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Tensor Satisfying Binary Law for the Equation Including the Trigonometric Function

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Abstract

I already reported that $A^{\mu}_{;\nu;\sigma;\lambda}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\,\partial x^{\sigma}\,\partial x^{\lambda}}+\cdots$ must be expressed in $A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\,\partial x^{\nu}\,\partial x^{\nu}}=A$ if $A^{\mu}_{;\nu;\sigma;\lambda}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\,\partial x^{\sigma}\,\partial x^{\lambda}}+\cdots$ was tensor satisfying Binary Law. I reported that $A^{\mu}=Sin(x^{\nu})$ was established from the search result of the property of this $\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\,\partial x^{\nu}\,\partial x^{\nu}}=A$. A trigonometric function is included in $A^{\mu}=Sin(x^{\nu})$ here, but the search about the equation of Tensor satisfying Binary Law including the trigonometric function isn't done. I report the search result about the equation of Tensor satisfying Binary Law including the trigonometric function in this article.

Keywords: tensor, covariant derivative

1. Introduction

I have already reported establishment of $A^{\mu}=Sin(x^{\nu})$. (Ichidayama, 2017. Property of $\cdots$) $A^{\mu}=Sin(x^{\nu})$ is an equation including the trigonometric function here. However, it isn't investigated Tensor satisfying Binary Law for the equation including the trigonometric function. I investigate Tensor satisfying Binary Law for the equation including the trigonometric function newly and report this result in this article.

2. Definition

Definition1. $\overline{x^{\mu}}\neq x^{\mu},\overline{x^{\nu}}\neq x^{\nu},\overline{x^{\mu}}=x^{\nu},\overline{x^{\nu}}=x^{\mu}$ is established.(Ichidayama, 2017, Introduction of …)

I named $\overline{x^{\mu}}\neq x^{\mu},\overline{x^{\nu}}\neq x^{\nu},\overline{x^{\mu}}=x^{\nu},\overline{x^{\nu}}=x^{\mu}$ "Binary Law".(Ichidayama, 2017, Introduction of …)

Definition2. If $\overline{x^{\mu}}\neq x^{\mu},\overline{x^{\nu}}\neq x^{\nu},\overline{x^{\mu}}=x^{\nu},\overline{x^{\nu}}=x^{\mu}$ is established, $x_{\nu}=x^{\mu}$ is established.(Ichidayama, 2017, Introduction of …)

Definition3. If $\overline{x^{\mu}}\neq x^{\mu},\overline{x^{\nu}}\neq x^{\nu},\overline{x^{\mu}}=x^{\nu},\overline{x^{\nu}}=x^{\mu}$ is established, $x_{\mu}=x^{\nu}$ is established.(Ichidayama, 2017, Introduction of …)

Definition4. If $\overline{x^{\mu}}\neq x^{\mu},\overline{x^{\nu}}\neq x^{\nu},\overline{x^{\mu}}=x^{\nu},\overline{x^{\nu}}=x^{\mu}$ is established, $x_{\nu}=-x_{\mu}$ is established.(Ichidayama, 2017, Introduction of …)

Definition5. If $\overline{x^{\mu}}\neq x^{\mu},\overline{x^{\nu}}\neq x^{\nu},\overline{x^{\mu}}=x^{\nu},\overline{x^{\nu}}=x^{\mu}$ is established, $x^{\nu}=-x^{\mu}$ is established.(Ichidayama, 2017, Introduction of …)

Definition6. If all coordinate systems $x^{\mu},x^{\nu},x^{\sigma},x^{\lambda},\cdots$ satisfies $\overline{x^{\mu}}\neq x^{\mu},\overline{x^{\nu}}\neq x^{\nu},\overline{x^{\mu}}=x^{\nu},\overline{x^{\nu}}=x^{\mu}$, all coordinate systems $x^{\mu},x^{\nu},x^{\sigma},x^{\lambda},\cdots$ shifts to only two of x^{μ},x^{ν}. (Ichidayama, 2017, Introduction of …)

Definition7. If $\overline{x^{\mu}}\neq x^{\mu},\overline{x^{\nu}}\neq x^{\nu},\overline{x^{\mu}}=x^{\nu},\overline{x^{\nu}}=x^{\mu}$ is established, $\frac{\partial^3 M^{\mu}}{\partial x^{\nu}\,\partial x^{\nu}\,\partial x^{\nu}}=M$ is established.(Ichidayama, 2023)

Definition8. If $\overline{x^{\mu}} \neq x^{\mu}, \overline{x^{\nu}} \neq x^{\nu}, \overline{x^{\mu}} = x^{\nu}, \overline{x^{\nu}} = x^{\mu}$ is established, $\frac{\partial m}{\partial x^{\nu}} = 0$ is established. m expresses Mass.

Definition9. The first-order covariant derivative of the covariant vector satisfied $M_{\mu;\nu} = \frac{\partial M_{\mu}}{\partial x^{\nu}} - M_{\tau}\Gamma^{\tau}_{\mu\nu} = \frac{\partial M_{\mu}}{\partial x^{\nu}} - M_{\tau}\frac{1}{2}g^{\epsilon\tau}\left(\frac{\partial g_{\mu\epsilon}}{\partial x^{\nu}} + \frac{\partial g_{\nu\epsilon}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\epsilon}}\right)$.(Fleisch, 2012)

Definition10. $x^{\mu} = \frac{\partial x^{\mu}}{\partial x^{\nu}} x^{\nu}$ is established.

Definition11. $-SinA = Sin(-A)$ is established. (Spiegel, 1968)
Definition12. $CosA = Cos(-A)$ is established. (Spiegel, 1968)

Definition13. $SinA + SinB = 2Sin\frac{(A+B)}{2}Cos\frac{(A-B)}{2}$ is established. (Spiegel, 1968)

Definition14. $\left(\vec{A}\right)^2 = \vec{A} \cdot \vec{A}$ is established. (Spiegel, 1968)

Definition15. $-Sin^{-1}A = Sin^{-1}(-A)$ is established. (Spiegel, 1968)
Definition16. $\mathrm{Sin}(n\pi) = 0$ is established. n expresses natural number here.

Definition17. $\int \frac{dX}{\sqrt{A^2 - X^2}} = \arcsin\left(\frac{X}{A}\right)$ is established. (Spiegel, 1968)

Definition18. $\mathrm{W}(A \rightarrow B) = -\mathrm{U} = \int_A^B \vec{F} \cdot d\vec{r}$ is established. (Kittel, Knight, Ruderman, 1975)

W expresses Work, U expresses Potential Energy, $\vec{F}$ expresses External force vector, and $\vec{r}$ expresses Displacement vector.
Definition19. $E = mc^2$ is established. (Taylor, 1975)
E expresses Energy, m expresses Mass, and c expresses Speed of light.

Definition20. The force that the nucleus attracts electron is expressed in $F = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2} = -\frac{k}{r^2}$. (Crawford, 1968)

r expresses distance between nucleus and the electron, ϵ_0 expresses dielectric constant, k expresses constant, Q expresses nuclear charge, q expresses electronic charge.

The force that binary proton repels is expressed in $F = \frac{1}{4\pi\epsilon_0}\frac{QQ}{r^2} = \frac{k'}{r^2}$. (Crawford, 1968)

r expresses distance between proton each, k' expresses constant.
Definition21. $y[x] = C[1]Cos[x] + C[2]Sin[x]$ is established as a solution of the equations of $y''[x] + y[x] = 0$. I obtained this calculation result by Wolfrem Mathematica 11.3.
y is function $y = f(x)$ which assumes x an independent variable.
Hypothesis1. $M \propto m, M = \epsilon m$ is established.

