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Property of Tensor Satisfying Binary Law 4

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Abstract

I have already reported "Property of Tensor Satisfying Binary Law 3". This article is the article that I revise contents of "Property of Tensor Satisfying Binary Law 3", and increased the report about new characteristics. I did not touch it about a contraction in the tensor which satisfied Binary Law in "Property of Tensor Satisfying Binary Law 3". I report a contraction in the tensor satisfying Binary Law in this article.

Keywords: tensor, covariant derivative

1. Introduction

I reported that I got $x_{\mu;\nu}=\frac{\partial x_\mu}{\partial x^\nu}-x_\tau\,\Gamma^{\ \tau}_{\mu\nu}, x^\mu_{;\nu}=\frac{\partial x^\mu}{\partial x^\nu}+x^\tau\,\Gamma^{\ \mu}_{\tau\nu},\cdots$ as the tensor which satisfied Binary Law. (Ichidayama, 2021) I report a revised edition in "Property of Tensor Satisfying Binary Law 3" (Ichidayama, 2021) about getting $M_{\mu;\nu}=\frac{\partial M_\mu}{\partial x^\nu}-M_\tau\,\Gamma^{\ \tau}_{\mu\nu}, M^\mu_{;\nu}=\frac{\partial M^\mu}{\partial x^\nu}+M^\tau\,\Gamma^{\ \mu}_{\tau\nu},\cdots$ as the tensor satisfying Binary Law in this article.

The contraction in the tensor satisfying Binary Law is different from the contraction in the tensor. This article introduces this new contraction method.

2. Definition

Definition1. $\overline{x^\mu}\neq x^\mu,\overline{x^\nu}\neq x^\nu,\overline{x^\mu}=x^\nu,\overline{x^\nu}=x^\mu$ is established.(Ichidayama, 2017)

I named $\overline{x^\mu}\neq x^\mu,\overline{x^\nu}\neq x^\nu,\overline{x^\mu}=x^\nu,\overline{x^\nu}=x^\mu$ "Binary Law".(Ichidayama, 2017)

Definition2. If $\overline{x^\mu}\neq x^\mu,\overline{x^\nu}\neq x^\nu,\overline{x^\mu}=x^\nu,\overline{x^\nu}=x^\mu$ is established, $x_\nu=x^\mu$ is established.(Ichidayama, 2017)

Definition3. If $\overline{x^\mu}\neq x^\mu,\overline{x^\nu}\neq x^\nu,\overline{x^\mu}=x^\nu,\overline{x^\nu}=x^\mu$ is established, $x_\mu=x^\nu$ is established.(Ichidayama, 2017)

Definition4. If $\overline{x^\mu}\neq x^\mu,\overline{x^\nu}\neq x^\nu,\overline{x^\mu}=x^\nu,\overline{x^\nu}=x^\mu$ is established, $x_\nu=-x_\mu$ is established.(Ichidayama, 2017)

Definition5. If $\overline{x^\mu}\neq x^\mu,\overline{x^\nu}\neq x^\nu,\overline{x^\mu}=x^\nu,\overline{x^\nu}=x^\mu$ is established, $x^\nu=-x^\mu$ is established.(Ichidayama, 2017)

Definition6. If all coordinate systems $x^\mu,x^\nu,x^\sigma,x^\lambda,\cdots$ satisfies $\overline{x^\mu}\neq x^\mu,\overline{x^\nu}\neq x^\nu,\overline{x^\mu}=x^\nu,\overline{x^\nu}=x^\mu$, all coordinate systems $x^\mu,x^\nu,x^\sigma,x^\lambda,\cdots$ shifts to only two of x^μ,x^ν. (Ichidayama, 2017)

Definition7. $g^\mu_\mu=1,g^\mu_\nu=0:(\mu\neq\nu)$ establishment.(Dirac, 1975)

Definition8. The first-order covariant derivative of the covariant vector satisfied $M_{\mu;\nu}=\frac{\partial M_\mu}{\partial x^\nu}-M_\tau\,\Gamma^{\ \tau}_{\mu\nu}=\frac{\partial M_\mu}{\partial x^\nu}-M_\tau\frac{1}{2}g^{\epsilon\tau}\left(\frac{\partial g_{\mu\epsilon}}{\partial x^\nu}+\frac{\partial g_{\nu\epsilon}}{\partial x^\mu}-\frac{\partial g_{\mu\nu}}{\partial x^\epsilon}\right)$.(Fleisch, 2012)

Definition9. The first-order covariant derivative of the contravariant vector satisfied $M^\mu_{;\nu}=\frac{\partial M^\mu}{\partial x^\nu}+M^\tau\,\Gamma^{\ \mu}_{\tau\nu}=\frac{\partial M^\mu}{\partial x^\nu}+M^\tau\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu}+\frac{\partial g_{\nu\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)$. (Fleisch, 2012)

Definition10. The second-order covariant derivative of the covariant vector satisfied $M_{\mu;\nu;\sigma} = \frac{\partial M_{\mu;\nu}}{\partial x^{\sigma}} - M_{\iota;\nu}\Gamma^{\iota}_{\mu\sigma} - M_{\mu;\iota}\Gamma^{\iota}_{\nu\sigma} = \frac{\partial}{\partial x^{\sigma}}\left(\frac{\partial M_{\mu}}{\partial x^{\nu}} - M_{\tau}\Gamma^{\tau}_{\mu\nu}\right) - \left(\frac{\partial M_{\iota}}{\partial x^{\nu}} - M_{\tau}\Gamma^{\tau}_{\iota\nu}\right)\Gamma^{\iota}_{\mu\sigma} - \left(\frac{\partial M_{\mu}}{\partial x^{\iota}} - M_{\tau}\Gamma^{\tau}_{\mu\iota}\right)\Gamma^{\iota}_{\nu\sigma} = \frac{\partial^2 M_{\mu}}{\partial x^{\nu}\partial x^{\sigma}} - \frac{\partial}{\partial x^{\sigma}}\left(M_{\tau}\frac{1}{2} g^{\epsilon\tau}\left(\frac{\partial g_{\mu\epsilon}}{\partial x^{\nu}} + \frac{\partial g_{\nu\epsilon}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\epsilon}}\right)\right) - \frac{\partial M_{\iota}}{\partial x^{\nu}}\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\mu\epsilon}}{\partial x^{\sigma}} + \frac{\partial g_{\sigma\epsilon}}{\partial x^{\mu}} - \frac{\partial g_{\mu\sigma}}{\partial x^{\epsilon}}\right) + M_{\tau}\frac{1}{2} g^{\epsilon\tau}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^{\nu}} + \frac{\partial g_{\nu\epsilon}}{\partial x^{\iota}} - \frac{\partial g_{\iota\nu}}{\partial x^{\epsilon}}\right)\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\mu\epsilon}}{\partial x^{\sigma}} + \frac{\partial g_{\sigma\epsilon}}{\partial x^{\mu}} - \frac{\partial g_{\mu\sigma}}{\partial x^{\epsilon}}\right) - \frac{\partial M_{\mu}}{\partial x^{\iota}}\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^{\sigma}} + \frac{\partial g_{\sigma\epsilon}}{\partial x^{\nu}} - \frac{\partial g_{\nu\sigma}}{\partial x^{\epsilon}}\right) + M_{\tau}\frac{1}{2} g^{\epsilon\tau}\left(\frac{\partial g_{\mu\epsilon}}{\partial x^{\iota}} + \frac{\partial g_{\iota\epsilon}}{\partial x^{\mu}} - \frac{\partial g_{\mu\iota}}{\partial x^{\epsilon}}\right)\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^{\sigma}} + \frac{\partial g_{\sigma\epsilon}}{\partial x^{\nu}} - \frac{\partial g_{\nu\sigma}}{\partial x^{\epsilon}}\right)$. (Fleisch, 2012)

Definition11. The second-order covariant derivative of the contravariant vector satisfied $M^{\mu}_{;\nu;\sigma} = \frac{\partial M^{\mu}_{;\nu}}{\partial x^{\sigma}} + M^{\iota}_{;\nu}\Gamma^{\mu}_{\iota\sigma} - M^{\mu}_{;\iota}\Gamma^{\iota}_{\nu\sigma} = \frac{\partial}{\partial x^\sigma}\left(\frac{\partial M^\mu}{\partial x^\nu} + M^\tau\Gamma^\mu_{\tau\nu}\right) + \left(\frac{\partial M^\iota}{\partial x^\nu} + M^\tau\Gamma^\iota_{\tau\nu}\right)\Gamma^\mu_{\iota\sigma} - \left(\frac{\partial M^\mu}{\partial x^\iota} + M^\tau\Gamma^\mu_{\tau\iota}\right)\Gamma^\iota_{\nu\sigma} = \frac{\partial^2 M^\mu}{\partial x^\nu\partial x^\sigma} + \frac{\partial}{\partial x^\sigma}\left(M^\tau\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu} + \frac{\partial g_{\nu\epsilon}}{\partial x^\tau} - \frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)\right) + \frac{\partial M^\iota}{\partial x^\nu}\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\iota} - \frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right) + M^\tau\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu} + \frac{\partial g_{\nu\epsilon}}{\partial x^\tau} - \frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\iota} - \frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right) - \frac{\partial M^\mu}{\partial x^\iota}\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\nu} - \frac{\partial g_{\nu\sigma}}{\partial x^\epsilon}\right) - M^\tau\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\iota} + \frac{\partial g_{\iota\epsilon}}{\partial x^\tau} - \frac{\partial g_{\tau\iota}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\nu} - \frac{\partial g_{\nu\sigma}}{\partial x^\epsilon}\right)$.(Fleisch, 2012)

Definition12. The third-order covariant derivative of the contravariant vector satisfied $M^\mu_{;\nu;\sigma;\lambda} = \frac{\partial M^\mu_{;\nu;\sigma}}{\partial x^\lambda} + M^\kappa_{;\nu;\sigma}\Gamma^\mu_{\kappa\lambda} - M^\mu_{;\kappa:\sigma}\Gamma^\kappa_{\nu\lambda} - M^\mu_{;\nu;\kappa}\Gamma^\kappa_{\sigma\lambda} = \frac{\partial}{\partial x^\lambda}\left\{\frac{\partial}{\partial x^\sigma}\left(\frac{\partial M^\mu}{\partial x^\nu} + M^\tau\Gamma^\mu_{\tau\nu}\right) + \left(\frac{\partial M^\iota}{\partial x^\nu} + M^\tau\Gamma^\iota_{\tau\nu}\right)\Gamma^\mu_{\iota\sigma}\left(\frac{\partial M^\mu}{\partial x^\iota} + M^\tau\Gamma^\mu_{\tau\iota}\right)\Gamma^\iota_{\nu\sigma}\right\} + \left\{\frac{\partial}{\partial x^\sigma}\left(\frac{\partial M^\kappa}{\partial x^\nu} + M^\tau\Gamma^\kappa_{\tau\nu}\right) + \left(\frac{\partial M^\iota}{\partial x^\nu} + M^\tau\Gamma^\iota_{\tau\nu}\right)\Gamma^\kappa_{\iota\sigma} - \left(\frac{\partial M^\kappa}{\partial x^\iota} + M^\tau\Gamma^\kappa_{\tau\iota}\right)\Gamma^\iota_{\nu\sigma}\right\}\Gamma^\mu_{\kappa\lambda} - \left\{\frac{\partial}{\partial x^\sigma}\left(\frac{\partial M^\mu}{\partial x^\kappa} + M^\tau\Gamma^\mu_{\tau\kappa}\right) + \left(\frac{\partial M^\iota}{\partial x^\kappa} + M^\tau\Gamma^\iota_{\tau\kappa}\right)\Gamma^\mu_{\iota\sigma} - \left(\frac{\partial M^\mu}{\partial x^\iota} + M^\tau\Gamma^\mu_{\tau\iota}\right)\Gamma^\iota_{\kappa\sigma}\right\}\Gamma^\kappa_{\nu\lambda} - \left\{\frac{\partial}{\partial x^\kappa}\left(\frac{\partial M^\mu}{\partial x^\nu} + M^\tau\Gamma^\mu_{\tau\nu}\right) + \left(\frac{\partial M^\iota}{\partial x^\nu} + M^\tau\Gamma^\iota_{\tau\nu}\right)\Gamma^\mu_{\iota\kappa} - \left(\frac{\partial M^\mu}{\partial x^\iota} + M^\tau\Gamma^\mu_{\tau\iota}\right)\Gamma^\iota_{\nu\kappa}\right\}\Gamma^\kappa_{\sigma\lambda} = \frac{\partial^3 M^\mu}{\partial x^\nu\partial x^\sigma\partial x^\lambda} + \frac{\partial^2}{\partial x^\sigma\partial x^\lambda}\left(M^\tau\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu} + \frac{\partial g_{\nu\epsilon}}{\partial x^\tau} - \frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)\right) + \frac{\partial}{\partial x^\lambda}\left(\frac{\partial M^\iota}{\partial x^\nu}\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\iota} - \frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right)\right) + \frac{\partial}{\partial x^\lambda}\left(M^\tau\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu} + \frac{\partial g_{\nu\epsilon}}{\partial x^\tau} - \frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\iota} - \frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right)\right) - \frac{\partial}{\partial x^\lambda}\left(\frac{\partial M^\mu}{\partial x^\iota}\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\nu} - \frac{\partial g_{\nu\sigma}}{\partial x^\epsilon}\right)\right) - \frac{\partial}{\partial x^\lambda}\left(M^\tau\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\iota} + \frac{\partial g_{\iota\epsilon}}{\partial x^\tau} - \frac{\partial g_{\tau\iota}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\nu} - \frac{\partial g_{\nu\sigma}}{\partial x^\epsilon}\right)\right) + \frac{\partial^2 M^\kappa}{\partial x^\nu\partial x^\sigma}\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^\lambda} + \frac{\partial g_{\lambda\epsilon}}{\partial x^\kappa} - \frac{\partial g_{\kappa\lambda}}{\partial x^\epsilon}\right) + \frac{\partial}{\partial x^\sigma}\left(M^\tau\frac{1}{2} g^{\epsilon\kappa}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu} + \frac{\partial g_{\nu\epsilon}}{\partial x^\tau} - \frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)\right)\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^\lambda} + \frac{\partial g_{\lambda\epsilon}}{\partial x^\kappa} - \frac{\partial g_{\kappa\lambda}}{\partial x^\epsilon}\right) + \frac{\partial M^\iota}{\partial x^\nu}\frac{1}{2} g^{\epsilon\kappa}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\iota} - \frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^\lambda} + \frac{\partial g_{\lambda\epsilon}}{\partial x^\kappa} - \frac{\partial g_{\kappa\lambda}}{\partial x^\epsilon}\right) + M^\tau\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\nu} + \frac{\partial g_{\nu\epsilon}}{\partial x^\tau} - \frac{\partial g_{\tau\nu}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\kappa}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\iota} - \frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^\lambda} + \frac{\partial g_{\lambda\epsilon}}{\partial x^\kappa} - \frac{\partial g_{\kappa\lambda}}{\partial x^\epsilon}\right) - \frac{\partial M^\kappa}{\partial x^\iota}\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\nu} - \frac{\partial g_{\nu\sigma}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^\lambda} + \frac{\partial g_{\lambda\epsilon}}{\partial x^\kappa} - \frac{\partial g_{\kappa\lambda}}{\partial x^\epsilon}\right) - M^\tau\frac{1}{2} g^{\epsilon\kappa}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\iota} + \frac{\partial g_{\iota\epsilon}}{\partial x^\tau} - \frac{\partial g_{\tau\iota}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\nu} - \frac{\partial g_{\nu\sigma}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^\lambda} + \frac{\partial g_{\lambda\epsilon}}{\partial x^\kappa} - \frac{\partial g_{\kappa\lambda}}{\partial x^\epsilon}\right) - \frac{\partial^2 M^\mu}{\partial x^\kappa\partial x^\sigma}\frac{1}{2} g^{\epsilon\kappa}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\lambda} + \frac{\partial g_{\lambda\epsilon}}{\partial x^\nu} - \frac{\partial g_{\nu\lambda}}{\partial x^\epsilon}\right) - \frac{\partial}{\partial x^\sigma}\left(M^\tau\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\kappa} + \frac{\partial g_{\kappa\epsilon}}{\partial x^\tau} - \frac{\partial g_{\tau\kappa}}{\partial x^\epsilon}\right)\right)\frac{1}{2} g^{\epsilon\kappa}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\lambda} + \frac{\partial g_{\lambda\epsilon}}{\partial x^\nu} - \frac{\partial g_{\nu\lambda}}{\partial x^\epsilon}\right) - \frac{\partial M^\iota}{\partial x^\kappa}\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\iota} - \frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\kappa}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\lambda} + \frac{\partial g_{\lambda\epsilon}}{\partial x^\nu} - \frac{\partial g_{\nu\lambda}}{\partial x^\epsilon}\right) - M^\tau\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\kappa} + \frac{\partial g_{\kappa\epsilon}}{\partial x^\tau} - \frac{\partial g_{\tau\kappa}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\iota} - \frac{\partial g_{\iota\sigma}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\kappa}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^\lambda} + \frac{\partial g_{\lambda\epsilon}}{\partial x^\nu} - \frac{\partial g_{\nu\lambda}}{\partial x^\epsilon}\right) + \frac{\partial M^\mu}{\partial x^\iota}\frac{1}{2} g^{\epsilon\iota}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^\sigma} + \frac{\partial g_{\sigma\epsilon}}{\partial x^\kappa} - \frac{\partial g_{\kappa\sigma}}{\partial x^\epsilon}\right)\frac{1}{2} g^{\epsilon\kappa}\Bigl(\frac{\partial g_{\nu\epsilon}}{\partial x^\lambda} +$

