B_J(r_{sh},t)$ is also expressed as the Lagrangian form $B_J(a_{sh},t)$; Note that the "Introduction" already states that due to the steady flow of the EIB fluid, the position vector $r(a,t)$ of the streamline cannot be an explicitfunction of t, thus $\partial B_J/\partial t = (c^2\rho(g)/4\pi)\,\{\partial[\partial(r_{sh}B_J)/(r_{sh}\partial r_{sh})]/\partial r_{sh}\}$ can be derived.

The above equation can be further changed to $(\partial B_J/\partial g)\,(dg/dt) = (c^2\rho(g)/4\pi)\,\{\partial[\partial(r_{sh}B_J)/(r_{sh}\partial r_{sh})]/\partial r_{sh}$. In this equation, it is known $\frac{dg}{dt} = J(t)^2$ from formula (4.2-1)2, and then according to Maxwell's equations $\partial(r_{sh}B_J)/(r_{sh}\partial r_{sh}) = 4\pi J/c$ in ring coordinate system, $J^2 = [\,(c/4\pi)\partial(r_{sh}B_J)/(r_{sh}\partial r_{sh})]^2$ is obtained, so resulting in $\partial B_J/[4\pi\rho(g)\partial g] = [\partial(r_{sh}B_J)/(r_{sh}\partial r_{sh})]^{-2}\{\partial[\partial(r_{sh}B_J)/(r_{sh}\partial r_{sh})]/\partial r_{sh}\}$.

4.3.4 Solving the Equation of Magnetic Field Motion

The solution of the equation of magnetic field motion is:

$$B_J(g)^2 = 8\pi\int_{g_{jz}}^{g}\rho(g)dg + B_J(g_{jz})^2 \tag{4.3-4}$$

where g_{jz} ～starting value of g (unchanged of state $g_{jz} = g_{jo}$, change of state stage $g_{jz} = g_{je}$).

Argument the above:

Firstly, plan to use the method of separation of variables to solve the equation of magnetic field motion. Let $B_J(r_{sh},g) = -B_{Jr}(r_{sh})B_{Jg}(g)$ for this reason. Substitute it into the original equation to obtain two equations $B_{Jg}dB_{Jg}/(4\pi\rho(g)dg) = \lambda^2$ and $[d(r_{sh}B_{Jr})/(r_{sh}dr_{sh})]^{-2}\{d[d(r_{sh}B_{Jr})/(r_{sh}dr_{sh})]/(B_{Jr}dr_{sh})\} = -\lambda^2$.

Secondly, solve equation $B_{Jg}dB_{Jg}/(4\pi\rho(g)dg) = \lambda^2$, change it to $B_{Jg}dB_{Jg} = 4\pi\lambda^2\rho(g)dg$ and integrate, obtain solution $B_{Jg} = \pm\lambda[8\pi\int_{g_{jo}}^{g}\rho(g)dg + B_J(g_{jz})^2]^{1/2}$.

Thirdly, solving equation $[d(r_{sh}B_{Jr})/(r_{sh}dr_{sh})]^{-2}\,[d(r_{sh}B_{Jr})/(r_{sh}dr_{sh})]/(B_{Jr}dr_{sh}) = -\lambda^2$. It can be changed to $[\frac{d(r_{sh}B_{Jr})}{dr_{sh}^2}]^{-2}\,\frac{r_{sh}^2 d}{(r_{sh}B_{Jr})dr_{sh}^2}[\frac{d(r_{sh}B_{Jr})}{dr_{sh}^2}] = -\lambda^2$. If let $r_{sh}^2 = x_1$ and $r_{sh}B_{Jr} = x_2$ then it changes to $(\frac{dx_2}{dx_1})^{-2}\,\frac{x_1 d}{x_2 dx_1}(\frac{dx_2}{dx_1}) = -\lambda^2$, and If let $\frac{dx_2}{dx_1} = x_3$ then it changes to $x_3^{-2}\,\frac{x_1 dx_3}{dx_1} = -\lambda^2 x_2$. Taking the derivative of x_1 on both sides obtains $\frac{d}{dx_1}(x_3^{-2}\,\frac{x_1 dx_3}{dx_1}) = -\lambda^2 x_3$, which can be changed to obtain $\frac{d}{r_{sho}^2\,d\ln(x_1/r_{sho}^2)}[-\frac{dx_3^{-1}}{d\ln(x_1/r_{sho}^2)}] = -\lambda^2(x_1/r_{sho}^2)x_3$.

In this equation, If again let $x_3^{-1} = x_4$ and $\ln(\frac{x_1}{r_{sho}^2}) = x_5$ then the original equation becomes $d^2x_4 / dx_5^2 - \lambda^2 r_{sho}^2 e^{x_5} / x_4 = 0$.

The above equation has solution, its form is as $x_4 = C^{(12)} e^{C^{(13)} x_5}$. Substitute it into the original equation and change it to $C^{(12)}[C^{(13)}]^2 e^{C^{(13)} x_5} - [\lambda^2 r_{sho}^2 / C^{(12)}] e^{x_5[1-C^{(13)}]} = 0$. At $C^{(13)} = 1/2$ and $C^{(12)} = 2\lambda r_{sho}$ both sides of the above equation are equal, so the solution is $x_4 = 2\lambda r_{sho} e^{x_5/2}$.

Restore to the original variable: according to $x_3^{-1} = x_4$ and $\ln(x_1 / r_{sho}^2) = x_5$, make $x_4 = 2\lambda r_{sho} e^{x_5/2}$ become $x_3^{-1} = 2\lambda\sqrt{x_1}$, and according to $dx_2 / dx_1 = x_3$, make $x_3^{-1} = 2\lambda\sqrt{x_1}$ become $dx_1 / \sqrt{x_1} = 2\lambda dx_2$; Integrating it obtains $2x_1^{1/2} = 2\lambda x_2 + C^{(14)}$; Note here that since the original equation is a second-order equation, only two integration constants are needed, and now there have been integration constants $C^{(12)} = 2\lambda r_{sho}$ and $C^{(13)} = 1/2$, so $C^{(14)}$ is not needed, thus obtain $x_1^{1/2} = \lambda x_2$; Based on this, $B_{Jr} = 1/\lambda$ is exported from $r_{sh}^2 = x_1$ and $r_{sh} B_{Jr} = x_2$.

Fourthly, in summary, the solution $B_J(r_{sh}, g) = -B_{Jr}(r_{sh}) B_{Jg}(g)$ of the equation of magnetic field motion becomes $B_J(g)^2 = 8\pi \int_{g_{jz}}^{g} \rho(g) dg + B_J(g_{jz})^2$.

4.4 The driving magnetic field $\mathrm{B}_{\mathrm{dr}}(t)$ is the driving energy source, which drives the ring target shell to contract inward. Therefore, in order to achieve stagnate, form hot spots and lead to fusion, must to ensure that the changes in $\mathrm{B}_{\mathrm{dr}}(t)$ follow a certain pattern.

In solid state, solid-liquid state, and liquid state, due to the incompressibility of the solid and liquid, the ring target shell has not yet deformed; In solid-liquid state, the gaseous substances produced will expand,but the expansion is limited by the Lorentz force; On the other hand, due to the rapid reaching of the maximum resistivity at this time, so the current is reduced to the minimum, which makes the Lorentz force insufficient to cause the ring target shell to shrink, thereby it is approximately considered no deformation; In the breakdown stage, plasma has formed; Due to the sudden increase in current, a strong Lorentz force is generated, causing the ring target shell to pinch DT; Therefore, the breakdown stage of the ring target shell corresponds to the implosion of DT.

Before the breakdown stage, the ring target shell has not yet deformed, the expression for the driving magnetic field at this time is

$$B_{dr}^2(g) = \frac{1}{2\pi}\left(\frac{L\delta r_{sho}}{\overline{R}^2}\right)^2 \int_{g_{jz}}^{g} \rho(g) dg + B_{dr}(g_{jz})^2 \tag{4.3-5)1}$$

In the derivation of the above formula, the following formula is also obtained

$$B_{dr}(g_{jz}) = -(L\delta r_{sho} / 4\pi \overline{R}^2) B_J(g_{jz}) \tag{4.3-5)2}$$

Note that the above formula only holds before the breakdown stage. The breakdown stage corresponds to an implosion; The ring target shell is pinched inward at this time, and the driving magnetic field expression is

$$B_{dr}(r_{sh}) = \frac{-c^2}{4\pi\, r_{sho}^3 u_{sho} \overline{R}} \int_{r_{sho}}^{r_{sh}} \rho_3(g) B_3(g) r_{sh}^2 dr_{sh} - \frac{L\delta r_{sho}}{4\pi r_{sho} \overline{R}^2} r_{sh} B_3(g) + \frac{1}{\overline{R}} \int_{r_{sho}}^{r_{sh}} B_3(g) dr_{sh} \tag{4.3-5)3}$$

where u_{sho} ~the starting speed of the ring target shell, L ~coefficient of self-inductance of ring target; The dimension of above formulas are in the Gaussian system.

Argument the above:

Firstly, the generalized Ohm's law should be represented as $J=(E+c^{-1}u_{sh}B_J)/\rho(g)$ in ring coordinate system. $E=E_{dr}+E'$ has been mentioned earlier, formula(4.3-2) is substituted to obtain $E=-(\overline{R}/c)\,d[B_{dr}(t)]/dt+E'$. In the equation, E' corresponds to the self induction electromotive force $\mathsf{E}_J=2\pi\overline{R}E'=-(L/c^2)\,d(2\pi r_{sh}\delta r_{sh}\cdot J)/dt$. Substitute this and formula(4.3-1)1 into the E' formula to obtain $E'=(-L\delta r_{sho}/c^2 r_{sho}\overline{R})\,d(r_{sh}{}^2J)/dt$, thereby obtain $E=(-\overline{R}/c)\,d[B_{dr}(t)]/dt+(-L\delta r_{sho}/c^2 r_{sho}\overline{R})\,d(r_{sh}{}^2J)/dt$.

According to formula(4.3-4), B_J is not an explicitfunction of r_{sh}, hence $\partial B_J/\partial r_{sh}=0$; Therefore, from the Maxwell's equations $\partial(r_{sh}B_J)/(r_{sh}\partial r_{sh})=4\pi J/c$ in the ring coordinate system, $(c/4\pi)\,B_J/r_{sh}=J$ is obtained; Apply this to obtain $E=\dfrac{-\overline{R}}{c}\dfrac{d[B_{dr}(t)]}{dt}+\dfrac{-L\delta r_{sho}}{4\pi c r_{sho}\overline{R}}\dfrac{d(r_{sh}B_J)}{dt}$ to the above formula, substitute back into the generalized Ohm's law formula to obtain $\dfrac{dB_{dr}(t)}{dr_{sh}}\dfrac{dr_{sh}}{dt}=\dfrac{-c}{\overline{R}}J\rho(g)-\dfrac{L\delta r_{sho}}{4\pi r_{sho}\overline{R}^2}\dfrac{d(r_{sh}B_J)}{dr_{sh}}\dfrac{dr_{sh}}{dt}+\dfrac{1}{\overline{R}}u_{sh}B_J$, substitute $\dfrac{c}{4\pi}\dfrac{B_J}{r_{sh}}=J$ into it again, obtain $dB_{dr}=\dfrac{-c^2}{4\pi\overline{R}}\dfrac{\rho(g)B_J}{r_{sh}u_{sh}}dr_{sh}-\dfrac{L\delta r_{sho}}{4\pi r_{sho}\overline{R}^2}d(r_{sh}B_J)+\dfrac{1}{\overline{R}}B_J dr_{sh}$.

Secondly, discuss $r_{sh}u_{sh}$ in the above equation. As mentioned in "4.3.1 Introduction": can considered that the ring target shell undergoes steady flow in an EIB, therefore, $r_{sh}u_{sh}$ is not an explicitfunction of t, so there is $\dfrac{\partial[r_{sh}u_{sh}]}{\partial t}=0$, then according to $\dfrac{d(r_{sh}u_{sh})}{dr_{sh}}=\dfrac{\partial(r_{sh}u_{sh})}{\partial r_{sh}}+\dfrac{\partial(r_{sh}u_{sh})}{\partial t}\dfrac{dt}{dr_{sh}}$ there is $\dfrac{d(r_{sh}u_{sh})}{dr_{sh}}=\dfrac{\partial(r_{sh}u_{sh})}{\partial r_{sh}}$.

Therefore, the continuity equation(2.2-6) can be written as $d\rho_{msh}/\rho_{msh}=(-d(r_{sh}u_{sh})/r_{sh})\,(dt/dr_{sh})$, but should note $dr_{sh}/dt=u_{sh}$: for implosion, $u_{sh}<0$ is the velocity flowing into the surface of the fluid element, however the positive direction of u_{sh} in the continuity equation points outward from the surface of the fluid element, so should be written as $\dfrac{d\rho_{msh}}{\rho_{msh}}=\dfrac{-d[r_{sh}(-u_{sh})]}{r_{sh}u_{sh}}$; Integrate it to obtain $r_{sh}u_{sh}=\dfrac{\rho_{msh}r_{sho}u_{sho}}{\rho_{msho}}$, where u_{sho} ～the starting speed of the ring target shell; Substitute formula(4.3-1)2 into this formula, pay attention to $\overline{r}_{sho}/\overline{r}_{sh}=r_{sho}/r_{sh}$, so $r_{sh}u_{sh}=r_{sho}{}^3u_{sho}/r_{sh}{}^2$ is obtained. Substitute it back to the original formula to obtain $dB_{dr}=\dfrac{-c^2}{4\pi\,r_{sho}{}^3u_{sho}\overline{R}}\rho(g)B_J r_{sh}{}^2dr_{sh}-\dfrac{L\delta r_{sho}}{4\pi r_{sho}\overline{R}^2}d(r_{sh}B_J)+\dfrac{1}{\overline{R}}B_J dr_{sh}$.

Thirdly, before the breakdown stage, since the ring target shell has not yet deformed, so $r_{sh}=const$, thereby the above equation becomes $dB_{dr}=-(L\delta r_{sho}/4\pi\overline{R}^2)dB_J$; Integrating it, $B_{dr}=-(L\delta r_{sho}/4\pi\overline{R}^2)B_J$ is obtained, and then result in $B_{dr}(g_{jz})=-(L\delta r_{sho}/4\pi\overline{R}^2)B_J(g_{jz})$. Substitute formula (4.3-4) into the previous formula to derive $B_{dr}{}^2=\dfrac{1}{2\pi}(\dfrac{L\delta r_{sho}}{\overline{R}^2})^2\displaystyle\int_{g_{jz}}^{g}\rho(g)dg+B_{dr}(g_{jz})^2$.

In the breakdown stage, since the breakdown stage corresponds to implosion, there should be $r_{3o}=r_{sho}$. Integrate the formula of dB_{dr} in " Secondly ", and apply formula (4.3-5)2 to $B_J(g_{3o})=B_J(g_{sho})$ to obtain $B_{dr}(r_{sh})=\dfrac{-c^2}{4\pi\,r_{sho}{}^3u_{sho}\overline{R}}\displaystyle\int_{r_{sho}}^{r_{sh}}\rho_3(g)B_3(g)r_{sh}{}^2dr_{sh}-\dfrac{L\delta r_{sho}}{4\pi r_{sho}\overline{R}^2}r_{sh}B_3(g)+\dfrac{1}{\overline{R}}\int_{r_{sho}}^{r_{sh}}B_3(g)dr_{sh}$.

Notes on formula (4.3-5)1,3

The above driving magnetic field(4.3-5)1,3 should be connected end-to-end in sequence, namely

$B_{dr}(g_{je}) = B_{dr}(g_{ko})$ and $B_{dr}(g_{ke}) = B_{dr}(g_{jo})$, where g_{jo}, g_{ko} ～the starting g value of j, k states, g_{je}, g_{ke} ～the ending g value of j, k states; And thus should be able to establish a connection between the $B_{dr}(g_{1o})$ value and the $B_{dr}(g_{3o})$ value. The $B_{dr}(g_{3o})$ value can be calculated using formula(4.3-7)2c below, and the starting value $B_{dr}(g_{1o})$ of the driving magnetic field can be calculated using the $B_{dr}(g_{3o})$ value.

Furthermore, according to formula (4.3-5)1, $B_{dr}{}^2(g)$ is an increasing function of g, so $B_{dr}{}^2(g_{3o})$ should belong to the peak of the driving magnetic field before the breakdown stage.

Further evolution of formula(4.3-5)3

According to Figure 1, if B_{dr} increases in the negative direction as shown in the figure, $\mathrm{B}_3(g)$ should be in the positive direction as shown in the figure, so formula(4.3-4) should be written as $B_3(g) = [8\pi \int_{g_{3o}}^{g} \rho_3(g)dg + B_3(g_{3o})^2]^{1/2}$; Substituting this and formulas(4.2-3)1,2 into formulas (4.3-5)3 obtains the following formula

$$B_{dr}(r_{sh}) = \boldsymbol{C_1}[-\boldsymbol{C_2}\int_{r_{sho}}^{r_{sh}} \boldsymbol{S}(g)\mathbf{A}(g) r_{sh}{}^2 dr_{sh} + \int_{r_{sho}}^{r_{sh}} \boldsymbol{S}(g) dr_{sh}] - \boldsymbol{C_3} r_{sh} B_3(g) \tag{4.3-6)1}$$

In the formula

$$\mathbf{A}(g) = A_{33}{}^{-(g-g_{3o})}, \quad \boldsymbol{S}(g) = \sqrt{1 + [B_3(g_{3o}) / \boldsymbol{C_1}\overline{R}]^2 - \mathbf{A}(g)} \tag{4.3-6)2a,b}$$

as well as

$$\boldsymbol{C_1} = \overline{R}^{-1}\{8\pi\rho_{3o}[g(t_{he}) - g(t_{3o})]/\ln(\rho_{3o}/\rho_{3e})\}^{1/2}, \quad \boldsymbol{C_2} = c^2\rho_{3o}/(4\pi r_{sho}{}^3 u_{sho}), \quad \boldsymbol{C_3} = L\delta r_{sho}/(4\pi r_{sho}\overline{R}^2) \tag{4.3-6)2c,d,e}$$

where u_{sho} ～the starting speed of the ring target shell.