M expresses $\frac{\partial^3 \mathrm{M}^{\mu}}{\partial \mathrm{x}^{\nu}\,\partial \mathrm{x}^{\nu}\,\partial \mathrm{x}^{\nu}} = \mathrm{M}$, ϵ expresses Proportional constant, and m expresses Mass.

3. Property $\frac{\partial^3 M^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}} = M: (M > 0)$

Proposition1. When all coordinate systems satisfies Binary Law, $\frac{\partial M}{\partial x^{\mu}} = 0, \frac{\partial M}{\partial x^{\nu}} = 0, \frac{\partial M}{\partial \varphi^{\mu}} = 0, \frac{\partial M}{\partial \varphi^{\nu}} = 0$ is established. M expresses $\frac{\partial^3 M^{\mu}}{\partial x^{\nu}\,\partial x^{\nu}\,\partial x^{\nu}} = M$.

Proof. I get

$$\frac{\partial M}{\partial x^{\nu}} = \epsilon \frac{\partial m}{\partial x^{\nu}} = 0 \tag{1}$$

from Hypothesis1,Definision8. I rewrite $\frac{\partial M}{\partial \varphi^{\nu}}$ and get

$$\frac{\partial M}{\partial \varphi^{\nu}} = \frac{\partial M}{\partial \varphi x^{\nu}} = \frac{1}{\varphi}\frac{\partial M}{\partial x^{\nu}}. \tag{2}$$

I get

$$\frac{\partial M}{\partial \varphi^{\nu}} = 0 \tag{3}$$

from (1),(2). I get

$$\frac{\partial M}{\partial x^{\mu}} = 0, \frac{\partial M}{\partial \varphi^{\mu}} = 0 \tag{4}$$

as μ, ν-inversion form of (1),(3).

Proposition2. When all coordinate systems satisfies Binary Law, $\frac{\partial^2 M^1}{\partial \dot{x}^1 \partial \dot{x}^1} = -M^1, \frac{\partial^2 M^2}{\partial \dot{x}^1 \partial \dot{x}^1} = -M^2, \dot{x}^1 = \dot{x}^2$ is established for $\frac{\partial^3 M^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} = M : M > 0$ if the number of the dimensions is 2.

Proof. I get

$$\frac{\partial^2 M^{\mu}}{\partial x^{\nu} \partial x^{\nu}} = \mathrm{M} \int \partial x^{\nu} = \mathrm{M} x^{\nu} \tag{5}$$

in consideration of Proposition1 for Definision7. Two next

$$\frac{\partial^2 M^{\mu}}{\partial x^{\nu} \partial x^{\nu}} = -M x^{\mu} = -M^{\mu}, \tag{6}$$

$$\frac{\partial^2 M^{\mu}}{\partial x^{\nu} \partial x^{\nu}} = M x_{\mu} = \frac{M}{x^{\mu}} = \frac{(M)^2}{M^{\mu}} \tag{7}$$

can rewrite (5) each using Definision3,Definision5,$M x^{\mu} = M^{\mu}$. I get (7) as $x_{\mu} = \frac{1}{x^{\mu}}$ here. I get

$$\frac{\partial^2 M^1}{\partial \dot{x}^1 \partial \dot{x}^1} = -M^1, \frac{\partial^2 M^1}{\partial \dot{x}^2 \partial \dot{x}^2} = -M^1,$$
$$\frac{\partial^2 M^2}{\partial \dot{x}^1 \partial \dot{x}^1} = -M^2, \frac{\partial^2 M^2}{\partial \dot{x}^2 \partial \dot{x}^2} = -M^2, \tag{8}$$

$$\frac{\partial^2 M^1}{\partial \dot{x}^1 \partial \dot{x}^1} = \frac{(M)^2}{M^1}, \frac{\partial^2 M^1}{\partial \dot{x}^2 \partial \dot{x}^2} = \frac{(M)^2}{M^1},$$
$$\frac{\partial^2 M^2}{\partial \dot{x}^1 \partial \dot{x}^1} = \frac{(M)^2}{M^2}, \frac{\partial^2 M^2}{\partial \dot{x}^2 \partial \dot{x}^2} = \frac{(M)^2}{M^2} \tag{9}$$

from (6),(7) if I assume a dimensional number 2. I get

$$\frac{\partial^2 M^1}{\partial \dot{x}^1 \partial \dot{x}^1} = \frac{\partial^2 M^1}{\partial \dot{x}^2 \partial \dot{x}^2}, \frac{\partial^2 M^2}{\partial \dot{x}^1 \partial \dot{x}^1} = \frac{\partial^2 M^2}{\partial \dot{x}^2 \partial \dot{x}^2} \tag{10}$$

from (8),(9). I get

$$\frac{\partial^2 M^1}{\partial \dot{x^1}\partial \dot{x^1}} = \frac{\partial^2 M^1}{\partial \dot{x^1}\partial \dot{x^1}} \ (false), \frac{\partial^2 M^2}{\partial \dot{x^1}\partial \dot{x^1}} = \frac{\partial^2 M^2}{\partial \dot{x^1}\partial \dot{x^1}} \ (false) \tag{11}$$

from (10) if I assume establishment of $\dot{x^1} = \dot{x^2} \ (false)$. Because (11) isn't established,

$$\dot{x^1} = \dot{x^2} \tag{12}$$

is established. I get

$$\frac{\partial^2 M^1}{\partial \dot{x^1}\partial \dot{x^1}} = -M^1, \frac{\partial^2 M^2}{\partial \dot{x^1}\partial \dot{x^1}} = -M^2, \tag{13}$$

$$\frac{\partial^2 M^1}{\partial \dot{x^1}\partial \dot{x^1}} = \frac{(M)^2}{M^1}, \frac{\partial^2 M^2}{\partial \dot{x^1}\partial \dot{x^1}} = \frac{(M)^2}{M^2} \tag{14}$$

in consideration of (12) for (8),(9).

Proposition.3 When all coordinate systems satisfies Binary Law, $M^1 = MSin(\dot{x^1}), M^2 = MSin(\dot{x^1})$ is established for $\frac{\partial^3 M^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu} = M : M > 0$ if the number of the dimensions is 2.

Proof. When $M > 0$ is established, I get

$$M^1 = C[1]Cos(\dot{x^1}) + C[2]Sin(\dot{x^1}),$$
$$M^2 = C[1]Cos(\dot{x^1}) + C[2]Sin(\dot{x^1}) \tag{15}$$

as a solution of the equations of (13) in consideration of Definision21. In addition, I do not deal in this article about (14). I get

$$M^1 = C[2]Sin(\dot{x^1}), M^2 = C[2]Sin(\dot{x^1}) \tag{16}$$

as $C[1] = 0$ for (15). I assume that

$$x^1 = Sin(\dot{x^1}) \ (false), x^2 = Sin(\dot{x^1}) \ (false) \tag{17}$$

is established. I rewrite (17) using $Mx^\mu = M^\mu$ and get

$$M^1 = MSin(\dot{x^1}) \ (false), M^2 = MSin(\dot{x^1}) \ (false). \tag{18}$$

I get

$$\frac{d^2 M^1}{d\dot{x^1} d\dot{x^1}} = -MSin(\dot{x^1}) = -M^1 \ (false),$$
$$\frac{d^2 M^2}{d\dot{x^1} d\dot{x^1}} = -MSin(\dot{x^1}) = -M^2 \ (false) \tag{19}$$