$\frac{\partial g_{\lambda\epsilon}}{\partial x^{\nu}} - \frac{\partial g_{\nu\lambda}}{\partial x^{\epsilon}}\Big) + M^{\tau}\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^{\iota}} + \frac{\partial g_{\iota\epsilon}}{\partial x^{\tau}} - \frac{\partial g_{\tau\iota}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^{\sigma}} + \frac{\partial g_{\sigma\epsilon}}{\partial x^{\kappa}} - \frac{\partial g_{\kappa\sigma}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\epsilon}}{\partial x^{\nu}} - \frac{\partial g_{\nu\lambda}}{\partial x^{\epsilon}}\right) -$

$\frac{\partial^2 M^{\mu}}{\partial x^{\nu}\partial x^{\kappa}}\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\sigma\epsilon}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\epsilon}}{\partial x^{\sigma}} - \frac{\partial g_{\sigma\lambda}}{\partial x^{\epsilon}}\right) - \frac{\partial}{\partial x^{\kappa}}\left(M^{\tau}\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^{\nu}} + \frac{\partial g_{\nu\epsilon}}{\partial x^{\tau}} - \frac{\partial g_{\tau\nu}}{\partial x^{\epsilon}}\right)\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\sigma\epsilon}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\epsilon}}{\partial x^{\sigma}} - \frac{\partial g_{\sigma\lambda}}{\partial x^{\epsilon}}\right) -$

$\frac{\partial M^{\iota}}{\partial x^{\nu}}\frac{1}{2}g^{\epsilon\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^{\kappa}} + \frac{\partial g_{\kappa\epsilon}}{\partial x^{\iota}} - \frac{\partial g_{\iota\kappa}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\sigma\epsilon}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\epsilon}}{\partial x^{\sigma}} - \frac{\partial g_{\sigma\lambda}}{\partial x^{\epsilon}}\right) - M^{\tau}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^{\nu}} + \frac{\partial g_{\nu\epsilon}}{\partial x^{\tau}} - \frac{\partial g_{\tau\nu}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\mu}\Big(\frac{\partial g_{\iota\epsilon}}{\partial x^{\kappa}} + \frac{\partial g_{\kappa\epsilon}}{\partial x^{\iota}} -$

$\frac{\partial g_{\iota\kappa}}{\partial x^{\epsilon}}\Big)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\sigma\epsilon}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\epsilon}}{\partial x^{\sigma}} - \frac{\partial g_{\sigma\lambda}}{\partial x^{\epsilon}}\right) + \frac{\partial M^{\mu}}{\partial x^{\iota}}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^{\kappa}} + \frac{\partial g_{\kappa\epsilon}}{\partial x^{\nu}} - \frac{\partial g_{\nu\kappa}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\sigma\epsilon}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\epsilon}}{\partial x^{\sigma}} - \frac{\partial g_{\sigma\lambda}}{\partial x^{\epsilon}}\right) + M^{\tau}\frac{1}{2}g^{\epsilon\mu}\Big(\frac{\partial g_{\tau\epsilon}}{\partial x^{\iota}} +$

$\frac{\partial g_{\iota\epsilon}}{\partial x^{\tau}} - \frac{\partial g_{\tau\iota}}{\partial x^{\epsilon}}\Big)\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\nu\epsilon}}{\partial x^{\kappa}} + \frac{\partial g_{\kappa\epsilon}}{\partial x^{\nu}} - \frac{\partial g_{\nu\kappa}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\sigma\epsilon}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\epsilon}}{\partial x^{\sigma}} - \frac{\partial g_{\sigma\lambda}}{\partial x^{\epsilon}}\right).$

Definition13. When the next conversion equation is established, $x^{\mu\nu}$ is contravariant components of a tensor of the second rank. (Fleisch, 2012) $x^{\mu\nu} = \frac{\partial x^{\mu}}{\partial x^{\sigma}}\frac{\partial x^{\nu}}{\partial x^{\lambda}}x^{\sigma\lambda}$

Definition14. When the next conversion equation is established, $x_{\mu\nu}$ is covariant components of a tensor of the second rank. (Fleisch, 2012) $x_{\mu\nu} = \frac{\partial x^{\sigma}}{\partial x^{\mu}}\frac{\partial x^{\lambda}}{\partial x^{\nu}}x_{\sigma\lambda}$

Definition15. When the next conversion equation is established, x^{μ}_{ν} is components of the mixed tensor of the second rank. (Fleisch, 2012) $x^{\mu}_{\nu} = \frac{\partial x^{\mu}}{\partial x^{\sigma}}\frac{\partial x^{\lambda}}{\partial x^{\nu}}x^{\sigma}_{\lambda}$

Definition16. When the next conversion equation is established, $x_{\mu\nu\sigma}$ is covariant components of a tensor of the third rank. (Fleisch, 2012) $x_{\mu\nu\sigma} = \frac{\partial x^{\lambda}}{\partial x^{\mu}}\frac{\partial x^{\iota}}{\partial x^{\nu}}\frac{\partial x^{\epsilon}}{\partial x^{\sigma}}x_{\lambda\iota\epsilon}$

Definition17. When the next conversion equation is established, $x^{\mu}_{\nu\sigma}$ is components of the mixed tensor of the third rank of the second rank covariant in the first rank contravariant. (Fleisch, 2012) $x^{\mu}_{\nu\sigma} = \frac{\partial x^{\mu}}{\partial x^{\lambda}}\frac{\partial x^{\iota}}{\partial x^{\nu}}\frac{\partial x^{\epsilon}}{\partial x^{\sigma}}x^{\lambda}_{\iota\epsilon}$

Definition18. When the next conversion equation is established, $x^{\mu}_{\nu\sigma\lambda}$ is components of the mixed tensor of the fourth rank of the third rank covariant in the first rank contravariant. (Fleisch, 2012) $x^{\mu}_{\nu\sigma\lambda} = \frac{\partial x^{\mu}}{\partial x^{\iota}}\frac{\partial x^{\epsilon}}{\partial x^{\nu}}\frac{\partial x^{\alpha}}{\partial x^{\sigma}}\frac{\partial x^{\beta}}{\partial x^{\lambda}}x^{\iota}_{\epsilon\alpha\beta}$

Definition19. $Y_{:\sigma} = Y_{,\sigma} = \frac{\partial Y}{\partial x^{\sigma}}$ is established for covariant differentiation of scalar Y. When X, Y are tensor each, $(XY)_{:\sigma} = X_{:\sigma}Y + XY_{:\sigma}$ is established for covariant differentiation of XY.(Dirac, 1975)

Definition20. $M^{\mu} = Mx^{\mu}$ is established. M^{μ}, x^{μ} are tensor of rank 1, and M is scalar here. $\frac{M^{\mu}}{x^{\mu}} = \frac{M^{\mu}_{;\nu}}{x^{\mu}_{;\nu}} = M$ is established. $M^{\mu}_{;\nu}, x^{\mu}_{;\nu}$ are tensor of rank 2. Notation; before the index with the bottom expresses covariant differentiation here.

Definition21. "Contraction" of a Mixed Tensor. - From any mixed tensor we may form a tensor whose rank is less by two, by equating an index of covariant with one of contravariant character, and summing with respect to this index ("contraction"). Thus, for example, from the mixed tensor of the fourth rank $A^{\sigma\tau}_{\mu\nu}$, we obtain the mixed tensor of the second rank, $A^{\tau}_{\nu} = A^{\mu\tau}_{\mu\nu}\left(\sum_{\mu} A^{\mu\tau}_{\mu\nu}\right)$, and from this, by a second contraction, the tensor of zero rank, $A = A^{\nu}_{\nu} = A^{\mu\nu}_{\mu\nu}$.(Einstein, 1916).

3. Covariant Derivative for the Vector (M^{μ}, M_{μ}) in Tensor Satisfying Binary Law

Proposition1. $M_{\mu;\nu} = \frac{\partial M_{\mu}}{\partial x^{\nu}}, M_{\mu}^{;\mu} = \frac{\partial M_{\mu}}{\partial x_{\mu}} - M_{\nu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^{\mu}}$ established in tensor satisfying Binary Law.

Proof. If all coordinate systems satisfy Binary Law, I get

$$M_{\mu;\nu} = \frac{\partial M_{\mu}}{\partial x^{\nu}} - M_{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\mu\nu}}{\partial x^{\nu}} + \frac{\partial g_{\nu\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\nu}}\right) \tag{1}$$

from Definision8. An existence position of the free index is unidentified in the second term of the right side of (1). This is problem. Then, I limit operation satisfying Binary Law to index μ in Definision8 and get

$$M_{\bar{\mu};\nu} = \frac{\partial M_{\bar{\mu}}}{\partial x^{\nu}} - M_{\tau}\frac{1}{2}g^{\epsilon\tau}\left(\frac{\partial g_{\bar{\mu}\epsilon}}{\partial x^{\nu}} + \frac{\partial g_{\nu\epsilon}}{\partial x^{\bar{\mu}}} - \frac{\partial g_{\bar{\mu}\nu}}{\partial x^{\epsilon}}\right). \tag{2}$$

I mark the index which I operated satisfying Binary Law here. Similarly, I limit operation satisfying Binary Law to index ν in (2) and get

$$M_{\bar{\mu};\bar{\nu}} = \frac{\partial M_{\bar{\mu}}}{\partial x^{\bar{\nu}}} - M_{\tau}\frac{1}{2}g^{\epsilon\tau}\left(\frac{\partial g_{\bar{\mu}\epsilon}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\bar{\mu}}} - \frac{\partial g_{\bar{\mu}\bar{\nu}}}{\partial x^{\epsilon}}\right). \tag{3}$$

And, if all coordinate systems satisfy Binary Law, I get

$$M_{\bar{\mu};\bar{\nu}} = \frac{\partial M_{\bar{\mu}}}{\partial x^{\bar{\nu}}} - M_{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar{\mu}\nu}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\mu}}} - \frac{\partial g_{\bar{\mu}\bar{\nu}}}{\partial x^{\nu}}\right) \tag{4}$$

from (3). The problem in (1) is solved in (4). In consideration of (2),(3),(4) mentioned above, I mark free index of Definision8 to solve this and get

$$M_{\bar{\mu};\bar{\nu}} = \frac{\partial M_{\bar{\mu}}}{\partial x^{\bar{\nu}}} - M_{\tau}\frac{1}{2}g^{\epsilon\tau}\left(\frac{\partial g_{\bar{\mu}\epsilon}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\bar{\mu}}} - \frac{\partial g_{\bar{\mu}\bar{\nu}}}{\partial x^{\epsilon}}\right). \tag{5}$$

If all coordinate systems satisfy Binary Law, I get

$$M_{\bar{\mu};\bar{\nu}} = \frac{\partial M_{\bar{\mu}}}{\partial x^{\bar{\nu}}} - M_{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar{\mu}\nu}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\mu}}} - \frac{\partial g_{\bar{\mu}\bar{\nu}}}{\partial x^{\nu}}\right) \tag{6}$$

from (5). I get

$$M_{\bar{\mu};\bar{\nu}} = \frac{\partial M_{\bar{\mu}}}{\partial x^{\bar{\nu}}} - M_{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\mu}}}\right) = \frac{\partial M_{\bar{\mu}}}{\partial x^{\bar{\nu}}} - M_{\nu}\frac{1}{2}\frac{\partial g_{\bar{\nu}}^{\nu}}{\partial x^{\bar{\mu}}} \tag{7}$$

from (6). (7) must rewrite it in

$$M_{\mu;\nu} = \frac{\partial M_{\mu}}{\partial x^{\nu}} - M_{\sigma}\frac{1}{2}\frac{\partial g_{\nu}^{\sigma}}{\partial x^{\mu}} \tag{8}$$

by (7) being a tensor equation. I get the conclusion that (8) doesn't satisfy Binary Law from Definision6. I get the conclusion that (8) isn't an equation of the tensor satisfying Binary Law because (8) doesn't satisfy Binary Law.