Formula (4.3-6)1is an integral about r_{sh}, but $\mathbf{A}(g)$ is a function of g, so must to derive the function $r_{sh} = r_{sh}(g)$. The expression for this function has been exported, as shown below

$$r_{sho}{}^2 - r_{sh}{}^2 = \boldsymbol{C_4}^{-1}\ln\{[B_3(g_{3o})^2\boldsymbol{C_5} + 1 - (\rho_{3e}/\rho_{3o})^z]/[B_3(g_{3o})^2\boldsymbol{C_5}\cdot(\rho_{3e}/\rho_{3o})^z]\} \tag{4.3-7)1}$$

In the formula, $z = (g - g_{3o})/[g(t_{he}) - g(t_o)]$ is a variable with range of values of $0 \le z \le 1$; and there are also:

$$\boldsymbol{C_4} = \frac{c^2\rho_{3o}A_{33o}}{4\pi r_{sho}{}^3|u_{sho}|}, \quad \boldsymbol{C_5} = \frac{1}{8\pi\rho_{3o}(g_{3e} - g_{3o})}\ln(\frac{\rho_{3o}}{\rho_{3e}}), \quad B_3(g_{3o})^2 = B_{Ju}{}^2\frac{A_{33o}-1}{A_{33o}}, \quad A_{33o} = B_3(g_{3o})^2\frac{\ln A_{33}}{8\pi\rho_{3o}} + 1 \tag{4.3-7)2a,b,c,d}$$

where $B_{Ju}{}^2$ ～given upper bound of magnetic field, and A_{33o} must be solved by the following equation

$$A_{33o}[c^2\rho_{3o}(r_{sho}{}^2 - r_{she}{}^2)/(4\pi r_{sho}{}^3|u_{sho}|)] = \ln\{(A_{33o} - \rho_{3e}/\rho_{3o})/[(A_{33o} - 1)\rho_{3e}/\rho_{3o}]\} \tag{4.3-7)2e}$$

Note that according to formula(4.3-6)1: $B_{dr}(g_{3o})^2 = [\boldsymbol{C_3} r_{sho} B_3(g_{3o})]^2$.

Argument the above

In the " Firstly" of argument of formulas (4.3-5)1,2, $cB_J/4\pi r_{sh} = J$ has been derived, substituting $J^2 = dg/dt$ derived from formula(4.2-1)2into this formula can lead to $(\frac{c}{4\pi})^2 B_J{}^2/r_{sh} = r_{sh}u_{sh}\,dg/dr_{sh}$; In addition,

$r_{sh}u_{sh} = r_{sho}{}^3 u_{sho} / r_{sh}{}^2$ has been derived in the “Secondly”of argument of formula(4.3-5)3, and substitute it into the above formula to obtain the differential equation $dg/dr_{sh} = [(c/4\pi)^2 / r_{sho}{}^3 u_{sho}] r_{sh} B_3{}^2$; Solving this equation obtains $r_{sho}{}^2 - r_{sh}{}^2 = \frac{2r_{sho}{}^3|u_{sho}|}{(c/4\pi)^2}\int_{g_{3o}}^{g}\frac{dg}{B_3(g)^2}$.

Calculating $\int_{g_{3o}}^{g}\frac{1}{B_J(g)^2}dg$ of the above formula: substitute formulas (4.2-3)1,2 into formula (4.3-4) to obtain $B_3(g)^2 = (8\pi\rho_{3o}/\ln A_{33})\,[1 - A_{33}{}^{-(g-g_{3o})}] + B_J(g_{3o})^2$; Substitute this into $\int_{g_{3o}}^{g}\frac{1}{B_J(g)^2}dg$ and let $A_{33o} = B_3(g_{3o})^2\frac{\ln A_{33}}{8\pi\rho_{3o}} + 1$, then can obtain $\int_{g_{3o}}^{g}\frac{dg}{B_3(g)^2} = \frac{1/(8\pi\rho_{3o})}{A_{33o}}\ln\left|\frac{A_{33o} - A_{33}{}^{-(g-g_{3o})}}{(A_{33o}-1)A_{33}{}^{-(g-g_{3o})}}\right|$.

Substitute the above formula back to the original formula of $r_{sho}{}^2 - r_{sh}{}^2$, note that as $\rho_{3o}/\rho_{3e} > 1$ and $g_{3e} - g_{3o} >> 1$ can prove $|A_{33o} - 1| = A_{33o} - 1$ and $|A_{33o}/\mathbf{A}(g) - 1| = A_{33o}/\mathbf{A}(g) - 1$, and let $\boldsymbol{C_4} = c^2\rho_{3o}A_{33o} / 4\pi r_{sho}{}^3|u_{sho}|$, then the original formula becomes $r_{sho}{}^2 - r_{sh}{}^2 = \boldsymbol{C_4}{}^{-1}\ln\{[A_{33o} - A_{33}{}^{-(g-g_{3o})}] / (A_{33o}-1)A_{33}{}^{-(g-g_{3o})}\}$.

The above formula also needs to satisfy $r_{sh}(g_{3e}) = r_{she}$. After substituting the above formula, equation $\boldsymbol{C_4}(r_{sho}{}^2 - r_{she}{}^2) = \ln\{[A_{33o} - A_{33}{}^{-(g_{3e}-g_{3o})}] / (A_{33o}-1)A_{33}{}^{-(g_{3e}-g_{3o})}\}$ can be obtained. Substituting formula (4.3-7)2a into it and using formula(4.2-3)2, an equation $A_{33o}\frac{c^2\rho_{3o}(r_{sho}{}^2 - r_{she}{}^2)}{4\pi r_{sho}{}^3|u_{sho}|} = \ln\frac{A_{33o} - \rho_{3e}/\rho_{3o}}{(A_{33o}-1)\,\rho_{3e}/\rho_{3o}}$ about A_{33o} can be obtained. After solving the equation to obtain the A_{33o} value, substitute A_{33o} into formula (4.3-7)2d, and substitute formulas (4.2-3)2,3 into this formula, finally obtain $B_3(g_{3o})^2 = B_{Ju}{}^2\frac{A_{33o}-1}{A_{33o}}$.

Firstly, can derive the expression for the unit mass Lorentz force as follows

$$\mathrm{f_{JB}} = -[r_{sh}B_J(g)^2 / 4\pi\rho_{msho}r_{sho}{}^2]\mathrm{e_r} \tag{4.3-8)1}$$

Secondly f_{JB} the breakdown stage, namely during the implosion of the ring target, as the driving force, the work done by f_{JB} during the implosion is:

$$E_{d2} = \int_{r_{ce}}^{r_{co}}|F_{JB}(r_c)|dr_c \quad , \quad |F_{JB}(r_c)| = \pi\bar{R}\int_{r_c}^{r_{sh}}B_3{}^2 dr \tag{4.3-8)2,3}$$

where $r_{sh} = r_c(1 + \delta r_{sho}/r_{co})$.

4.5 Appendix

The driving energy E_{d2} during the implosion has the following approximate values

$$E_{d2} \approx (r_{sho}/\bar{r}_{sho})\,(V_{sho}\overline{B_3{}^2}/8\pi) \tag{4.3-8)4}$$

where V_{sho} ～the initial volume of the ring target; The average value of $B_3^{\;2}(r)$ during the implosion in the formula is

$$\overline{B_3^{\;2}} = \frac{1}{g_{he} - g_{3o}} \int_{g_{3o}}^{g_{he}} B_3(g)^2 dg \tag{4.3-8)4'}$$

Before the breakdown stage, change is from the solid state to the liquid - gas state. As the ring target shell has not yet deformed, its volume is V_{sho}; During this period, the energy density $B_{dr}^{\;2}/8\pi$ of magnetic field $B_{dr}(g)$ increases to $B_{dr}(g_{3o})^2/8\pi$ at $g = g_{3o}$, so at this moment, the energy outputed from the driving magnetic field reaches $E_{d1} = V_{sho}B_{dr}(g_{3o})^2/8\pi$. After this, the implosion process corresponding to the breakdown stage is entered; So at this moment, the energy outputed from the driving magnetic field is

$$E_{d1} = V_{sho}B_{dr}(g_{3o})^2/8\pi \tag{4.3-8)5}$$

The total driving energy should be

$$E_d = E_{d1} + E_{d2} = (V_{sho}/8\pi)\,[B_{dr}(g_{3o})^2 + (r_{sho}/\overline{r}_{sho})\overline{B_3^{\;2}}] \tag{4.3-8)6}$$

Argumenting formulas(4.3-8)2,3,4,5:

In the breakdown stage, inside the ring target shell, take a annular volume element $dV_{sh} = S_{sh}dr$ that is concentric with the ring target, then the Lorentz force exerted on the volume element is $f_{JB}\rho_{msh}S_{sh}dr$. The Lorentz force on the entire ring target shell is $F_{JB} = \int_{r_c}^{r_{sh}} f_{JB}\rho_{msh}S_{sh}dr$. Substitute formulas(4.3-8)1 and (2.2-5)2into this formula, and substitute formula(4.3-1)2 into this formula, paying attention to $\frac{\overline{r}_{sho}}{\overline{r}_{sh}} = \frac{r_{sho}}{r_{sh}}$, then $F_{JB} = -\pi\overline{R}\int_{r_c}^{r_{sh}} B_3^{\;2}dr$ is obtained.

The total amount of work done by force F_{JB} during the implosion is $E_{d2} = \int_{r_{she}}^{r_{sho}} |F_{JB}| dr_{sh}$; The following approximate calculation can be made for E_{d2}: during the implosion, within the interval $[r_c, r_{sh}]$, the average value of $B_3(r)^2$ is Q6, thereby resulting in: $|F_{JB}| = \pi\overline{R}\int_{r_c}^{r_{sh}} B_3^{\;2}dr = \frac{\pi\overline{R}(r_{sh} - r_c)}{r_{sh} - r_c}\int_{r_c}^{r_{sh}} B_3^{\;2}dr = \pi\overline{R}(r_{sh} - r_c)\overline{B_3(r)^2}$.

Furthermore, during the implosion, within the interva $[g_{3o}, g_{he}]$, the average value of $B_3(g)^2$ is $\overline{B_3^{\;2}} = \frac{1}{g_{he} - g_{3o}}\int_{g_{3o}}^{g_{he}} B_3(g)^2 dg$; Use $\overline{B_3^{\;2}}$ to approximate $\overline{B_3(r)^2}$, thus $|F_{JB}(r_c)| \approx \pi\overline{R}\delta r_{sh}\overline{B_3^{\;2}}$ is obtained. Substitute this into the E_{d2} formula, and according to formula(4.3-1)1, $\delta r_{sh} = (r_{sh}/r_{sho})\delta r_{sho}$ is obtained, so resulting in $E_{d2} \approx \frac{\pi\overline{R}\delta r_{sho}}{2r_{sho}}\overline{B_3^{\;2}}r_{sho}^{\;2}(1 - \frac{r_{she}^{\;2}}{r_{sho}^{\;2}})$. Additionally, due to $r_{co} >> r_{ce}$, thereby $E_{d2} \approx \frac{r_{sho}}{\overline{r}_{sho}}\frac{4\pi^2\overline{r}_{sho}\overline{R}\delta r_{sho}}{8\pi}\overline{B_3^{\;2}}$ is obtained, where $\overline{r}_{sho} = (r_{sho} + r_{co})/2$; In this formula, as the initial volume of the ring target shell is $V_{sho} = 2\pi^2 r_{sho}^{\;2}\overline{R} - 2\pi^2 r_{co}^{\;2}\overline{R} = 4\pi^2\overline{r}_{sho}\overline{R}\delta r_{sho}$, therefore, obtain $E_{d2} \approx r_{sho}V_{sho}\overline{B_3^{\;2}}/\overline{r}_{sho}\cdot 8\pi$ from the original formula.

Thirdly, argumenting for formulas (4.2-3)2,3

Integrate formula(4.2-3)1to obtain $W_3(g_{he})=\int_{g_{3o}}^{g_{he}}\rho_3(g)\,\mathrm{dg}=\frac{\rho_{3o}}{\ln A_{33}}(1-\frac{1}{A_{33}{}^{g_{he}-g_{3o}}})$; The dimension of this formula is $\frac{erg}{cm^3}$, indicating that the physical meaning of $W(g_{he})=\int_{g_{3o}}^{g_{he}}\rho_3(g)\,\mathrm{dg}$ is the energy density at $g=g_{he}$; As g_{he} corresponds to time t_{he}, so $W(g_{he})$ is the energy density at the end of the stagnate ; But the energy comes from the magnetic field $B_J(g)^2$; According to formula(4.3-4),the $B_J(g)^2/8\pi$ value shows an increasing trend, so $B_J(g)^2$ should reach the upper limit of $B_{Ju}{}^2$ at $g=g(r_{he})=g_{3e}$,which is $\frac{B_{Ju}{}^2}{8\pi}=\int_{g_{3o}}^{g_{3e}}\rho(g)dg+B_J(g_{3o})^2$, while $\int_{g_{3o}}^{g_{3e}}\rho(g)dg$ in the formula is $W(g_{he})=\int_{g_{3o}}^{g_{he}}\rho_3(g)\,\mathrm{dg}$. thereby $(\rho_{3o}/\ln A_{33})\,[1-A_{33}{}^{-(g_{he}-g_{3o})}]=B_{Ju}{}^2/8\pi-B_3(g_{3o})^2/8\pi$. According to formula (4.2-3)1 there is $A_{33}=(\rho_{3o}/\rho_{3e})^{1/(g_{he}-g_{3o})}$, substitute this into the above formula, and due to $\rho_{3o}>>\rho_{3e}$, thereby leads to $g_{he}-g_{3o}\approx[B_{Ju}{}^2-B_3(g_{3o})^2]\,[\ln(\rho_{3o}/\rho_{3e})/(8\pi\rho_{3o})]$.

Fourthly, regarding the coefficient of self-inductance L of the ring target in formula (4.3-5)2, refer to literature Xisen Pang and Yu Ke, 1994-03), can be calculated approximately according to the coefficient of self-inductance of circle conducting wire with a radius $\overline{R}$ and a wire radius r_{sho}, its formula is $L=\mu_o\sqrt{\overline{R}(\overline{R}-r_{sho})}[\,(2/k-k)K-2E/k]+\mu\overline{R}/4$, where μ_o ～ permeability of vacuum , μ ～ the magnetic permeability of the wire material, K , E ～value of complete elliptic integral of the first kind, second kind, $k^2=4\overline{R}(\overline{R}-r_{sho})/[\,(2\overline{R}-r_{sho})^2]$.

Fifthly, B_{dr} can be represented as a function $B_{dr}=B_{dr}(t)$ of t using formula(4.3-5)3and parameter equation $\begin{cases} r=r(a_{ho},\xi) \\ t=t(\xi) \end{cases}$.

5. Stability of Implosive Fluid

5.1 Origin of Fluid Instability

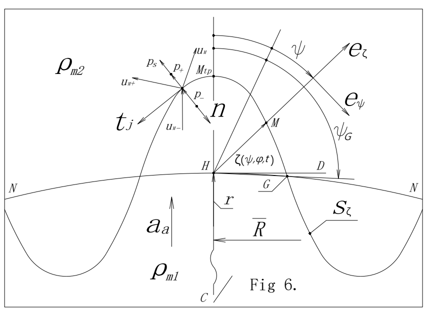


Fig 6.

As shown in Figure 6, the fluid is divided into two parts by the $N-N$ face in a confined space, the density is ρ_{m1} and ρ_{m2} respectively, $\rho_{m1}<\rho_{m2}$, and the fluid moves with an acceleration of $\mathrm{a_a}=a_a(r)\mathrm{e_r}$. $\mathrm{a_a}$ is perpendicular to the $N-N$ face, so there is a unit mass inertial force $\mathrm{f_a}=-a_a(r)\mathrm{e_r}$ in the volume V; In addition, there is also the unit mass Lorentz force $\mathrm{f_{JB}}$ in the ring target shell, this $\mathrm{f_a}+\mathrm{f_{JB}}=\mathrm{f}$ is the body force; Since $\mathrm{f_a}$ and $\mathrm{f_{JB}}$ are not related to θ and ϕ, according to formula(2.2-2)3 there is $\nabla\times(\mathrm{f_a}+\mathrm{f_{JB}})=0$, so there is the potential energy U_f that leads to $\mathrm{f_a}+\mathrm{f_{JB}}=-\nabla U_f$.