from (18). I get the conclusion that

$$x^1 = Sin(\dot{x^1}), x^2 = Sin(\dot{x^1}) \tag{20}$$

is established as (19) is not established from (13). I get

$$M^1 = C[2]x^1, M^2 = C[2]x^2 \tag{21}$$

from (16),(20). I get

$$C[2] = \frac{Mx^1}{x^1} = \frac{Mx^2}{x^2} = M \tag{22}$$

as $Mx^\mu = M^\mu$ in (21). I get

$$M^1 = MSin(\dot{x^1}), M^2 = MSin(\dot{x^1}) \tag{23}$$

from (16),(22). I get

$$M^\mu = MSin(x^\nu) \tag{24}$$

from (23) in consideration of (12). Similarly, I get

$$M^1 = C[1]Cos(\dot{x^1}), M^2 = C[1]Cos(\dot{x^1}) \tag{25}$$

as $C[2] = 0$ for (15). I assume that

$$x^1 = Cos(\dot{x^1})\ (false), x^2 = Cos(\dot{x^1})\ (false) \tag{26}$$

is established. I rewrite (26) using $Mx^{\mu} = M^{\mu}$ and get

$$M^1 = MCos(\dot{x^1})\ (false), M^2 = MCos(\dot{x^1})\ (false). \tag{27}$$

I get

$$\frac{d^2M^1}{d\dot{x^1}d\dot{x^1}} = -MCos(\dot{x^1}) = -M^1\ (false),$$

$$\frac{d^2M^2}{d\dot{x^1}d\dot{x^1}} = -MCos(\dot{x^1}) = -M^2\ (false) \tag{28}$$

from (27). I get the conclusion that

$$x^1 = Cos(\dot{x^1}), x^2 = Cos(\dot{x^1}) \tag{29}$$

is established as (28) is not established from (13). I get

$$M^1 = C[1]x^1, M^2 = C[1]x^2 \tag{30}$$

from (25),(29). I get

$$C[1] = \frac{Mx^1}{x^1} = \frac{Mx^2}{x^2} = M \tag{31}$$

as $Mx^{\mu} = M^{\mu}$ in (30). I get

$$M^1 = MCos(\dot{x^1}), M^2 = MCos(\dot{x^1}) \tag{32}$$

from (25),(31). I get

$$M^{\mu} = MCos(x^{\nu}) \tag{33}$$

from (32) in consideration of (12).

4. Tensor Satisfying BinaryLaw for the Equation Including the Trigonometric Function

Proposition.4 When all coordinate system satisfies Binary Law, $ASin(x^{\nu}) = Sin(Ax^{\nu})$ is established.
Proof. I get

$$x^{\mu} = Sin(x^{\nu}) \tag{34}$$

from (24) in consideration of $Mx^{\mu} = M^{\mu}$. I get

$$A^{\mu} = Sin(A^{\nu}) \tag{35}$$

from (34) as $x^{\mu} \to A^{\mu}, x^{\nu} \to A^{\nu}$ in all coordinate system x^{μ}, x^{ν}. I rewrite (35) using $Ax^{\mu} = A^{\mu}, Ax^{\nu} = A^{\nu}$ and get

$$x^{\mu} = \frac{1}{A} Sin(Ax^{\nu}). \tag{36}$$

I get

$$Sin(x^{\nu}) = \frac{1}{A} Sin(Ax^{\nu}) \tag{37}$$

from (34),(36). I rewrite (37) and get

$$ASin(x^{\nu}) = Sin(Ax^{\nu}). \tag{38}$$

Proposition.5 When all coordinate system satisfies Binary Law, $ACos(x^{\nu}) = Cos(Ax^{\nu})$ is established.
Proof. I get

$$x^{\mu} = Cos(x^{\nu}) \tag{39}$$

from (33) in consideration of $Mx^{\mu} = M^{\mu}$. I get

$$A^{\mu} = Cos(A^{\nu}) \tag{40}$$

from (33) as $x^{\mu} \to A^{\mu}, x^{\nu} \to A^{\nu}$ in all coordinate system x^{μ}, x^{ν}. I rewrite (40) using $Ax^{\mu} = A^{\mu}, Ax^{\nu} = A^{\nu}$ and get

$$x^{\mu} = \frac{1}{A}Cos(Ax^{\nu}). \tag{41}$$

I get

$$Cos(x^{\nu}) = \frac{1}{A}Cos(Ax^{\nu}) \tag{42}$$

from (39),(41). I rewrite (42) and get

$$ACos(x^{\nu}) = Cos(Ax^{\nu}). \tag{43}$$

Proposition.6 When all coordinate system satisfies Binary Law, $ASin(x^{\nu})Cos(x^{\nu}) = Sin(Ax^{\nu})Cos(Ax^{\nu})$ is established.

Proof. I get

$$x^{\mu}x^{\mu} = Sin(x^{\nu})Cos(x^{\nu}) = x^{\mu\mu} \tag{44}$$

as the product of (34) and (39). I rewrite

$$x^{\mu}x^{\mu} = \frac{\partial x^{\mu}x^{\mu}}{\partial x^{\nu}x^{\nu}}x^{\nu}x^{\nu} \tag{45}$$

and get

$$\sqrt{x^{\mu}x^{\mu}} = \frac{\partial\sqrt{x^{\mu}x^{\mu}}}{\partial\sqrt{x^{\nu}x^{\nu}}}\sqrt{x^{\nu}x^{\nu}},$$

$$x^{\mu} = \frac{\partial x^{\mu}}{\partial x^{\nu}}x^{\nu}. \tag{46}$$

As (46) establish from Definision10, (45) establish. I rewrite (45) and get

$$x^{\mu}x^{\mu} = \frac{\partial x^{\mu}}{\partial x^{\nu}}x^{\nu}x^{\nu}. \tag{47}$$

$x^{\mu}x^{\mu}, x^{\nu}x^{\nu}$ is contravariant tensor of rank 1 than (47). Therefore, I get

$$A^{\mu\mu} = Sin(A^{\nu})Cos(A^{\nu}) \tag{48}$$

from (44) as $x^{\mu\mu} \to A^{\mu\mu}, x^{\nu} \to A^{\nu}$ in all coordinate system x^{μ}, x^{ν}. I rewrite (48) using $Ax^{\mu\mu} = A^{\mu\mu}, Ax^{\nu} = A^{\nu}$ and get

$$x^{\mu\mu} = \frac{1}{A}Sin(Ax^{\nu})Cos(Ax^{\nu}). \tag{49}$$

I get

$$Sin(x^{\nu})Cos(x^{\nu}) = \frac{1}{A}Sin(Ax^{\nu})Cos(Ax^{\nu}) \tag{50}$$

from (44),(49). I rewrite (50) and get

$$ASin(x^{\nu})Cos(x^{\nu}) = Sin(Ax^{\nu})Cos(Ax^{\nu}). \tag{51}$$

Proposition.7 When all coordinate system satisfies Binary Law, $ASin^{-1}(x^{\mu}) = Sin^{-1}(Ax^{\mu})$ is established.

Proof. I get

$$x^{\nu} = Sin^{-1}(x^{\mu}) \tag{52}$$

from (34). I get

$$A^{\nu} = Sin^{-1}(A^{\mu}) \tag{53}$$

from (52) as $x^{\mu} \to A^{\mu}, x^{\nu} \to A^{\nu}$ in all coordinate system x^{μ}, x^{ν}. I rewrite (53) using $Ax^{\mu} = A^{\mu}, Ax^{\nu} = A^{\nu}$ and get

$$x^{\nu} = \frac{1}{A}Sin^{-1}(Ax^{\mu}). \tag{54}$$

I get

$$Sin^{-1}(x^{\mu}) = \frac{1}{A} Sin^{-1}(Ax^{\mu}) \tag{55}$$

from (52),(54). I rewrite (55) and get

$$ASin^{-1}(x^{\mu}) = Sin^{-1}(Ax^{\mu}). \tag{56}$$

Proposition.8 When all coordinate system satisfies Binary Law, $SinA^{\nu} + SinB^{\nu} = (SinA^{\nu} + SinB^{\nu})Cos(A - B)x^{\nu}$ is established.