I rewrite one existing index ν in each term of (8) in index μ using Definision2 and get

$$M_{\mu}^{;\mu} = \frac{\partial M_{\mu}}{\partial x_{\mu}} - M_{\sigma}\frac{1}{2}\frac{\partial g^{\sigma\mu}}{\partial x^{\mu}} = \frac{\partial M_{\mu}}{\partial x_{\mu}} - M_{\nu}\frac{1}{2}\frac{\partial g^{\nu\mu}}{\partial x^{\mu}}. \tag{9}$$

I get the conclusion that (9) does satisfy Binary Law from Definision6. I get the conclusion that (9) is an equation of the tensor satisfying Binary Law because (9) does satisfy Binary Law. I rewrite one existing index ν in each term of (8) in index μ using Definision4 and get

$$-M_{\mu;\mu} = -\frac{\partial M_{\mu}}{\partial x^{\mu}} + M_{\sigma}\frac{1}{2}\left(\frac{\partial g_{\mu}^{\sigma}}{\partial x^{\mu}}\right) = -\frac{\partial M_{\mu}}{\partial x^{\mu}} + M_{\nu}\frac{1}{2}\left(\frac{\partial g_{\mu}^{\nu}}{\partial x^{\mu}}\right). \tag{10}$$

I get the conclusion that (10) does satisfy Binary Law from Definision6. I get the conclusion that (10) is an equation of the tensor satisfying Binary Law because (10) does satisfy Binary Law. I get

$$-M_{\mu;\mu}=-\frac{\partial M_{\mu}}{\partial x^{\mu}} \tag{11}$$

in consideration of Definision7 for (10). Because the second term of the right side of (11) doesn't exist,

$$M_{\mu;\nu}=\frac{\partial M_{\mu}}{\partial x^{\nu}} \tag{12}$$

can rewrite (11) using Definision4. In addition, $M_{\mu;\nu}$ can't rewrite $M_{\mu}^{;\mu}$ of (9) using Definision2 because the second term of the right side exists in (9).

Proposition2. $M^{\mu}_{;\nu}=\frac{\partial M^{\mu}}{\partial x^{\nu}}$ is established in tensor satisfying Binary Law.

Proof. I mark free index of Definision9 by the same reason which I expressed in Proposition1 and get

$$M^{\bar{\mu}}_{;\bar{\nu}}=\frac{\partial M^{\bar{\mu}}}{\partial x^{\bar{\nu}}}+M^{\tau}\frac{1}{2}g^{\epsilon\bar{\mu}}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^{\bar{\nu}}}+\frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\tau}}-\frac{\partial g_{\tau\bar{\nu}}}{\partial x^{\epsilon}}\right). \tag{13}$$

If all coordinate systems satisfy Binary Law, I get

$$M^{\bar{\mu}}_{;\bar{\nu}}=\frac{\partial M^{\bar{\mu}}}{\partial x^{\bar{\nu}}}+M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\left(\widehat{\frac{\partial g_{\nu\nu}}{\partial x^{\bar{\nu}}}}+\frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\right) \tag{14}$$

from (13). I put a mark in (14) to express the difference in computation sequence. I get

$$M^{\bar{\mu}}_{;\bar{\nu}}=\frac{\partial M^{\bar{\mu}}}{\partial x^{\bar{\nu}}}+M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\left(\widehat{\frac{\partial g_{\nu\nu}}{\partial x^{\bar{\nu}}}}\right)=\frac{\partial M^{\bar{\mu}}}{\partial x^{\bar{\nu}}}+M^{\nu}\frac{1}{2}\widehat{\frac{\partial g^{\bar{\mu}}_{\nu}}{\partial x^{\bar{\nu}}}}, \tag{15}$$

$$M^{\bar{\mu}}_{;\bar{\nu}}=\frac{\partial M^{\bar{\mu}}}{\partial x^{\bar{\nu}}}+M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\left(\frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}}\right)=\frac{\partial M^{\bar{\mu}}}{\partial x^{\bar{\nu}}}+M^{\nu}\frac{1}{2}\frac{\partial g^{\bar{\mu}}_{\bar{\nu}}}{\partial x^{\nu}} \tag{16}$$

from (14). (15),(16) must rewrite it in

$$M^{\mu}_{;\nu}=\frac{\partial M^{\mu}}{\partial x^{\nu}}+M^{\sigma}\frac{1}{2}\widehat{\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu}}}, \tag{17}$$

$$M^{\mu}_{;\nu}=\frac{\partial M^{\mu}}{\partial x^{\nu}}+M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}} \tag{18}$$

by (15),(16) being a tensor equation. I get the conclusion that (17),(18) doesn't satisfy Binary Law from Definision6. I get the conclusion that (17),(18) isn't an equation of the tensor satisfying Binary Law because (17),(18) doesn't satisfy Binary Law.

I rewrite one existing index ν in each term of (17),(18) in index μ using Definision4 and get

$$-M^{\mu}_{;\mu}=-\frac{\partial M^{\mu}}{\partial x^{\mu}}-M^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)=-\frac{\partial M^{\mu}}{\partial x^{\mu}}-M^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right), \tag{19}$$

$$-M^{\mu}_{;\mu}=-\frac{\partial M^{\mu}}{\partial x^{\mu}}-M^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)=-\frac{\partial M^{\mu}}{\partial x^{\mu}}-M^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right). \tag{20}$$

I get the conclusion that (19),(20) does satisfy Binary Law from Definision6. I get the conclusion that (19),(20) is an equation of the tensor satisfying Binary Law because (19),(20) does satisfy Binary Law. I get

$$-M^{\mu}_{;\mu}=-\frac{\partial M^{\mu}}{\partial x^{\mu}} \tag{21}$$

in consideration of Definision7 for (19),(20). Because the second term of the right side of (21) doesn't exist,

$$M^{\mu}_{;\nu}=\frac{\partial M^{\mu}}{\partial x^{\nu}} \tag{22}$$

can rewrite (21) using Definision4. I rewrite one existing index ν in each term of (17),(18) in index μ using

Definision2 and get

$$M^{\mu;\mu}=\frac{\partial M^{\mu}}{\partial x_{\mu}}+M^{\sigma}\frac{1}{2}\left(\frac{\partial g_{\sigma}^{\mu}}{\partial x_{\mu}}\right)=\frac{\partial M^{\mu}}{\partial x_{\mu}}+M^{\nu}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\mu}}{\partial x_{\mu}}\right), \tag{23}$$

$$M^{\mu;\mu}=\frac{\partial M^{\mu}}{\partial x_{\mu}}+M^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)=\frac{\partial M^{\mu}}{\partial x_{\mu}}+M^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\nu}}\right). \tag{24}$$

I get the conclusion that (23),(24) does satisfy Binary Law from Definision6. I get the conclusion that (23),(24) is an equation of the tensor satisfying Binary Law because (23),(24) does satisfy Binary Law.

Proposition3. $M_{\mu;\nu;\nu}=\frac{\partial M_{\mu}}{\partial x^{\nu}\,\partial x^{\nu}}$ is established in tensor satisfying Binary Law.

Proof. I mark free index of Definision10 by the same reason which I expressed in Proposition1 and get

$$\begin{aligned}M_{\bar\mu;\bar\nu;\bar\sigma}&=\frac{\partial^2 M_{\bar\mu}}{\partial x^{\bar\nu}\,\partial x^{\bar\sigma}}-\frac{\partial}{\partial x^{\bar\sigma}}\left(M_{\tau}\frac{1}{2}g^{\epsilon\tau}\left(\frac{\partial g_{\bar\mu\epsilon}}{\partial x^{\bar\nu}}+\frac{\partial g_{\bar\nu\epsilon}}{\partial x^{\bar\mu}}-\frac{\partial g_{\bar\mu\bar\nu}}{\partial x^{\epsilon}}\right)\right)-\frac{\partial M_{\iota}}{\partial x^{\bar\nu}}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\bar\mu\epsilon}}{\partial x^{\bar\sigma}}+\frac{\partial g_{\bar\sigma\epsilon}}{\partial x^{\bar\mu}}-\frac{\partial g_{\bar\mu\bar\sigma}}{\partial x^{\epsilon}}\right)+M_{\tau}\frac{1}{2}g^{\epsilon\tau}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^{\bar\nu}}+\frac{\partial g_{\bar\nu\epsilon}}{\partial x^{\iota}}-\right.\\&\left.\frac{\partial g_{\iota\bar\nu}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\bar\mu\epsilon}}{\partial x^{\bar\sigma}}+\frac{\partial g_{\bar\sigma\epsilon}}{\partial x^{\bar\mu}}-\frac{\partial g_{\bar\mu\bar\sigma}}{\partial x^{\epsilon}}\right)-\frac{\partial M_{\bar\mu}}{\partial x^{\iota}}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\bar\nu\epsilon}}{\partial x^{\bar\sigma}}+\frac{\partial g_{\bar\sigma\epsilon}}{\partial x^{\bar\nu}}-\frac{\partial g_{\bar\nu\bar\sigma}}{\partial x^{\epsilon}}\right)+M_{\tau}\frac{1}{2}g^{\epsilon\tau}\left(\frac{\partial g_{\bar\mu\epsilon}}{\partial x^{\iota}}+\frac{\partial g_{\iota\epsilon}}{\partial x^{\bar\mu}}-\frac{\partial g_{\bar\mu\iota}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\bar\nu\epsilon}}{\partial x^{\bar\sigma}}+\right.\\&\left.\frac{\partial g_{\bar\sigma\epsilon}}{\partial x^{\bar\nu}}-\frac{\partial g_{\bar\nu\bar\sigma}}{\partial x^{\epsilon}}\right).\end{aligned} \tag{25}$$

If all coordinate systems satisfy Binary Law, I get

$$\begin{aligned}M_{\bar\mu;\bar\nu;\bar\nu}&=\frac{\partial^2 M_{\bar\mu}}{\partial x^{\bar\nu}\,\partial x^{\bar\nu}}-\frac{\partial}{\partial x^{\bar\nu}}\left(M_{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\mu\nu}}{\partial x^{\bar\nu}}+\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\mu}}-\frac{\partial g_{\bar\mu\bar\nu}}{\partial x^{\nu}}\right)\right)-\frac{\partial M_{\nu}}{\partial x^{\bar\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\mu\nu}}{\partial x^{\bar\nu}}+\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\mu}}-\frac{\partial g_{\bar\mu\bar\nu}}{\partial x^{\nu}}\right)+M_{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar\nu}}+\right.\\&\left.\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\bar\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\mu\nu}}{\partial x^{\bar\nu}}+\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\mu}}-\frac{\partial g_{\bar\mu\bar\nu}}{\partial x^{\nu}}\right)-\frac{\partial M_{\bar\mu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\bar\nu\nu}}}{\partial x^{\bar\nu}}+\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}-\frac{\partial g_{\bar\nu\bar\nu}}{\partial x^{\nu}}\right)+M_{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\mu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\bar\mu}}-\right.\\&\left.\frac{\partial g_{\bar\mu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\bar\nu\nu}}}{\partial x^{\bar\nu}}+\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}-\frac{\partial g_{\bar\nu\bar\nu}}{\partial x^{\nu}}\right)\end{aligned} \tag{26}$$

from (25). I put a mark in (26) to express the difference in computation sequence. I get

$$\begin{aligned}M_{\bar\mu;\bar\nu;\bar\nu}&=\frac{\partial^2 M_{\bar\mu}}{\partial x^{\bar\nu}\,\partial x^{\bar\nu}}-\frac{\partial}{\partial x^{\bar\nu}}\left(M_{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\mu}}\right)\right)-\frac{\partial M_{\nu}}{\partial x^{\bar\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\mu}}\right)+M_{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\mu}}\right)-\frac{\partial M_{\bar\mu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\bar\nu\nu}}}{\partial x^{\bar\nu}}\right)+\\&M_{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\bar\mu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\bar\nu\nu}}}{\partial x^{\bar\nu}}\right)=\frac{\partial^2 M_{\bar\mu}}{\partial x^{\bar\nu}\,\partial x^{\bar\nu}}-\frac{\partial}{\partial x^{\bar\nu}}\left(M_{\nu}\frac{1}{2}\frac{\partial g_{\bar\nu}^{\nu}}{\partial x^{\bar\mu}}\right)-\frac{\partial M_{\nu}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\partial g_{\bar\nu}^{\nu}}{\partial x^{\bar\mu}}+M_{\nu}\frac{1}{2}\frac{\widehat{\partial g_{\nu}^{\nu}}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\partial g_{\bar\nu}^{\nu}}{\partial x^{\bar\mu}}-\frac{\partial M_{\bar\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g_{\bar\nu}^{\nu}}}{\partial x^{\bar\nu}}+M_{\nu}\frac{1}{2}\frac{\partial g_{\nu}^{\nu}}{\partial x^{\bar\mu}}\frac{1}{2}\frac{\widehat{\partial g_{\bar\nu}^{\nu}}}{\partial x^{\bar\nu}},\end{aligned} \tag{27}$$

