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Single Parameter Matching of Ordinary and Novel (Dark Matter) Particles From Their Physical Equalities to Their Physical Differences

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Abstract

Novel (dark matter) particles, while known to exist, refuse to show up explicitly. Theoretical approaches within the Standard Model (SM) as for example, looking for the dark photon with Feynman diagrams, in the process $\gamma\gamma \longrightarrow e^+e^-$, is still inconclusive (Xu, I. et al., 2022). However, empirical-like methods can give the proof about the existence of dark matter, see for instance (Clowe, D. et al., 2006). Hence it is reasonable trying to understand as to why ordinary and novel (dm) particles differ so much from each other. This we wish to do with solutions of the bicubic equation for particle limiting velocities (Šoln, J., 2014-2022). Once we have the solutions for novel and ordinary particle limiting velocities from (Šoln, J. 2021.1.2, 2022), we first establish, with the help of evolutionary congruent parameters, ordinary z_1 and novel z_2, satisfying $z_1 \preceq 1$and $z_2 \succeq 1$,the smooth matching point of equal values for ordinary and novel particles at $z_1 = z_2 = 1$. At this point the limiting velocities and other physical quantities of ordinary and novel particles have equal values, which can be also characterized by $z_1\times z_2 = 1$; this, consistent with Discriminants of ordinary and novel limiting velocity solutions, is extended everywhere, so that $z_2 = 1/ z_1$. the novel particle limiting velocity solutions reveal congruent angle α, contained now in z_1 and z_2, and as such can also serve as another evolutionary parameter. The smooth matching point is now $\alpha = \pi/2$. If physically equivalent ordinary and novel particles move away from this point to $\alpha \neq \pi/2$, they will physically be different from each other. In other words, the novel particle is in $z_2 \succeq 1$ territory, and the ordinary particle is in $z_1 \preceq 1$ territory and direct interactions are likely impossible. With this formalism, we investigate physical differences between ordinary and novel particles, when moving away from $\alpha = \pi/2$. In tis article, we largely are dealing with high energy leptons together with relevant photons with congruent parameter ranges of $0 \prec \alpha \preceq \pi/2$, $0 \prec z_1 \preceq 1, \infty \succ z_2 \succeq 1$. In fact due to a large interest in photons, here, within this formalism, we evaluate very precisely limiting velocities for the ordinary and novel photons. From these evaluations, we deduce numerically that congruent angles of novel and ordinary photons are related through the quantum jump $\alpha(\gamma_N) = 2\alpha(\gamma)$, which is verified also for other particles. Hence, the general quantum jump between congruent angles of limiting velocities associated with ordinary and novel particles is $\alpha(x_N) = \alpha(x)$, where $x = \gamma, e, \nu$, etc. The congruent angle quantum jump connects every ordinary particle, such as electron e, or neutrino ν,respectively, to novel electron e_N and novel neutrino ν_N. This, definitively is a rather simple way to identify novel particles. All that one needs is to find them.

Keywords: limiting velocity, energy, congruent parameter, novel, dark matter, particle, quantum jump

1. Introduction

The Standard Model (SM) is based on four stable particles, photon, electron, neutrino and proton. The attempts to incorporate novel (dm) particles into the SM usually yield nebulous results. Even for ingenious proposals like this one to look for dark photon with Feynman diagrams in the reaction $\gamma\gamma \rightarrow e^+e^-$ (Xu, I. et al., 2022), one does not know what results to expect. As long as one stays just with the novel (dm) particles, the gravitational studies and empirical like methods can give the proof of the existence of dark matter (Clowe, D. et al., 2006; Battaglieri, M. et al., 2017; Fillippi, A. et al., 2022). Hence it is reasonable trying to understand as to why ordinary and novel (dm) particles differ so much from each other. Namely, when applying the SM to the dark matter, people find that ordinary particles are restricted to interact with dark matter, as for example, by exclusion limits for dark matter (Romanenko, A. et al., 2023), or constraints on Sub-GeV Dark matter (Arnquist, I. et al., 2023), and kinetic decoupling from dark matter (LIu. Y, et al., 2023). There seem to be particular interest in novel (dm) particles that can be seen as replicas of ordinary articles. Good examples are the Search for Light Dark Photon (Yeong, G-K., et al., 2013) and Tracking Down the Origin of Neutrino Mass (Gehrlein, J., et al., 2018).

Here we wish to establish through solutions of the bicubic equation for particle limiting velocities (Šoln, J., 2014-2022) the relationship between ordinary and novel (dm) particles. In this pursuit,the most useful solutions, however, are the ones from (Šoln, J., 2021.1.2, 2022). which here will be somewhat notational modified. We continue by first, with the help of evolutionary congruent parameters ordinary z_1 and novel z_2 , satisfying $z_1 \preceq 1$ and $z_2 \succeq 1$,to establish he smooth matching point at $z_1 = z_2 = 1$, where the the ordinary and novel particles have equal values in limiting velocities,energies and the like. The smooth matching point also satisfy $z_1.z_2 = 1$, which we extend to everywhere: $z_1.z_2 = 1 \rightarrow z_2 = 1/z_1$ and which satisfy the discriminant D requirements for ordinary and novel particle limiting velocity solutions. With this, it appears that only one is needed to keep evolutionary track between the ordinary and novel particles. However, one may still get entangled between z_1 and z_2. However, in the novel particle limiting velocity solutions, the new congruent angle α reveals itself, contained both in z_1 and z_2. Now the smooth matching point is simply $\alpha = \pi/2$.The range of α is: $(0...\pi/2)$. Moving α away from $\pi/2$, one can see how the properties of ordinary and novel (dm) particle physical quantities change from each other. Particularly interesting is the possibility that to one ordinary particle there correspond double novel (dark), particles.