Proof. I get

$$SinA^1 + SinB^1 = 2Sin\frac{(A^1 + B^1)}{2} Cos\frac{(A^1 - B^1)}{2},$$

$$SinA^2 + SinB^2 = 2Sin\frac{(A^2+B^2)}{2} Cos\frac{(A^2-B^2)}{2}, \cdots \tag{57}$$

from Definision13. I get

$$SinA^{\nu} + SinB^{\nu} = 2Sin\frac{(A^{\nu} + B^{\nu})}{2} Cos\frac{(A^{\nu} - B^{\nu})}{2}$$

$$= 2Sin\frac{(A+B)x^{\nu}}{2} Cos\frac{(A-B)x^{\nu}}{2} \tag{58}$$

from (57). I get

$$SinA^{\nu} + SinB^{\nu} = 2Sin\frac{(A + B)x^{\nu}}{2} Cos\frac{(A - B)x^{\nu}}{2}$$

$$= Sin(A + B)x^{\nu} Cos(A - B)x^{\nu} \tag{59}$$

from (58) using (51). I get

$$SinA^{\nu} + SinB^{\nu} = (A + B)Sinx^{\nu} Cos(A - B)x^{\nu}$$

$$= (ASinx^{\nu} + BSinx^{\nu})Cos(A - B)x^{\nu}$$

$$= (SinA^{\nu} + SinB^{\nu})Cos(A - B)x^{\nu} \tag{60}$$

from (59) using (38).

5. Force in the Tensor Satisfying Binary Law

Proposition.9 When all coordinate system satisfies Binary Law, $M' = M\mathrm{Cos}(\varphi^{\nu}), M' = \mathrm{M}\frac{1}{B}\mathrm{Cos}(B\varphi^{\mu}) : \mathrm{B} = \frac{1}{\sqrt{1-(\varphi^{\mu})^2}}$ is established for $\frac{\partial^3 M^{\mu}}{\partial x^{\nu}\, \partial x^{\nu}\, \partial x^{\nu}} = M : M > 0.$

Proof. I get

$$M(x^{\mu} + \varphi^{\mu}) = MSin(x^{\nu} + \varphi^{\nu}) \tag{61}$$

from (24) using $\mathrm{M}x^{\mu} = M^{\mu}$ as $x^{\mu} \to (x^{\mu} + \varphi^{\mu}), x^{\nu} \to (x^{\nu} + \varphi^{\nu})$. I add (24) to (61) and get

$$M(x^{\mu} + \varphi^{\mu}) + Mx^{\mu} = MSin(x^{\nu} + \varphi^{\nu}) + MSin(x^{\nu}). \tag{62}$$

I rewrite right-hand side of (62) in consideration of (60) and get

$$M\big(Sin(x^{\nu} + \varphi^{\nu}) + Sin(x^{\nu})\big)$$

$$\mathrm{M}\big(Sin(x^{\nu} + \varphi^{\nu}) + Sin(x^{\nu})\big)\mathrm{Cos}(\varphi^{\nu}),$$

$$\mathrm{M} = \mathrm{MCos}(\varphi^{\nu}). \tag{63}$$

I express (63) in

$$M' = M\mathrm{Cos}(\varphi^{\nu}) \tag{64}$$

to distinguish M of the both sides of (63). I rewrite left-hand side of (62) in consideration of (60) and get

$$M(x^{\mu}+\varphi^{\mu})+Mx^{\mu}=M\left(SinSin^{-1}(x^{\mu}+\varphi^{\mu})+SinSin^{-1}(x^{\mu})\right)$$
$$=M\left(SinSin^{-1}(x^{\mu}+\varphi^{\mu})+SinSin^{-1}(x^{\mu})\right)Cos\left(Sin^{-1}(x^{\mu}+\varphi^{\mu})-Sin^{-1}(x^{\mu})\right),$$
$$M=MCos\left(Sin^{-1}(x^{\mu}+\varphi^{\mu})-Sin^{-1}(x^{\mu})\right). \tag{65}$$

I get

$$M=MCos\left(Sin^{-1}(1+\varphi)x^{\mu}-Sin^{-1}(x^{\mu})\right)$$
$$=MCos\left((1+\varphi)Sin^{-1}(x^{\mu})-Sin^{-1}(x^{\mu})\right)$$
$$=MCos\left(Sin^{-1}(x^{\mu})+\varphi Sin^{-1}(x^{\mu})-Sin^{-1}(x^{\mu})\right)$$
$$\mathrm{M}=\mathrm{MCos}\left(\mathrm{Sin}^{-1}(\varphi x^{\mu})\right)=\mathrm{MCos}\left(\mathrm{Sin}^{-1}(\varphi^{\mu})\right) \tag{66}$$

using (56) from (65). I express (66) in

$$M^{'}=\mathrm{MCos}\left(\mathrm{Sin}^{-1}(\varphi^{\mu})\right) \tag{67}$$

to distinguish M of the both sides of (66). I get

$$\frac{\partial Sin^{-1}(\varphi^{\mu})}{\partial\varphi^{\mu}}=\frac{1}{\sqrt{1-(\varphi^{\mu})^{2}}} \tag{68}$$

as $A=1, X\rightarrow\varphi^{\mu}$ for Definision17. I get

$$\frac{\partial Sin^{-1}(\varphi^{\mu})}{\partial\varphi^{\mu}}=\frac{1}{\sqrt{1-(\varphi^{\mu})^{2}}}=\frac{1}{\sqrt{1-\varphi^{\mu}\cdot\varphi^{\mu}}}=Scalar \tag{69}$$

from (68) in consideration of Definision14. I decide to express (69) in

$$\frac{\partial Sin^{-1}(\varphi^{\mu})}{\partial\varphi^{\mu}}=\frac{1}{\sqrt{1-(\varphi^{\mu})^{2}}}=B. \tag{70}$$

I get

$$Sin^{-1}(\varphi^{\mu})=B\int\partial\varphi^{\mu}=B\varphi^{\mu} \tag{71}$$

from (70). I get

$$M'=MCos(B\varphi^{\mu}) \tag{72}$$

from (67),(71). I get

$$M^{'}=M\mathrm{Cos}(\varphi^{\mu}) \tag{73}$$

as μ,ν-inversion form of (64). I get

$$MCos(\varphi^{\mu})=MCos(B\varphi^{\mu}) \tag{74}$$

from (72),(73). The establishment of (74) is impossible here. Therefore, I rewrite (72) as $MCos(B\varphi^{\mu})\rightarrow M\frac{1}{B}Cos(B\varphi^{\mu})$ in consideration of (42) and get

$$M'=M\frac{1}{B}Cos(B\varphi^{\mu}). \tag{75}$$

Proposition.10 When all coordinate system satisfies Binary Law, $F_{\nu}=-\frac{\partial M}{\partial x^{\nu}}$ is established.