$$\begin{aligned}M_{\bar\mu;\bar\nu;\bar\nu}&=\frac{\partial^2 M_{\bar\mu}}{\partial x^{\bar\nu}\,\partial x^{\bar\nu}}-\frac{\partial}{\partial x^{\bar\nu}}\left(M_{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\mu}}\right)\right)-\frac{\partial M_{\nu}}{\partial x^{\bar\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\mu}}\right)+M_{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\mu}}\right)-\frac{\partial M_{\bar\mu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)+\\&M_{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\bar\mu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)=\frac{\partial^2 M_{\bar\mu}}{\partial x^{\bar\nu}\,\partial x^{\bar\nu}}-\frac{\partial}{\partial x^{\bar\nu}}\left(M_{\nu}\frac{1}{2}\frac{\partial g_{\bar\nu}^{\nu}}{\partial x^{\bar\mu}}\right)-\frac{\partial M_{\nu}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\partial g_{\bar\nu}^{\nu}}{\partial x^{\bar\mu}}+M_{\nu}\frac{1}{2}\frac{\partial g_{\bar\nu}^{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g_{\bar\nu}^{\nu}}{\partial x^{\bar\mu}}-\frac{\partial M_{\bar\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g_{\bar\nu}^{\nu}}{\partial x^{\bar\nu}}+M_{\nu}\frac{1}{2}\frac{\partial g_{\nu}^{\nu}}{\partial x^{\bar\mu}}\frac{1}{2}\frac{\partial g_{\bar\nu}^{\nu}}{\partial x^{\bar\nu}}\end{aligned} \tag{28}$$

from (26). (27),(28) must rewrite it in

$$M_{\mu;\nu;\nu}=\frac{\partial^2 M_{\mu}}{\partial x^{\nu}\,\partial x^{\nu}}-\frac{\partial}{\partial x^{\nu}}\left(M_{\sigma}\frac{1}{2}\frac{\partial g_{\nu}^{\sigma}}{\partial x^{\mu}}\right)-\frac{\partial M_{\sigma}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g_{\nu}^{\sigma}}{\partial x^{\mu}}+M_{\sigma}\frac{1}{2}\frac{\widehat{\partial g_{\sigma}^{\sigma}}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g_{\nu}^{\sigma}}{\partial x^{\mu}}-\frac{\partial M_{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g_{\nu}^{\sigma}}}{\partial x^{\nu}}+M_{\sigma}\frac{1}{2}\frac{\partial g_{\sigma}^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\widehat{\partial g_{\nu}^{\sigma}}}{\partial x^{\nu}}, \tag{29}$$

$$M_{\mu;\nu;\nu}=\frac{\partial^2 M_{\mu}}{\partial x^{\nu}\,\partial x^{\nu}}-\frac{\partial}{\partial x^{\nu}}\left(M_{\sigma}\frac{1}{2}\frac{\partial g_{\nu}^{\sigma}}{\partial x^{\mu}}\right)-\frac{\partial M_{\sigma}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g_{\nu}^{\sigma}}{\partial x^{\mu}}+M_{\sigma}\frac{1}{2}\frac{\partial g_{\nu}^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g_{\nu}^{\sigma}}{\partial x^{\mu}}-\frac{\partial M_{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g_{\nu}^{\sigma}}{\partial x^{\nu}}+M_{\sigma}\frac{1}{2}\frac{\partial g_{\sigma}^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g_{\nu}^{\sigma}}{\partial x^{\nu}} \tag{30}$$

by (27),(28) being a tensor equation. I get the conclusion that (29),(30) doesn't satisfy Binary Law from Definision6. I get the conclusion that (29),(30) isn't an equation of the tensor satisfying Binary Law because (29),(30) doesn't satisfy Binary Law.

I rewrite two existing index ν in each term of (29),(30) in index μ using Definision4 and get

$$M_{\mu;\mu;\mu}=\frac{\partial^2 M_\mu}{\partial x^\mu\,\partial x^\mu}-\frac{\partial}{\partial x^\mu}\left(M_\sigma\frac{1}{2}\frac{\partial g_\mu^\sigma}{\partial x^\mu}\right)-\frac{\partial M_\sigma}{\partial x^\mu}\frac{1}{2}\frac{\partial g_\mu^\sigma}{\partial x^\mu}+M_\sigma\frac{1}{2}\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\frac{1}{2}\frac{\partial g_\mu^\sigma}{\partial x^\mu}-\frac{\partial M_\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g_\mu^\sigma}{\partial x^\mu}+M_\sigma\frac{1}{2}\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\frac{1}{2}\frac{\partial g_\mu^\sigma}{\partial x^\mu}=\frac{\partial^2 M_\mu}{\partial x^\mu\,\partial x^\mu}-\frac{\partial}{\partial x^\mu}\left(M_\nu\frac{1}{2}\frac{\partial g_\mu^\nu}{\partial x^\mu}\right)-\frac{\partial M_\nu}{\partial x^\mu}\frac{1}{2}\frac{\partial g_\mu^\nu}{\partial x^\mu}+M_\nu\frac{1}{2}\frac{\partial g_\nu^\nu}{\partial x^\mu}\frac{1}{2}\frac{\partial g_\mu^\nu}{\partial x^\mu}-\frac{\partial M_\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g_\mu^\nu}{\partial x^\mu}+M_\nu\frac{1}{2}\frac{\partial g_\nu^\nu}{\partial x^\mu}\frac{1}{2}\frac{\partial g_\mu^\nu}{\partial x^\mu},\tag{31}$$

$$M_{\mu;\mu;\mu}=\frac{\partial^2 M_\mu}{\partial x^\mu\,\partial x^\mu}-\frac{\partial}{\partial x^\mu}\left(M_\sigma\frac{1}{2}\frac{\partial g_\mu^\sigma}{\partial x^\mu}\right)-\frac{\partial M_\sigma}{\partial x^\mu}\frac{1}{2}\frac{\partial g_\mu^\sigma}{\partial x^\mu}+M_\sigma\frac{1}{2}\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g_\mu^\sigma}{\partial x^\mu}-\frac{\partial M_\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g_\mu^\sigma}{\partial x^\mu}+M_\sigma\frac{1}{2}\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\frac{1}{2}\frac{\partial g_\mu^\sigma}{\partial x^\mu}=\frac{\partial^2 M_\mu}{\partial x^\mu\,\partial x^\mu}-\frac{\partial}{\partial x^\mu}\left(M_\nu\frac{1}{2}\frac{\partial g_\mu^\nu}{\partial x^\mu}\right)-\frac{\partial M_\nu}{\partial x^\mu}\frac{1}{2}\frac{\partial g_\mu^\nu}{\partial x^\mu}+M_\nu\frac{1}{2}\frac{\partial g_\mu^\nu}{\partial x^\nu}\frac{1}{2}\frac{\partial g_\mu^\nu}{\partial x^\mu}-\frac{\partial M_\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g_\mu^\nu}{\partial x^\mu}+M_\nu\frac{1}{2}\frac{\partial g_\nu^\nu}{\partial x^\mu}\frac{1}{2}\frac{\partial g_\mu^\nu}{\partial x^\mu}.\tag{32}$$

I get the conclusion that (31),(32) does satisfy Binary Law from Definision6. I get the conclusion that (31),(32) is an equation of the tensor satisfying Binary Law because (31),(32) does satisfy Binary Law. I get

$$M_{\mu;\mu;\mu}=\frac{\partial^2 M_\mu}{\partial x^\mu\,\partial x^\mu}\tag{33}$$

in consideration of Definision7 for (31),(32). Because the second term of the right side of (33) doesn't exist,

$$M_{\mu;\nu;\nu}=\frac{\partial^2 M_\mu}{\partial x^\nu\,\partial x^\nu}\tag{34}$$

can rewrite (33) using Definision4. I rewrite two existing index ν in each term of (29),(30) in index μ using Definision2 and get

$$M_\mu^{;\mu;\mu}=\frac{\partial^2 M_\mu}{\partial x_\mu\,\partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(M_\sigma\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^\mu}\right)-\frac{\partial M_\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^\mu}+M_\sigma\frac{1}{2}\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^\mu}-\frac{\partial M_\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu}+M_\sigma\frac{1}{2}\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu}=\frac{\partial^2 M_\mu}{\partial x_\mu\,\partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(M_\nu\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^\mu}\right)-\frac{\partial M_\nu}{\partial x_\mu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^\mu}+M_\nu\frac{1}{2}\frac{\partial g_\nu^\nu}{\partial x_\mu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^\mu}-\frac{\partial M_\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_\mu}+M_\nu\frac{1}{2}\frac{\partial g_\nu^\nu}{\partial x^\mu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_\mu},\tag{35}$$

$$M_\mu^{;\mu;\mu}=\frac{\partial^2 M_\mu}{\partial x_\mu\,\partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(M_\sigma\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^\mu}\right)-\frac{\partial M_\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^\mu}+M_\sigma\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^\mu}-\frac{\partial M_\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu}+M_\sigma\frac{1}{2}\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu}=\frac{\partial^2 M_\mu}{\partial x_\mu\,\partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(M_\nu\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^\mu}\right)-\frac{\partial M_\nu}{\partial x_\mu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^\mu}+M_\nu\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^\nu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^\mu}-\frac{\partial M_\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_\mu}+M_\nu\frac{1}{2}\frac{\partial g_\nu^\nu}{\partial x^\mu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_\mu}.\tag{36}$$

I get the conclusion that (35),(36) does satisfy Binary Law from Definision6. I get the conclusion that (35),(36) is an equation of the tensor satisfying Binary Law because (35),(36) does satisfy Binary Law.

Proposition4. $M^\mu_{;\nu;\nu}=\frac{\partial^2 M^\mu}{\partial x^\nu\,\partial x^\nu}$ is established in tensor satisfying Binary Law.

Proof. I mark free index of Definision11 by the same reason which I expressed in Proposition1 and get

$$M^{\bar\mu}_{;\bar\nu;\bar\sigma}=\frac{\partial^2 M^{\bar\mu}}{\partial x^{\bar\nu}\,\partial x^{\bar\sigma}}+\frac{\partial}{\partial x^{\bar\sigma}}\left(M^\tau\frac{1}{2}g^{\epsilon\bar\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^{\bar\nu}}+\frac{\partial g_{\bar\nu\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\bar\nu}}{\partial x^\epsilon}\right)\right)+\frac{\partial M^\iota}{\partial x^{\bar\nu}}\frac{1}{2}g^{\epsilon\bar\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^{\bar\sigma}}+\frac{\partial g_{\bar\sigma\epsilon}}{\partial x^\iota}-\frac{\partial g_{\iota\bar\sigma}}{\partial x^\epsilon}\right)+M^\tau\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^{\bar\nu}}+\frac{\partial g_{\bar\nu\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\bar\nu}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\bar\mu}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^{\bar\sigma}}+\frac{\partial g_{\bar\sigma\epsilon}}{\partial x^\iota}-\frac{\partial g_{\iota\bar\sigma}}{\partial x^\epsilon}\right)-\frac{\partial M^{\bar\mu}}{\partial x^\iota}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\bar\nu\epsilon}}{\partial x^{\bar\sigma}}+\frac{\partial g_{\bar\sigma\epsilon}}{\partial x^{\bar\nu}}-\frac{\partial g_{\bar\nu\bar\sigma}}{\partial x^\epsilon}\right)-M^\tau\frac{1}{2}g^{\epsilon\bar\mu}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^\iota}+\frac{\partial g_{\iota\epsilon}}{\partial x^\tau}-\frac{\partial g_{\tau\iota}}{\partial x^\epsilon}\right)\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\bar\nu\epsilon}}{\partial x^{\bar\sigma}}+\frac{\partial g_{\bar\sigma\epsilon}}{\partial x^{\bar\nu}}-\frac{\partial g_{\bar\nu\bar\sigma}}{\partial x^\epsilon}\right).\tag{37}$$