Because U_f is the potential energy per unit mass, the potential energy of the two fluids per unit volume are

$U_f \rho_{m1}$ and $U_f \rho_{m2}$, respectively. Due to $\rho_{m1} < \rho_{m2}$, so there is $U_f \rho_{m1} < U_f \rho_{m2}$. Therefore, there is a potential energy difference on the $N-N$ face, which makes the $N-N$ face unstable. If the force $\mathbf{f}$ points towards the ρ_{m1} fluid, once there is external disturbance, the ρ_{m2} fluid will flow to the direction of decreasing potential energy along the force $\mathbf{f}$, thus, $N-N$ protrudes towards ρ_{m1} fluid, forming a " pike" shaped protrusion, at the same time, in order to fill the gap caused by the "spike", the $N-N$ face also retracts towards the ρ_{m2} fluid to form a "bubble" shaped depression, thus the $N-N$ face will form a concave convex disturbance surface. If the increase of this concave convex causes the $N-N$ face to be damaged, instability will occur.

In summary, if the body force $\mathbf{f}$ points towards a thin fluid, instability may occur; According to (Atzeni, & Meyer-ter-Vehn, 2008), this instability is known as the "Rayleigh Taylor instability (RTI)". For the implosion discussed in this article, RTI may occur in the following three situations:

Firstly, the ring target shell shrinks inward until the starting of stagnate. At this point, $\mathbf{a}_a$ points towards the center of the ring target shell cross-section, making $\mathbf{f}_a$ point in the opposite direction of motion; While force $\mathbf{f}_{JB}$ points toward the center, but as the driving force $\mathbf{f}_{JB}$ there should be $f_{JB} > f_a$, so that the resultant force $\mathbf{f}$ at the outer interface of the ring target shell points toward the center; But the density of the substance outside the shell is much smaller than the density of the substance inside, therefore, $\mathbf{f}$ points towards a dense fluid, thus, there will be no instability at the external interface of the ring target shell.

Secondly, at the starting of stagnate, as $\mathbf{f}$ at the internal interface of the ring target shell also points towards the center, that is, from the ring target shell to DT ice, the density of the former > the density of the latter. Therefore, the internal boundary of the ring target shell, instability may occur.

Thirdly, from the starting of stagnate to the end of stagnate, the velocity of the the center DT gas is all reduced to zero, $\mathbf{a}_a$ points in the opposite direction of the motion, making $\mathbf{f}_a$ point from DT ice to DT gas. Pay attention to the center DT gas there is no $\mathbf{f}_{JB}$ present, so disturbance will occur at the interface between DT ice and DT gas; Due to the need to form hot spot, so must limit the peak disturbance at this time, to ensure that the hot spot is not damaged.

5.2 Disturbance Face and Its Neighboring Fluid Conditions

As shown in Figure 6, disturbance ξ occurred on the $N-N$ face thus forming interface S_ζ; Now establish the coordinate system at the intersection H of the axis of symmetry of the disturbance wave peak and the $N-N$ face; With $\boldsymbol{\xi}$ as the position vector of S_ζ face, and thus the S_ζ face equation is $\xi = \xi(\psi, \phi, t)$, and its implicit equation is $S_\zeta(\xi, \psi, \phi, t) = \xi - \xi(\psi, \phi, t)$. When a series of values for S_ζ are given, $S_\xi = \xi - \xi(\psi, \phi, t)$ forms a family of equipotential surface related to S_ζ.

The fluid in the neighborhood of the disturbance face can be approximated as an incompressible and irrotational fluid.For the disturbance velocity $\mathbf{u}_\zeta$, there is a potential U_ς that causes $\mathbf{u}_M = \nabla U_\varsigma$, and the following equation exists

$$\nabla^2 U_\varsigma = 0 \tag{5.1-1}$$

Argument the above:

Firstly, the fluid in the neighborhood of S_ζ face is approximately incompressible flow. If the change amount that ξ occurred during time Δt is $\Delta\xi$, and the change amount of position vector r of the face $N-N$ is Δr ,then $\Delta\xi$ should be limited to tiny quantity compared to Δr, namely $\Delta\xi << \Delta \mathrm{r}$; Δr can be represented as $\Delta r = c_s \Delta t$, c_s is the sound velocity under the current fluid condition. Therefore, within the same Δt, there is $d\xi/dt = u_M << c_s$ due to $\Delta\xi << \Delta \mathrm{r}$, where u_M ～disturbance velocity.

Furthermore, the fluid density ρ_m in this topic should not be an explicitfunction of t, otherwise ρ_m will change only due to t changing, but the pressure, volume, and temperature remain unchanged, hence $\partial\rho_m/\partial t = 0$.

In addition, substituting $\rho_m = \dfrac{\gamma p}{c_s^2}$ into the continuity equation (2.2-6) obtains $\dfrac{d\rho_m}{dt} + \dfrac{\gamma p}{c_s^2}\dfrac{\partial(ru_\zeta)}{r\partial r} = 0$; if in the neighborhood of face S_ζ, regarding r can considered $c_s^{\,2} \approx const$, then there can be $\frac{d\rho_m}{dt} + \gamma p \frac{\partial}{r\partial r}(\frac{ru_M}{c_s^{\,2}}) = 0$.

Due to $u_M << c_s$, there is $\frac{u_M}{c_s^{\,2}} \approx 0$, so infer $\frac{d\rho_m}{dt} \approx 0$; Applying Euler operators(2.2-4)2 to this equation obtains $d\rho_m/dt = \partial\rho_m/\partial t + u\partial\rho_m/\partial r$, so $\partial\rho_m/\partial r \approx 0$ due to $\partial\rho_m/\partial t = 0$; In addition according to the continuity equation $d\rho_m/dt + \rho_m\nabla\cdot u = 0$, since $d\rho_m/dt \approx 0$, due to $\rho_m \neq 0$, $\nabla\cdot \mathbf{u}_\zeta \approx 0$ can also be inferred.

From $\partial\rho_m/\partial t = 0$ and $\partial\rho_m/\partial r \approx 0$ above, in the neighborhood of face $N-N$, ρ_m can be approximated as a constant for spacetime, so this fluid can be approximated as an incompressible flow.

Secondly, the fluid in the neighborhood of S_ζ face is an irrotational fluid

The fluid in the neighborhood of S_ζ face satisfies the following three conditions of Kelvin's circulation theorem can be regarded as an ideal fluid, this has been mentioned earlier.

(2) ρ_m is only a function of pressure p, in the small space to the neighbourhood of the S_ζ face, can be approximated as an isentropic process, then resulting in $p/\rho_m^{\,\gamma} \approx const$.

(3) There is potential energy for the body force acting on the fluid; According to $\mathbf{f} = -\nabla U_f$ mentioned above, therefore this point is established.

So, then according to the Kelvin's circulation theorem, the velocity circulation of the fluid is conserved with respect to time; Because the velocity circulation of the fluid at the starting of the implosion is zero, thus, the velocity circulation in the subsequent process remains zero, so there is $\nabla\times\mathbf{u_M} = 0$; Based on this, the existence of potential U_ς leads to $\mathbf{u_M} = \nabla U_\varsigma$, substituting this into $\nabla\cdot\mathbf{u_M} \approx 0$ obtains $\nabla^2 U_\varsigma = 0$.

5.3 Solving Equation $\nabla^2 U_\varsigma = 0$

5.3.1 Introduction

In a closed space, waves can only form the standing wave with unchanged positions of "crest - trough pair"; Therefore, the "spike - bubble pair" generated by the disturbance ξ in the ring target shell must be distributed in the form of standing wave, and their function is

$$\xi(\psi,\phi,t) = \varsigma(\psi,\phi)\varsigma_t(t) \tag{5.2-1}$$

where $\varsigma(\psi,\phi)$ and $\varsigma_t(t)$ are two unrelated functions, $\varsigma(\psi,\phi)$ represents the spatial distribution of the amplitude of the "spike-bubble pair ", $\varsigma_t(t)$ represents the variation of the amplitude over time.

The solution U_ς of equation $\nabla^2 U_\varsigma = 0$ should correspond to disturbance ξ, therefore U_ς should also have a form similar to equation (5.2-1).

In the above article, the coordinate system has been established at the intersection H of the symmetry axis of the disturbance wave peak and the $N-N$ face. Now only discuss this crest of wave separately, and the conclusions obtained can be extended periodically; Using $\boldsymbol{\xi}$ as the position vector of S_ζ face, the length in $\mathbf{e}_\psi$ direction is measured according to $\Delta L_\psi = \xi\Delta\psi$, the length in $\mathbf{e}_\phi$ direction is measured according to $L_\phi = \phi\bar{R}$; In addition, as mentioned above,the ring target shell is equivalent to a circle conducting wire with a radius of $\bar{R}$, so equation $\nabla^2 U_\varsigma = \partial(\xi\bar{R}\,\partial U_\varsigma/\partial\xi)/(\xi\bar{R}\partial\xi) + \partial(\bar{R}\partial U_\varsigma/\partial L_\psi)/(\bar{R}\partial L_\psi) + \partial^2 U_\varsigma/\partial L_\phi^{\,2} = 0$ can be written as $\partial(\xi\partial U_\varsigma/\partial\xi)/(\xi\partial\xi) + \partial^2 U_\varsigma/\partial L_\psi^{\,2} + \partial^2 U_\varsigma/\partial L_\phi^{\,2} = 0$ using the Laplace operator(2.2-3) in the ring coordinate system.

Plan to use the method of separation of variables to solve the above equation, so let $U_\varsigma(\zeta, L_\theta, L_\phi, t) = C(t)U_{\varsigma r}(\zeta)U_{\varsigma\theta}(L_\theta)U_{\varsigma\phi}(L_\phi)$ for this, and thus the following equations $U_{\varsigma\xi}^{\;-1} d(\xi dU_{\varsigma\xi}/d\zeta)/(\xi d\xi) = 2(mk)^2$, $U_{\varsigma\psi}^{\;-1} d^2 U_{\varsigma\psi}/dL_\psi^{\,2} = -(mk)^2$, and $U_{\varsigma\phi}^{\;-1} d^2 U_{\varsigma\phi}/dL_\phi^{\,2} = -(mk)^2$ are obtained, where $m = 1,2,3\cdots$.

5.3.2 The Solution of the Equation

The solutions of $\frac{d^2U_{\varsigma\psi}}{dL_\psi{}^2}\frac{1}{U_{\varsigma\psi}}=-(mk)^2$ and $\frac{d^2U_{\varsigma\phi}}{dL_\phi{}^2}\frac{1}{U_{\varsigma\phi}}=-(mk)^2$ with respect to wave number mk are as follows:

$$U_{\varsigma\psi}^{(mk)}=\cos(mkL_\psi+\alpha_\psi),\quad U_{\varsigma\phi}^{(mk)}=\cos(mkL_\phi+\alpha_\phi),\quad k=n_r/r_{ho}=n_R/\overline{R} \tag{5.3-1)1,2,3}$$

where r_{ho} ～starting radius of DT ice inner surface, L_ψ and L_ϕ are the arc lengths along the circumference $2\pi r_{ho}$ and $2\pi\overline{R}$, $n_r\ge 2$ and $n_R\ge 2$ are a pair of minimum coprime positive integer, making $\overline{R}/r_{ho}=n_R/n_r$; The arc length $L_\psi{}'$ along the circumference $2\pi r$ must be measured according to $L_\psi{}'/r=L_\psi/r_{ho}=\vartheta$, where ϑ ～central angle.

Argument the above:

Substituting formulas (5.3-1)1,2 into the original equation can verify that it is the solution of the equation. $2\pi/mk$ is an arc length period along the circumference $2\pi r_{ho}$ or $2\pi\overline{R}$, so $2\pi r_{ho}/(2\pi/mk)$ and $2\pi\overline{R}/(2\pi/mk)$ are the number of periods along the circumference $2\pi r_{ho}$ and $2\pi\overline{R}$, respectively. In order for the "spike -bubble pair" to be distributed in standing wave form, positive integers n_r and n_R must exist, making $2\pi r_{ho}/(2\pi/mk)=n_r$ and $2\pi\overline{R}/(2\pi/mk)=n_R$; Can choose n_r and n_R as follows: reducting $\overline{R}/r_{ho}$ into a pair of minimum coprime positive integers $n_r\ge 2$ and $n_R\ge 2$, resulting in $\overline{R}/r_{ho}=n_R/n_r$.

Measure the arc length $L_\psi{}'$ along the circumference $2\pi r$ according to $L_\psi{}'/r=L_\psi/r_{ho}=\vartheta$, this can make the number of period along circumference $2\pi r$ same as the number of period along circumference $2\pi r_{ho}$.

The solution of $\frac{d}{\zeta d\zeta}[\zeta\frac{dU_{\varsigma r}}{d\zeta}]\frac{1}{U_{\varsigma r}}=(mk_r)^2$ with respect to wave number mk is as follows

$$U_{\varsigma r}^{(mk)}=C^{(15)}e^{-\sqrt{2}mk\xi}[\ln(2\sqrt{2}mk\xi)+\sum_{n=1}^{\infty}\frac{(2\sqrt{2}mk\xi)^n}{n\times n!}+C^{(16)}] \tag{5.3-2}$$

where $C^{(j)}$ ～constant, $j=1,2,3\cdots$.

Argument the above:

The original equation can be transformed into $d(dU_{\varsigma r}/d\ln\zeta)/d\ln\zeta=2(mk_r)^2\zeta^2U_{\varsigma r}$, let $\ln\zeta=z$ then it becomes $d^2U_{\varsigma r}/dz^2-2(mk)^2e^{2z}U_{\varsigma r}=0$; Apply operator $d()/dz=D$ to this equation, the original equation becomes $[D^2-2(mk)^2e^{2z}]U_{\varsigma r}=0$. Solving this equation using the method of operator obtains equation (5.3-2).

The general solution of equation $\nabla^2U_\varsigma=0$ that satisfies central symmetry and convergence is $U_\varsigma=\sum_{m=1}^{\infty}C_m(t)U_{\varsigma r}^{(mk)}U_{\varsigma\psi}^{(mk)}U_{\varsigma\phi}^{(mk)}$, the average value of this general solution has an upper bound $\overline{U}_{\varsigma u}$. $\overline{U}_{\varsigma u}$ will be used as the estimation of the solution in the following article, and for simplicity, it is still referred to as the solution, and still recorded as U_ς. The equation is

$$U_\varsigma(\xi,\psi,\phi,t)=C_1(t)U_{SP}(\xi,\psi,\phi) \tag{5.3-3)1}$$

in the equation

$$U_{SP}(\xi,\psi,\phi)=e^{-\sqrt{2}k\xi}\ln(\xi/\zeta_{\varsigma o})\cos(kL_\psi+\alpha_\psi)\cos(kL_\phi+\alpha_\phi) \tag{5.3-3)2}$$

Argument the above:

Superposition the solutions $U_\varsigma^{(mk)}=U_{\varsigma r}^{(mk)}U_{\varsigma\theta}^{(mk)}U_{\varsigma\phi}^{(mk)}$ of all wave numbers to obtain the general solution $U_\varsigma=\sum_{m=1}^{\infty}C_m(t)U_{\varsigma r}^{(mk)}U_{\varsigma\psi}^{(mk)}U_{\varsigma\phi}^{(mk)}$; Substitute equations (5.3-1)1,2 and (5.3-2) into this equation to obtain

$$U_\varsigma=\sum_{m=1}^{\infty}C_m e^{-\sqrt{2}mk\xi}[\ln(2\sqrt{2}mk\xi)+\sum_{n=1}^{\infty}[(2\sqrt{2}mk\xi)^n/n\times n!]+C^{(16)}]\cdot\cos(mkL_\psi+\alpha_\psi)\cos(mkL_\phi+\alpha_\phi).$$

$e^{-\sqrt{2}mk\xi}[\ln(2\sqrt{2}mk\xi)+\sum_{n=1}^{\infty}[(2\sqrt{2}mk\xi)^n/n\times n!]+C^{(16)}]=h_m$ in the above equation is the disturbance peak with wave number mk; If the average disturbance force $\bar{f}_\zeta$ causes the disturbance to reach the peak value h_m, then the work done by force $\bar{f}_\zeta$ is $\bar{f}_\zeta h_m$.

Due to there are mn_r or mn_R peaks on circumference $2\pi r$ or $2\pi\bar{R}$ for the disturbance with wave number mk, and thus the disturbance energy is $mn_r\bar{f}_\zeta h_m$ or $mn_R\bar{f}_\zeta h_m$. From this, can see that under the same disturbance energy, the larger the wave number mk, the smaller the peak h_m; Therefore at $m=1$, h_m should take the maximum value h_1.

The previous equation expresses the superposition effect of many disturbances. The result is that the spikes and bubbles of various peaks fuse with each other, resulting in a decrease in the peak of the higher peak, while an increase in the peak of the lower peak, thereby tending to an average value. The value can be calculated using the weighted average $\bar{U}_\varsigma=\lim_{n\to\infty}(\frac{1}{n})\sum_{m=1}^{n}C_m h_m\cos(mkL_\psi+\alpha_\psi)\cos(mkL_\phi+\alpha_\phi)$, thereby there is $\bar{U}_\varsigma<C_1h_1\lim_{n\to\infty}\frac{1}{n}\sum_{m=1}^{n}\cos(mkL_\psi+\alpha_\psi)\cos(mkL_\phi+\alpha_\phi)$.