Proof.

$$\vec{F}=\overrightarrow{e^{\nu}}F_{\nu}, d\vec{r}=\overrightarrow{e_{\nu}}dx^{\nu} \tag{76}$$

is established. I rewrite Definision18 using (76) and get

$$\mathrm{U}=-\int\overrightarrow{\mathrm{e}^{\nu}}\mathrm{F}_{\nu}\cdot\overrightarrow{\mathrm{e}_{\nu}}\mathrm{d}x^{\nu}=-\int\left(\overrightarrow{e^{\nu}}\cdot\overrightarrow{e_{\nu}}\right)F_{\nu}\,dx^{\nu}=-\int F_{\nu}dx^{\nu}. \tag{77}$$

I get (77) as $\overrightarrow{e^{\nu}}\cdot\overrightarrow{e_{\nu}}=1$ here. I get

$$F_{\nu}=-\frac{\partial U}{\partial x^{\nu}} \tag{78}$$

from (77). I get

$$U=\frac{c^{2}}{\epsilon}M \tag{79}$$

as E→U from Definision19,Hypothesis1. I get

$$U = M \tag{80}$$

as $\frac{c^2}{\epsilon} = 1$ for (79). In addition, I rewrite Hypothesis1 using $\frac{c^2}{\epsilon} = 1$ and get

$$M = mc^2. \tag{81}$$

I get

$$F_\nu = -\frac{\partial M}{\partial x^\nu} \tag{82}$$

from (78), (80).

Proposition.11 When all coordinate system satisfies Binary Law, $F_\nu{}' = MSin - (\varphi^\nu), F_\mu{}' = -M\frac{1}{B}Sin(B\varphi^\mu): B = \frac{1}{\sqrt{1-(\varphi^\mu)^2}}$ is established for $\frac{\partial^3 M^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu} = M: M > 0$.

Proof. I get

$$F_\nu = -\frac{\partial M}{\partial \varphi^\nu} \tag{83}$$

as $x^\nu \to \varphi x^\nu = \varphi^\nu$ from (82). I get

$$F_\nu{}' = -\frac{\partial M'}{\partial \varphi^\nu} = -\frac{\partial MCos(\varphi^\nu)}{\partial \varphi^\nu}$$

$$= -\frac{\partial M}{\partial \varphi^\nu}Cos(\varphi^\nu) - \frac{M\,\partial Cos(\varphi^\nu)}{\partial \varphi^\nu} = MSin(\varphi^\nu). \tag{84}$$

from (64),(83),Proposition1. If $F_\nu{}' = 0$ is established,

$$\varphi^\nu = \pi n \tag{85}$$

is established in consideration of (84),Definision16. I get

$$\dot{F_1}' = \mathrm{MSin}(\dot{\varphi}^1), \tag{86}$$

$$\dot{\varphi}^1 = \pi n \tag{87}$$

from (84),(85) if I assume dimensionality 1. I show figure of (86) in Figure 1.

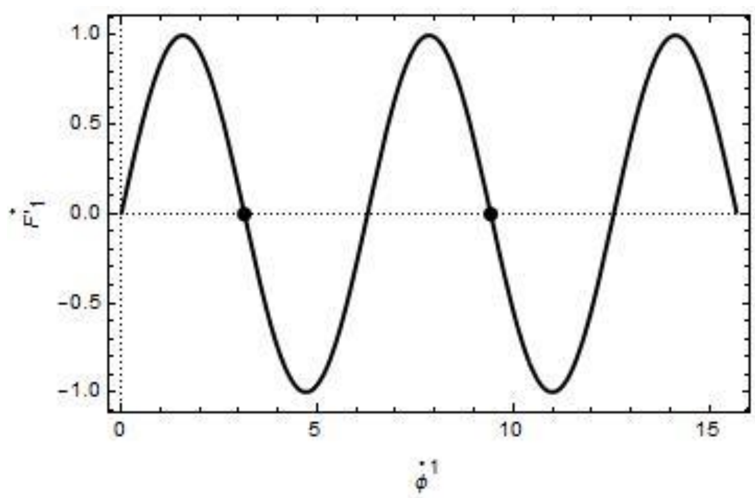

Figure 1. Plot for variable $\dot{\varphi}^1$ of $\dot{F_1}' = MSin(\dot{\varphi}^1): (M = 1)$. The black dot expresses $\dot{\varphi}^1$ satisfying $\dot{F_1}' = 0$, and this is negative divergence point in the field of force more.

The value of $\dot{\varphi}^1$ satisfying $\dot{F_1}' = 0$ exists innumerably according to (87). I show this in

$$\dot{\varphi}^1 = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, \cdots. \tag{88}$$

In Fig.1, Positive divergence point in the field of force exists in (88). I show this in

$$\dot{\varphi}^1 = 0, 2\pi, 4\pi, \cdots. \tag{89}$$

Negative divergence point in the field of force exists in (88). I show this in

$$\dot{\varphi}^1 = \pi, 3\pi, 5\pi, \cdots. \tag{90}$$

I think about wave moving in circle length of radius r. Circle length of radius r as $2\pi r$. Wave length of the wave as λ. The phase length of the wave as φ.

$$2\pi r - n\lambda = \varphi \tag{91}$$

is established here. When $\varphi \neq 0$ is established, φ of the wave is only out of φ every time for each $2\pi r$. As overlap between the wave which φ is different each occurs, I get

$$Sin(x) + Sin(x + \varphi). \tag{92}$$

(92) accords with right-hand side of (62). As λ is constant, I get

$$\varphi \propto r \tag{93}$$

from (91). I also get

$$\dot{\varphi}^1 = 2\pi \dot{r}^1 - n\lambda \tag{94}$$

by consideration of (93) as $\varphi \to \dot{\varphi}^1$ for (91). I get

$$\dot{F_1}' = MSin(2\pi \dot{r}^1 - n\lambda) = MSin(2\pi \dot{r}^1) \tag{95}$$

from (86) as (94). I show figure of (95) in Figure 2.

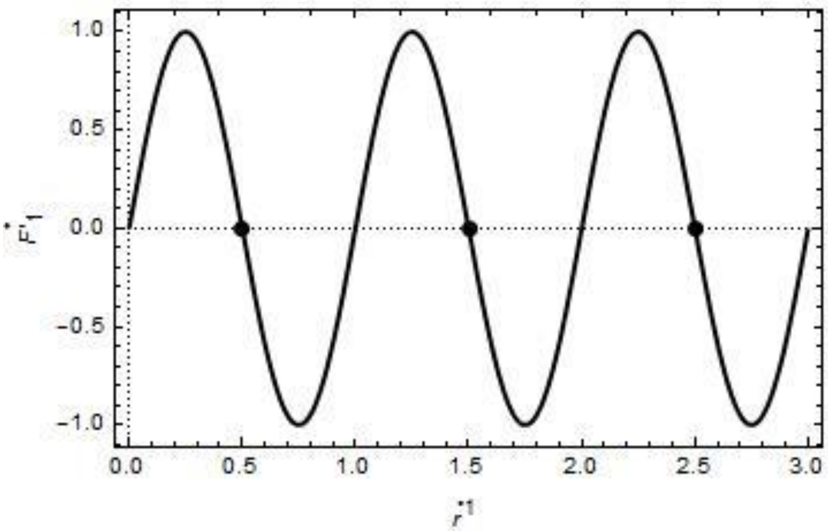

Figure 2. Plot for variable $\dot{r}^1$ of $\dot{F_1}' = MSin(2\pi \dot{r}^1)$: $(M = 1)$. The black dot expresses $\dot{r}^1$ satisfying $\dot{F_1}' = 0$, and this is negative divergence point in the field of force more.

When binary particle with the opposite charge is located each as distance $\dot{r}^1$. Force $\dot{F_1}'$ which one particle receives is obtained as

$$\dot{F_1}' = -\frac{k}{(\dot{r}^1)^2} \tag{96}$$

in consideration of Definision20. I show figure of (96) in Figure 3.