The bicubic equation for particle limiting velocities was derived by upgrading the usual relativistic kinematics in (Šoln, J., 2014). Here, in order to be specific, we need next to each other the solutions of the bicubic equation for ordinary and novel particle limiting velocities, denoted for now simply by c:

$$\left(\frac{c^2}{v^2}\right)^3 - \left(\frac{E}{mv^2}\right)^2 \left(\frac{c^2}{v^2}\right) + \left(\frac{E}{mv^2}\right)^2 = 0 \tag{1}$$

where c, E, m, and v^2, are respectively, limiting velocity, particle energy, real particle mass and particle velocity squared. First, by relating the energy E to the real evolutionary congruent parameter z,one can go directly to the solutions of equation (1) as the discriminant for (1) assumes simpler form (Šoln, J., 2021.1.2-2022):

$$\begin{aligned} E &= \frac{3\sqrt{3}mv^2}{2z}, z = \frac{3\sqrt{3}mv^2}{2E}; \\ D &= \frac{1}{4}\left(\frac{E}{mv^2}\right)^4 \left[1 - \frac{4}{27}\left(\frac{E}{mv^2}\right)^2\right] = \left(\frac{27}{8z^3}\right)^2 (z+1)(z-1) \end{aligned} \tag{2}$$

At this point we designate congruent parameters with kind of particle limiting velocity solutions from (1):

$$\begin{aligned} Ordinary\; particles &: D \preceq 0, z_1 \preceq 1, E = \frac{3\sqrt{3}mv^2}{2z_1} \\ Novel(dm)\; particles &: D \succeq 0, z_2 \succeq 1, E = \frac{3\sqrt{3}mv^2}{2z_2} \end{aligned} \tag{3}$$

where from now on, ordinary and novel (dm) have each its own congruent parameter, respectively, z_1 and z_2 We immediately see that when $z_1 = z_2 = 1$ $D = 0$ and both limiting velocity solutions for ordinary and novel particles shrink to the same limiting velocity solutions. That is why we call the point $z_1 = z_2$ the smooth matching point with velocity solutions smoothly becoming equal. By looking at D^2 one sees that the smooth point can also be characterized by $z_1 z_2 = 1$. However, in order to maintain $z_1 \preceq 1$ and $z_2 \succeq 1$ we extend for $D \leq, \geq 0, : z_1.z_2 = 1$ to everywhere:

$$Smooth\; matching\; point : D = 0, z_1 z_2 = 1; Evertwhere : D \leq, \geq 0, z_1 z_2 = 1, z_2 = \frac{1}{z_1} \tag{4}$$

As already indicated, the congruent angle α directly follows from the limiting velocity solutions for novel (dm) particles. Besides, as compared to (Šoln, J., 2019-2022), here we slightly change enumeration of solutions, to make easier their comparisons. To further facilitate this comparisons, we first give mathematical preliminary. In Section 2 we give, simple and illustrative examples of dealing with limiting velocities for ordinary and novel particles. They will be very useful when in Section 3 we start applying this formalism to evaluations of limiting velocities of ordinary and novel massive photons. Our formalism is unconformable with massless photon which theoretical requires the congruent angle $\alpha(\gamma) = 0$. Besides, both (Itzykson, C. and Zuber, J.-B., 1980 on page 138) and (Lin, H.-L. et al., 2023) point out that from localized fast radio bursts, the mass of the ordinary photon should be $m(\gamma) \simeq 6 \times 10^{-48} g \simeq 3 \times 10^{-15} eV$, so that our ordinary photon, with small mass will have congruent angle $\alpha(\gamma) \succ 0$ which also makes $0 \prec z_1 \preceq 1$ so that energy E in (3) is finite and not infinite.

2. Next to Each Other the Bicubic Equation Limited Velocity Solutions for Ordinary and Novel (Dark Matter) Particles

We start with a mathematical preliminary, most of which can be found in (Šoln, J., 2021.1.2, 2022). Namely, the congruent angle α follows from the novel particle limiting velocity solutions, and here, consistent with (4), we wish to indicate that the economical way to write the limiting velocity solutions for both ordinary and novel particles is to emphasize congruent angle α as an evolutionary parameter:

$$\alpha = 2\tan^{-1}\left(\tan\left(\frac{1}{2}\sin^{-1}\frac{1}{z_2}\right)\right)^{\frac{1}{3}} = 2\tan^{-1}\left(\tan\left(\frac{1}{2}\sin^{-1}z_1\right)\right)^{\frac{1}{3}}, \quad (5.1)$$

$$z_1(\alpha) = \frac{1}{z_2(\alpha)} = \sin\left[2\tan^{-1}\left(\tan(\frac{\alpha}{2}\right)^3\right], \quad (5.2)$$

$$z_1(\alpha) = \frac{1}{z_2(\alpha)} = \frac{\sin^3(\alpha)}{4-3\sin^2(\alpha)}. \quad (5.3)$$

The congruent angle α can be transformed to another angle and the most useful transformation is $\alpha \to -\alpha$ This transformation, in view of (5.3) can change one limiting velocity solution into another, as we shall see shortly.

Next we first write down, the limiting velocity solutions for ordinary particles, both in the usual form (Šoln, J., 2021.1.2,2022) an in the form emphasizing the congruent angle α :

$$\frac{c_1^2(\alpha)}{v^2} = \frac{3}{z_1(\alpha)}\sin\left[\frac{1}{3}(\pi-\sin^{-1}z_1(\alpha)\right] \succ 0, \quad (6.1)$$

$$\frac{c_2^2(\alpha)}{v^2} = -\frac{3}{z_1(\alpha)}\sin\left[\frac{1}{3}(\pi+\sin^{-1}z_1(\alpha)\right] \prec 0, \quad (6.2)$$

$$\frac{c_3^2(\alpha)}{v^2} = \frac{3}{z_1(\alpha)}\sin\left[\frac{1}{3}\sin^{-1}z_1(\alpha)\right] \succ 0. \quad (6.3)$$

Utilizing relations (5) we rewrite (6.1,2,3) as follows:

$$C(\alpha) = \frac{\sqrt{3}}{2}\cos\left[\frac{1}{3}\sin^{-1}z_1(\alpha)\right] = \frac{\sqrt{3}}{2}\cos\left[\frac{2}{3}\tan^{-1}\left(\tan(\frac{\alpha}{2}\right)^3\right], \quad (6.4)$$

$$S(\alpha) = \frac{1}{2}\sin\left[\frac{1}{3}\sin^{-1}z_1(\alpha)\right] = \frac{1}{2}\sin\left[\frac{2}{3}\tan^{-1}\left(\tan(\frac{\alpha}{2}\right)^3\right], \quad (6.5)$$

$$\frac{c_1^2(\alpha)}{v^2} = \frac{3}{z_1(\alpha)}\left[C(\alpha)-S(\alpha)\right] = \frac{3\left[C(\alpha)-S(\alpha)\right]}{\sin\left[2\tan^{-1}\left(\tan(\frac{\alpha}{2}\right)^3\right]} \quad (6.6)$$

$$\frac{c_2^2(\alpha)}{v^2} = -\frac{3}{z_1(\alpha)}\left[C(\alpha)+S(\alpha)\right] = -\frac{3\left[C(\alpha)+S(\alpha)\right]}{\sin\left[2\tan^{-1}\left(\tan(\frac{\alpha}{2}\right)^3\right]} \quad (6.7)$$

$$\frac{c_3^2(\alpha)}{v^2} = \frac{3}{z_1(\alpha)}2S(\alpha) = \frac{6S(\alpha)}{\sin\left[2\tan^{-1}\left(\tan(\frac{\alpha}{2}\right)^3\right]} \quad (6.8)$$