The above equation involves the peak at point H in Figure 6. Due to symmetry can discuss $mkL_\psi+\alpha_\psi$ and $mkL_\phi+\alpha_\phi$ only within the interval $[0,\pi/2]$, within this interval, $\cos(mkL_\psi+\alpha_\psi)$ and $\cos(mkL_\phi+\alpha_\phi)$ are decreasing functions for m. Therefore there is $\cos(mkL_\psi+\alpha_\psi)\cos(mkL_\phi+\alpha_\phi)<\cos(kL_\psi+\alpha_\psi)\cos(kL_\phi+\alpha_\phi)$, so there is $\sum_{m=1}^{n}\cos(mkL_\psi+\alpha_\psi)\cos(mkL_\phi+\alpha_\phi)<n\cdot\cos(kL_\psi+\alpha_\psi)\cos(kL_\phi+\alpha_\phi)$, thereby can infer $\bar{U}_\varsigma<C_1h_1\cos(kL_\psi+\alpha_\psi)\cos(kL_\phi+\alpha_\phi)$.

Further discussing h_1 in the above formula, there is $\sum_{n=1}^{\infty}\frac{(2\sqrt{2}k\zeta)^n}{n\times n!}<\sum_{n=1}^{\infty}\frac{(2\sqrt{2}k)^n}{n\times n!}$ due to $\xi<<1$, resulting in $h_1<\frac{1}{e^{\sqrt{2}k\xi}}[\ln(2\sqrt{2}k\xi)+\sum_{n=1}^{\infty}\frac{(2\sqrt{2}k)^n}{n\times n!}+C^{(16)}]$. Can prove that $\sum_{n=1}^{\infty}\frac{(2\sqrt{2}k)^n}{n\times n!}$ convergences to a constant, and if the constant term is uniformly written as $C^{(16)}$, then there is $\bar{U}_\varsigma<C_1[e^{-\sqrt{2}k\zeta}\ln(C^{(16)}k\zeta)]\cos(kL_\psi+\alpha_\psi)\cos(kL_\phi+\alpha_\phi)$.

Determine $C^{(16)}$ in the above formula. From $\mathbf{u_M}=\nabla U_\varsigma$ know that U_ς has the dimension $[U_\varsigma]=[erg\cdot s/g]$ namely $[U_\varsigma/s]=[erg/g]$. This indicates that the physical meaning of U_ς/s is: U_ς per unit time is the unit mass disturbance energy, so if there is no disturbance, then there should be $U_\varsigma/s=0$; So, if there is an initial manufacturing error $\zeta_{\varsigma o}$, then in order to achieve $[U_\varsigma/s]_{\xi=\zeta_{\varsigma o}}=0$, i.e. $\bar{U}_\varsigma<C_1[e^{-\sqrt{2}k\xi}\ln(C^{(16)}k\xi)\cos(kL_\psi+\alpha_\psi)\cos(kL_\phi+\alpha_\phi)]_{\xi=\zeta_{\varsigma o}}=0$, there must be $C^{(16)}=(k\zeta_{\varsigma o})^{-1}$.

In summary, there is an upper bound $U_{SP}(\xi,\psi,\phi)=e^{-\sqrt{2}k\xi}\ln(\xi/\zeta_{\varsigma o})\cos(kL_\psi+\alpha_\psi)\cos(kL_\phi+\alpha_\phi)$ that makes $\bar{U}_\varsigma<C_1(t)U_{SP}(\xi,\psi,\phi)$, so as a valuation, it can be approximated as $U_\varsigma=C_1(t)U_{SP}(\xi,\psi,\phi)$.

5.4 Boundary Condition and Initial Condition at the Disturbance Interface

Motion Boundary Condition

Mark the subscripts of the fluid parameters in Figure 6 as follows: corresponding to ρ_{m2} add "+", corresponding to ρ_{m1} add "-"; The point $M_\pm$ in the neighbourhood of point M on the S_ζ face forms the $S_\pm$ face. In the disturbance, since S_ζ is the interface between fluids ρ_{m2} and ρ_{m1}, so neither fluid should cross the S_ζ face; So the normal component $u_{Mn\pm}$ of the velocity $\mathbf{u}_{M\pm}$ of the fluid element at point $M_\pm$, must be equal to the normal component u_{Mn} of the velocity $\mathbf{u}_M$ of the fluid element at M point on the S_ζ face, that is $u_{Mn\pm} = u_{Mn}$; But $(\mathbf{u}_{M\pm} - \mathbf{u}_M)\cdot\mathbf{n} = 0$, $\mathbf{n}$ ~normal unit vector, therefore $(\mathbf{u}_{M\pm} - \mathbf{u}_M)\cdot\mathbf{n} = 0$ is required. Based on this, the following motion boundary condition can be derived

$$u_{\zeta\psi}(\partial\xi(\psi,\phi,t)/\partial L_\psi) + u_{\zeta\phi}(\partial\xi(\psi,\phi,t)/\partial L_\phi) = 0 \tag{5.4-1}$$

The above speed $\mathbf{u}_M$ is represented as $\mathbf{u}_M = d\boldsymbol{\xi}/dt = u_{\zeta\zeta}\mathbf{e}_\zeta + u_{\zeta\psi}\mathbf{e}_\psi + u_{\zeta\phi}\mathbf{e}_\phi$ in the coordinate system H.

Argument the above:

For equipotential surface $S_\xi = \zeta - \xi(\psi,\phi,t)$ there is $\mathbf{n} = \dfrac{-\nabla S_\zeta}{|\nabla S_\zeta|}$, so there is $(\mathbf{u}_{M\pm} - \mathbf{u}_M)\cdot\dfrac{-\nabla S_\zeta}{|\nabla S_\zeta|} = 0$, thereby there is $\dfrac{\partial S_\zeta}{\partial t} + \mathbf{u}_{M\pm}\cdot\nabla S_\zeta - \dfrac{\partial S_\zeta}{\partial t} - \mathbf{u}_M\cdot\nabla S_\zeta = 0$, this equation should have $\dfrac{\partial S_\pm}{\partial t} \to \dfrac{\partial S_\zeta}{\partial t}$ and $\nabla S_\pm \to \nabla S_\zeta$ at $M_\pm \to M$, thus can be written as $\dfrac{\partial S_\pm}{\partial t} + \mathbf{u}_{M\pm}\cdot\nabla S_\pm - \dfrac{\partial S_\zeta}{\partial t} - \mathbf{u}_M\cdot\nabla S = 0$; Apply Euler operators(2.2-4)1 to this equation to obtain $\dfrac{dS_\pm}{dt} = 0$ and $\dfrac{dS_\zeta}{dt} = 0$, according to the second equation, $\dfrac{dS_\zeta}{dt} = \dfrac{\partial S_\zeta}{\partial t} + \mathbf{u}_M\cdot\nabla S_\zeta = 0$ is obtained.

Substitute the implicit equation $S_\xi(\xi,\psi,\phi,t) = \xi - \xi(\psi,\phi,t)$ of face S_ζ into the above equation to obtain $\dfrac{\partial[\xi - \xi(\psi,\phi,t)]}{\partial t} + \mathbf{u}_M\cdot\nabla[\xi - \xi(\psi,\phi,t)] = 0$, and then in the coordinate system H use the ring coordinate gradient $\nabla = \dfrac{\partial}{\partial\xi}\mathbf{e}_\xi + \dfrac{\partial}{\partial L_\psi}\mathbf{e}_\psi + \dfrac{\partial}{\partial L_\phi}\mathbf{e}_\phi$, leads to $u_{\zeta\psi}\dfrac{\partial\xi(\psi,\phi,t)}{\partial L_\psi} + u_{\zeta\phi}\dfrac{\partial\xi(\psi,\phi,t)}{\partial L_\phi} = 0$, this is the Motion boundary condition.

Dynamic boundary condition

As shown in Figure 6, during motion, the fluids on both sides of interface S_ζ always come into contact with each other without separation. Therefore, the resultant force of fluid interaction at face S_ζ should be zero, i.e. $p_s + p_+ + p_- = 0$. Where p_s ~the surface tension of face S_ζ, where p_- and p_+ ~the fluid pressure acting on the ρ_{m1} side and ρ_{m2} side of face S_ζ at the point M, respectively. For implosion, p_s can be omitted due to $p_s << p_+,\ p_-$, thereby there is $p_+ + p_- = 0$.

From this, the following dynamic boundary condition can be derived

$$\partial\mathbf{u}_M/\partial t + \nabla(u_M^2/2) - \mathbf{f} = 0 \tag{5.4-2}$$

where $\mathbf{f} = \mathbf{f}_a + \mathbf{f}_{JB}$ ~body force, $\mathbf{f}_a$ ~inertia force per unit mass, $\mathbf{f}_{JB}$ ~Lorentz force per unit mass.

Argument the above:

Due to the tiny amount of the viscous force compared to the internal pressure of the implosive fluid, it can be omitted; According to Bernoulli's principle $\partial U_\varsigma/\partial t + u_M^2/2 + p/\rho_m + U_f = C^{(17)}$, where p ~ fluid internal pressure; U_f ~the potential energy of body force $\mathbf{f}$, namely $-\nabla U_f = \mathbf{f}$; Or the equation can be written as $p = \rho_m[C^{(17)} - \partial U_\varsigma/\partial t - u_M^2/2 - U_f]$.

Substituting the above equation into $p_+ + p_- = 0$ obtains $[\rho_{m2}C_+^{(17)} + \rho_{m1}C_-^{(17)}] - [\rho_{m2}(\dfrac{\partial U_{\varsigma+}}{\partial t} + \dfrac{u_{M+}^2}{2}) + \rho_{m1}(\dfrac{\partial U_{\varsigma-}}{\partial t} + \dfrac{u_{M-}^2}{2})] - (\rho_{m2} + \rho_{m1})U_f = 0$,

due to in the neighbourhood of point M, so there are $\partial U_{\varsigma+}/\partial t \to \partial U_\varsigma/\partial t$, $\partial U_{\varsigma-}/\partial t\partial t \to \partial U_\varsigma/\partial t$ and $u_{M+} \to u_M$, $u_{M-} \to u_M$, therefore there is $\partial U_\varsigma/\partial t + u_M^2/2 + U_f = [\rho_{m2}C_+^{(17)} + \rho_{m1}C_-^{(17)}]/(\rho_{m2} + \rho_{m1})$.

In the neighbourhood of the disturbance face, because can be approximated as an incompressible fluid, so in the above equation, there are $\rho_{m1} \approx const$ and $\rho_{m2} \approx const$, thereby there is $[\rho_{m2}C_+^{(17)} + \rho_{m1}C_-^{(17)}]/(\rho_{m2} + \rho_{m1}) \approx const$.

If $\rho_{m2} >> \rho_{m1}$, then $const = C_+^{(17)}$, thus deriving $\partial U_\varsigma/\partial t + u_M{}^2/2 + U_f = C_+^{(17)}$.

Perform the ∇ operation on both sides of the above equation to obtain $\nabla(\partial U_\varsigma/\partial t) + \nabla(u_M{}^2/2) + \nabla U_f = \nabla C_+^{(17)}$, and apply formula (2.2-1)3 and $\nabla U_\varsigma = \mathbf{u_M}$ to this. Thereby, the dynamic boundary condition for disturbance is derived as $\partial \mathbf{u_M}/\partial t + \nabla(u_M{}^2/2) - \mathbf{f} = 0$.

Initial condition, regarding (5.2-1) equation $\xi(\psi,\phi,t) = \varsigma(\psi,\phi)\varsigma_t(t)$, there are the following initial condition:

$$[\varsigma(\psi,\phi)]_{\substack{L_\psi=0\\L_\phi=0}} = \zeta_{\varsigma o},\quad \varsigma_t(t_{\varsigma o}) = 1,\quad [\varsigma(\psi,\phi)]_{\substack{L_\psi=0\\L_\phi=\pi/2k}} = 0,\quad [\varsigma(\psi,\phi)]_{kL_\psi=\psi_G} = 2r\sin[\pi/(4kr_{ho})] \tag{5.4-3)1,2,3,4}$$

In the formulas:

$$\psi_G = \pi(1+1/2n_r)/2 \tag{5.4-3)5}$$

where $\zeta_{\varsigma o}$ ~the initial manufacturing error of the cross-sectional radius of the ring target, $t_{\varsigma o}$ ~the starting time of the disturbance; For implosion, the starting time of disturbance is t_o; For stagnate, the starting time of the disturbance is $t=0$.

Argument the above:

Due to the initial manufacturing erro $\zeta_{\varsigma o}$ of the cross-sectional radius of the ring target, so the disturbance at time $t_{\varsigma o}$ has an initial value $\zeta_{\varsigma o}$; This $\zeta_{\varsigma o}$ should be the vertex of the disturbance crest or trough.If corresponding to the disturbance peak at time $t_{\varsigma o}$, let $L_\psi = 0$, $L_\phi = 0$, then there is $\zeta_{\varsigma o} = [\xi(\psi,\phi,t)]_{\substack{L_\psi=0,L_\phi=0\\t=t_{\varsigma o}}}$.

Substituting equation(5.2-1) into this formula obtains $[\varsigma(\psi,\phi)]_{\substack{L_\psi=0\\L_\phi=0}} = \zeta_{\varsigma o}$ and $\varsigma_t(t_{\varsigma o}) = 1$.

As shown in Figure 6, since the disturbance amplitude reaches its peak at Mtp points of $L_\psi = 0$ and $L_\phi = 0$, then the amplitude should decrease when leaving this point. At the 1/4 cycle from the Mtp point along the circumference $2\pi\dot{R}$, this should be the intersection of the crest and trough, the disturbance amplitude at this point should be zero, hence $[\varsigma(\psi,\phi)]_{\substack{L_\psi=0\\L_\phi=\pi/2k}} = 0$.

As shown in Figure 6, since the width of the crest or trough of wave along the circumference $2\pi r$ is the half cycle along the arc length. Therefore, regardless of the value of kL_ϕ, the arc length $\widetilde{HG}$ along the circumference $2\pi r$ should always be equal to 1/4 cycle. If $\overline{HG}$ is the chord length corresponding to $\widetilde{HG}$ then there is $[\varsigma(\psi,\phi)]_{kL_\psi=\psi_k} = \overline{HG}$, where ψ_G is the ψ angle value corresponding to point G; The $\overline{HG}$ value can be determined as follows: if the arc length $\widetilde{HG}$ corresponds to the central angle ϑ of the circumference $2\pi r$ then the chord length $\overline{HG} = 2r\sin(\vartheta/2)$, so there is $[\varsigma(\psi,\phi)]_{kL_\psi=\psi_k} = 2r\sin(\vartheta/2)$; But for the circle $2\pi r_{ho}$, its central angle is $\vartheta = \pi/(2kr_{ho})$, so there is $[\varsigma(\psi,\phi)]_{kL_\psi=\psi_k} = 2r\sin[\pi/(4kr_{ho})]$.

As shown in Figure 6, $\psi_G = \pi/2 + \vartheta/2 = \pi(1+1/2kr_{ho})/2$ can be inferred from ΔHCG, substituting formula (5.3-1)3 into this formula obtains $\psi_G = \pi(1+1/2n_r)/2$.

5.5 Basic Formula for Calculating the Crest Value ξ_p of Disturbance From the General Solution of U_ς

5.5.1 General Solution Must Comply With Motion Boundary Condition

Meeting the motion boundary condition requires ξ to satisfy equation $(\partial\xi/\partial L_\psi)\sin(kL_\psi+\alpha_\psi)\cos(kL_\phi+\alpha_\phi) + (\partial\xi/\partial L_\phi)\cos(kL_\psi+\alpha_\psi)\sin(kL_\phi+\alpha_\phi) = 0$, its solution is

$$\xi(r,\psi,\phi,t) = \varsigma_t(t)\cdot\zeta_{\varsigma o}\{B(r) - A(r)\sin(\psi_k - kL_\psi)/\sin[\alpha_\phi(r) + kL_\phi]\} \tag{5.5-1)1}$$

In the equation:

$$A(r) = [B-1]\sin\alpha_\phi/\sin\psi_k,\quad B(r) = (2r/\zeta_{\varsigma o})\sin(\pi/4kr_{ho}),\quad \alpha_\phi(r) = arctg[B/(B-1)] \tag{5.5-1)2,3,4}$$

Argument the above:

Substituting equation (5.3-3)1 into $\mathbf{u_M} = \nabla U_\varsigma$ obtains three equations $C_1 \partial U_{SP}/\partial\xi = u_{\zeta\zeta}$, $C_1 \partial U_{SP}/\partial L_\psi = u_{\zeta\psi}$, and $C_1 \partial U_{SP}/\partial L_\phi = u_{\zeta\phi}$.Substituting the latter two equations into the motion boundary condition (5.4-1) obtains $\frac{\partial U_{SP}}{\partial L_\psi}\frac{\partial \xi}{\partial L_\psi} + \frac{\partial U_{SP}}{\partial L_\phi}\frac{\partial \xi}{\partial L_\phi} = 0$, and then substituting the general solution(5.3-3)2 into this equation obtains $(\partial\xi/\partial L_\psi)\sin(kL_\psi+\alpha_\psi)\cos(kL_\phi+\alpha_\phi)+(\partial\xi/\partial L_\phi)\cos(kL_\psi+\alpha_\psi)\sin(kL_\phi+\alpha_\phi)=0$; And also substitute the above equation into equation(5.2-1) to obtain equation $\frac{\partial\varsigma}{\partial L_\psi}\sin(kL_\psi+\alpha_\psi)\cos(kL_\phi+\alpha_\phi)+\frac{\partial\varsigma}{\partial L_\phi}\cos(kL_\psi+\alpha_\psi)\sin(kL_\phi+\alpha_\phi)=0$ about $\varsigma(\psi,\phi)$. Using the method of separation of variables to solve the above equation, the general solution obtained is $\varsigma(\psi,\phi) = A\zeta_{\varsigma o}\sin(kL_\psi+\alpha_\psi)/\sin(kL_\phi+\alpha_\phi) + D\zeta_{\varsigma o}\sin(kL_\phi+\alpha_\phi)/\sin(kL_\psi+\alpha_\psi)+C$, where A, B, C ～constants. This solution should satisfy the initial conditions (5.4-3)1,3,4 formulas.