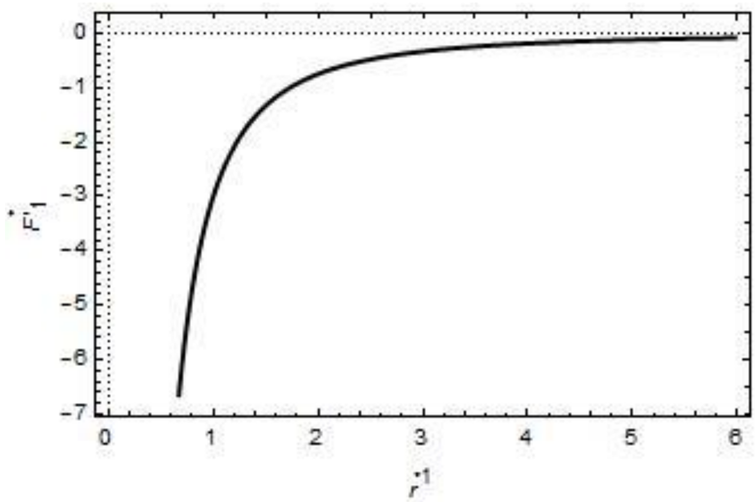

Figure 3. Plot for variable $\dot{r^1}$ of $\dot{F_1}' = -\frac{k}{(\dot{r^1})^2}:(k = 3)$

I add (96) to (95) and get

$$\dot{F_1}' = MSin(2\pi\dot{r^1}) - \frac{k}{(\dot{r^1})^2}. \tag{97}$$

I show figure of (97) in Figure 4.

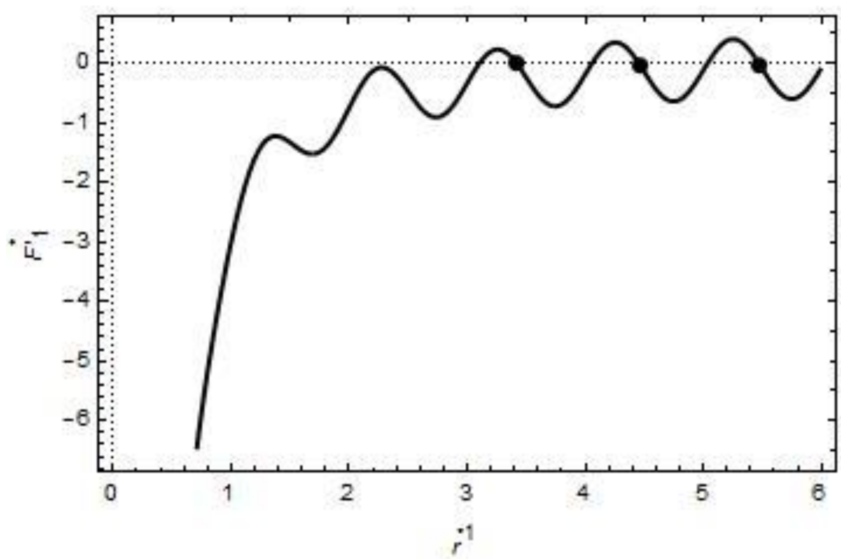

Figure 4. Plot for variable $\dot{r^1}$ of $\dot{F_1}' = MSin(2\pi\dot{r^1}) - \frac{k}{(\dot{r^1})^2}:(M = 0.511, k = 3)$. The black dot expresses $\dot{r^1}$ satisfying $\dot{F_1}' = 0$, and this is negative divergence point in the field of force more.

I get

$$F_\mu = -\frac{\partial M}{\partial \varphi^\mu} \tag{98}$$

than μ, ν-inversion form of (83). I get

$$F_{\mu}{}' = -\frac{\partial M'}{\partial \varphi^{\mu}} = -\frac{\partial M\frac{1}{B}Cos(B\varphi^{\mu})}{\partial \varphi^{\mu}} \tag{99}$$

from (75),(98). I rewrite (99) in consideration of μ,ν-inversion form of (42) and get

$$F_{\mu}{}' = -\frac{\partial M\frac{1}{B}Cos(B\varphi^{\mu})}{\partial \varphi^{\mu}} = -\frac{\partial MCos(\varphi^{\mu})}{\partial \varphi^{\mu}}. \tag{100}$$

I get

$$F_{\mu}{}' = -\frac{\partial MCos(\varphi^{\mu})}{\partial \varphi^{\mu}}$$

$$= -\frac{\partial M}{\partial \varphi^{\mu}}Cos(\varphi^{\mu}) - \frac{M\,\partial Cos(\varphi^{\mu})}{\partial \varphi^{\mu}} = \mathrm{MSin}(\varphi^{\mu}). \tag{101}$$

from (98),(100), Proposition1. I rewrite (101) in consideration of μ,ν-inversion form of (37) and get

$$F_{\mu}{}' = M\frac{1}{B}Sin(B\varphi^{\mu}). \tag{102}$$

If $F_{\mu}{}' = 0$ is established,

$$B\varphi^{\mu} = n\pi \tag{103}$$

is established in consideration of (102),Definision16. I get

$$F_{1}{}' = M\sqrt{1-(\varphi^{1})^{2}}Sin\left(\frac{\varphi^{1}}{\sqrt{1-(\varphi^{1})^{2}}}\right), \tag{104}$$

$$\frac{(\varphi^{1})^{2}}{1-(\varphi^{1})^{2}} = n\pi, \varphi^{1} = \sqrt{\frac{n\pi}{1+n\pi}} \tag{105}$$

from (70),(102),(103) if I assume dimensionality 1. I show figure of (104) in Figure 5, Figure 6, Figure 7.

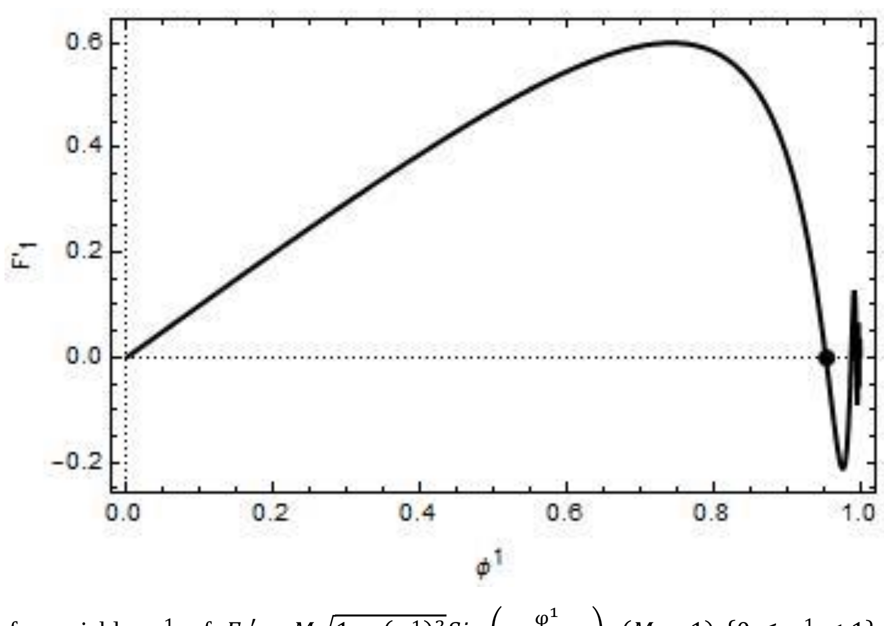

Figure 5. Plot for variable φ^{1} of $F_{1}{}' = M\sqrt{1-(\varphi^{1})^{2}}Sin\left(\frac{\varphi^{1}}{\sqrt{1-(\varphi^{1})^{2}}}\right):(M=1),\{0 \le \varphi^{1} < 1\}$. The black dot expresses φ^{1} satisfying $F_{1}{}' = 0$, and this is negative divergence point in the field of force more.