Already with ordinary particle solutions (6.1) and (6.2) we see that transforrmation $\alpha \to -\alpha$ interchanges (6.1) $\leftrightarrows$(6.2) as $c_1^2(\pm\alpha) = c_2^2(\mp\alpha)$, while (6.3) and (6.8) remain invariant. $c_3^2(-\alpha) = c_3^2(\alpha)$. The similar things one concludes with solutions (6.6,7,8) This means that. $c_3(\alpha)$ is favored with which to define a photon as the fastest speed particle. The solutions (6) satisfy the Cardano's relation : $c_1^2(\alpha) + c_2^2(\alpha) + c_3^2(\alpha) = 0$ (Šoln, J., 2014-2022).

Now, we continue with writing down the novel (dark matter) particle limiting velocity solutions, with interchanged indices from (Šoln, J., 2021.1.2-2022), $2 \leftrightarrows 3$:

$$\frac{c_{1,3}^2(\alpha)}{v^2} = \frac{3}{2z_2(\alpha)}\csc 2\tan^{-1}\left(\tan\left(\frac{1}{2}\sin^{-1}\left(\frac{1}{z_2(\alpha)}\right)\right)\right)^{\frac{1}{3}},$$
$$\pm i\frac{3\sqrt{3}}{2z_2(\alpha)}ctn2\tan^{-1}\left(\tan\left(\frac{1}{2}\sin^{-1}\left(\frac{1}{z_2(\alpha)}\right)\right)\right)^{\frac{1}{3}}; \quad (7.1,3)$$

$$\frac{c_2^2(\alpha)}{v^2} = -\frac{3}{z_2(\alpha)}\csc 2\tan^{-1}\left(\tan\left(\frac{1}{2}\sin^{-1}\left(\frac{1}{z_2(\alpha)}\right)\right)\right)^{\frac{1}{3}}. \quad (7.2)$$

The solutions (6.1,2,3) and (7.1,3; 2) are good examples of consistent self-generating evolutionary system, with z_1, z_2 and α being evolution parameters. Comparison of solutions (7) with relations (5) changes (7) to:

$$\frac{c^2_{1,3}(\alpha)}{v^2} = \frac{3\left[1 \pm i\sqrt{3}\cos(\alpha)\right]}{2z_2(\alpha)\sin(\alpha)} = \frac{Rc^2_{1,3}(\alpha)}{v^2} + i\frac{Ic^2_{1,3}(\alpha)}{v^2}, \tag{8.1}$$

$$\frac{Rc^2_{1,3}(\alpha)}{v^2} = \frac{3}{2z_2(\alpha)\sin(\alpha)},\ \frac{Ic^2_{1,3}(\alpha)}{v^2} = \pm\frac{3\sqrt{3}\cos(\alpha)}{2z_2(\alpha)\sin(\alpha)}, \tag{8.2}$$

$$\frac{c^2_2(\alpha)}{v^2} = -\frac{3}{z_2(\alpha)\sin(\alpha)}; \tag{8.3}$$

$$Rc^2_1(\alpha) = Rc^2_3(\alpha),\ Ic^2_1(\alpha) = -Ic^2_3(\alpha), c^2_2(\alpha) = -2Rc^2_{1,3}(\alpha) \tag{8.4}$$

$$Rc_{1,3}(\alpha) = \sqrt{Rc^2_{1,3}(\alpha)},\ \ Ic_{1,3}(\alpha) = \sqrt{Ic^2_{1,3}(\alpha)} \tag{8.5}$$

We can rewrite relations (8.1,2,3,4,5) to better emphasize the dependence on the congruent angle α :

$$\frac{c^2_{1,3}(\alpha)}{v^2} = \frac{Rc^2_{1,3}(\alpha)}{v^2} + i\frac{Ic^2_{1,3}(\alpha)}{v^2}, \tag{8.6}$$

$$\frac{Rc^2_{1,3}(\alpha)}{v^2} = \frac{3\sin\left[2\tan^{-1}\left(\tan(\frac{\alpha}{2})^3\right]}{2\sin(\alpha)}, \tag{8.7}$$

$$\frac{Ic^2_{1,3}(\alpha)}{v^2} = \pm\frac{\sqrt{3}3\sin\left[2\tan^{-1}\left(\tan(\frac{\alpha}{2})^3\right]\cos(\alpha)}{2\sin(\alpha)} = \pm\sqrt{3}\frac{Rc^2_{1,3}(\alpha)}{v^2}\cos(\alpha), \tag{8.8}$$

$$\frac{c^2_2(\alpha)}{v^2} = -\frac{3\sin\left[2\tan^{-1}\left(\tan(\frac{\alpha}{2})^3\right]}{\sin(\alpha)} = -2\frac{Rc^2_{1,3}(\alpha)}{v^2} \tag{8.9}$$

At this point, we wish to emphasize the number of Cardano's relations, which follow from (8.1,...,9)

$$\frac{Rc^2_3(\alpha)}{v^2} + \frac{Rc^2_1(\alpha)}{v^2} + \frac{c^2_2(\alpha)}{v^2} = 0, \tag{8.10}$$

$$\frac{Ic^2_{,3}(\alpha)}{v^2} + \frac{Ic^2_{,1}(\alpha)}{v^2} = 0, \tag{8.11}$$

$$\frac{c^2_3(\alpha)}{v^2} + \frac{c^2_1(\alpha)}{v^2} + \frac{c^2_2(\alpha)}{v^2} = 0. \tag{8.12}$$

The importance of these relations (8.10,11,12) comes from the fact that in each group the elements are complement physically to each other and, as such, can contribute to enclosed physical picture.. For instance, $c^2_2(\alpha)/v^2$ complements physically $Rc^2_{3,1}(\alpha)/v^2$, by giving maximal real novel particle velocities Similarly $c^2_{3,1}(\alpha)/v^2$, $c^2_2(\alpha)/v^2$ and $Ic^2_{,3,1}(\alpha)/v$ together give unphysical fudge novel particle velocities that will discussed later.