Substitute the general solution $\varsigma(\psi,\phi)$ into the initial condition (5.4-3)4 to obtain $A\zeta_{\varsigma o}\frac{\sin(\psi_G+\alpha_\psi)}{\sin(kL_\phi+\alpha_\phi)} + D\zeta_{\varsigma o}\cdot\frac{\sin(kL_\phi+\alpha_\phi)}{\sin(\psi_G+\alpha_\psi)} + C = 2r\sin(\frac{\pi}{4kr_{ho}})$. To make this equation independent of the value of $kL_\phi+\alpha_\phi$, can only take $\psi_G = -\alpha_\psi$ and $D=0$, thus $\varsigma(r,\psi,\phi) = -A\zeta_{\varsigma o}\frac{\sin(\psi_G - kL_\psi)}{\sin(\alpha_\phi+kL_\phi)} + 2r\sin(\frac{\pi}{4kr_{ho}})$ is obtained.

Substitute the above equation into the initial condition(5.4-3)1 to obtain $A=[(2r/\zeta_{\varsigma o})\sin(\pi/4kr_{ho})-1]\sin\alpha_\phi/\sin\psi_G$; And substitute this formula into the original equation to obtain $\varsigma(\psi,\phi)=\zeta_{\varsigma o}\{-[(2r/\zeta_{\varsigma o})\sin(\pi/4kr_{ho})-1]\sin\alpha_\phi\sin(\psi_G-kL_\psi)/\sin\psi_G\sin(\alpha_\phi+kL_\phi)+(2r/\zeta_{\varsigma o})\sin(\pi/4kr_{ho})\}$, the formula should meet the initial condition (5.4-3)3, and thus derive $\alpha_\phi = arctg\{\frac{2r}{\zeta_{\varsigma o}}\sin(\frac{\pi}{4kr_{ho}})\Big/[\frac{2r}{\zeta_{\varsigma o}}\sin(\frac{\pi}{4kr_{ho}})-1]\}$.

In summary, substituting the formula of $\varsigma(r,\psi,\phi)$ back to equation (5.2-1) obtains $\xi(r,\psi,\phi,t)=\varsigma_t(t)\cdot\zeta_{\varsigma o}\{B(r)-A(r)\sin(\psi_k-kL_\psi)/\sin[\alpha_\phi(r)+kL_\phi]\}$.

5.5.2 General Solution Must Comply With Dynamic Boundary Condition

The disturbance crest corresponds to the general solution $[U_\varsigma]_{\substack{\xi=\zeta_{\varsigma o}\\ \psi=0,\phi=0}} = C_1(t)U_{SP}(\zeta_{\varsigma o},0,0)$. In order to make that the general solution satisfies the dynamic boundary condition, $C_1(t)$ in equation(5.3-3)1 must satisfy the following equation

$$dC_H(t)/dt - 2C_H(t)^2/\zeta_{\varsigma o} + f(r) = 0 \tag{5.5-2)1}$$

where $C_H(t) = C_1(t)\cdot\cos\alpha_\psi\cos\alpha_\phi\Big/\zeta_{\varsigma o}e^{\sqrt{2}k\zeta_{\varsigma o}}$.

Integrate the equation (5.5-2)1 $\frac{1}{r_{he}-r_o}\int_{r_o}^{r_{he}}\frac{dC_H(t)}{dt}dr - \frac{1}{r_{he}-r_o}\int_{r_o}^{r_{he}}C_H(t)^2\frac{2}{\zeta_{\varsigma o}}dr + \frac{1}{r_{he}-r_o}\int_{r_o}^{r_{he}}f(r)dr = 0$, where $\overline{f} = \frac{1}{r_{he}-r_o}\int_{r_o}^{r_{he}}f(r)dr = 0$, $\overline{C_H(t)^2} = \frac{1}{r_{he}-r_o}\int_{r_o}^{r_{he}}C_H(t)^2\frac{2}{\zeta_{\varsigma o}}dr$ and $\overline{\frac{dC_H(t)}{dt}} = \frac{1}{r_{he}-r_o}\int_{r_o}^{r_{he}}\frac{dC_H(t)}{dt}dr$ are all the average values related to r. Below, $\overline{dC_H(t)}/dt$ and $\overline{C_H(t)^2}$ are still recorded as $dC_H(t)/dt$ and $C_H(t)^2$, and thus the original equation is transformed into

$$dC_H(t)/dt - 2C_H(t)^2/\zeta_{\varsigma o} + \overline{f} = 0 \tag{5.5-2)2}$$

Argumenting equation (5.5-2)1

Firstly, the dynamic boundary condition (5.4-2) can be expanded to $\frac{\partial}{\partial t}(u_{\zeta\zeta}\mathbf{e}_\zeta + u_{\zeta\psi}\mathbf{e}_\psi + u_{\zeta\phi}\mathbf{e}_\phi) + \nabla(\frac{u_M^2}{2}) + f\cos\psi\mathbf{e}_\varsigma - f\sin\psi\mathbf{e}_\psi = 0$ in the coordinate system H. When proving equation(5.5-1), obtained three equations: $C_1 \partial U_{SP} / \partial \xi = u_{\zeta\zeta}$, $C_1 \partial U_{SP} / \partial L_\psi = u_{\zeta\psi}$, and $C_1 \partial U_{SP} / \partial L_\phi = u_{\zeta\phi}$, substitute these three equations into the previous equation, thereby three component equations $\frac{dC_1}{dt}\frac{\partial U_{SP}}{\partial \xi}d\xi + \frac{1}{2}\frac{\partial u_M{}^2}{\partial \xi}d\xi = -f\cos\psi d\xi$, $\frac{dC_1}{dt}\frac{\partial U_{SP}}{\partial L_\psi}dL_\psi + \frac{1}{2}\frac{\partial u_M{}^2}{\partial L_\psi}dL_\psi = f\sin\psi dL_\psi$ and $(dC_1/dt)\,(\partial U_{SP}/\partial L_\phi)dL_\phi + (\partial u_M{}^2 / 2\partial L_\phi)dL_\phi = 0$ are obtained; Add the three equations together to obtain $(dC_1/dt)dU_{SP} + du_M{}^2/2 = f(-\cos\psi d\xi + \sin\psi dL_\psi)$.

Because the discussion of this article is about disturbance crest, and the L_ψ , L_ϕ values corresponding to the disturbance crest are $L_\psi = 0$ and $L_\phi = 0$, thereby $[\cos\psi(d\xi/dU_{SP}) - \sin\psi(dL_\psi/dU_{SP})]_{\substack{\psi=0\\ \phi=0}} = [d\xi/dU_{SP}]_{\substack{\psi=0\\ \phi=0}} = P_1$, and $0.5[du_M{}^2/dU_{SP}]_{\substack{\psi=0\\ \phi=0}} = P_2$ can be set, so above equation corresponding to the disturbance peak can be written as $dC_1(t)/dt = -f(r)P_1 - P_2$.

Secondly, calculate $P_1 = [d\xi/dU_{SP}]_{\substack{\psi=0\\ \phi=0}}$ in the above equation. For this use formula (5.3-3)2 to calculate $\frac{dU_{SP}}{d\xi} = \frac{\partial U_{SP}}{\partial \xi} + \frac{\partial U_{SP}}{\partial L_\psi}\frac{dL_\psi}{d\xi} + \frac{\partial U_{SP}}{\partial L_\phi}\frac{dL_\phi}{d\xi}$, and then use formulas (5.5-1)1 and (5.2-1) to calculate $\frac{dL_\psi}{d\xi} = (\frac{\partial \xi}{\partial L_\psi})^{-1}$ and $\frac{dL_\phi}{d\xi} = (\frac{\partial \xi}{\partial L_\phi})^{-1}$ (Note: Here L_ψ, L_ψ, and r are independent of each other) in this equation. Then, substitute this and equations(5.5-1)2,3 into the original equation, obtain $[\frac{dU_{SP}}{d\xi}]_{\substack{L_\psi=0\\ L_\phi=0}} = \frac{e^{-\sqrt{2}k\zeta_{\varsigma o}}}{\zeta_{\varsigma o}}\cos\alpha_\phi\{[-\sqrt{2}k\zeta_{\varsigma o}\ln(\xi/\zeta_{\varsigma o}) + 1]\cos\alpha_\psi + \frac{-\xi_o\ln(\xi/\zeta_{\varsigma o})\sin\psi_k}{\varsigma_t(t)[2r\sin(\frac{\pi}{4kr_{ho}}) - \xi_o]}(\frac{\sin\alpha_\psi}{\cos\psi_k} + \frac{tg^2\alpha_\phi}{tg\psi_k})\}$ at $L_\psi = 0$ and $L_\phi = 0$. There is $\lim_{\zeta_{\varsigma o}\to 0}[\zeta_{\varsigma o}\ln(\zeta_{\varsigma o}/\xi)] = 0$ due to $\xi_o << 1$ and $[\xi]_{\zeta_{\varsigma o}=0} = \zeta_{\varsigma o}$, so after calculation, $P_1 = [d\xi/dU_{SP}]_{\substack{L_\psi=0\\ L_\phi=0}} \approx \zeta_{\varsigma o}e^{\sqrt{2}k\zeta_{\varsigma o}} / \cos\alpha_\psi\cos\alpha_\phi$ is obtained.

Thirdly, calculate $P_2 = 0.5[du_M{}^2/dU_{SP}]_{\substack{\psi=0\\ \phi=0}} = 0.5[d(u_{\zeta\zeta}{}^2 + u_{\zeta\psi}{}^2 + u_{\zeta\phi}{}^2)/dU_{SP}]_{\substack{\psi=0\\ \phi=0}}$ in equation $dC_1(t)/dt = -f(r)P_1 - P_2$.

When proving equation(5.5-1), three formulas $C_1\frac{\partial U_{SP}}{\partial \xi} = u_{\zeta\zeta}$, $C_1\frac{\partial U_{SP}}{\partial L_\psi} = u_{\zeta\psi}$ and $C_1\frac{\partial U_{SP}}{\partial L_\phi} = u_{\zeta\phi}$ were obtained, substitute these three formulas into the previous formula, obtain $[\frac{du_M{}^2}{dU_{SP}}]_{\substack{\psi=0\\ \phi=0}} = C_1{}^2\{\frac{d}{dU_{SP}}[(\frac{\partial U_{SP}}{\partial \xi})^2 + (\frac{\partial U_{SP}}{\partial L_\psi})^2 + (\frac{\partial U_{SP}}{\partial L_\phi})^2]\}_{\substack{\psi=0\\ \phi=0}}$.

Regarding $[d(\partial U_{SP}/\partial\xi)^2/dU_{SP}]_{\substack{\psi=0\\ \phi=0}} = 2[(\partial U_{SP}/\partial\xi)d(\partial U_{SP}/\partial\xi)/d\xi]_{\substack{\psi=0\\ \phi=0}}[d\xi/dU_{SP}]_{\substack{\psi=0\\ \phi=0}}$ in the above equation, since $\zeta = \zeta_{\varsigma o}$ is present at $\psi = 0$ and $\phi = 0$, and in the numerator omit the tiny quantity containing $\xi_o << 1$, then $[\partial U_{SP}/\partial\xi]_{\substack{\psi=0\\ \phi=0}} = \zeta_{\varsigma o}{}^{-1}e^{-\sqrt{2}k\zeta_{\varsigma o}}\cos\alpha_\psi\cos\alpha_\phi$ and $[d(\partial U_{SP}/\partial\xi)/d\xi]_{\substack{\psi=0\\ \phi=0}} \approx -\zeta_{\varsigma o}{}^{-2}e^{-\sqrt{2}k\zeta_{\varsigma o}}\cos\alpha_\psi\cos\alpha_\phi$ can be derived by using formula(5.3-3)2. Substituting these two formulas and $[dU_{SP}/d\xi]_{\substack{L_\psi=0\\ L_\phi=0}} = \zeta_{\varsigma o}{}^{-1}e^{-\sqrt{2}k\zeta_{\varsigma o}}\cos\alpha_\psi\cos\alpha_\phi$ calculated earlier into the original formula obtains $[d(\partial U_{SP}/\partial\xi)^2/dU_{SP}]_{\substack{\psi=0\\ \phi=0}} = -2\zeta_{\varsigma o}{}^{-2}e^{-\sqrt{2}k\zeta_{\varsigma o}}\cos\alpha_\psi\cos\alpha_\phi$.

Regarding $\{\frac{d}{dU_{SP}}[(\frac{\partial U_{SP}}{\partial L_\psi})^2 + (\frac{\partial U_{SP}}{\partial L_\phi})^2]\}_{\substack{\psi=0\\ \phi=0}}$ in the original equation, where

$[\frac{d}{dU_{SP}}(\frac{\partial U_{SP}}{\partial L_\psi})^2]_{\substack{\psi=0\\ \phi=0}} = [\frac{2\partial U_{SP}}{\partial L_\psi}\frac{d}{dL_\psi}(\frac{\partial U_{SP}}{\partial L_\psi})\frac{dL_\psi}{dU_{SP}}]_{\substack{\psi=0\\ \phi=0}}$, there is $\zeta = \zeta_{\varsigma o}$ at $\psi = 0$ and $\phi = 0$, $[\frac{\partial U_{SP}}{\partial L_\psi}]_{\substack{\psi=0\\ \phi=0}} = 0$ can be calculated using formula (5.3-3)2, thus $[\frac{d}{dU_{SP}}(\frac{\partial U_{SP}}{\partial L_\psi})^2]_{\substack{\psi=0\\ \phi=0}} = 0$ can be derived, and $[\frac{d}{dU_{SP}}(\frac{\partial U_{SP}}{\partial L_\phi})^2]_{\substack{\psi=0\\ \phi=0}} = 0$ can be derived similarly, thus $\{d[(\partial U_{SP}/\partial L_\psi)^2 + (\partial U_{SP}/\partial L_\phi)^2]/dU_{SP}\}_{\substack{\psi=0\\ \phi=0}} = 0$ can be derived; Substitute the calculated formulas into P_2, then obtain $P_2 = -2C_1(t)^2\zeta_{\varsigma o}{}^{-2}e^{-\sqrt{2}k\zeta_{\varsigma o}}\cos\alpha_\psi\cos\alpha_\phi$.

Fourthly, substitute the derived P_1 and P_2 into the original equation $dC_1(t)/dt = -f(r)P_1 - P_2$, and let $\zeta_{\varsigma o}{}^{-1}e^{-\sqrt{2}k\zeta_{\varsigma o}}\cos\alpha_\psi\cos\alpha_\phi C_1(t) = C_H(t)$, then equation $\frac{dC_H(t)}{dt} - C_H(t)^2\frac{2}{\zeta_{\varsigma o}} + f(r) = 0$ is obtained, this is equation (5.5-2)1.

Solve equation (5.5-2)2 $dC_H(t)/dt - 2C_H(t)^2/\zeta_{\varsigma o} + \bar{f} = 0$, Its basic solution is:

$$C_H = \sqrt{\bar{f}\zeta_{\varsigma o}/2}\left|[e^{-\omega(t+C_{ic1})}+1]/[e^{-\omega(t+C_{ic1})}-1]\right|,\quad \omega = \sqrt{8\bar{f}/\zeta_{\varsigma o}} \tag{5.5-3)1,2}$$

where C_{ic1} ～integral constant.

Argument the above:

Let $dC_H/dt = z$, then the equation is transformed into $z + \bar{f} = 2C_H{}^2/\zeta_{\varsigma o}$, namely $C_H = \pm\sqrt{(\zeta_{\varsigma o}/2)\,[z+\bar{f}]}$; Discuss the symbol of this equation: according to (5.3-3)1 equation $U_\varsigma = C_1(t)U_{SP}(\xi,\psi,\phi)$, note that the dimension of velocity potential U_ς is $[erg\cdot s/g]$, its meaning is: the unit mass energy of a disturbance during a certain period of time, this energy should not be negative, so $C_1(t)$, namely $C_H(t)$, alsow should not be negative. Thereby there should be $C_H = \sqrt{\zeta_{\varsigma o}[z+\bar{f}]/2}$.

Take the derivative on both sides of the above equation, it can be transformed into $\sqrt{8/\zeta_{\varsigma o}}dt = dz/(z\sqrt{z+\bar{f}})$, integrating this equation, $z = 4\bar{f}e^{-\omega(t+C_{ic1})}/[e^{-\omega(t+C_{ic1})}-1]^2$ is obtained, where already set $\omega = \sqrt{8\bar{f}/\zeta_{\varsigma o}}$. Since the disturbance amplitude only increases over time and does not decrease, so when integrating, $dC_H(t)/dt = z > 0$ is taken.

Combining $z = 4\bar{f}e^{-\omega(t+C_{ic1})}/[e^{-\omega(t+C_{ic1})}-1]^2$ with equation (5.5-2)2, thus the basic solution $C_H = \sqrt{0.5\bar{f}\zeta_{\varsigma o}}\left|[e^{-\omega(t+C_{ic1})}+1]/[e^{-\omega(t+C_{ic1})}-1]\right|$ is obtained.