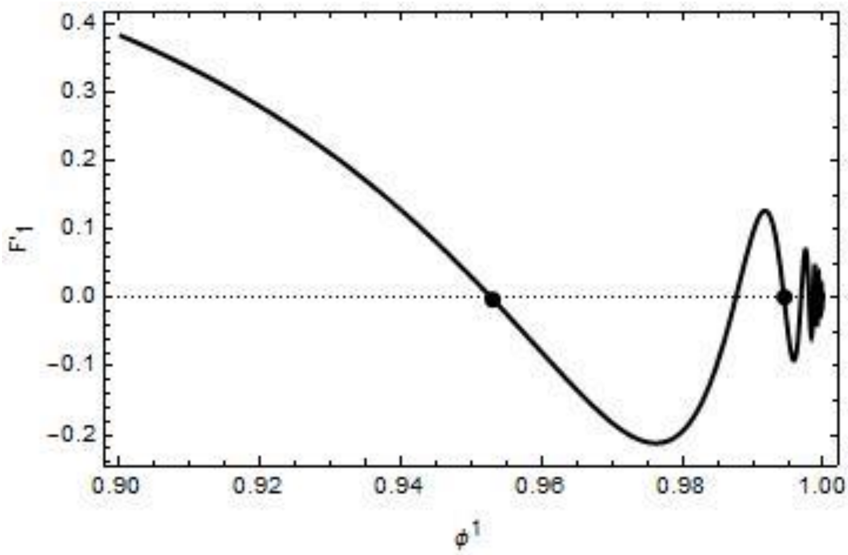

Figure 6. Plot for variable φ^1 of $F_1{}' = M\sqrt{1-(\varphi^1)^2}Sin\left(\frac{\varphi^1}{\sqrt{1-(\varphi^1)^2}}\right) : (M = 1), \{0.9 \leq \varphi^1 < 1\}$. The black dot expresses φ^1 satisfying $F_1{}' = 0$, and this is negative divergence point in the field of force more.

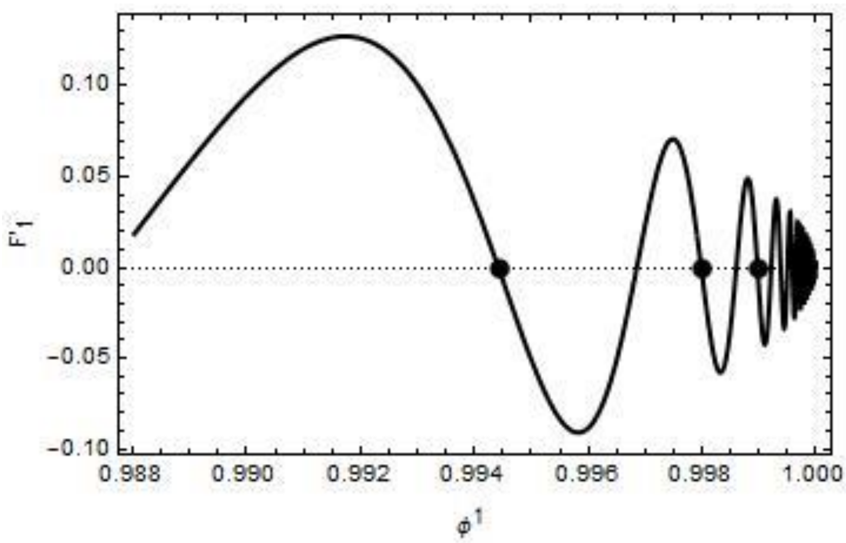

Figure 7. Plot for variable φ^1 of $F_1{}' = M\sqrt{1-(\varphi^1)^2}Sin\left(\frac{\varphi^1}{\sqrt{1-(\varphi^1)^2}}\right) : (M = 1), \{0.988 \leq \varphi^1 < 1\}$. The black dot expresses φ^1 satisfying $F_1{}' = 0$, and this is negative divergence point in the field of force more.

The value of φ^1 satisfying $F_1{}' = 0$ exists innumerably according to (105). I show this in

$$\varphi^1 = 0, \sqrt{\frac{\pi}{1+\pi}}, \sqrt{\frac{2\pi}{1+2\pi}}, \sqrt{\frac{3\pi}{1+3\pi}}, \sqrt{\frac{4\pi}{1+4\pi}}, \sqrt{\frac{5\pi}{1+5\pi}}, \cdots. \tag{106}$$

In Figure 5, Figure 6, Figure 7, Positive divergence point in the field of force exists in (106). I show this in

$$\varphi^1 = 0, \sqrt{\frac{2\pi}{1+2\pi}}, \sqrt{\frac{4\pi}{1+4\pi}}, \cdots. \tag{107}$$

Negative divergence point in the field of force exists in (106). I show this in

$$\varphi^1 = \sqrt{\frac{\pi}{1+\pi}}, \sqrt{\frac{3\pi}{1+3\pi}}, \sqrt{\frac{5\pi}{1+5\pi}}, \cdots. \tag{108}$$

When binary particle with the same charge is located each as distance φ^1. Force $F_1{}'$ which one particle receives is obtained as

$$F_1{}' = \frac{k}{(\varphi^1)^2} \tag{109}$$

in consideration of Definision20. I show figure of (109) in Figure 8.

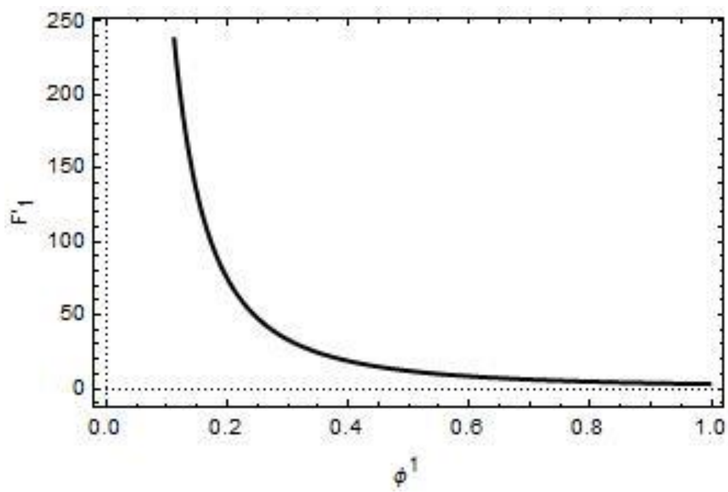

Figure 8. Plot for variable φ^1 of $F_1{}' = -\frac{k}{(\varphi^1)^2}: (k = 3), \{0 < \varphi^1 < 1\}$

I add (109) to (104) and get

$$F_1{}' = M\sqrt{1-(\varphi^1)^2}Sin\left(\frac{\varphi^1}{\sqrt{1-(\varphi^1)^2}}\right) + \frac{k}{(\varphi^1)^2}. \tag{110}$$

I show figure of (110) in Figure 9, Figure 10.