One can easily verify, that these limiting velocity squares are invariant under the transformation $\alpha \to -\alpha$, making the novel particles more stable than the ordinary ones. Furthermore, the novel (dark matter) particles, as we shall see, appear in doubles. So we may have more than one novel photon. Next, using relations (5), (6) an (8), we wish to verify the smooth matching point between the ordinary and novel particle limiting velocity squares at $\alpha = \pi/2$, where one should notice that there are no imaginary contributions for novel particle limiting velocity squares. Next, in the Tables 1. Through 5, we give simple mathematical examples that as soon as the congruent angle α moved from $\alpha = \pi/2$ to $\alpha = \pi/3$, the limiting velocities of ordinary and novel particles move from their equalities to the distinct inequalities.

Table 1. Ordinary particle limiting velocity squares at smooth matching point of $z_1 = 1, \alpha = \pi/2$:

$c^2_1(\alpha)/v^2$	$c^2_2(\alpha)/v^2$	$c^2_3(\alpha)/v^2$
1.5	−3	1.5

Table 2. Novel particle limiting velocity squares at smooth matching point of $z_2 = 1, \alpha = \pi/2$:

$c^2_1(\alpha)/v^2 = Rc^2_1(\alpha)/v^2 + i0$	$c^2_2(\alpha)/v^2$	$c^2_3(\alpha)/v^2 = Rc^2_3(\alpha)/v^2 + i0$
1.5	−3	1.5

Clearly, at the smooth matching point of $\alpha = \pi/2$, limiting velocity squares of ordinary and novel particles have the same values. To show how fast these values diverge away from $\alpha/2$, we now take $\alpha = \pi/3$:

Table 3. Ordinary particle limiting velocity squares at $z_1 = 0.371, \alpha = \pi/3$:

$$\begin{pmatrix} c_1^2(\alpha)/v^2 & c_2^2(\alpha)/v^2 & c_3^2(\alpha)/v^2 \\ 6.436 & -7.458 & 1.022 \end{pmatrix}$$

Table 4. Novel (dm) particle limiting velocity squares at $z_2 = 1/z_1 = 2.695, \alpha = \pi/3$:

$$\begin{pmatrix} \begin{array}{c} c_1^2(\alpha)/v^2 \\ = Rc_1^2(\alpha)/v^2 + iIc_1^2(\alpha)/v^2 \end{array} & c_2^2(\alpha)/v^2 & \begin{array}{c} c_3^2(\alpha)/v^2 \\ = Rc_3^2(\alpha)/v^2 + iIc_3^2(\alpha)/v^2 \end{array} \\ 0.643 + i0.557 & -1.286 & 0.643 - i0.557 \end{pmatrix}$$

It is immediately evident to the large differences in limiting velocities between the matched ordinary and novel particles as we move away from $\alpha = \pi/2$ to $\alpha = \pi/3$. Next, we wish to show that the same thing happens with the Global "pseudo-relativistic" kinetic energies from (2). The "relativistic"-limiting velocity particle energies will be dealt with separately.

Table 5. The ordinary and novel particle"pseudo-relativistic" energies as functions of α

$E(z_1(\alpha)) = \frac{3\sqrt{3}mv^2}{2}(1/z_1)$, $E(z_2(\alpha)) = \frac{3\sqrt{3}mv^2}{2}(1/z_2)$:

$$\begin{pmatrix} \alpha : & \pi/2 & \pi/2.25 & \pi/2.3 & \pi/2.5 & \pi/2.75 & \pi/3 & \pi/3.25 \\ 1/z_1 : & 1 & 1.142 & 1.198 & 1.495 & 2.016 & 2.694 & 3.531 \\ 1/z_2 : & 1 & 0.876 & 0.835 & 0.669 & 0.496 & 0.371 & 0.283 \end{pmatrix}$$

3. Evaluation of Limiting Velocities for Ordinary Particles, Each With Established Congruent Angle α, and Novel Particles Each With Congruent Angle α_N , Obtained by Quantum Jump $\alpha_N = 2\alpha$ From Matched Ordinary Particle

Here we shall be dealing with establishing the congruent α angles for ordinary particles as well as α_N for matched novel particles, both in the range of $0 \prec \alpha, \alpha_N \prec \pi/2$. This by itself is rather restrictive and, from practical point of view, is imposed by energy expressions in (3); particularly so, that in addition to the emphasis on the congruent angles α andα_N we also have to have knowledge of congruent parameters $z_1(\alpha), z_2(\alpha)$ and $z_1(\alpha_N), z_2(\alpha_N)$. For novel matched particle, this knowledge is important when we are trying to evaluate the velocities of these novel particles as seen by ordinary particles.

3.1 Details of limiting velocity evaluations of ordinary particles: photon γ, muon neutrino ν and electron e , which will be matched in next section 3.2 with novel photon γ_N, muon neutrino ν_N and electron e_N . All the ordinary particles in consideration here are massive including photon γ. If ordinary photon were massless then to get a reasonable limiting velocity one would have to have $z_1 = 0$, which according to (3) would make the energy infinite We prefer to take the new notion that ordinary photon has tiny mass of $m_\gamma \simeq 6 \times 10^{-48}g = 4.5 \times 10^{-15}eV$ (Lin, H.-L, Tang, L. Zou, R., 2023), (Itzykson, C. and Zuber, J.-B., 1980 on page 138), The mass of the muon neutrino is $m_{\nu,\mu} = 1.01 \times 10^{-33}g = 0.76eV$, while of the electron $m_e = 0.68 \times 10^{-27}g = 0.51MeV$. Of major interest here is of course $c_3^2(\alpha(\gamma))/v^2 \approx 1$. As it can be seen from (Šoln, J., 2021.1.2-2022), $c_1^2(\alpha)/v^2$ could, in principle, be also reduced to unity with, very very small congruent angle α and very small,congruent parameter $z_1(\alpha)$ with the help of very powerful computing capabilities. For now we stick with,the $c_3^2(\alpha)/v^2$ whose unity is caused by the velocity of light c. We find it to be the most economical to to deal with $c_3^2(\alpha)/v^2$, not only for ordinary photon γ, but also for neutrino ν and electron e when trying to connect them to novel γ_N ,ν_N and e_N.