Regarding $\bar{f}$ in the basic solution

The body force $\mathrm{f} = \mathrm{f}_a + \mathrm{f}_{JB}$ includes unit mass inertial force $\mathrm{f}_a = -\mathrm{a}_a$ and unit mass Lorentz force f_{JB}. Where f_a should be discussed in two stages: firstly, exists in the ring target shell from starting of the implosion to starting of stagnate; Secondly, exists in the DT gas of center from starting of stagnate to end of stagnate.

In the first stage, due to implosion, the overall system is in a speed increase state, and $\bar{f}_a$ can be estimated as follows

$$\bar{f}_a = |a_a| \approx [|u(a,\xi_\infty)| - |u_{co}|]/|t_o| \tag{5.5-4)1}$$

Where u_{co} ～starting speed of the outer surface of DT ice, $|u(a,\xi_\infty)|$ is calculated using the(2.5-3)2 formula.

In the second stage, due to stagnate, the overall system is in a speed reduction state, and $\bar{f}_a$ can be estimated as follows

$$\bar{f}_a = |a_a| \approx |u(a,\xi_\infty)|/t_{he} \tag{5.5-4)2}$$

where $|u(a,\xi_\infty)|$ is also calculated using the (2.5-3)2 formula.

The formula for calculating the Lorentz force per unit mass is

$$\bar{f}_{JB} \approx -\pi\bar{R}\delta r_{sho}\overline{B_3}^2 \big/ 2M_{sh} \tag{5.5-4)3}$$

where M_{sh} ～the mass of the ring target shell.

Argumenting formula (5.5-4)3:

Regarding the unit mass Lorentz force, according to formula(4.3-8)1, the unit mass Lorentz force on the entire target ring shell is $f_{JB} = \frac{-\pi\bar{R}}{M_{sh}}\int_{r_c}^{r_{sh}} B_3{}^2 dr = \frac{-\pi\bar{R}}{M_{sh}}\frac{r_{sh}-r_c}{r_{sh}-r_c}\int_{r_c}^{r_{sh}} B_3{}^2 dr$, where $\overline{B_3(r)^2} = \frac{1}{r_{sh}-r_c}\int_{r_c}^{r_{sh}} B_3(r)^2 dr$ is the average value of $B_3(r)^2$; When deriving formula(4.3-8)4, the average value $\overline{B_3}^2 = \frac{1}{g_{he}-g_{3o}}\int_{g_{3o}}^{g_{he}} B_3(g)^2 dg$ was used to approximately replace $\overline{B_3(r)^2}$, thus $f_{JB} = -\pi\bar{R}\delta r_{sh}\overline{B_3}^2 \big/ M_{sh}$ can be derived.

Substitute the above formula into $\bar{f}_{JB} = \frac{1}{r_{he}-r_o}\int_{r_o}^{r_{he}} f_{JB} dr$, note that f_{JB} only exists in the (r_{she}, r_{sho}) interval, so there is $\bar{f}_{JB} = \frac{-\pi\bar{R}\overline{B_3}^2}{(r_{she}-r_{sho})M_{sh}}\int_{r_{sho}}^{r_{she}} \delta r_{sh} dr_{sh}$. According to formula(4.3-1)2, $\bar{f}_{JB} = \frac{-\pi\bar{R}\delta r_{sho}\overline{B_3}^2}{M_{sh}}\frac{r_{she}/r_{sho}+1}{2}$ can be derived, due to $r_{sho} >> r_{she}$, there is $\bar{f}_{JB} \approx -\pi\bar{R}\delta r_{sho}\overline{B_3}^2 \big/ 2M_{sh}$.

5.5.3 According to the description in 5.5.1 and 5.5.2, the basic formula for calculating the crest value of disturbance ξ, i.e. the value ξ_p at $L_\psi = 0$ and $L_\phi = 0$, can be derived, as follows

$$\xi_p = (\zeta_{\varsigma o}/4)\ln\{C_{ic2}e^{\omega(t+C_{ic1})}[1-e^{-\omega(t+C_{ic1})}]^2\} \tag{5.5-5}$$

where C_{ic2} ～integral constant.

Argument the above:

Because $|\mathrm{d}\xi/dt| = |u_M| = \sqrt{u_{\zeta\zeta}{}^2 + u_{\zeta\psi}{}^2 + u_{\zeta\phi}{}^2}$, substitute the three equations $C_1\,\partial U_{SP}/\partial\xi = u_{\zeta\zeta}$, $C_1\,\partial U_{SP}/\partial L_\psi = u_{\zeta\psi}$ and $C_1\,\partial U_{SP}/\partial L_\phi = u_{\zeta\phi}$ btained from proving formula (5.5-1) into it, and integrate it to obtain $|\xi| = \int C_1(t)\sqrt{(\frac{\partial U_{SP}}{\partial\xi})^2 + (\frac{\partial U_{SP}}{\partial L_\psi})^2 + (\frac{\partial U_{SP}}{\partial L_\phi})^2}dt$; At $L_\psi = 0$ and $L_\phi = 0$, there is $\xi_p = [\int C_1(t)\sqrt{(\frac{\partial U_{SP}}{\partial\xi})^2 + (\frac{\partial U_{SP}}{\partial L_\psi})^2 + (\frac{\partial U_{SP}}{\partial L_\phi})^2}dt]_{\substack{L_\psi=0\\ L_\phi=0}}$;

Substitute $[\frac{\partial U_{SP}}{\partial\xi}]_{\substack{\psi=0\\ \phi=0}} = e^{-\sqrt{2}k\zeta_{\varsigma o}}\cos\alpha_\psi\cos\alpha_\phi/\zeta_{\varsigma o}$, $[\frac{\partial U_{SP}}{\partial L_\psi}]_{\substack{\psi=0\\ \phi=0}} = 0$, $[\frac{\partial U_{SP}}{\partial L_\phi}]_{\substack{\psi=0\\ \phi=0}} = 0$ obtained from proving formula (5.5-2)2, as well as the $C_H(t) = C_1(t)\cdot\cos\alpha_\psi\cos\alpha_\phi \big/ \zeta_{\varsigma o}e^{\sqrt{2}k\zeta_{\varsigma o}}$ already set above, into this formula, thereby obtain $\xi_p = \int C_H(t)dt$.

In addition, according to the formula (5.5-1)1 there is $\xi_p = [\xi]_{\substack{L_\psi=0\\ L_\phi=0}} = \varsigma_t(t)\cdot\zeta_{\varsigma o}\{B - A[\sin(\psi_k - kL_\psi)]/[\sin[\alpha_\phi + kL_\phi]]\}_{\substack{L_\psi=0\\ L_\phi=0}}$, and substituting formula(5.5-1)2into it can obtain $\xi_p = \varsigma_t(t)\zeta_{\varsigma o}$; Combine this formula with the previous formula, and substitute the basic solution (5.5-3)1into it, thereby derive $\varsigma_t(t) = \sqrt{\frac{\bar{f}}{2\zeta_{\varsigma o}}}\int\left|\frac{e^{-\omega(t+C_{ic1})}+1}{e^{-\omega(t+C_{ic1})}-1}\right|dt$.

In the above formula, because $\zeta_{\varsigma o}$ is a tiny amount and $\bar{f}$ is a huge quantity, thereby $\omega = \sqrt{8\bar{f}/\zeta_{\varsigma o}} >> 1$, and according to umerical computation, there is $\omega t >> 1$, so $e^{\omega(t+C_{ic1})} >> 1$; Therefore, the previous formula should be $\varsigma_t(t) = \sqrt{\bar{f}/2\zeta_{\varsigma o}}\int[1+e^{-\omega(t+C_{ic1})}]/[1-e^{-\omega(t+C_{ic1})}]dt$; Integrating it, and substituting into $\xi_p = \varsigma_t(t)\zeta_{\varsigma o}$, thereby the basic formula for the crest value $\xi_p=(\zeta_{\varsigma o}/4)\ln\{C_{ic2}e^{\omega(t+C_{ic1})}[1-e^{-\omega(t+C_{ic1})}]^2\}$ is obtained.

5.6 Determine the Integral Constant and Corresponding Formula in the Basic Equation

5.6.1 During the time interval $t_o\le t\le 0$, the body force f points towards the center of the ring target shell, the formula for expressing the disturbance at the interface of the ring target is

$$\xi_p(t)=(\zeta_{\varsigma o}/4)\ln\{e^4[C_u e^{-\omega(t-t_o)}-1]^2/(C_u-1)^2e^{-\omega(t-t_o)}\},\quad C_u=[|u(a,1)|\sqrt{2/\bar f\zeta_{\varsigma o}}-1]/[|u(a,1)|\sqrt{2/\bar f\zeta_{\varsigma o}}+1] \qquad (5.6\text{-}1)1,2$$

Argument the above:

Firstly, at the starting time t_o of the implosion, the disturbance starting speed $u_{\zeta\zeta}$ at the crest should be the starting speed $|u(a,1)|$ of the implosion; When deriving equation(5.5-1)1, $C_1(t)\partial U_{SP}/\partial\xi=u_{\zeta\zeta}$ was derived, so there must be $[C_1(t)\partial U_{SP}/\partial\xi]_{\substack{\psi=0,\phi=0\\ t=t_o}}=|u(a,1)|$. But $[\partial U_{SP}/\partial\xi]_{\substack{\psi=0\\ \phi=0}}=\zeta_{\varsigma o}{}^{-1}e^{-\sqrt{2}k\zeta_{\varsigma o}}\cos\alpha_\psi\cos\alpha_\phi$ has already been derived above, thus there is $C_1(t_o)=\zeta_{\varsigma o}e^{\sqrt{2}k\zeta_{\varsigma o}}|u(a,1)|/(\cos\alpha_\psi\cos\alpha_\phi)$; Thereby there is $C_H(t_o)=|u(a,1)|$ due to $C_H(t)=C_1(t)\cdot\cos\alpha_\psi\cos\alpha_\phi/\zeta_{\varsigma o}e^{\sqrt{2}k\zeta_{\varsigma o}}$.

Substitute the basic solution(5.5-3)1into the above formula,and note that $e^{\omega(t+C_{ic1})}>>1$ has already been mentioned above, so there is $\sqrt{\frac{\bar f\zeta_{\varsigma o}}{2}}\frac{1+e^{-\omega(t_o+C_{ic1})}}{1-e^{-\omega(t_o+C_{ic1})}}=|u(a,1)|$, from this, $C_{ic1}=\frac{1}{\omega}\ln\left|\frac{|u(a,1)|\sqrt{2/\bar f\zeta_{\varsigma o}}+1}{|u(a,1)|\sqrt{2/\bar f\zeta_{\varsigma o}}-1}\right|-t_o$ can be obtained; Substituting this back into the basic formula(5.5-5) of ξ_p, then $\xi_p=(\zeta_{\varsigma o}/4)\ln\{C_{ic2}[1-C_u e^{-\omega(t-t_o)}]^2/C_u e^{-\omega(t-t_o)}\}$ is obtained, where have set $C_u=[|u(a,1)|\sqrt{2/\bar f\zeta_{\varsigma o}}-1]/[|u(a,1)|\sqrt{2/\bar f\zeta_{\varsigma o}}+1]$.

Secondly, according to the initial conditions(5.4-3)2 and $\xi_p=\varsigma_t(t)\zeta_{\varsigma o}$, the previous formula becomes $\zeta_{\varsigma o}=(\zeta_{\varsigma o}/4)\ln[C_{ic2}(C_u-1)^2/C_u]$, thus obtaining $C_{ic2}=e^4C_u/(C_u-1)^2$. Substituting this formula back to the original formula obtains $\xi_p(t)=(\zeta_{\varsigma o}/4)\ln\{e^4[C_u e^{-\omega(t-t_o)}-1]^2/(C_u-1)^2e^{-\omega(t-t_o)}\}$.

5.6.2 At the starting moment $t=0$ of stagnate, the disturbance starting speed $u_{\zeta\zeta}$ at the crest should be the stagnate speed $|u(a,\infty)|$; So, from the same principle as the previous derivation of formulas (5.6-1)1,2, the formula for expressing the disturbance at the interface of the center DT gas in the time interval $0\le t\le t_{he}$ can be derived as follows

$$\xi_p(t)=(\zeta_{\varsigma o}/4)\ln[e^{4+\omega t}(C_v e^{-\omega t}-1)^2/(C_v-1)^2],\quad C_v=[|u(a,\infty)|\sqrt{2/\bar f\zeta_{\varsigma o}}-1]/[|u(a,\infty)|\sqrt{2/\bar f\zeta_{\varsigma o}}+1] \qquad (5.7\text{-}1)1,2$$

5.7 Stability Criteria

As mentioned earlier, the following two places must limit the disturbance crest: firstly, on the interface inside the ring target shell at the starting of stagnate, in order to avoid instability, must ensure that both the ring target shell and DT ice layer are not damaged by disturbance. So the stability criterion is

$$\xi_p(0)<\delta r_{sh}(0),\quad \xi_p(0)<\delta r_i(0) \qquad (5.8\text{-}1)1,2$$

Secondly, at the end of the stagnate, on the center DT gas surface, in order to avoid instability, must ensure that hot spot is not damaged by disturbance. So the stability criterion is

$$\xi_p(t_{he})<r_{he} \qquad (5.8\text{-}1)3$$

6. Energy Gain

6.1 Foreword

For nuclear fusion devices with practical value, the energy released by fusion should exceed the driving energy; For inertial fusion devices, its application value is measured by "energy gain"; According to literature (Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008), the energy gain G_a is defined as

$$G_a \equiv E_{fus}/E_d \tag{6.1-1}$$

where E_d ～driving energy; This E_d causes the ring target to pinch, resulting in an increase in the internal energy of "DT"; E_{fus} ～the energy released due to fusion.

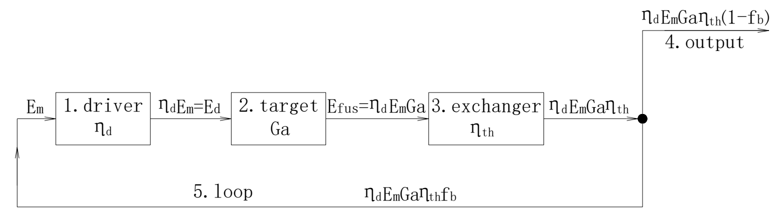


Fig 7.

Inertial fusion devices should be combined with other necessary devices to form an energy system, only then can work properly; Figure 7 shows the energy flow diagram of the inertial confinement fusion energy system, where the input energy E_m is inputted into a driver with the conversion efficiency of η_d and the output is $E_d = \eta_d E_m$; E_d and the gain G_a follow formula(6.1-1) $E_{fus} = E_d G_a$ ..., can write the energy flow thereby as $E_m = \eta_d E_m G_a \eta_{th} f_b$, where η_{th} ～the conversion efficiency of heat exchangers, f_b ～Feedback quantity; Thus, $1 = \eta_d G_a \eta_{th} f_b$ is obtained, namely $G_a = 1/\eta_d \eta_{th} f_b$.

In literature (Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008), $f_b < 0.25$, $\eta_d < 0.1\sim0.33$ and $\eta_{th} < 0.4$ are taken, then from the above formula can calculate that: the threshold that energy gain must reach is $G_a = 30\sim100$.

6.2 Exploring the Functional Relationship Between G_a and Driving Energy E_d

6.2.1 Introduction

According to the principle of conservation of mass, the total installed amount of DT fuel $M_\Sigma = M_c + M_h$ in the ring target should always be maintained as a constant, where M_c ～DT ice mass, M_h ～mass of the center DT gas,and M_c and M_h also being constant. When given a M_Σ value, there should be a functional relationship $G_a = G_a(E_d, M_\Sigma)$ between G_a and E_d ; On the $E_d \sim G_a$ plane, a curve can be drawn using function $G_a = G_a(E_d, M_\Sigma)$, and the following understanding can be given to this curve, as shown in Figure 8:

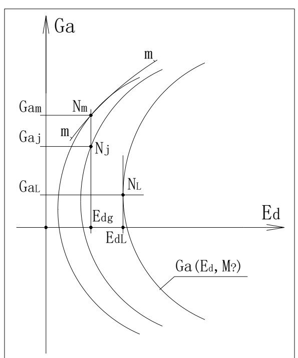


Fig 8.

Firstly, for a given total mass M_Σ of DT fuel, due to differences in the size of the ring target structure, causing the different values of driving energy E_d , but there is surely a minimum value E_{dL} in the E_d value; Due to E_{dL} being a minimum,there is surely a U-shaped bend of the $G_a = G_a(E_d, M_\Sigma)$ curve at point $N_L(E_{dL}, G_{aL})$, and the tangent of the $G_a = G_a(E_d, M_\Sigma)$ curve at point Q8 is parallel to the G_a axis.

Secondly, when changing the given value of M_Σ , the position of the $G_a(E_d, M_\Sigma)$ curve on the $E_d \sim G_a$ plane will change, resulting in a cluster of $G_a(E_d, M_\Sigma)$ curves.