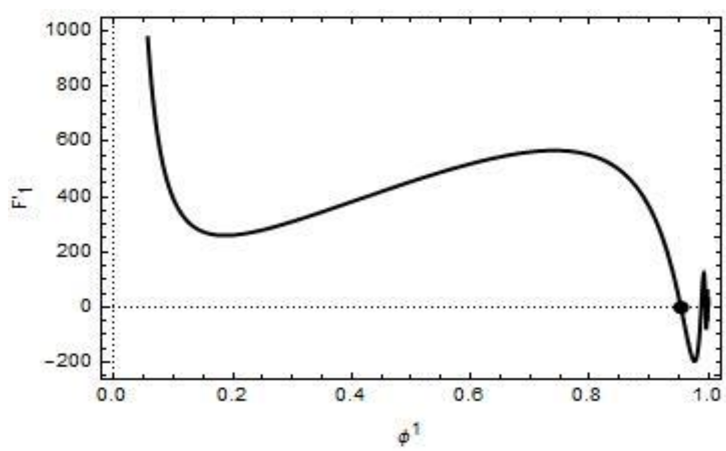

Figure 9. Plot for variable φ^1 of $F_1{}' = M\sqrt{1-(\varphi^1)^2}Sin\left(\frac{\varphi^1}{\sqrt{1-(\varphi^1)^2}}\right) + \frac{k}{(\varphi^1)^2}: (M = 938.3, k = 3), \{0 < \varphi^1 < 1\}$. The black dot expresses φ^1 satisfying $F_1{}' = 0$, and this is negative divergence point in the field of force more.

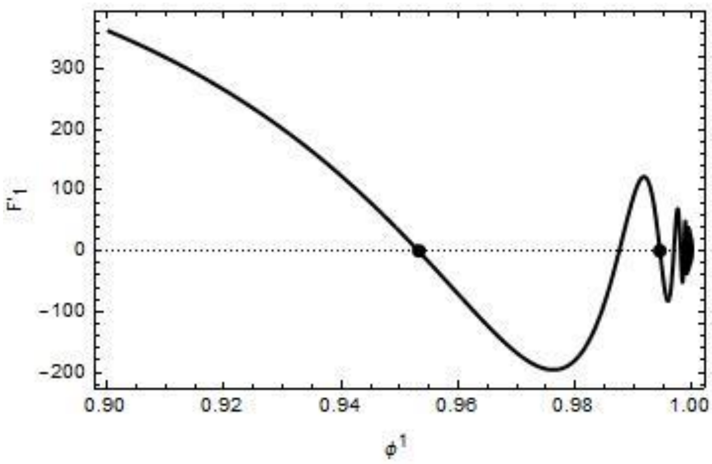

Figure 10. Plot for variable φ^1 of $F_1{}' = M\sqrt{1-(\varphi^1)^2}Sin\left(\frac{\varphi^1}{\sqrt{1-(\varphi^1)^2}}\right) + \frac{k}{(\varphi^1)^2} : (M = 938.3, k = 3), \{0.9 \leq \varphi^1 < 1\}$. The black dot expresses φ^1 satisfying $F_1{}' = 0$, and this is negative divergence point in the field of force more.

6. Property of the Tensor Satisfying Binary Law 4

I was not able to report Proposition12 in Property of the tensor satisfying Binary Law 4(Ichidayama, 2023). Thus, I report Proposition12 in this article.

Proposition12 The index which is free index remains free index when Binary Law is satisfied

Proof. I put a mark in the index which is free index of Definision9 and get

$$M_{\bar{\mu};\bar{\nu}} = \frac{\partial M_{\bar{\mu}}}{\partial x^{\bar{\nu}}} - M_\tau \frac{1}{2} g^{\epsilon\tau}\left(\frac{\partial g_{\bar{\mu}\epsilon}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\bar{\mu}}} - \frac{\partial g_{\bar{\mu}\bar{\nu}}}{\partial x^{\epsilon}}\right). \tag{111}$$

If all coordinate system satisfies Binary Law for (111), I get

$$M_{\bar{\mu};\bar{\nu}} = \frac{\partial M_{\bar{\mu}}}{\partial x^{\bar{\nu}}} - M_\nu \frac{1}{2} g^{\nu\nu}\left(\frac{\partial g_{\bar{\mu}\nu}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\mu}}} - \frac{\partial g_{\bar{\mu}\bar{\nu}}}{\partial x^{\nu}}\right). \tag{112}$$

Index with the mark of the first terms of the right-hand side of (112) is free index. Thus, I get the conclusion that index with the mark of the second terms of the right-hand side of (112) is free index. In other words, "The index which is free index remains free index when Binary Law is satisfied" establish.

7. Discussion

About Figure 4

When binary particle with the opposite charge is located each as distance $\dot{r}^1$. It is decided that I assume binary particle with atomic nucleus and electron each. The domain of $\dot{r}^1$ is $\{0 < \dot{r}^1 \leq \infty\}$ in (97). In other words, the field of force exists to infinity. The black dot expresses $\dot{r}^1$ satisfying $\dot{F_1}{}' = 0$, and this is negative divergence point in the field of force more. And these exist innumerably. Only four points are displayed in Fig.4 here. The electron satisfies $\dot{F_1}{}' = 0$ in $\dot{r}^1$ of the black dot, and the momentum does not change. When the electron satisfies $\dot{F_1}{}' \neq 0$ any place other than $\dot{r}^1$ of the black dot, the momentum changes. If electron moves from black dot point only to $\pm d\dot{r}^1$, the direction of force $\dot{F_1}{}' \neq 0$ which electron receives intends to be pulled back to a black dot. It is thought that the electron continues vibrating in the range of $\pm d\dot{r}^1$ for $\dot{r}^1$ of the black dot. As the electron has charge, an electromagnetic wave is caused by vibration. And the electron will stay in $\dot{r}^1$ of the black dot because energy dissipation occurs.

When it was only (97), the force to act on electron in atom was recognized. I report (95) as force to act on electron in atom in this article other than (97). When electron stays in steady orbit in atom, the force that electron receives must be zero. Force (97) is denied by existence of force (95). Thus, the force that electron receives can become the zero. In other words, it is interpretability that electron stays in steady orbit in atom.

About Figure 9, Figure10

When binary particle with the equal charge is located each as distance φ^1. It is decided that I suppose binary particle to be proton each. The domain of φ^1 is $\{0 < \varphi^1 \leq \infty\}$ in (109). In other words, the field of force exists to infinity. In contrast, The domain of φ^1 is $\{0 \leq \varphi^1 < 1\}$ in (104). In other words, the outreach of the field of force is limited. The black dot expresses φ^1 satisfying $F_1{}' = 0$, and this is negative divergence point in the field of force more. And these exist innumerably. Only one points are displayed in Fig.9 here. The proton satisfies $F_1{}' = 0$ in φ^1 of the black dot, and the momentum does not change. When the proton satisfies $F_1{}' \neq 0$ any place other than φ^1 of the black dot, the momentum changes. If proton moves from black dot point only to $\pm d\varphi^1$, the direction of force $F_1{}' \neq 0$ which proton receives intends to be pulled back to a black dot. It is thought that the proton continues vibrating in the range of $\pm d\varphi^1$ for φ^1 of the black dot. As the proton has charge, an electromagnetic wave is caused by vibration. And the proton will stay in φ^1 of the black dot because energy dissipation occurs.

Competing interests

Any competition does not happen between others for the publication of this article.

Informed consent

Obtained.

Ethics approval

The Publication Ethics Committee of the Canadian Center of Science and Education.

The journal and publisher adhere to the Core Practices established by the Committee on Publication Ethics (COPE).

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Not commissioned; externally double-blind peer reviewed.

Data availability statement

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

Data sharing statement

No additional data are available.

Open access

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