We start with the photon γ. First we need to find the best match of congruent angle $\alpha(\gamma)$ for the photon $\gamma$with maximal squared limiting velocity $c_3^2(\alpha(\gamma))$ This, with ordinary particle limiting velocity solution, either from (6.1,2,3) or (6.4,5,6,7,8). through trial and error,we find to be $\alpha(\gamma) = 0.617 = 0.393\ \pi/2$ with corresponding congruent parameters $z_1(0.617) = 0.0646572$, $z_2(0.617) = 15.466181647$. Now, specifically, from (6.4) and (6.5) we find $S(\alpha(\gamma)) = 0.01078854$ and $C(\alpha(\gamma)) = 0.8658238531$,with which from (6.6,7,8), using this $\alpha(\gamma) = 0.617$, we find additional limiting velocity solutions $c_1^2(\alpha(\gamma))$ and $c_2^2(\alpha(\gamma))$ accompanying $c_3^2(\alpha(\gamma))$. The full expressions of limiting velocity values are all listed in Table 6.

Table 6. The values of limiting velocities for ordinary photon γ together with the photon congruent parameters

$\alpha(\gamma) = 0.617 \quad z_1(\alpha(\gamma)) = 0.0646572 \quad z_2(\alpha(\gamma)) = 15.466229$

$$\begin{pmatrix} c_1^2(\alpha(\gamma))/v^2 & c_2^2(\alpha(\gamma))/v^2 & c_3^2(\alpha(\gamma))/v^2 \\ 39.672517 & -40.673539 & 1.001145 \end{pmatrix}$$

One can verify the correctness of solutions by adding them: $c_1^2(\alpha(\gamma)//v^2)+c_2^2(\alpha(\gamma)//v^2)+c_3^2(\alpha(\gamma)//v^2)=1.23\times10^{-4}\approx0$, satisfying the Cardano's relation (for details, see (Šoln, J., 2014).

Next, with the help from (Šoln, J., 2021.1.2), we do the same thing with muon neutrino ν and electron e of finding their best fits in limiting velocities within respective squared limiting velocities $c_3^2(\alpha(\nu,\mu))$ and $c_3^2(\alpha(e))$Simply, using the limiting velocity solution, either from (6.1,2,3) or (6.4,5,6,7,8), through trial and error,we find from the best fit for maximal $c_3^2(\alpha(\nu))$ that the ordinary muon neutrino congruent angle is $\alpha(\nu)=0.6175=0.393\ \pi/2$ with corresponding $\alpha(\nu)=0.6175$ and $\alpha(e)=0.618$, respectively. These,with other squared limiting velocities, will be given in Table 7 and Table 8, respectively.

For muon neutrino ν, as for the photon γ case, we are looking for the best value of the congruent angle $\alpha(\nu)$ which gives the best fit for maximal squared limiting velocity $c_3^2(\alpha(\nu))$. Simply, using the limiting velocity solution, either from (6.1,2,3) or (6.4,5,6,7,8), through trial and error,we find from the best fit for maximal $c_3^2(\alpha(\nu))$ that the ordinary muon neutrino congruent angle is $\alpha(\nu)=0.6175=0.393\ \pi/2$ which yields the congruent parameters $z_1(0.6175)=0.064825$, $z_2(0.6175)=15.426237$. Next, from (6.4) and (6.5) with this $\alpha(\nu)$ we find $S(\alpha(\nu))=0.010811$ and $C(\alpha(\nu))=0.865823$,with which from (6.6,7,8) we also find $c_1^2(\alpha(\nu))$ and $c_2^2(\alpha(\nu))$. The full values of all limiting velocities are listed in Table 7.

Table 7. The values of limiting velocities for ordinary muon neutrino ν together with the neutrino congruent parameters

$\alpha(\nu)=0.6175 \quad z_1(\alpha(\nu))=0.064825 \quad z_2(\alpha(\nu))=15.426237$

$$\begin{pmatrix} c_1^2(\alpha(\nu))/v^2 & c_2^2(\alpha(\nu))/v^2 & c_3^2(\alpha(\nu))/v^2 \\ 39.568623 & -40.569256 & 1.000632 \end{pmatrix}$$

By summing up the limiting velocity solutions from the Table 7: $c_1^2(\alpha(\nu))/v^2)+c_2^2(\alpha(\nu))/v^2)+c_3^2(\alpha(\nu))/v^2)=-5.2796\times10^{-7}\approx0$, one sees the Cardano's relation being well satisfied.

The last in the collection of leptons as the ordinary particles is the electron e. Here we are also looking for congruent angle $\alpha(e)$ that gives best fit for maximal squared limiting velocity $c_3^2(\alpha(e)$. As shown earlier, with the applications of formal solutions either from (6.1,2,3) or (6.4,5,6,7,8) through trial and error,we find from the best fit for maximal $c_3^2(\alpha(e))$ that the congruent angle is $\alpha(e)=0.618=0.3934\pi/2$ with congruent parameters $z_1(0.618)=0.06499$, $z_2(0.618)=15.3864$. .Next, from (6.4) and (6.5) with this $\alpha(e)$ we find $S(\alpha(e))=0.010839$ and $C(\alpha(e))=0.865821$,with which from (6.6,7,8) we also find $c_1^2(\alpha(e))$ and $c_2^2(\alpha(e))$. The full values of all limiting velocities are listed in Table 8.

Table 8. The values of limiting velocities for ordinary electron e together with the electron congruent parameters

$\alpha(e)=0.618 \quad z_1(\alpha(e))=0.06499 \quad z_2(\alpha(\nu))=15.3864$

$$\begin{pmatrix} c_1^2(\alpha(e))/v^2 & c_2^2(\alpha(e))/v^2 & c_3^2(\alpha(e))/v^2 \\ 39.465565 & -40.569256 & 1.000646 \end{pmatrix}$$

By summing up the limiting velocity solutions from the Table 8: $c_1^2(\alpha(e))/v^2)+c_2^2(\alpha(e))/v^2)+c_3^2(\alpha(e))/v^2)=-2.3338\times10^{-7}\approx0$, one sees again that the Cardano's relation being well satisfied.