Thirdly, as shown in Fig 8, for a given $E_d = E_{dg}$ value, a straight line $\overline{E_{dg}N_j}$ parallel to the G_a axis can be drawn. $\overline{E_{dg}N_j}$ intersects multiple curves within the $G_a(E_d, M_\Sigma)$ cluster, the G_a value at the intersection point N_j is $G_{aj}(E_{dg}, M_\Sigma)$. Due to the different M_Σ values, make the intersection N_j different, resulting in different $G_{aj}(E_{dg}, M_\Sigma)$ values, thus obtaining a set of G_a values; But in physical meaning, for a certain driving energy E_{dg}, it is impossible to obtain an infinite gain G_a. So there must be an upper limit G_{am} in $G_{aj}(E_{dg}, M_\Sigma)$; Let the intersection point corresponding to G_{am} is N_m, changing the value of E_{dg}, the coordinates (E_{dg}, G_{am}) of N_m will change accordingly, thus draw a trajectory line $m\tilde{m}$; The G_a value at each point on this curve is all the upper limit value G_{am}.

Fluid parameters after stagnate, according to literature (Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008), for strong compression in ICF, Fermi fluid may be involved,where the average particle spacing is smaller than the de Broglie wavelength; According to literature (JialuanXu, Shangxian, 1981), i.e.the following discriminant formula holds $\hbar/(mk_BT_h)^{1/2} > n_n^{-1/3}$, where $\hbar$ ～Planck constant, m～particle mass, k_BT_h ～the temperature inside the center DT gas after the stagnate, n_n ～particle quantity density; For the DT ice of equimolar, the number density of electrons and ions both are $n_n = \rho_m/2.5m_p$, so for electrons, the discriminant is $(\rho_{mc}/2.5m_p)\,[\hbar/(m_ek_BT_h)^{1/2}]^3 > 1$; At the end of the stagnate, the estimate of magnitude orders for ρ_{mc} and k_BT_{he} are: can reach $10^5[g\cdot cm^{-3}]$ and $10^8[K] = 1.4\times10^{-8}[erg]$ respectively; Based on this, can calculated that the left side of discriminant >1, indicating that there indeed is an electron Fermi fluid after the stagnate.

Based on the above, the calculation should be carried out in the quantum domain, but the results obtained above were all obtained in the classical field. Here, plan to make corrections to the relevant fluid parameters. Literature(Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008) provides the following approximate calculation: still using DT ice the mass density ρ_{mc} calculated after stagnate, introduce $p_{\deg}$ to express the pressure of electron Fermi fluid,for the Fermi pressure p_{ic}^f of DT ice after stagnate there are two formulas $p_{\deg} = \mathbf{A}_r\rho_{mc}^{5/3}$ and $p_{ic}^f = a_f p_{\deg}$, where $\mathbf{A}_r$ ～constant, for equimolarDT $\mathbf{A}_r = 2.17\times10^{-1}[(MJ/g)/(g/cm^3)^{2/3}]$, a_f ～constant; Accordingto (Atzeni, & Meyer-ter-Vehn, 2008), in the current target design, the average value of a_f is $1.5\sim4$; The combining of these two formulas results in $p_{ic}^f = a_f\mathbf{A}_f\rho_{mc}^{5/3}$, thus obtain

$$\rho_{mc} = (a_f\mathbf{A}_r)^{-3/5}p_{ic}^{f\,3/5} \tag{6.2-1)a}$$

According to the ideal gas law $T_{ce} = p_{ic}^f/R_g\rho_{mce}$ during stagnate, substituting $p_{ic}^f = a_f\mathbf{A}_f\rho_{mce}^{5/3}$ into it can obtain the DT ice temperature during stagnate

$$T_{ce} = a_f\mathbf{A}_f\rho_{mce}^{2/3}/R_g \tag{6.2-1)b}$$

6.2.2 Deriving E_{fus}

E_{fus} can be calculated using the following formula

$$E_{fus} = N_{fus}Q_{DT\Sigma} = \Gamma N_{DTO}Q_{DT\Sigma} = \Gamma M_\Sigma q_{DT} \tag{6.2-2}$$

where N_{fus} ～the number of "DT pair" that actually undergo fusion within the inertial constraint time, $Q_{DT\Sigma}$ ～ the whole fusion energy of a single "DT pair", $\Gamma = N_{fus}/N_{DTO}$ ～the combustion efficiency, N_{DTO} ～the total number of "DT pair" participating in fusion, q_{DT} ～the unit mass fusion energy of "DT pairs".

Regarding Γ

According to literature(Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008), for inertial fusion where most of the fuel will be burned, there exists the following approximate formula (Fraley. et. ai, 1974) $\Gamma \approx 1/(1 + H_B/H_c)$, where

$H_c \equiv \rho_{mc}(r_c - r_h)$, ρ_{mc} ～DT ice mass density, r_c ～DT ice outer surface radius, r_h ～the center DT gas radius, $H_B[g \cdot cm^{-2}]$ is referred to as "combustion parameters".

Among them, H_c can be written as $H_c = (M_c/V_c)\,(r_c - r_h)$, the volume enclosed by r_c in the formula is based on formula(2.2-5)1 is $V_c = 2\pi^2 r_c{}^2 \overline{R}$,so there is $H_c = (M_c/2\pi^2 r_c \overline{R})\,(1 - r_h/r_c)$ .In this formula, r_h/r_c is expressed as $r_h/r_c = r_o/r_{co} = const$ according to formula (2.5-4)1, and due to $M_c = const$, thereby H_c is the decreasing function of r_c.

According to $\Gamma \approx 1/(1 + H_B/H_c)$, Γ is an increasing function of H_c, so when r_c decreases due to implosion, Γ will increase; According to $[\Gamma]_{H_c \to \infty} \approx [1/(1 + H_B/H_c)]_{H_c \to \infty} = 1$, Γ has an upper bound; So when r_c decreases to a certain value, Γ will reach its peak Γ_u, seting H_c corresponding to Γ_u is H_{cu}, when reaching peak,can prove that Γ has a asymptotic value

$$\Gamma_u = 0.5(H_{cu}/H_B)^{1/2} \tag{6.2-3}$$

Argument the above:

According to (Atzeni, & Meyer-ter-Vehn, 2008), the energy E_{fus} released by a single micro fusion must be limited to $E_{fus} < \kappa \cdot 10^{10} J$, $\kappa < 10$. The fuel loading amount M_Σ also needs to be limited to ensure that micro fusion is carried out several times per second in the reaction chamber without damaging the equipment. This article takes $E_{fus} \le 8 \times 10^4 MJ$ and limits $M_\Sigma \le 1.60[g]$. Estimating that the amount of fuel involved in fusion accounts for 0.3 of the total fuel installed. Known that complete combustion of $1[mg]$ DT releases 337 MJ of fusion energy, the peak value of Γ is $\Gamma \le \Gamma_u = E_{fus}/0.3 \times 337 M_\Sigma \approx 0.5$; So there must be $\Gamma_u = [(H_c/H_B)/(1 + H_c/H_B)]_{H_c \to H_{cu}} = (H_{cu}/H_B)/(1 + H_{cu}/H_B) \approx 0.5$, thereby when Γ reaching its peak,there is $H_{cu} \approx H_B$.

Based on the above, can infer that: when Γ approaches its peak, there can be an asymptotic form $\Gamma = 0.5(H_c/H_B)^\kappa$,making $[0.5(H_c/H_B)^\kappa]_{H_c \to H_B} = [H_c/(H_c + H_B)]_{H_c \to H_B}$ hold, Where κ ～ Undetermined constant; determine κ : let $H_c/H_B = y$ then there is $[0.5y^\kappa]_{y \to 1} = [y/(1+y)]_{y \to 1}$,this formula is a indeterminate form regarding κ. By taking the logarithm of both sides of the formula and using the L'Hôpital's rule, $\kappa = 0.5$ can be obtained. From this, can infer that: when Γ reaches its peak, there is a asymptotic value $\Gamma_u = 0.5(H_{cu}/H_B)^{\frac{1}{2}}$.

Regarding H_B, its value can be approximated as

$$H_B \approx 7.69[g \cdot cm^{-2}] \tag{6.2-4}$$

Argument the above:

According to Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008), H_B can be calculated using the following formula: $H_B = 8 c_T \overline{m}_{DT} / \overline{v\sigma}(T_{he})$, where c_T ～the isothermal sound speed in the center DT gas at end moment t_{he} of stagnate, $T_{he}[KeV]$ ～the DT temperature in the center DT gas at end moment of stagnate.

Substitute the average reaction rate formula (1.1-2)1into the above formula, and let $y = T_{he}/k_3$ and $k_5 = 8\sqrt{5 k_3 m_p k_B}\,/k_1$, then there is $H_B = k_5 y^{1/2} e^{k_2 |\ln y|^{2.13}}$; In the $10 \le T_{he} \le 64.2 KeV$ domain the following will discuss the formula, here the original formula should be written as $H_B = k_5 y^{1/2} e^{k_2(-\ln y)^{2.13}}$; The reason for setting the discussion domain in this way, according to literature (Stefano Atzeni, Jürgen Meyer-ter-Vehn, 2008), the temperature of ICF increases from $0.5 KeV$ to $10 KeV$ during stagnate and combustion, and can then reach $100 KeV$. The discussion domain set here basically covers this region.

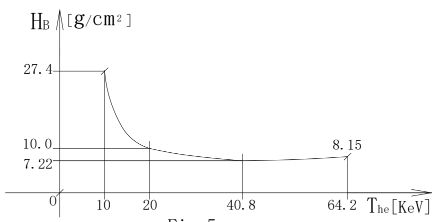


Fig 5.

Substituting all relevant data into $H_B = k_5 y^{1/2} e^{k_2(-\ln y)^{2.13}}$, the graph can be drawn as shown in Figure 5, the graph has a minimum value $H_{B\min} = 7.22$ of H_B at $T_{he} = 40.8[KeV]$. From the graph, can see that at $T_{he} > 3.00\times 10$ the graph of H_B approximates a straight line parallel to the T_{he} axis in the interval $7.22 \le H_B \le 8.15$; The following article approximates the H_B value as $H_B = (7.22 + 8.15)/2$, namely $H_B \approx 7.69[g\cdot cm^{-2}]$.

The formula for Q1 is:

$$H_{cu} = [p_{ic}^{f\ 1/10} \big/ (a_f \mathbf{A}_r)^{3/5}]\ (E_d \big/ 3\pi^2 \bar{R})^{1/2} [1 - r_{he} / r_{ce}] \tag{6.2-5}$$

Argument the above:

The peak H_{cu} should correspond to the end moment of stagnate, and according to the definition formula, there is $H_{cu} = \rho_{mce} r_{ce} (1 - r_{he} / r_{ce})$; Discussing r_{ce} in this formula: due to the pinch of the ring target caused by the driving energy E_d, ultimately leads to an increase in the internal energy of DT, so E_d can be written as $E_d = \varepsilon_c \rho_{mce} V_c + \varepsilon_h \rho_{mhe} V_h$, where the specific internal energy is $\varepsilon_c = 1.5\, p_{ce} / \rho_{mce}$, $\varepsilon_h = 1.5\, p_{he} / \rho_{mhe}$, there should be $p_{he} \approx p_{ce} = p_{ic}^f$ when reaching thermal equilibrium, hence $E_d = 1.5 p_{ic}^f (V_c + V_h) = 1.5 p_{ic}^f V$. Using formula(2.2-5)1 to this formula, $r_{ce} = (E_d \big/ 3\pi^2 \bar{R} p_{ic}^f)^{1/2}$ is obtained.

Substituting the above formula and equation(6.2-1)back to the original formula H_{cu} then obtain to $H_{cu} = [p_{ic}^{f\ 1/10} \big/ (a_f \mathbf{A}_r)^{3/5}]\ (E_d \big/ 3\pi^2 \bar{R})^{1/2} [1 - r_{he} / r_{ce}]$.

The formula for M_Σ is

$$M_\Sigma = (2E_d/3)[p_{ic}^{f\ -2/5} \big/ (a_f \mathbf{A}_r)^{3/5}]\ [1 - (r_{he} / r_{ce})^2] \tag{6.2-6}$$

Argument the above:

The total installed amount of ring target fuel is $M_\Sigma = \rho_{mce} V_c + \rho_{mhe} V_h$, due to $\rho_{mce} >> \rho_{mhe}$, there is $M_\Sigma \approx M_c = \rho_{mce} V_c$, where V_c is the volume of DT ice layer. Using the ring target volume formula (2.2-5)1, $M_\Sigma = 2\pi^2 \rho_{mce} \bar{R} r_{ce}{}^2 [1 - (r_{he} / r_{ce})^2]$ can be derived; Substitute formula (6.2-1)a and $r_{ce} = (E_d \big/ 3\pi^2 \bar{R} p_{ic}^f)^{1/2}$ into the above formula to derive formula (6.2-6), where $r_{ce} = (E_d \big/ 3\pi^2 \bar{R} p_{ic}^f)^{1/2}$ obtained from proving formula (6.2-5).

In summary, can conclude that Q1 at the end moment of stagnate is

$$E_{fusu} = (\mathcal{K}_G \big/ a_f{}^{9/10})(E_d{}^{8/5} \big/ \bar{R}^{3/5}) z^{7/10} (1 - z)^{1/2} (1 - z^2) \tag{6.2-7}$$

where $\mathcal{K}_G = q_{DT} \big/ 3^{8/5} \pi^{6/5} H_B{}^{1/2} \mathbf{A}_r{}^{9/10} \mathcal{F}_{DT}{}^{7/10}$, $z = r_{he} / r_{ce}$.

Argument the above:

At the end moment of stagnate, H reaches the peak H_{cu}, causing Γ to also reach peak Γ_u. So according to formula (6.2-2), make E_{fus} also reach peak $E_{fusu} = \Gamma_u M_\Sigma q_{DT}$; Substitute formula(6.2-3)into this formula to obtain $E_{fusu} = 0.5 M_\Sigma q_{DT} (H_{cu} / H_B)^{1/2}$.

Substituting formulas (6.2-5) and (6.2-6) into the above formula obtains

$E_{fusu}=\frac{q_{DT}}{3^{5/4}(\pi H_B)^{1/2}\bar{R}^{1/4}(a_f\mathbf{A}_r)^{9/10}}\cdot E_d^{5/4}p_{ic}^{f\ -7/20}[(1-\frac{r_{he}}{r_{ce}})]^{1/2}[1-(\frac{r_{he}}{r_{ce}})^2]$; Discuss p_{ic}^f in this formula: for this the definition formula $\mathcal{F}_{DT}\equiv p_{he}r_{he}$ involved in ignition criterion (3.3-7)3 is used, where p_{he} should be the Fermi pressure p_{ic}^f of DT ice at the end of stagnate, hence $r_{he}=\mathcal{F}_{DT}/p_{ic}^f$; According to $r_{ce}=(E_d/3\pi^2\bar{R}p_{ic}^f)^{1/2}$ that is obtained from proving formula(6.2-5), there is $r_{he}/r_{ce}=(\mathcal{F}_{DT}/p_{ic}^f)/(E_d/3\pi^2\bar{R}p_{ic}^f)^{1/2}$, thus $p_{ic}^f=3(\pi\mathcal{F}_{DT})^2(\bar{R}/E_d)\,(r_{he}/r_{ce})^{-2}$ is obtained; Substitute this formula into the original formula, and let $\mathcal{K}_G=\frac{q_{DT}}{3^{8/5}\pi^{6/5}H_B^{\ 1/2}\mathbf{A}_r^{\ 9/10}\mathcal{F}_{DT}^{\ 7/10}}$ and $z=\frac{r_{he}}{r_{ce}}$, then obtain $E_{fusu}=\mathcal{K}_G\frac{1}{a_f^{\ 9/10}}\frac{E_d^{\ 8/5}}{\bar{R}^{3/5}}z^{7/10}(1-z)^{1/2}(1-z^2)$.

6.2.3 Deriving the Maximum Value G_{am} of G_a When Givened the Driving Energy E_d

The expression for G_{am} is

$$G_{am}=(3.938\times10^{-6}/a_f^{\ 9/10})\,(E_d/\bar{R})^{3/5} \tag{6.2-8)1}$$

The dimension of E_d in the formula is [erg].

The structural parameters of the above formula must satisfy the following formula

$$r_o/r_{co}=r_{he}/r_{ce}=0.396 \tag{6.2-8)2}$$

Argument the above:

Substitute formula (6.2-7) into formula (6.1-1), obtain the expression of E_{fusu} corresponding to peak G_a as $G_a(z)=(\mathcal{K}_G/a_f^{\ 9/10})\,(E_d/\bar{R})^{3/5}z^{7/10}(1-z)^{1/2}(1-z^2)$; When E_d is given, this formula is only a function of z. So to obtain the extreme value of $G_a(z)$, the extremum of $f_1(z)=z^{7/10}(1-z)^{1/2}(1-z^2)$ must be calculated; the function $f_1(z)$ has the following values: $f_1(0)=0$ and $f_1(1)=0$,and there is $0\le z\le1$ within interval $f_1(z)\ge0$, therefore there must exist a maximum value of $f_1(z)$; To obtain this maximum value, calculate $df_1(z)/dz=0$ to obtain equation $32z^2+5z-7=0$, solving this equation obtains a positive root $r_{he}/r_{ce}=z_m=0.396$, according to equation (2.5-4)1, $r_o/r_{co}=0.396$ can be further derived from this formula; Substituting it into the original formula of $G_a(z)$ obtains $G_{am}=(0.343\mathcal{K}_G/a_f^{\ 9/10})\,(E_d/\bar{R})^{3/5}$, substituting the value of $\mathcal{K}_G$ into this equation obtains equation (6.2-8)1.