If all coordinate systems satisfy Binary Law, I get

$$M^{\bar\mu}_{;\bar\nu;\bar\nu}=\frac{\partial^2 M^{\bar\mu}}{\partial x^{\bar\nu}\,\partial x^{\bar\nu}}+\frac{\partial}{\partial x^{\bar\nu}}\left(M^\nu\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar\nu}}+\frac{\partial g_{\bar\nu\nu}}{\partial x^\nu}-\frac{\partial g_{\nu\bar\nu}}{\partial x^\nu}\right)\right)+\frac{\partial M^\nu}{\partial x^{\bar\nu}}\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar\nu}}+\frac{\partial g_{\bar\nu\nu}}{\partial x^\nu}-\frac{\partial g_{\nu\bar\nu}}{\partial x^\nu}\right)+M^\nu\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar\nu}}+\frac{\partial g_{\bar\nu\nu}}{\partial x^\nu}-\frac{\partial g_{\nu\bar\nu}}{\partial x^\nu}\right)\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar\nu}}+\frac{\partial g_{\bar\nu\nu}}{\partial x^\nu}-\frac{\partial g_{\nu\bar\nu}}{\partial x^\nu}\right)-\frac{\partial M^{\bar\mu}}{\partial x^\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\bar\nu\nu}}}{\partial x^{\bar\nu}}+\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}-\frac{\partial g_{\bar\nu\bar\nu}}{\partial x^\nu}\right)-M^\nu\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^\nu}+\frac{\partial g_{\nu\nu}}{\partial x^\nu}-\right.$$

$$\left.\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}}+\frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}}-\frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\right) \tag{38}$$

from (37). I put a mark in (38) to express the difference in computation sequence. I get

$$M^{\bar{\mu}}_{;\bar{\nu};\bar{\nu}}=\frac{\partial^2 M^{\bar{\mu}}}{\partial x^{\bar{\nu}}\,\partial x^{\bar{\nu}}}+\frac{\partial}{\partial x^{\bar{\nu}}}\left(M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\right)\right)+\frac{\partial M^{\nu}}{\partial x^{\bar{\nu}}}\frac{1}{2}g^{\nu\bar{\mu}}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\right)+M^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\right)\frac{1}{2}g^{\nu\bar{\mu}}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\right)-\frac{\partial M^{\bar{\mu}}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}}\right)-$$

$$M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}}\right)=\frac{\partial^2 M^{\bar{\mu}}}{\partial x^{\bar{\nu}}\,\partial x^{\bar{\nu}}}+\frac{\partial}{\partial x^{\bar{\nu}}}\left(M^{\nu}\frac{1}{2}\frac{\widehat{\partial g^{\bar{\mu}}_{\nu}}}{\partial x^{\bar{\nu}}}\right)+\frac{\partial M^{\nu}}{\partial x^{\bar{\nu}}}\frac{1}{2}\frac{\widehat{\partial g^{\bar{\mu}}_{\nu}}}{\partial x^{\bar{\nu}}}+M^{\nu}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\nu}}}{\partial x^{\bar{\nu}}}\frac{1}{2}\frac{\widehat{\partial g^{\bar{\mu}}_{\nu}}}{\partial x^{\bar{\nu}}}-\frac{\partial M^{\bar{\mu}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar{\nu}}}}{\partial x^{\bar{\nu}}}-M^{\nu}\frac{1}{2}\frac{\widehat{\partial g^{\bar{\mu}}_{\nu}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar{\nu}}}}{\partial x^{\bar{\nu}}}, \tag{39}$$

$$M^{\bar{\mu}}_{;\bar{\nu};\bar{\nu}}=\frac{\partial^2 M^{\bar{\mu}}}{\partial x^{\bar{\nu}}\,\partial x^{\bar{\nu}}}+\frac{\partial}{\partial x^{\bar{\nu}}}\left(M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\left(\frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}}\right)\right)+\frac{\partial M^{\nu}}{\partial x^{\bar{\nu}}}\frac{1}{2}g^{\nu\bar{\mu}}\left(\frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}}\right)+M^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\bar{\mu}}\left(\frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}}\right)-\frac{\partial M^{\bar{\mu}}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}}\right)-$$

$$M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}}\right)=\frac{\partial^2 M^{\bar{\mu}}}{\partial x^{\bar{\nu}}\,\partial x^{\bar{\nu}}}+\frac{\partial}{\partial x^{\bar{\nu}}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\bar{\mu}}_{\bar{\nu}}}{\partial x^{\nu}}\right)+\frac{\partial M^{\nu}}{\partial x^{\bar{\nu}}}\frac{1}{2}\frac{\partial g^{\bar{\mu}}_{\bar{\nu}}}{\partial x^{\nu}}+M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\bar{\nu}}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\bar{\mu}}_{\bar{\nu}}}{\partial x^{\nu}}-\frac{\partial M^{\bar{\mu}}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar{\nu}}}{\partial x^{\bar{\nu}}}-M^{\nu}\frac{1}{2}\frac{\partial g^{\bar{\mu}}_{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar{\nu}}}{\partial x^{\bar{\nu}}} \tag{40}$$

from (38). (39),(40) must rewrite it in

$$M^{\mu}_{;\nu;\nu}=\frac{\partial^2 M^{\mu}}{\partial x^{\nu}\,\partial x^{\nu}}+\frac{\partial}{\partial x^{\nu}}\left(M^{\sigma}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\nu}}\right)+\frac{\partial M^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\nu}}+M^{\sigma}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\sigma}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\nu}}-\frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}-M^{\sigma}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}, \tag{41}$$

$$M^{\mu}_{;\nu;\nu}=\frac{\partial^2 M^{\mu}}{\partial x^{\nu}\,\partial x^{\nu}}+\frac{\partial}{\partial x^{\nu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right)+\frac{\partial M^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}+M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}-\frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}-M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}} \tag{42}$$

by (39),(40) being a tensor equation. I get the conclusion that (41),(42) doesn't satisfy Binary Law from Definision6.

I get the conclusion that (41),(42) isn't an equation of the tensor satisfying Binary Law because (41),(42) doesn't satisfy Binary Law.

I rewrite two existing index ν in each term of (41),(42) in index μ using Definision4 and get

$$M^{\mu}_{;\mu;\mu}=\frac{\partial^2 M^{\mu}}{\partial x^{\mu}\,\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)+\frac{\partial M^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}+M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}-\frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}-M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}=\frac{\partial^2 M^{\mu}}{\partial x^{\mu}\,\partial x^{\mu}}+$$

$$\frac{\partial}{\partial x^{\mu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)+\frac{\partial M^{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}+M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}-\frac{\partial M^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}-M^{\nu}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}, \tag{43}$$

$$M^{\mu}_{;\mu;\mu}=\frac{\partial^2 M^{\mu}}{\partial x^{\mu}\,\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)+\frac{\partial M^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}+M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}-\frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}-M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}=\frac{\partial^2 M^{\mu}}{\partial x^{\mu}\,\partial x^{\mu}}+$$

$$\frac{\partial}{\partial x^{\mu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)+\frac{\partial M^{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}+M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}-\frac{\partial M^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}-M^{\nu}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}. \tag{44}$$

I get the conclusion that (43),(44) does satisfy Binary Law from Definision6. I get the conclusion that (43),(44) is an equation of the tensor satisfying Binary Law because (43),(44) does satisfy Binary Law. I get

$$M^{\mu}_{;\mu;\mu}=\frac{\partial^2 M^{\mu}}{\partial x^{\mu}\,\partial x^{\mu}} \tag{45}$$

in consideration of Definision7 for (43),(44). Because the second term of the right side of (45) doesn't exist,

$$M^{\mu}_{;\nu;\nu}=\frac{\partial^2 M^{\mu}}{\partial x^{\nu}\,\partial x^{\nu}} \tag{46}$$

can rewrite (45) using Definision4. I rewrite two existing index ν in each term of (41),(42) in index μ using Definision2 and get

$$
\begin{aligned}
M^{\mu;\mu;\mu} = {} & \frac{\partial^2 M^{\mu}}{\partial x_{\mu}\partial x_{\mu}} + \frac{\partial}{\partial x_{\mu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right) + \frac{\partial M^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}} + M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}} - \frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} - M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} = \frac{\partial^2 M^{\mu}}{\partial x_{\mu}\partial x_{\mu}} + \\
& \frac{\partial}{\partial x_{\mu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x_{\mu}}\right) + \frac{\partial M^{\nu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x_{\mu}} + M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\nu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x_{\mu}} - \frac{\partial M^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} - M^{\nu}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}},
\end{aligned}
\tag{47}
$$

$$
\begin{aligned}
M^{\mu;\mu;\mu} = {} & \frac{\partial^2 M^{\mu}}{\partial x_{\mu}\partial x_{\mu}} + \frac{\partial}{\partial x_{\mu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right) + \frac{\partial M^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}} + M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}} - \frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} - M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} = \frac{\partial^2 M^{\mu}}{\partial x_{\mu}\partial x_{\mu}} + \\
& \frac{\partial}{\partial x_{\mu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right) + \frac{\partial M^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}} + M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}} - \frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} - M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}}.
\end{aligned}
\tag{48}
$$

I get the conclusion that (47),(48) does satisfy Binary Law from Definision6. I get the conclusion that (47),(48) is an equation of the tensor satisfying Binary Law because (47),(48) does satisfy Binary Law.

Proposition5. $M^{\mu}_{;\nu;\nu;\nu} = \frac{\partial^3 M^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}$ is established in tensor satisfying Binary Law.

Proof. I mark free index of Definision12 by the same reason which I expressed in Proposition1 and get

$$
\begin{aligned}
M^{\bar{\mu}}_{;\bar{\nu};\bar{\sigma};\bar{\lambda}} = {} & \frac{\partial^3 M^{\bar{\mu}}}{\partial x^{\bar{\nu}}\partial x^{\bar{\sigma}}\partial x^{\bar{\lambda}}} + \frac{\partial^2}{\partial x^{\bar{\sigma}}\partial x^{\bar{\lambda}}}\left(M^{\tau}\frac{1}{2}g^{\epsilon\bar{\mu}}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\tau}} - \frac{\partial g_{\tau\bar{\nu}}}{\partial x^{\epsilon}}\right)\right) + \frac{\partial}{\partial x^{\bar{\lambda}}}\left(\frac{\partial M^{\iota}}{\partial x^{\bar{\nu}}}\frac{1}{2}g^{\epsilon\bar{\mu}}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^{\bar{\sigma}}} + \frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\iota}} - \frac{\partial g_{\iota\bar{\sigma}}}{\partial x^{\epsilon}}\right)\right) + \\
& \frac{\partial}{\partial x^{\bar{\lambda}}}\left(M^{\tau}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\tau}} - \frac{\partial g_{\tau\bar{\nu}}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\bar{\mu}}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^{\bar{\sigma}}} + \frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\iota}} - \frac{\partial g_{\iota\bar{\sigma}}}{\partial x^{\epsilon}}\right)\right) - \frac{\partial}{\partial x^{\bar{\lambda}}}\left(\frac{\partial M^{\bar{\mu}}}{\partial x^{\iota}}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\bar{\sigma}}} + \frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\sigma}}}{\partial x^{\epsilon}}\right)\right) - \\
& \frac{\partial}{\partial x^{\bar{\lambda}}}\left(M^{\tau}\frac{1}{2}g^{\epsilon\bar{\mu}}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^{\iota}} + \frac{\partial g_{\iota\epsilon}}{\partial x^{\tau}} - \frac{\partial g_{\tau\iota}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\bar{\sigma}}} + \frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\sigma}}}{\partial x^{\epsilon}}\right)\right) + \frac{\partial^2 M^{\kappa}}{\partial x^{\bar{\nu}}\partial x^{\bar{\sigma}}}\frac{1}{2}g^{\epsilon\bar{\mu}}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\kappa}} - \frac{\partial g_{\kappa\bar{\lambda}}}{\partial x^{\epsilon}}\right) + \\
& \frac{\partial}{\partial x^{\bar{\sigma}}}\left(M^{\tau}\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\tau}} - \frac{\partial g_{\tau\bar{\nu}}}{\partial x^{\epsilon}}\right)\right)\frac{1}{2}g^{\epsilon\bar{\mu}}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\kappa}} - \frac{\partial g_{\kappa\bar{\lambda}}}{\partial x^{\epsilon}}\right) + \frac{\partial M^{\iota}}{\partial x^{\bar{\nu}}}\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^{\bar{\sigma}}} + \frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\iota}} - \frac{\partial g_{\iota\bar{\sigma}}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\bar{\mu}}\Big(\frac{\partial g_{\kappa\epsilon}}{\partial x^{\bar{\lambda}}} + \\
& \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\kappa}} - \frac{\partial g_{\kappa\bar{\lambda}}}{\partial x^{\epsilon}}\Big) + M^{\tau}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\tau}} - \frac{\partial g_{\tau\bar{\nu}}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^{\bar{\sigma}}} + \frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\iota}} - \frac{\partial g_{\iota\bar{\sigma}}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\bar{\mu}}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\kappa}} - \frac{\partial g_{\kappa\bar{\lambda}}}{\partial x^{\epsilon}}\right) - \\
& \frac{\partial M^{\kappa}}{\partial x^{\iota}}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\bar{\sigma}}} + \frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\sigma}}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\bar{\mu}}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\kappa}} - \frac{\partial g_{\kappa\bar{\lambda}}}{\partial x^{\epsilon}}\right) - M^{\tau}\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^{\iota}} + \frac{\partial g_{\iota\epsilon}}{\partial x^{\tau}} - \frac{\partial g_{\tau\iota}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\iota}\Big(\frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\bar{\sigma}}} + \frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\bar{\nu}}} - \\
& \frac{\partial g_{\bar{\nu}\bar{\sigma}}}{\partial x^{\epsilon}}\Big)\frac{1}{2}g^{\epsilon\bar{\mu}}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\kappa}} - \frac{\partial g_{\kappa\bar{\lambda}}}{\partial x^{\epsilon}}\right) - \frac{\partial^2 M^{\bar{\mu}}}{\partial x^{\kappa}\partial x^{\bar{\sigma}}}\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\lambda}}}{\partial x^{\epsilon}}\right) - \frac{\partial}{\partial x^{\bar{\sigma}}}\Big(M^{\tau}\frac{1}{2}g^{\epsilon\bar{\mu}}\Big(\frac{\partial g_{\tau\epsilon}}{\partial x^{\kappa}} + \frac{\partial g_{\kappa\epsilon}}{\partial x^{\tau}} - \\
& \frac{\partial g_{\tau\kappa}}{\partial x^{\epsilon}}\Big)\Big)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\lambda}}}{\partial x^{\epsilon}}\right) - \frac{\partial M^{\iota}}{\partial x^{\kappa}}\frac{1}{2}g^{\epsilon\bar{\mu}}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^{\bar{\sigma}}} + \frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\iota}} - \frac{\partial g_{\iota\bar{\sigma}}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\lambda}}}{\partial x^{\epsilon}}\right) - M^{\tau}\frac{1}{2}g^{\epsilon\iota}\Big(\frac{\partial g_{\tau\epsilon}}{\partial x^{\kappa}} + \\
& \frac{\partial g_{\kappa\epsilon}}{\partial x^{\tau}} - \frac{\partial g_{\tau\kappa}}{\partial x^{\epsilon}}\Big)\frac{1}{2}g^{\epsilon\bar{\mu}}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^{\bar{\sigma}}} + \frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\iota}} - \frac{\partial g_{\iota\bar{\sigma}}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\lambda}}}{\partial x^{\epsilon}}\right) + \frac{\partial M^{\bar{\mu}}}{\partial x^{\iota}}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^{\bar{\sigma}}} + \frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\kappa}} - \frac{\partial g_{\kappa\bar{\sigma}}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\kappa}\Big(\frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\bar{\lambda}}} + \\
& \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\lambda}}}{\partial x^{\epsilon}}\Big) + M^{\tau}\frac{1}{2}g^{\epsilon\bar{\mu}}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^{\iota}} + \frac{\partial g_{\iota\epsilon}}{\partial x^{\tau}} - \frac{\partial g_{\tau\iota}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\kappa\epsilon}}{\partial x^{\bar{\sigma}}} + \frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\kappa}} - \frac{\partial g_{\kappa\bar{\sigma}}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\lambda}}}{\partial x^{\epsilon}}\right) - \\
& \frac{\partial^2 M^{\bar{\mu}}}{\partial x^{\bar{\nu}}\partial x^{\kappa}}\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\bar{\sigma}}} - \frac{\partial g_{\bar{\sigma}\bar{\lambda}}}{\partial x^{\epsilon}}\right) - \frac{\partial}{\partial x^{\kappa}}\left(M^{\tau}\frac{1}{2}g^{\epsilon\bar{\mu}}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\tau}} - \frac{\partial g_{\tau\bar{\nu}}}{\partial x^{\epsilon}}\right)\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\bar{\sigma}}} - \frac{\partial g_{\bar{\sigma}\bar{\lambda}}}{\partial x^{\epsilon}}\right) - \\
& \frac{\partial M^{\iota}}{\partial x^{\bar{\nu}}}\frac{1}{2}g^{\epsilon\bar{\mu}}\left(\frac{\partial g_{\iota\epsilon}}{\partial x^{\kappa}} + \frac{\partial g_{\kappa\epsilon}}{\partial x^{\iota}} - \frac{\partial g_{\iota\kappa}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\bar{\sigma}}} - \frac{\partial g_{\bar{\sigma}\bar{\lambda}}}{\partial x^{\epsilon}}\right) - M^{\tau}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\tau\epsilon}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\tau}} - \frac{\partial g_{\tau\bar{\nu}}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\bar{\mu}}\Big(\frac{\partial g_{\iota\epsilon}}{\partial x^{\kappa}} + \frac{\partial g_{\kappa\epsilon}}{\partial x^{\iota}} - \\
& \frac{\partial g_{\iota\kappa}}{\partial x^{\epsilon}}\Big)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\bar{\sigma}}} - \frac{\partial g_{\bar{\sigma}\bar{\lambda}}}{\partial x^{\epsilon}}\right) + \frac{\partial M^{\bar{\mu}}}{\partial x^{\iota}}\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\kappa}} + \frac{\partial g_{\kappa\epsilon}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\kappa}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\bar{\sigma}}} - \frac{\partial g_{\bar{\sigma}\bar{\lambda}}}{\partial x^{\epsilon}}\right) + M^{\tau}\frac{1}{2}g^{\epsilon\bar{\mu}}\Big(\frac{\partial g_{\tau\epsilon}}{\partial x^{\iota}} + \\
& \frac{\partial g_{\iota\epsilon}}{\partial x^{\tau}} - \frac{\partial g_{\tau\iota}}{\partial x^{\epsilon}}\Big)\frac{1}{2}g^{\epsilon\iota}\left(\frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\kappa}} + \frac{\partial g_{\kappa\epsilon}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\kappa}}{\partial x^{\epsilon}}\right)\frac{1}{2}g^{\epsilon\kappa}\left(\frac{\partial g_{\bar{\sigma}\epsilon}}{\partial x^{\bar{\lambda}}} + \frac{\partial g_{\bar{\lambda}\epsilon}}{\partial x^{\bar{\sigma}}} - \frac{\partial g_{\bar{\sigma}\bar{\lambda}}}{\partial x^{\epsilon}}\right).
\end{aligned}
\tag{49}
$$