3.2 Details of novel particles limiting velocity evaluations for photon γ_N, neutrino ν_N, electron e_N.With every congruent angle quantum jump: $\alpha\implies\alpha_N=2\alpha$, the evaluations of limiting velocities for novel particles: photon γ_N, novel muon neutrino ν_N and novel electron e_N will automatically chose the maximal squared limiting velocity values for $Rc_{1,3}^2(\alpha_N)$ and $c_2^2(\alpha_N)$. This is quite different from the ordinary particle cases. The novel particle limiting velocity solutions consist of real and imaginary parts, as can be seen from relations (8.1,2,3,4,5) and (8.6,7,8,9). The real parts of limiting velocity solutions for novel particles follow either from (8.2,3) or directly from (8.7) and (8.9). The imaginary parts follow from (8.2) and (8.9). Relations (8.8) and (8.9) relate imaginary parts to real parts of solutions. The suggestion is to first evaluate real parts as imaginary parts are numerically expressible in terms of real parts. One should be aware that for matched ordinary and novel particles,despite the quantum jump, both,ordinary congruent α and novel congruent α_N should independently assume $\pi/2$ value in order to assume equal physical values at matching point $\pi/2$.

We start with the novel photon γ_N. The quantum jump from ordinary γ congruent angle $\alpha(\gamma)=0.617$ to novel photon γ_N congruent angle, yields the value of $\alpha(\gamma_N)=2\times0.617,=1.234$, or $\alpha(\gamma_N)=0.7856\ \pi/2$.The corresponding congruent parameters are: $z_1(1.234)=0.632726889)$, $z_2(1.234)=1.579098573$. With these congruent parameters, either from relations (8.1,2.3.4.5) or directly from (8.6,7,8,9), we evaluate $R(\alpha(\gamma_N))c_{3,1}^2/v^2$, $c_2^2(\alpha(\gamma_N))/v^2$, $I(\alpha(\gamma_N))c_{3,1}^2/v^2$ and $c_{3,1}^2(\alpha(\gamma_N))/v^2$. Their values are given in Table 9.

Table 9. The values of limiting velocities for novel photon γ_N together with the congruent parameters

$\alpha(\gamma_N) = 1.234, z_1(\alpha(\gamma_N)) = 0.632726889), z_2(\alpha(\gamma_N)) = 1.579098573.$

$$\begin{pmatrix} R(\alpha(\gamma_N))c_{3,1}^2/v^2 & c_2^2(\alpha(\gamma_N))/v^2 & I(\alpha(\gamma_N))c_{3,1}^2/v^2 & c_{3,1}^2(\alpha(\gamma_N))/v^2 \\ 1.0065 & -2 & \mp 0.572 & 1.0065 \mp i0.572 \end{pmatrix}$$

We can make different limiting velocity sums, each of them satisfying Cardano's relation of zero value: (a):$2R(\alpha(\gamma_N))c_{3,1}{}^2/v^2 + c_2^2(\alpha(\gamma_N))/v^2 \approx 0$,(b): $I(\alpha(\gamma_N))c_3^2/v^2 + I(\alpha(\gamma_N))c_1^2/v^2 \approx 0$ and (c): $c_3^2(\alpha(\gamma_N))/v^2 + c_1^2(\alpha(\gamma_N))/v^2 + c_2^2(\alpha(\gamma_N))/v^2 \approx 0$. For example, the importance of relation (a) is in the fact that these three velocities in it are real and measurable.

As we see from the Table 9, $R(\alpha(\gamma_N))c_{3,1}^2 \approx v^2 \approxeq \overline{c}^2((\gamma_N)$, is the maximum real physical squared velocity of the novel photon γ_N. What we do not know is: What is its numerical value? We can find this value from the point of view of ordinary particles. Namely, since $\alpha(\gamma_N) = 0.7856\ \pi/2$ is smaller than $\pi/2$, we may for this endeavor force γ_N to be ordinary-like particle and then from (6.3) or (6.8) to evaluate

$$\frac{c_3^2(\alpha(\gamma_N))}{Rc_{3,1}^2(\alpha(\gamma_N))} = \frac{c^2}{\overline{c}^2(\alpha(\gamma_N))} = \frac{6S\left((\alpha(\gamma_N)\right)}{z_1(\alpha((\gamma_N))} = \frac{6 \times 0.113302996}{0.632726889} = 1.0744252825, \tag{9.1}$$

$$\overline{c}^2((\alpha(\gamma_N)) = 0.9307301459\ c^2 \preceq\ c^2 \tag{9.2}$$

It is gratifying that the Special Theory of Relativity is not violated. In fact, $\overline{c}^2(\gamma_N)$ is practically the same as c^2. However, we still should take into account the imaginary part in the complete solution $c_{3,1}^2(\alpha(\gamma_N))/v^2 = 1.0065 \mp i0.572$ as shown n Table 9. Hence, starting from $\overline{c}^2((\alpha(\gamma_N))$, we introduce the fudge maximum velocity squared $\overline{c}_f^2(\gamma_N)$ from which, after taking absolute value we deduce average fudge velocity squared $\prec \overline{c}_f^2((\gamma_N) \succ = 1.07224\ c^2$ as detailed in (9.3) to (9.6):

$$\frac{c_3^2(\alpha(\gamma_N))}{c_{3,1}^2(\alpha(\gamma_N))} = \frac{c^2}{\overline{c}^2(\gamma_N)(1 \mp i\sqrt{3}\cos\alpha(\gamma_N)} = \frac{c^2}{\overline{c}^2(\gamma_N)(1 \mp i0.572)} \tag{9.3}$$

$$\frac{c_3^2(\alpha(\gamma_N))}{c_{3,1}^2(\alpha(\gamma_N))} = \frac{c^2}{\overline{c}_f^2((\gamma_N)} : \overline{c}_f^2((\gamma_N) = \overline{c}^2((\gamma_N)(1 \mp i0.572) \tag{9.4}$$

$$\left[\overline{c}_f^2(\gamma_N)\right]^2 = \overline{c}^4((\gamma_N)\left[1 \mp i0.572\right]^2 = 0.9307301459^2 c^4 \times 1.3272 = 1.149698c^4 \tag{9.5}$$

$$\prec\ \overline{c}_f^2((\gamma_N) \succ\ = 1.07224\ c^2 \tag{9.6}$$

Surprisingly, this average fudge maximal velocity $\prec \overline{c}_f^2((\gamma_N) \succ$ barely violates Special Theory of Relativity, even being unphysical.