6.3 Estimating the Ring Target Size Range and Driving Energy Value Based on the Expected G_{am} Value

The initial volume V_{sho} of the ring target shell

If the upper bound of B_J is B_{Ju}, then the upper bound of its energy density is $B_{Ju}^{\ 2}/8\pi$; Due to the magnetic field act on the ring target shell, so the upper bound of the driving energy provided by the magnetic field can be estimated as $E_d\le V_{sho}B_{Ju}^{\ 2}/8\pi$; Substitute this formula into formula (6.2-8)1 to obtain $G_{am}\le(3.938\times10^{-6}/a_f^{\ 9/10})\,(V_{sho}B_{Ju}^{\ 2}/8\pi\bar{R})^{3/5}$, thereby for the expected gain value G_{am} there must be

$$V_{sho}\ge(8\pi\bar{R}/B_{Ju}^{\ 2})\,(a_f^{\ 9/10}G_{am}/3.938\times10^{-6})^{5/3} \tag{6.2-9}$$

The initial volume V_{io} of the ring target DT ice layer

As mentioned earlier: must to limit the loaded quantity of fuel to $M_\Sigma\le1.60[g]$, so that micro fusion can occur several times per second without damaging the reaction chamber; So $\rho_{mco}V_{io}\le1.6[g]$ must be met, therefore due to $\rho_{mco}=0.224[g\cdot cm^{-3}]$, must have

$$V_{io}\le7.143[cm^3] \tag{6.2-10}$$

The required ring target volume threshold was estimated based on the expected G_{am} value using formulas (6.2-9,10),this can further determine the structural size of the ring target.

7. Examples

The following is an example to verify the feasibility of the proposed programme in this article.

7.1 Firstly, using numerical methods to solve the first order differential equation (2.4-4)1 $dU/dC = \Delta_1(U, C)/\Delta_2(U, C)$, the numerical function $U = U[C, C^{(1)}]$ corresponding to the curve $P_6\tilde{P}_2$ of solution can be obtained, while the curve $P_2\tilde{P}_4$ of solution has been proven to be a straight line $\overline{P_2P_4}$ In the previous text; When taking $\alpha = 0.69$ draw the curve of solution graph as shown in Figure 3; In the solving, $U(C = 1/\alpha) = U(\xi = 1) = 0.9827036$ is obtain at $C = 1/\alpha$.

7.2 Determine the Initial Parameters of the Ring Target and the Size During Stagnate

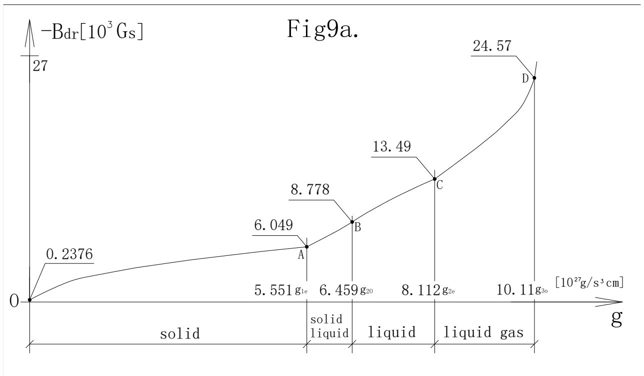
The initial volume of DT ice layer should be constrained by formula (6.2-10) $V_{io} \le 7.143[cm^3]$, the initial volume of the ring target shell should be constrained by formula (6.2-9) $V_{sho} \ge (8\pi\bar{R}/B_{Ju}{}^2)\ (a_f{}^{9/10}G_{am}/3.938\times10^{-6})^{5/3}$. take $a_f = 1.5$, $B_{Ju} \le 2.650\times10^6[Gs]$ and $G_{am} \ge 75$, the following structural parameters are obtained under this constraint (as shown in Figure 1): $r_o = r_{ho} = 0.2[cm]$ is taken first, and then the initial dimensions of the ring target are calculated as $r_{co} = 0.5050[cm]$, $r_{sho} = 1.005[cm]$ and $\bar{R} = 1.605[cm]$; The initial volume of DT ice layer is $V_{io} = 6.812[cm^3]$, and the initial volume of ring target shell is $V_{sho} = 23.92[cm^3]$; The DT gas loading amount is $M_{ho} = 6.336\times10^{-4}[g]$, the total DT fuel loading amount is $M_\Sigma = 1.527[g]$.

At the starting of stagnate, using formula(2.5-1)6, $a_{ho} = 2.191\times10^{-3}[cm]$ can be obtained, subsequently $r_{sh}(0) \approx 7.702\times10^{-3}[cm]$, $r_c(0) = 5.533\times10^{-3}[cm]$ and $\delta r_{sh}(0) = 2.169\times10^{-3}[cm]$ obtained, and using $\delta r_i(0) = r_c(0) - a_{ho}$ obtained $\delta r_i(0) = 3.342\times10^{-3}[cm]$.

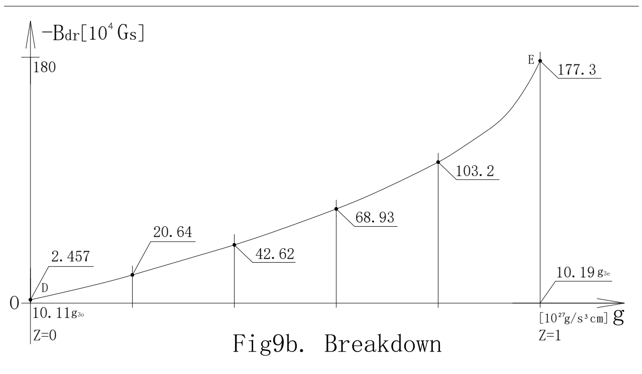
At the end of stagnate, the hot spot radius $r_{he} = 1.392\times10^{-4}[cm]$ can be obtained using formula (2.5-1)1.

7.3 Determining the Driving Magnetic Field

Since the peak value of the driving magnetic field before the breakdown stage is $B_{dr}{}^2(g_{3o})$, based on $B_{Ju} \le 2.650\times10^6[Gs]$ can estimate $B_3(g_{3o})$ as $B_3(g_{3o})^2 \approx 6.733\times10^{12}[Gs]^2$. Due to that between each driving magnetic field segment should be connected to each other end-to-end, namely $B_{dr}(g_{je}) = B_{dr}(g_{ko})$ and $B_{dr}(g_{ke}) = B_{dr}(g_{jo})$. Thereby establishing the relationship between $B_{dr}(g_{3o})$ and $B_{dr}(g_{1o})$ values to obtain the appropriate starting value $B_{dr}(g_{1o})$ for the driving magnetic field. This article obtained $B_{dr}(g_{1o})^2 = 51209.95[Gs]^2$. Then, using formula (4.3-5)1that describes the driving magnetic field before the breakdown stage, obtain respectively: the functions that drive the magnetic field in the solid state, solid- liquid state, liquid state and liquid- gas state are $B_{dr1}(g) \approx -[6.583\times10^{-21}\times g + 51209.95]^{1/2}[Gs]$, $B_{dr12}(g) = -\{\frac{1}{2\pi}(\frac{L\delta r_{sho}}{\bar{R}^2})^2[\frac{2\rho_{1e}}{\sqrt{A_{12}}}(\sqrt{F_{12}-g_{1e}} - \sqrt{F_{12}-g}) + \rho_{1o}g_{1e}] + B_{dr}(g_{1o})^2\}^{1/2}$, $B_{dr2}(g) = -\{\frac{1}{2\pi}(\frac{L\delta r_{sho}}{\bar{R}^2})^2[\rho_{2o}(g - g_{2o}) + \frac{2\rho_{1e}}{\sqrt{A_{12}}}(\sqrt{F_{12}-g_{1e}} - \sqrt{F_{12}-g_{2o}}) + \rho_{1o}g_{1e}] + B_{dr}(g_{1o}$, $B_{dr23}(g) = -\{\frac{1}{2\pi}(\frac{L\delta r_{sho}}{\bar{R}^2})^2[\frac{2\rho_{2e}}{\sqrt{A_{23}}}(\sqrt{F_{23}-g_{2e}} - \sqrt{F_{23}-g}) + \frac{2\rho_{1e}}{\sqrt{A_{12}}}(\sqrt{F_{12}-g_{1e}} - \sqrt{F_{12}-g_{2o}}) + \rho_{2o}(g_{2e} - g_{2o}) + \rho_{1o}g_{1e}] + B_{dr}(g_{1o})^2\}^{1/2}$, and the driving magnetic field waveform curve $OABC\tilde{D}$ before the breakdown section was plotted in Figure 9a using these functional formulas.



Using the expressions(4.3-6)1,(4.3-7)1for the driving magnetic field in the breakdown stage, numerical calculations are performed to obtain the numerical function $B_{dr}(z)$ regarding z , where $z=(g-g_{3o})/(g_{3e}-g_{3o})$, and the waveform curve $D\tilde{E}$ of the driving magnetic field in the breakdown stage is drew in Figure 9b using this function.



The curves $OABC\tilde{D}$ and $D\tilde{E}$ in Figure 9a,b are connected at point $B_{dr}(g_{3o})$, forming the driving magnetic field waveform curve starting from $B_{dr}(g_{1o})$. The driving magnetic field $B_{dr}=B_{dr}(g)$ obtained above is a function regarding g , the function of the driving magnetic field regarding time t can also be obtained using formula(4.3-7)1 and parameter equation $\begin{cases} r=r(a,\xi) \\ \xi=(r/r_o)\Big/\left|t/t_o\right|^{\alpha} \end{cases}$. To save the length, this article does not further discuss this.

7.4 Performance Parameters of the Ring Target

According to calculations: after the implosion, enters stagnate at moment $\left|t_o\right|=5.882\times10^{-6}[s]$, and after the starting of stagnate completes stagnate at moment $\left|t_{he}\right|=1.562\times10^{-10}[s]$. The total driving energy required for fusion: first the average value $\overline{B_3^{\ 2}}=6.606\times10^{12}[Gs]^2$ must be calculated using formula (4.3-8)4', and then $E_{d1}=4.023\times10^{12}[erg]$ and $E_{d2}\approx8.369\times10^{12}[erg]$ can be calculated using formulas (4.3-8)4,5, thereby the total driving energy $E_d=E_{d1}+E_{d2}\approx1.239\times10^{13}[erg]=1.239[MJ]$ can be calculated. The energy gain obtained by fusion: calculated using formula (6.2-8)1, the energy gain is $G_a\approx148$.

7.5 To Achieve Fusion, the Ring Target Should Meet Various Criteria

After the stagnate, $\overline{\rho}_{mhe}=1032[g/cm^3]$ can be calculated using formula (2.5-5)3,and thus $\overline{\rho}_{mhe}r_{he}=0.1437$ can be calculated; And using formula (6.2-1)b calculate the hot spot temperature at the end of stagnate as $T_{he} \approx 25.41[KeV] = 2.949\times10^{8}[K]$; Additionally, $\rho_{mce}/\overline{\rho}_{mhe} = \rho_{mco}/\rho_{mho} = 0.224/5\times10^{-4} = 448$ can be obtained. Substitute the above data into the condition for all DT to participate in and complete fusion within the inertial constraint time ～formula (3.3-4), and self heating condition (3.2-6)1, ignition criterion (3.3-7)1, then the left side of formula (3.3-4) becomes $\overline{\rho}_{mhe} r_{he} T_{he} - 2.26\times10^{-21}/(M_{ho} T_{he}{}^{1/2}) = 3.651 > 0$, the left side of formula(3.2-6)1becomes $A(T_{he})(\rho_{mhe} r_{he})^2 + B(T_{he})(\rho_{mhe} r_{he}) - C(T_{he}) = 6.561\times10^{21} > 0$, and the left side of formula (3.3-7)1 becomes $\overline{\rho}_{mhe} r_{he} T_{he} (\rho_{mce}/\rho_{mhe})^{1/2} - 1.2 T_{he}{}^{2}/(T_{he}{}^{3/2} - 3.4) = 71.07 > 0$.

From the above results: the conditions for all DT to participate in and complete fusion, self heating condition, and ignition criterion are all met, that is: the ring target with the selected parameters can achieve fusion.

7.6 Stability in Fusion

7.6.1 Stability of Ring Target Shell and DT Ice Layer

$|u(a,\xi_\infty)| = 2.186\times10^{5}[cm/s]$ and $|u_{co}| = u(a,1) = 2.305\times10^{4}[cm/s]$ can be obtained using formulas(2.5-3)2,3, thereby the average inertial force $\overline{f}_a \approx 3.325\times10^{10}[cm/s^2]$ can be obtained using formula (5.5-4)1. The average Lorentz force $\overline{f}_{JB} = -3.506\times10^{10}[dyn/g]$ per unit mass can be obtained using formula (5.5-4)3; Thereby the average body force from starting of the implosion of ring target shell to the starting of stagnate is obtained as $\overline{f} = |\overline{f}_a + \overline{f}_{JB}| = 1.81\times10^{9}[dyn/g]$.

Calculate the disturbance peak $\xi_p(0)$ of the ring target shell at the starting of stagnate needs to use formula(5.6-1)1,substitute $\overline{f} = 1.81\times10^{9}[dyn/g]$, $|t_o| = 5.882\times10^{-6}[s]$ and $u(a,1) = 2.305\times10^{4}[cm/s]$ into this formula, and take the initial manufacturing erro of the surface radius inside the ring target shell as $\zeta_{\varsigma o} < 1\times10^{-3}[mm]$, thereby obtain $\xi_p(0) = 2.052\times10^{-3}[cm]$. Comparing this with the thickness $\delta r_{sh}(0) = 2.169\times10^{-3}[cm]$ of the ring target shell and the DT thickness $\delta r_i(0) = 3.342\times10^{-3}[cm]$ of the ice layer at the starting moment of stagnate: If the initial manufacturing erro of the surface radius inside the ring target shell is $\zeta_{\varsigma o} < 1\times10^{-3}[mm]$, then the stability criterion (5.8-1)1,2 $\xi_p(0) < \delta r_{sh}(0)$ and $\xi_p(0) < \delta r_i(0)$ are satisfied.

7.6.2 Stability of Hot Spot

Using formula (5.5-4)2, the average body force inside the central DT gas can be obtained as $\overline{f} = \overline{f}_a \approx 1.399\times10^{15}[dyn/g]$.

Calculating the disturbance peak $\xi_p(t_{he})$ at the hot spot at the end of stagnate needs to use formula (5.6-1)1, substitute $\overline{f} = 1.399\times10^{15}[dyn/g]$, $t_{he} = 1.562\times10^{-10}[s]$ and $|u(a,\xi_\infty)| = 2.186\times10^{5}[cm/s]$, into this formula, and take the initial manufacturing erro of the surface radius inside the DT ice layer as $\zeta_{\varsigma o} < 9\times10^{-4}[mm]$, thereby obtain $\xi_p(t_{he}) = 1.324\times10^{-4}[cm]$.

Comparing this with the hot spot radius $r_{he} = 1.392\times10^{-4}[cm]$ at the end of stagnate: if the initial manufacturing erro $\zeta_{\varsigma o} < 0.9\times10^{-3}[mm]$ of the inner surface radius of the DT ice layer, then the stability criterion (5.8-1)3 $\xi_p(t_{he}) < r_{he}$ is satisfied.

7.6.3 In summary, the ring target of the selected parameters can maintain stability in fusion.

8. Conclusion

So far this article has derived the relevant formulas required for ICF driven by a strong pulse magnetic field, and used these formulas to calculate an example. The results show that the selected ring target parameters can meet various detection criteria, thus stably achieving DT fusion and obtaining a high energy gain of $G_a \approx 148$. There

are still the following issues that need to be further solved in this programme:
Firstly, the peak value of the driving magnetic field is relatively high, reaching 177.3 [T] at the end of the breakdown stage. At present, this belongs to the ultra strong magnetic field.

Secondly, the manufacturing accuracy of the ring target is required to be relatively high, the initial manufacturing erro of the inner surface radius of the DT ice layer is required to reach $\zeta_{\varsigma o} < 0.9\times10^{-3}[mm]$.
But there have been reports that the current world record for stable ultra strong magnetic fields is 45 [T] held by the United States, while the world record for non stable ultra strong magnetic fields is 2800[T]held by Russia. China is building a "world's strongest pulsed magnetic field device" that can generate 110 [T] magnetic flux density.In addition, there have been reports on micro nano 3D printing devices based on the principle of new surface projection micro lithography technology.So, the above issues can be solved with the development of high-tech.

The proposed programme in this article has obvious advantages compared to existing programmes. The current programme proposed of using high-energy short pulse laser or high-energy particle beam pulse heating is difficult to achieve uniform energy flow irradiation on the target surface, so resulting in instability during implosion due to asymmetric flow, this causes tearing of the target, making fusion impossible to complete; Therefore, the current solution needs to make the driving device more complex, such as using the hohlraum target and so on. However, the pulse magnetic field used in this article can act symmetrically and uniformly on the ring target. So in implosion, ring target can perform symmetric flow without instability.

Furthermore, the existing programme will form a coronal region due to the gasification of the spherical shell during implosion, which is particularly severe for hohlraum target. This coronal region will affect the transparency of the radiation energy flow, thereby reducing the input efficiency of the driving energy, which is not conducive to obtaining high energy gain. But for the pulse magnetic field used in this article, there is no such coronal region, which is very beneficial for improving energy gain.

In summary, after comparison, the following conclusions can be drawn: the proposed programme of "using a strong pulse magnetic field to drive ICF" in this article, It is a feasible and promising for development technical method.

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Data sharing statement

No additional data are available.

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