If all coordinate systems satisfy Binary Law, I get

$$M^{\bar{\mu}}_{;\bar{\nu};\bar{\nu};\bar{\nu}} = \frac{\partial^3 M^{\bar{\mu}}}{\partial x^{\bar{\nu}}\,\partial x^{\bar{\nu}}\,\partial x^{\bar{\nu}}} + \frac{\partial^2}{\partial x^{\bar{\nu}}\,\partial x^{\bar{\nu}}}\Bigg(M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg)\Bigg) + \frac{\partial}{\partial x^{\bar{\nu}}}\Bigg(\frac{\partial M^{\nu}}{\partial x^{\bar{\nu}}}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg)\Bigg) +$$

$$\frac{\partial}{\partial x^{\bar{\nu}}}\Bigg(M^{\nu}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\epsilon}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg)\Bigg) - \frac{\partial}{\partial x^{\bar{\nu}}}\Bigg(\frac{\partial M^{\bar{\mu}}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg)\Bigg) -$$

$$\frac{\partial}{\partial x^{\bar{\nu}}}\Bigg(M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}} + \frac{\partial g_{\nu\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg)\Bigg) + \frac{\partial^2 M^{\nu}}{\partial x^{\bar{\nu}}\,\partial x^{\bar{\nu}}}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg) +$$

$$\frac{\partial}{\partial x^{\bar{\nu}}}\Bigg(M^{\nu}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg)\Bigg)\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg) + \frac{\partial M^{\nu}}{\partial x^{\bar{\nu}}}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} +$$

$$\frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg) + M^{\nu}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg) -$$

$$\frac{\partial M^{\nu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg) - M^{\nu}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}} + \frac{\partial g_{\nu\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} -$$

$$\frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg) - \frac{\partial^2 M^{\bar{\mu}}}{\partial x^{\nu}\,\partial x^{\bar{\nu}}}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg) - \frac{\partial}{\partial x^{\bar{\nu}}}\Bigg(M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}} + \frac{\partial g_{\nu\nu}}{\partial x^{\nu}} -$$

$$\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\bigg)\Bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg) - \frac{\partial M^{\nu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg) - M^{\nu}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}} +$$

$$\frac{\partial g_{\nu\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg) + \frac{\partial M^{\bar{\mu}}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} +$$

$$\frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg) + M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}} + \frac{\partial g_{\nu\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg) -$$

$$\frac{\partial^2 M^{\bar{\mu}}}{\partial x^{\bar{\nu}}\,\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg) - \frac{\partial}{\partial x^{\nu}}\Bigg(M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg)\Bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg) -$$

$$\frac{\partial M^{\nu}}{\partial x^{\bar{\nu}}}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}} + \frac{\partial g_{\nu\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg) - M^{\nu}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\bar{\nu}}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}} + \frac{\partial g_{\nu\nu}}{\partial x^{\nu}} -$$

$$\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg) + \frac{\partial M^{\bar{\mu}}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\nu}} + \frac{\partial g_{\nu\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg) + M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}} +$$

$$\frac{\partial g_{\nu\nu}}{\partial x^{\nu}} - \frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\nu}} + \frac{\partial g_{\nu\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}} + \frac{\partial g_{\bar{\nu}\nu}}{\partial x^{\bar{\nu}}} - \frac{\partial g_{\bar{\nu}\bar{\nu}}}{\partial x^{\nu}}\bigg). \quad (50)$$

from (49). I put a mark in (50) to express the difference in computation sequence. I get

$$M^{\bar{\mu}}_{;\bar{\nu};\bar{\nu};\bar{\nu}} = \frac{\partial^3 M^{\bar{\mu}}}{\partial x^{\bar{\nu}}\,\partial x^{\bar{\nu}}\,\partial x^{\bar{\nu}}} + \frac{\partial^2}{\partial x^{\bar{\nu}}\,\partial x^{\bar{\nu}}}\Bigg(M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg)\Bigg) + \frac{\partial}{\partial x^{\bar{\nu}}}\Bigg(\frac{\partial M^{\nu}}{\partial x^{\bar{\nu}}}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg)\Bigg) +$$

$$\frac{\partial}{\partial x^{\bar{\nu}}}\Bigg(M^{\nu}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg)\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg)\Bigg) - \frac{\partial}{\partial x^{\bar{\nu}}}\Bigg(\frac{\partial M^{\bar{\mu}}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}}\bigg)\Bigg) - \frac{\partial}{\partial x^{\bar{\nu}}}\Bigg(M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}}\bigg)\Bigg) +$$

$$\frac{\partial^2 M^{\nu}}{\partial x^{\bar{\nu}}\,\partial x^{\bar{\nu}}}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg) + \frac{\partial}{\partial x^{\bar{\nu}}}\Bigg(M^{\nu}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg)\Bigg)\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg) + \frac{\partial M^{\nu}}{\partial x^{\bar{\nu}}}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg)\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg) +$$

$$M^{\nu}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg)\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg) - \frac{\partial M^{\nu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}}\bigg)\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg) -$$

$$M^{\nu}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}}\bigg)\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg) - \frac{\partial^2 M^{\bar{\mu}}}{\partial x^{\nu}\,\partial x^{\bar{\nu}}}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}}\bigg) - \frac{\partial}{\partial x^{\bar{\nu}}}\Bigg(M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}}\bigg)\Bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}}\bigg) -$$

$$\frac{\partial M^{\nu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}}\bigg) - M^{\nu}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}}\bigg) + \frac{\partial M^{\bar{\mu}}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}}\bigg) +$$

$$M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}}\bigg) - \frac{\partial^2 M^{\bar{\mu}}}{\partial x^{\bar{\nu}}\,\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}}\bigg) - \frac{\partial}{\partial x^{\nu}}\Bigg(M^{\nu}\frac{1}{2}g^{\nu\bar{\mu}}\bigg(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar{\nu}}}\bigg)\Bigg)\frac{1}{2}g^{\nu\nu}\bigg(\frac{\widehat{\partial g_{\bar{\nu}\nu}}}{\partial x^{\bar{\nu}}}\bigg) -$$

$$\frac{\partial M^{\nu}}{\partial x^{\bar\nu}}\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\bar\nu\nu}}}{\partial x^{\bar\nu}}\right)-M^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\bar\nu}}\right)\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\bar\nu\nu}}}{\partial x^{\bar\nu}}\right)+\frac{\partial M^{\bar\mu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\bar\nu\nu}}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\bar\nu\nu}}}{\partial x^{\bar\nu}}\right)+$$

$$M^{\nu}\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\widehat{\partial g_{\nu\nu}}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\bar\nu\nu}}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\widehat{\partial g_{\bar\nu\nu}}}{\partial x^{\bar\nu}}\right)=\frac{\partial^{3}M^{\bar\mu}}{\partial x^{\bar\nu}\partial x^{\bar\nu}\partial x^{\bar\nu}}+\frac{\partial^{2}}{\partial x^{\bar\nu}\partial x^{\bar\nu}}\left(M^{\nu}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\bar\nu}}\right)+\frac{\partial}{\partial x^{\bar\nu}}\left(\frac{\partial M^{\nu}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\bar\nu}}\right)+$$

$$\frac{\partial}{\partial x^{\bar\nu}}\left(M^{\nu}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\nu}}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\bar\nu}}\right)-\frac{\partial}{\partial x^{\bar\nu}}\left(\frac{\partial M^{\bar\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}\right)-\frac{\partial}{\partial x^{\bar\nu}}\left(M^{\nu}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}\right)+\frac{\partial^{2}M^{\nu}}{\partial x^{\bar\nu}\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\bar\nu}}+\frac{\partial}{\partial x^{\bar\nu}}\left(M^{\nu}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\nu}}}{\partial x^{\bar\nu}}\right)\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\bar\nu}}+$$

$$\frac{\partial M^{\nu}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\nu}}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\bar\nu}}+M^{\nu}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\nu}}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\nu}}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\bar\nu}}-\frac{\partial M^{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\bar\nu}}-M^{\nu}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\nu}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\bar\nu}}-\frac{\partial^{2}M^{\bar\mu}}{\partial x^{\nu}\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}-$$

$$\frac{\partial}{\partial x^{\bar\nu}}\left(M^{\nu}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\nu}}\right)\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}-\frac{\partial M^{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}-M^{\nu}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\nu}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}+\frac{\partial M^{\bar\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}+M^{\nu}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}-$$

$$\frac{\partial^{2}M^{\bar\mu}}{\partial x^{\bar\nu}\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}-\frac{\partial}{\partial x^{\nu}}\left(M^{\nu}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\bar\nu}}\right)\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}-\frac{\partial M^{\nu}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}-M^{\nu}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}+\frac{\partial M^{\bar\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}+M^{\nu}\frac{1}{2}\frac{\widehat{\partial g^{\bar\mu}_{\nu}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\nu}_{\bar\nu}}}{\partial x^{\bar\nu}}, \tag{51}$$

$$M^{\bar\mu}_{;\bar\nu;\bar\nu;\bar\nu}=\frac{\partial^{3}M^{\bar\mu}}{\partial x^{\bar\nu}\partial x^{\bar\nu}\partial x^{\bar\nu}}+\frac{\partial^{2}}{\partial x^{\bar\nu}\partial x^{\bar\nu}}\left(M^{\nu}\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)\right)+\frac{\partial}{\partial x^{\bar\nu}}\left(\frac{\partial M^{\nu}}{\partial x^{\bar\nu}}\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)\right)+\frac{\partial}{\partial x^{\bar\nu}}\left(M^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)\right)-$$

$$\frac{\partial}{\partial x^{\bar\nu}}\left(\frac{\partial M^{\bar\mu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)\right)-\frac{\partial}{\partial x^{\bar\nu}}\left(M^{\nu}\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)\right)+\frac{\partial^{2}M^{\nu}}{\partial x^{\bar\nu}\partial x^{\bar\nu}}\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)+$$

$$\frac{\partial}{\partial x^{\bar\nu}}\left(M^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)+\frac{\partial M^{\nu}}{\partial x^{\bar\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)+M^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)-$$

$$\frac{\partial M^{\nu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)-M^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)-\frac{\partial^{2}M^{\bar\mu}}{\partial x^{\nu}\partial x^{\bar\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)-$$