Next on the agenda is the novel neutrino ν_N. The quantum jump from ordinary ν congruent angle $\alpha(\nu) = 0.6175$ to novel neutrino ν_N congruent angle, yields the value of $\alpha(\nu_N) = 2 \times 0.6175, = 1.235$, or $\alpha(\nu_N) = 0.7862\ \pi/2$. The corresponding congruent parameters are: $z_1(1.235) = 0.63483$, $z_2(1.235) = 1.57523$. These congruent parameters when applied either to relations (8.1,2.3.4.5), or directly to (8.6,7,8,9), will evaluate $R(\alpha(\nu_N))c_{3,1}^2/v^2$, $c_2^2(\alpha(\nu_N))/v^2$, $I(\alpha(\nu_N))c_{3,1}^2/v^2$ and $c_{3,1}^2(\alpha(\nu_N))/v^2$. Their values are given in Table 10:

Table 10. The values of limiting velocities for novel neutrino ν_N together with the congruent parameters

$\alpha(\nu_N) = 1.235, z_1(\alpha(\nu_N)) = 0.63483, z_2(\alpha(\nu_N)) = 1.57523.$

$$\begin{pmatrix} R(\alpha(\nu_N))c_{3,1}^2/v^2 & c_2^2(\alpha(\nu_N))/v^2 & I(\alpha(\nu_N))c_{3,1}^2/v^2 & c_{3,1}^2(\alpha(\nu_N))/v^2 \\ 1.0086 & -2.01 & \mp 0.571 & 1.0086 \mp i0.571 \end{pmatrix}$$

Again, as we see from the Table 10, $R(\alpha(\nu_N))c_{3,1}^2 \approx v^2 = \overline{c}^2(\nu_N)$, the maximum real physical squared velocity of the novel neutrino ν_N. What is its numerical value? As before for γ_N, we can find this value from the point of view of ordinary particles. Since $\alpha(\nu_N) = 0.7862\ \pi/2$ is smaller than $\pi/2$, so for this endeavor we may force ν_N to be ordinary-like particle. Then from (6.3) or (6.8) evaluate

$$\frac{c_3^2(\alpha(\nu_N))}{Rc_{3,1}^2(\alpha(\nu_N))} = \frac{c^2}{\overline{c}^2(\alpha(\nu_N))} = \frac{6S\left((\alpha(\nu_N)\right)}{z_1(\alpha(\nu_N))} = \frac{0.68178022.}{0.634831} = 1.0739556602, \tag{9.7}$$

$$\overline{c}^2((\alpha(\nu_N)) = 0.93113713821\ c^2 \preceq\ c^2 \tag{9.8}$$

Again, It is gratifying that the Special Theory of Relativity is not violated. as $\overline{c}^2((\alpha\nu_N))$ is practically the same as c^2. However, we still take into account the imaginary part in the complete solution $c^2_{3,1}(\alpha(\nu_N))/v^2 = 1.0086 \mp i0.571$ as shown in Table 10. Hence, starting from $\overline{c}^2((\alpha(\nu_N))$, we introduce the fudge maximum velocity squared $\overline{c}^2_f((\nu_N))$ from which, after taking absolute value, we deduce average fudge velocity squared $\prec \overline{c}^2_f((\nu_N) \succ$ as indicated in (9.11,12):

$$\frac{c^2_3(\alpha(\nu_N))}{c^2_{3,1}((\alpha(\nu_N))} = \frac{c^2}{Rc^2_{3,1}(\nu_N)(1 \mp i\sqrt{3}\cos\alpha(\nu_N)} = \frac{c^2}{\overline{c}^2(\nu_N)(1 \mp i0.571)} \tag{9.9}$$

$$\frac{c^2_3(\alpha(\nu_N))}{c^2_{3,1}((\alpha(\nu_N))} = \frac{c^2}{\overline{c}^2_f(\nu_N)} : \overline{c}^2_f(\nu_N) = \overline{c}^2((\nu_N)(1 \mp i0.571) \tag{9.10}$$

$$\left[\overline{c}^2_f((\nu_N)\right]^2 = \overline{c}^4(\nu_N)\left[1 \mp i0.571\right]^2 = 0.93113713821^2 c^4 \times 1.33 = 1.1531 c^4 \tag{9.11}$$

$$\prec \overline{c}^2_f((\nu_N) \succ\ = 1.074\, c^2 \tag{9.12}$$

This average fudge maximal velocity $\prec \overline{c}^2_f((\nu_N) \succ$ barely violates Special Theory of Relativity, although it is again unphysical.

Next on the agenda is the novel electron e_N. The quantum jump from ordinary e congruent angle $\alpha(e) = 0.618$ to novel electron e_N congruent angle, yields the value of $\alpha(e_N) = 2 \times 0.618, = 1.236$, or $\alpha(e_N) = 0.7869\ \pi/2$. The corresponding congruent parameters are $z_1(1.236) = 0.63639$, $z_2(1.236) = 1.571364$. These congruent parameters when applied either to relations (8.1,2,3,4,5), or directly to (8.6,7,8,9), will evaluate $R(\alpha(e_N))c^2_{3,1}/v^2$, $c^2_2(\alpha(e_N))/v^2$, $I(\alpha(e_N))c^2_{3,1}/v^2$ and $c^2_{3,1}(\alpha(e_N))/v^2$. Their values are given in Table 11.