$$\frac{\partial}{\partial x^{\bar\nu}}\left(M^{\nu}\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)-\frac{\partial M^{\nu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)-M^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)+$$

$$\frac{\partial M^{\bar\mu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)+M^{\nu}\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)-\frac{\partial^{2}M^{\bar\mu}}{\partial x^{\bar\nu}\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)-$$

$$\frac{\partial}{\partial x^{\nu}}\left(M^{\nu}\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)-\frac{\partial M^{\nu}}{\partial x^{\bar\nu}}\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)-M^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)+$$

$$\frac{\partial M^{\bar\mu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\bar\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)+M^{\nu}\frac{1}{2}g^{\nu\bar\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\bar\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\bar\nu\nu}}{\partial x^{\bar\nu}}\right)=\frac{\partial^{3}M^{\bar\mu}}{\partial x^{\bar\nu}\partial x^{\bar\nu}\partial x^{\bar\nu}}+\frac{\partial^{2}}{\partial x^{\bar\nu}\partial x^{\bar\nu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}\right)+$$

$$\frac{\partial}{\partial x^{\bar\nu}}\left(\frac{\partial M^{\nu}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}\right)+\frac{\partial}{\partial x^{\bar\nu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}\right)-\frac{\partial}{\partial x^{\bar\nu}}\left(\frac{\partial M^{\bar\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}\right)-\frac{\partial}{\partial x^{\bar\nu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}\right)+\frac{\partial^{2}M^{\nu}}{\partial x^{\bar\nu}\partial x^{\bar\nu}}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}+$$

$$\frac{\partial}{\partial x^{\bar\nu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}+\frac{\partial M^{\nu}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}+M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}-\frac{\partial M^{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}-M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}-\frac{\partial^{2}M^{\bar\mu}}{\partial x^{\nu}\partial x^{\bar\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\nu}}-$$

$$\frac{\partial}{\partial x^{\bar\nu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}-\frac{\partial M^{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}-M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}+\frac{\partial M^{\bar\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}+M^{\nu}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}-\frac{\partial^{2}M^{\bar\mu}}{\partial x^{\bar\nu}\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}-$$

$$\frac{\partial}{\partial x^{\nu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}-\frac{\partial M^{\nu}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}-M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}+\frac{\partial M^{\bar\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}+M^{\nu}\frac{1}{2}\frac{\partial g^{\bar\mu}_{\bar\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\bar\nu}}{\partial x^{\bar\nu}} \tag{52}$$

from (50). (51),(52) must rewrite it in

$$M^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^{3}M^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}+\frac{\partial^{2}}{\partial x^{\nu}\partial x^{\nu}}\left(M^{\sigma}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\nu}}\right)+\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial M^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\nu}}\right)+\frac{\partial}{\partial x^{\nu}}\left(M^{\sigma}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\sigma}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\nu}}\right)-\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}\right)-$$

$$\frac{\partial}{\partial x^{\nu}}\left(M^{\sigma}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}\right)+\frac{\partial^{2}M^{\sigma}}{\partial x^{\nu}\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\nu}}+\frac{\partial}{\partial x^{\nu}}\left(M^{\sigma}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\sigma}}}{\partial x^{\nu}}\right)\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\nu}}+\frac{\partial M^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\sigma}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\nu}}+M^{\sigma}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\sigma}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\sigma}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\nu}}-$$

$$\frac{\partial M^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\nu}}-M^{\sigma}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\sigma}}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\nu}}-\frac{\partial^{2}M^{\mu}}{\partial x^{\sigma}\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}-\frac{\partial}{\partial x^{\nu}}\left(M^{\sigma}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\sigma}}\right)\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}-\frac{\partial M^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}-$$

$$M^{\sigma}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\sigma}}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}+\frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\sigma}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}+M^{\sigma}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\sigma}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}-\frac{\partial^{2}M^{\mu}}{\partial x^{\nu}\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}-\frac{\partial}{\partial x^{\sigma}}\left(M^{\sigma}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\nu}}\right)\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}-$$

$$\frac{\partial M^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}-M^{\sigma}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\sigma}}}{\partial x^{\nu}}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}+\frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}+M^{\sigma}\frac{1}{2}\frac{\widehat{\partial g^{\mu}_{\sigma}}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\sigma}}\frac{1}{2}\frac{\widehat{\partial g^{\sigma}_{\nu}}}{\partial x^{\nu}}, \tag{53}$$

$$M^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^{3}M^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}+\frac{\partial^{2}}{\partial x^{\nu}\partial x^{\nu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right)+\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial M^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right)+\frac{\partial}{\partial x^{\nu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right)-\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}\right)-$$

$$\frac{\partial}{\partial x^{\nu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}\right)+\frac{\partial^{2}M^{\sigma}}{\partial x^{\nu}\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}+\frac{\partial}{\partial x^{\nu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\right)\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}+\frac{\partial M^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}+M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}-$$

$$\frac{\partial M^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}-M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}-\frac{\partial^{2}M^{\mu}}{\partial x^{\sigma}\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}-\frac{\partial}{\partial x^{\nu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}-\frac{\partial M^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}-$$

$$M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}+\frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}+M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}-\frac{\partial^{2}M^{\mu}}{\partial x^{\nu}\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}-\frac{\partial}{\partial x^{\sigma}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right)\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}-$$

$$\frac{\partial M^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}-M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}+\frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}+M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}} \tag{54}$$

by (51),(52) being a tensor equation. I get the conclusion that (53),(54) doesn't satisfy Binary Law from Definision6.

I get the conclusion that (53),(54) isn't an equation of the tensor satisfying Binary Law because (53),(54) doesn't satisfy Binary Law.

I rewrite three existing index ν in each term of (53),(54) in index μ using Definision4 and get

$$-M^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^{3}M^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial^{2}}{\partial x^{\mu}\partial x^{\mu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)-\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial M^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)-\frac{\partial}{\partial x^{\mu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)+\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)+$$

$$\frac{\partial}{\partial x^{\mu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)-\frac{\partial^{2}M^{\sigma}}{\partial x^{\mu}\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}-\frac{\partial}{\partial x^{\mu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}-\frac{\partial M^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}-M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}+$$

$$\frac{\partial M^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}+M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}+\frac{\partial^{2}M^{\mu}}{\partial x^{\sigma}\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}+\frac{\partial M^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}+$$

$$M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}-\frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}-M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}+\frac{\partial^{2}M^{\mu}}{\partial x^{\mu}\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}+$$

$$\frac{\partial M^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}+M^{\sigma}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}-\frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}-M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}=-\frac{\partial^{3}M^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-$$

$$\frac{\partial^{2}}{\partial x^{\mu}\partial x^{\mu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)-\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial M^{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)-\frac{\partial}{\partial x^{\mu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)+\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial M^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}\right)+\frac{\partial}{\partial x^{\mu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}\right)-$$

$$\frac{\partial^{2}M^{\nu}}{\partial x^{\mu}\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}-\frac{\partial}{\partial x^{\mu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}-\frac{\partial M^{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}-M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}+\frac{\partial M^{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}+$$

$$M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}+\frac{\partial^{2}M^{\mu}}{\partial x^{\nu}\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}+\frac{\partial M^{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}+M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}-$$

$$\frac{\partial M^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}-M^{\nu}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}+\frac{\partial^{2}M^{\mu}}{\partial x^{\mu}\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}+\frac{\partial}{\partial x^{\nu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}+\frac{\partial M^{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}+$$

$$M^{\nu}\frac{1}{2}\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}-\frac{\partial M^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}-M^{\nu}\frac{1}{2}\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}, \tag{55}$$

$$\begin{aligned}
-M^{\mu}_{;\mu;\mu;\mu} = {} & -\frac{\partial^3 M^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu} - \frac{\partial^2}{\partial x^\mu\partial x^\mu}\left(M^\sigma \frac{1}{2} \frac{\partial g^\mu_\mu}{\partial x^\sigma}\right) - \frac{\partial}{\partial x^\mu}\left(\frac{\partial M^\sigma}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\sigma}\right) - \frac{\partial}{\partial x^\mu}\left(M^\sigma\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\sigma}\right) + \frac{\partial}{\partial x^\mu}\left(\frac{\partial M^\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu}\right) + \\
& \frac{\partial}{\partial x^\mu}\left(M^\sigma\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu}\right) - \frac{\partial^2 M^\sigma}{\partial x^\mu\partial x^\mu}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\sigma} - \frac{\partial}{\partial x^\mu}\left(M^\sigma\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\right)\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\sigma} - \frac{\partial M^\sigma}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\sigma} - M^\sigma\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\sigma} + \\
& \frac{\partial M^\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\sigma} + M^\sigma\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\sigma} + \frac{\partial^2 M^\mu}{\partial x^\sigma\partial x^\mu}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu} + \frac{\partial}{\partial x^\mu}\left(M^\sigma\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu} + \frac{\partial M^\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu} + \\
& M^\sigma\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu} - \frac{\partial M^\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu} - M^\sigma\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu} + \frac{\partial^2 M^\mu}{\partial x^\mu\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu} + \frac{\partial}{\partial x^\sigma}\left(M^\sigma\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\sigma}\right)\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu} + \\
& \frac{\partial M^\sigma}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu} + M^\sigma\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu} - \frac{\partial M^\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu} - M^\sigma\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\sigma_\mu}{\partial x^\mu} = -\frac{\partial^3 M^\mu}{\partial x^\mu\partial x^\mu\partial x^\mu} - \\
& \frac{\partial^2}{\partial x^\mu\partial x^\mu}\left(M^\nu\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\nu}\right) - \frac{\partial}{\partial x^\mu}\left(\frac{\partial M^\nu}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\nu}\right) - \frac{\partial}{\partial x^\mu}\left(M^\nu\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\nu}\right) + \frac{\partial}{\partial x^\mu}\left(\frac{\partial M^\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu}\right) + \frac{\partial}{\partial x^\mu}\left(M^\nu\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu}\right) - \\
& \frac{\partial^2 M^\nu}{\partial x^\mu\partial x^\mu}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\nu} - \frac{\partial}{\partial x^\mu}\left(M^\nu\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\nu}\right)\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\nu} - \frac{\partial M^\nu}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\nu} - M^\nu\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\nu} + \frac{\partial M^\nu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\nu} + \\
& M^\nu\frac{1}{2}\frac{\partial g^\nu_\nu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\nu} + \frac{\partial^2 M^\mu}{\partial x^\nu\partial x^\mu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu} + \frac{\partial}{\partial x^\mu}\left(M^\nu\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu} + \frac{\partial M^\nu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu} + M^\nu\frac{1}{2}\frac{\partial g^\nu_\nu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu} - \\
& \frac{\partial M^\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu} - M^\nu\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu} + \frac{\partial^2 M^\mu}{\partial x^\mu\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu} + \frac{\partial}{\partial x^\nu}\left(M^\nu\frac{1}{2}\frac{\partial g^\mu_\mu}{\partial x^\nu}\right)\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu} + \frac{\partial M^\nu}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu} + \\
& M^\nu\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu} - \frac{\partial M^\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\nu}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu} - M^\nu\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^\nu_\nu}{\partial x^\mu}\frac{1}{2}\frac{\partial g^\nu_\mu}{\partial x^\mu}.
\end{aligned} \tag{56}$$

I get the conclusion that (55),(56) does satisfy Binary Law from Definision6. I get the conclusion that (55),(56) is an equation of the tensor satisfying Binary Law because (55),(56) does satisfy Binary Law. I get

$$-M^{\mu}_{;\mu;\mu;\mu} = -\frac{\partial^3 M^\mu}{\partial x^\mu\partial x^\mu\partial x^\mu} \tag{57}$$

in consideration of Definision7 for (55),(56). Because the second term of the right side of (57) doesn't exist,

$$M^{\mu}_{;\nu;\nu;\nu} = \frac{\partial^3 M^\mu}{\partial x^\nu\partial x^\nu\partial x^\nu} \tag{58}$$

can rewrite (57) using Definision4. I rewrite three existing index ν in each term of (53),(54) in index μ using Definision2 and get

$$\begin{aligned}
M^{\mu;\mu;\mu;\mu} = {} & \frac{\partial^3 M^\mu}{\partial x_\mu\partial x_\mu\partial x_\mu} + \frac{\partial^2}{\partial x_\mu\partial x_\mu}\left(M^\sigma\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_\mu}\right) + \frac{\partial}{\partial x_\mu}\left(\frac{\partial M^\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_\mu}\right) + \frac{\partial}{\partial x_\mu}\left(M^\sigma\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_\mu}\right) - \frac{\partial}{\partial x_\mu}\left(\frac{\partial M^\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu}\right) - \\
& \frac{\partial}{\partial x_\mu}\left(M^\sigma\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu}\right) + \frac{\partial^2 M^\sigma}{\partial x_\mu\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_\mu} + \frac{\partial}{\partial x_\mu}\left(M^\sigma\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\right)\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_\mu} + \frac{\partial M^\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_\mu} + M^\sigma\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_\mu} - \\
& \frac{\partial M^\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_\mu} - M^\sigma\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_\mu} - \frac{\partial^2 M^\mu}{\partial x^\sigma\partial x_\mu}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu} - \frac{\partial}{\partial x_\mu}\left(M^\sigma\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu} - \frac{\partial M^\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu} - \\
& M^\sigma\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu} + \frac{\partial M^\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu} + M^\sigma\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu} - \frac{\partial^2 M^\mu}{\partial x_\mu\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu} - \frac{\partial}{\partial x^\sigma}\left(M^\sigma\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x_\mu}\right)\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu} - \\
& \frac{\partial M^\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu} - M^\sigma\frac{1}{2}\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu} + \frac{\partial M^\mu}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu} + M^\sigma\frac{1}{2}\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_\mu} = \frac{\partial^3 M^\mu}{\partial x_\mu\partial x_\mu\partial x_\mu} + \\
& \frac{\partial^2}{\partial x_\mu\partial x_\mu}\left(M^\nu\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x_\mu}\right) + \frac{\partial}{\partial x_\mu}\left(\frac{\partial M^\nu}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x_\mu}\right) + \frac{\partial}{\partial x_\mu}\left(M^\nu\frac{1}{2}\frac{\partial g^\nu_\nu}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x_\mu}\right) - \frac{\partial}{\partial x_\mu}\left(\frac{\partial M^\mu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_\mu}\right) - \frac{\partial}{\partial x_\mu}\left(M^\nu\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_\mu}\right) + \\
& \frac{\partial^2 M^\nu}{\partial x_\mu\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x_\mu} + \frac{\partial}{\partial x_\mu}\left(M^\nu\frac{1}{2}\frac{\partial g^\nu_\nu}{\partial x_\mu}\right)\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x_\mu} + \frac{\partial M^\nu}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\nu_\nu}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x_\mu} + M^\nu\frac{1}{2}\frac{\partial g^\nu_\nu}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\nu_\nu}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x_\mu} - \frac{\partial M^\nu}{\partial x^\nu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_\mu}\frac{1}{2}\frac{\partial g^\mu_\nu}{\partial x_\mu} -
\end{aligned}$$