Table 11. The values of limiting velocities for electron e_N. with the congruent parameters

$\alpha(e_N) = 1.236, z_1(\alpha(e_N)) = 0.63639, z_2(\alpha(e_N)) = 1.571364.$:

$R(\alpha(e_N))c^2_{3,1}/v^2$	$c^2_2(\alpha(e_N))/v^2$	$I(\alpha(e_N))c^2_{3,1}/v^2$	$c^2_{3,1}(\alpha(e_N))/v^2$
1.0107	−2.0214	∓0.5731	$1.0107 \mp i0.5731$

From Table 11, we have that $R(\alpha(e_N))c^2_{3,1} \approx v^2 = \overline{c}^2(e_N)$ the maximum real physical squared velocity of the novel electron e_N. What What is its numerical value? Since $\alpha(e_N) = 0.393431\ \pi/2$ is smaller than $\pi/2$, we may force e_N to be ordinary-like particle for finding this numerical value. Then from (6.3) or (6.8) we have

$$\frac{c^2_3(\alpha(e_N))}{Rc^2_{3,1}(\alpha(\nu_N))} = \frac{c^2}{\overline{c}^2(\alpha(e_N))} = \frac{6S\left((\alpha(e_N)\right)}{z_1(\alpha(e_N))} = \frac{0.6837465134.}{0.6363897949} = 1.0744146417, \tag{9.13}$$

$$\overline{c}^2((\alpha(e_N)) = 0.930739363731811\, c^2 \preceq\ c^2 \tag{9.14}$$

We see again that there is no violation of Special Theory of Relativity, as $\overline{c}^2(e_N)$ is practically the same as c^2. We wish to take into account the imaginary part in the complete solution $c^2_{3,1}(\alpha(e_N))/v^2 = 1.0107 \mp i0.5731$ as shown in Table 11. Hence, starting from $\overline{c}^2((\alpha(\nu_N))$, we introduce the fudge maximum velocity squared $\overline{c}^2_f((\nu_N))$ from which, after taking absolute value, we deduce average fudge velocity squared $\prec \overline{c}^2_f((\nu_N) \succ$ as indicated in (9.18):

$$\frac{c^2_3(\alpha(e_N))}{c^2_{3,1}((\alpha(e_N))} = \frac{c^2}{Rc^2_{3,1}(e_N)(1 \mp i\sqrt{3}\cos\alpha(e_N)} = \frac{c^2}{\overline{c}^2((e_N)(1.0107 \mp i0.5731)}, \tag{9.15}$$

$$\frac{c^2_3(\alpha(e_N))}{c^2_{3,1}((\alpha(e_N))} = \frac{c^2}{\overline{c}^2_f(e_N)} : \overline{c}^2_f((e_N) = \overline{c}^2((e_N)(1 \mp i0.5731) \tag{9.16}$$

$$\left[\overline{c}^2_f(e_N)\right]^2 = \overline{c}^4(e_N)\left[1 \mp i0.5731\right]^2 = 1.150798502121 c^4 \tag{9.17}$$

$$\left[\overline{c}^2_f(e_N)\right]^2 = \prec \overline{c}^2_f((\nu_N) \succ^2,\ \prec \overline{c}^2_f((\nu_N) \succ = 1.07275276841\, c^2. \tag{9.18}$$

Surprisingly, although being unphysical, the average e_N fudge maximal square velocity $\prec \overline{c}^2_f((\nu_N) \succ$ barely violates Special Theory of Relativity.

Based on the ordinary and novel particle maximum velocity evaluations, we assume the velocity of light c to be the maximum possible velocity in the Universe. Hopefully, this analysis of the matched ordinary and novel particles should be helpful for pursuing novel particles detections, even if novel particles are all dark mater particles.

4. Additional Remarks, Discussion and Conclusion

We, as yet, do not know the masses of novel leptons γ_N, ν_N and e_N as opposed to the ordinary leptons γ, ν and e whose masses are known quite well, as shown at the beginning of 3.1. Never the less, due to the fact that the congruent angle of a novel particle α_N is obtained by quantum jump from an ordinary particle congruent angel α, $\alpha_N = 2\alpha$, which allows qualitative observation the difference between energies of ordinary and novel particle.

As an example, we make comparison between energies of ordinary photon γ and novel photon γ_N at their maximal velocities, which we take to be c for either of them. With parameters from Table 6, for ordinary photon γ. and Table 9. for novel photon γ_N, we evaluate:

$$E(\gamma, c) = \frac{3\sqrt{3}m_\gamma c^2}{2z_1(\alpha(\gamma))} = \frac{3\sqrt{3}}{2 \times 0.0646572} m_\gamma c^2 = 40.18\ m_\gamma c^2, \tag{10.1}$$

$$E(\gamma_N, c) = \frac{3\sqrt{3}m_{\gamma_N} c^2}{2z_2(\alpha(\gamma_N))} = \frac{3\sqrt{3}z_1(\alpha(\gamma_N))}{2} m_{\gamma_N} c^2 = \frac{3\sqrt{3} \times 0.632726889}{2} m_{\gamma_N} c^2 = 1.644\ m_{\gamma_N} c^2 \tag{10.2}$$

In the case of ordinary photon γ and novel photon γ_N, the energies at the maximal velocity c show noticeable difference when expressed in terms of their masses. Of course to know how big their difference is we would have to know the value of m_{γ_N} versus $m\gamma$. If for example, it happens that the total energies are equal: $E(\gamma_N, c) = E(\gamma, c)$, then one easily sees their mass energy values through $m_{\gamma_N} c^2 \approx 24 m_\gamma c^2$ Of occurs,in general, with trial and error through comparison of different physical quantities of γ and γ_N , one might be able to determine the relationship between their masses.

Novel (dark) photon is taking more and more place in the literature discussions. The reason being that the ordinary photon plays a central role in the so called Standard Model. So, the "natural" thing is to simply add another novel (dark) photon to the Standard Model. For example, in "Search for Light Dark Photon in Forward Experiments at the LHC", (Yeong, G.-K. at al., 2023) propose to tackle the search for new dark photon by extending the standard model (SM) with the additional U(1) gauge field which with another U(1) gauge field, after proper redefinitions yield the massless electromagnetic photon gauge field A_μ and massive dark photon gauge field A'_μ. This new dark photon is actually, in our description an ordinary particle and the experimental results should be able to tell weather it is necessary.

In conclusion, we may say that ordinary and novel particles once they leave the smooth point of $\alpha = \pi/2$ actually retain still some,connection. The proof is the quantum jump of ordinary, particle α to novel particle $\alpha_N = 2\alpha$, when away from $\pi/2$. The further pursuit should utilize the effects of α-quantum jump on other limiting velocities besides of c_3^2 to $Rc_{3,1}^2$which here,was emphasized in order to solidly establish footing for ordinary and novel photons. Further worth probably should be to deal extensively with calculating the inertial scaling factors with energies and inertial masses from bicubic equation limiting velocity solutions for ordinary and novel particles as shown in (Šoln, J., 2022).

If the novel particles were to be discovered that would not be the end of it.

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