$$M^{\nu}\frac{1}{2}\frac{\partial g_{\nu}^{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g_{\nu}^{\mu}}{\partial x_{\mu}} - \frac{\partial^2 M^{\mu}}{\partial x^{\nu}\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} - \frac{\partial}{\partial x_{\mu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} - \frac{\partial M^{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g_{\nu}^{\mu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} - M^{\nu}\frac{1}{2}\frac{\partial g_{\nu}^{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g_{\nu}^{\mu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} +$$

$$\frac{\partial M^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g_{\nu}^{\nu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} + M^{\nu}\frac{1}{2}\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g_{\nu}^{\nu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} - \frac{\partial^2 M^{\mu}}{\partial x_{\mu}\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} - \frac{\partial}{\partial x^{\nu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g_{\nu}^{\mu}}{\partial x_{\mu}}\right)\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} - \frac{\partial M^{\nu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} -$$

$$M^{\nu}\frac{1}{2}\frac{\partial g_{\nu}^{\nu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} + \frac{\partial M^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} + M^{\nu}\frac{1}{2}\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}}, \tag{59}$$

$$M^{\mu;\mu;\mu;\mu} = \frac{\partial^3 M^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}} + \frac{\partial^2}{\partial x_{\mu}\partial x_{\mu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right) + \frac{\partial}{\partial x_{\mu}}\left(\frac{\partial M^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right) + \frac{\partial}{\partial x_{\mu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right) - \frac{\partial}{\partial x_{\mu}}\left(\frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}}\right) -$$

$$\frac{\partial}{\partial x_{\mu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g_{\sigma}^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}}\right) + \frac{\partial^2 M^{\sigma}}{\partial x_{\mu}\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}} + \frac{\partial}{\partial x_{\mu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}} + \frac{\partial M^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}} + M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}} -$$

$$\frac{\partial M^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}} - M^{\sigma}\frac{1}{2}\frac{\partial g_{\sigma}^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}} - \frac{\partial^2 M^{\mu}}{\partial x^{\sigma}\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} - \frac{\partial}{\partial x_{\mu}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g_{\sigma}^{\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} - \frac{\partial M^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} -$$

$$M^{\sigma}\frac{1}{2}\frac{\partial g_{\sigma}^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} + \frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} + M^{\sigma}\frac{1}{2}\frac{\partial g_{\sigma}^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} - \frac{\partial^2 M^{\mu}}{\partial x_{\mu}\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} - \frac{\partial}{\partial x^{\sigma}}\left(M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} -$$

$$\frac{\partial M^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g_{\sigma}^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} - M^{\sigma}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g_{\sigma}^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} + \frac{\partial M^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g_{\sigma}^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} + M^{\sigma}\frac{1}{2}\frac{\partial g_{\sigma}^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g_{\sigma}^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\sigma}}{\partial x_{\mu}} = \frac{\partial^3 M^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}} +$$

$$\frac{\partial^2}{\partial x_{\mu}\partial x_{\mu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\nu}}\right) + \frac{\partial}{\partial x_{\mu}}\left(\frac{\partial M^{\nu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\nu}}\right) + \frac{\partial}{\partial x_{\mu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\nu}}\right) - \frac{\partial}{\partial x_{\mu}}\left(\frac{\partial M^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}}\right) - \frac{\partial}{\partial x_{\mu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}}\right) +$$

$$\frac{\partial^2 M^{\nu}}{\partial x_{\mu}\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\nu}} + \frac{\partial}{\partial x_{\mu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\nu}} + \frac{\partial M^{\nu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\nu}} + M^{\nu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\nu}} - \frac{\partial M^{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\nu}} -$$

$$M^{\nu}\frac{1}{2}\frac{\partial g_{\nu}^{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\nu}} - \frac{\partial^2 M^{\mu}}{\partial x^{\nu}\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} - \frac{\partial}{\partial x_{\mu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} - \frac{\partial M^{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} - M^{\nu}\frac{1}{2}\frac{\partial g_{\nu}^{\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} +$$

$$\frac{\partial M^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} + M^{\nu}\frac{1}{2}\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} - \frac{\partial^2 M^{\mu}}{\partial x_{\mu}\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} - \frac{\partial}{\partial x^{\nu}}\left(M^{\nu}\frac{1}{2}\frac{\partial g^{\mu\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} - \frac{\partial M^{\nu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} -$$

$$M^{\nu}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} + \frac{\partial M^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g_{\nu}^{\nu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}} + M^{\nu}\frac{1}{2}\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\frac{1}{2}\frac{\partial g_{\nu}^{\nu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\mu\nu}}{\partial x_{\mu}}. \tag{60}$$

I get the conclusion that (59),(60) does satisfy Binary Law from Definision6. I get the conclusion that (59),(60) is an equation of the tensor satisfying Binary Law because (59),(60) does satisfy Binary Law.

4. Voluntary Contraction

Proposition6. $\frac{\partial^3 M^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}} = M$ is established in tensor satisfying Binary Law.

Proof. $\frac{\partial^3 M^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}$ is a tensor satisfying Binary Law than Proposition5. Furthermore, $\frac{\partial^3 M^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}$ is equivalent in a component of the tensor satisfying Binary Law of rank 0 than (94). I get

$$\frac{\partial^3 M^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}} = \frac{\partial^3 M x^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}} = M\frac{\partial^3 x^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}} \tag{61}$$

from $\frac{\partial^3 M^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}$ in consideration of (63),(67). I rewrite (61) using Definision3,Definision4 and get

$$\frac{\partial^3 M^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}} = M\frac{\partial^3 x^{\mu}}{(-\partial x^{\mu})(-\partial x^{\mu})\partial x^{\nu}}$$

$$= \mathrm{M}\frac{\partial^3 x^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\nu}} = \mathrm{M}\frac{\partial^3 x^{\mu}}{\partial x^{\mu}\partial x_{\nu}\partial x^{\nu}}$$

$$= M\frac{\partial^2}{\partial x_\nu \partial x^\nu} = M\frac{\partial x^\nu}{\partial x^\nu} = M. \tag{62}$$

Proposition7. $\frac{\partial M}{\partial x^\nu} = 0, \frac{\partial M}{\partial x^\mu} = 0$ is established in tensor satisfying Binary Law.

Proof. I get

$$M^\mu = Mx^\mu, \tag{63}$$

$$M^\mu_{;\nu} = Mx^\mu_{;\nu} \tag{64}$$

from Definision20. I get

$$M^\mu_{;\nu} = M_{;\nu}x^\mu + Mx^\mu_{;\nu} = \frac{\partial M}{\partial x^\nu}x^\mu + Mx^\mu_{;\nu} \tag{65}$$

as covariant differentiation of (63) in consideration of Definision19. I get

$$\frac{\partial M}{\partial x^\nu}x^\mu = 0 \tag{66}$$

from (64),(65). As x^μ is any tensor, I get

$$\frac{\partial M}{\partial x^\nu} = 0 \tag{67}$$

from (66). μ, ν-reverses (67) and gets

$$\frac{\partial M}{\partial x^\mu} = 0. \tag{68}$$

I get

$$\frac{\partial M}{\partial \dot{x}^1} = 0, \frac{\partial M}{\partial \dot{x}^2} = 0, \frac{\partial M}{\partial x^1} = 0, \frac{\partial M}{\partial x^2} = 0 \tag{69}$$

from (67),(68) if I assume a dimensional number 2.

Proposition8. $\frac{\partial^4 M^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu \partial x^\nu} = 0$ is established in tensor satisfying Binary Law.

Proof. I get

$$\frac{\partial^4 M^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu \partial x^\nu} = 0 \tag{70}$$

from (62),(67). $\frac{\partial M}{\partial x^\nu}$ is covariant components of a vector than Definision19. On the other hand, $\frac{\partial M}{\partial x^\nu}$ is unchangeable if all coordinate system satisfies Binary Law. Therefore, I get the conclusion that $\frac{\partial M}{\partial x^\nu}$ is a component of the tensor satisfying Binary Law. As $\frac{\partial M}{\partial x^\nu}$ is tensor satisfying Binary Law, I get the conclusion that $\frac{\partial^4 M^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu \partial x^\nu}$ is tensor satisfying Binary Law than (62),(67).

5. Coordinate Transformations Equation in Tensor Satisfying Binary Law and the Voluntary Contraction for Them

Proposition9. When all coordinate systems satisfy Binary Law, $x^{\mu\nu} = \frac{\partial x^\mu}{\partial x^\nu}\frac{\partial x^\nu}{\partial x^\mu}x^{\nu\mu} = x^{\nu\mu}$ is established for $x^{\mu\nu}$ contravariant components of a tensor satisfying Binary law of the second rank.

Proof. I mark free index of Definision13 by the same reason which I expressed in Proposition1 and get

$$x^{\bar{\mu}\bar{\nu}} = \frac{\partial x^{\bar{\mu}}}{\partial x^{\sigma}} \frac{\partial x^{\bar{\nu}}}{\partial x^{\lambda}} x^{\sigma\lambda}. \tag{71}$$

If all coordinate systems satisfy Binary Law, I get

$$x^{\bar{\mu}\bar{\nu}} = \frac{\partial x^{\bar{\mu}}}{\partial x^{\nu}} \frac{\partial x^{\bar{\nu}}}{\partial x^{\nu}} x^{\nu\nu} \tag{72}$$

from (71). As I compare $x^{\nu\nu}$ with $x^{\bar{\mu}\bar{\nu}}$ here and become same about the second index each, it is problem. I think that I change $x^{\nu\nu}$ to $x^{\nu\mu}$ in order to avoid this. Therefore I rewrite dummy index ν in μ for (72) and get

$$x^{\mu\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu}} \frac{\partial x^{\nu}}{\partial x^{\mu}} x^{\nu\mu} = x^{\nu\mu}. \tag{73}$$

I get the conclusion that (73) does satisfy Binary Law from Definision6. I get the conclusion that (73) is an equation of the tensor satisfying Binary Law because (73) does satisfy Binary Law.

I rewrite (73) using Definision2,Definision3 and get

$$x_{\mu}^{\mu} = x_{\nu}^{\nu}. \tag{74}$$

I get the conclusion that $x^{\mu\nu}$ and $x^{\nu\mu}$ are equivalent in a component of the tensor satisfying Binary Law of rank 0 than (73),(74).

Proposition10. When all coordinate systems satisfy Binary Law, $x_{\mu\nu} = \frac{\partial x^{\nu}}{\partial x^{\mu}} \frac{\partial x^{\mu}}{\partial x^{\nu}} x_{\nu\mu} = x_{\nu\mu}$ is established for $x_{\mu\nu}$ contravariant components of a tensor satisfying Binary law of the second rank.

Proof. I mark free index of Definision14 by the same reason which I expressed in Proposition1 and get

$$x_{\bar{\mu}\bar{\nu}} = \frac{\partial x^{\sigma}}{\partial x^{\bar{\mu}}} \frac{\partial x^{\lambda}}{\partial x^{\bar{\nu}}} x_{\sigma\lambda}. \tag{75}$$

If all coordinate systems satisfy Binary Law, I get

$$x_{\bar{\mu}\bar{\nu}} = \frac{\partial x^{\nu}}{\partial x^{\bar{\mu}}} \frac{\partial x^{\nu}}{\partial x^{\bar{\nu}}} x_{\nu\nu} \tag{76}$$

from (75). As I compare $x_{\nu\nu}$ with $x_{\bar{\mu}\bar{\nu}}$ here and become same about the second index each, it is problem. I think that I change $x_{\nu\nu}$ to $x_{\nu\mu}$ in order to avoid this. Therefore I rewrite dummy index ν in μ for (76) and get

$$x_{\mu\nu} = \frac{\partial x^{\nu}}{\partial x^{\mu}} \frac{\partial x^{\mu}}{\partial x^{\nu}} x_{\nu\mu} = x_{\nu\mu}. \tag{77}$$

I get the conclusion that (77) does satisfy Binary Law from Definision6. I get the conclusion that (77) is an equation of the tensor satisfying Binary Law because (77) does satisfy Binary Law.

I rewrite (77) using Definision2,Definision3 and get

$$x_{\mu}^{\mu} = x_{\nu}^{\nu}. \tag{78}$$

I get the conclusion that $x_{\mu\nu}$ and $x_{\nu\mu}$ are equivalent in a component of the tensor satisfying Binary Law of rank 0 than (77),(78).

Proposition11. When all coordinate systems satisfy Binary Law, $x_{\nu}^{\mu} = \frac{\partial x^{\mu}}{\partial x^{\nu}} \frac{\partial x^{\mu}}{\partial x^{\nu}} x_{\mu}^{\nu}$ is established for x_{ν}^{μ} components of a mixed tensor satisfying Binary law of the second rank.

Proof. I mark free index of Definision15 by the same reason which I expressed in Proposition1 and get

$$x_{\bar{\nu}}^{\bar{\mu}} = \frac{\partial x^{\bar{\mu}}}{\partial x^{\sigma}} \frac{\partial x^{\lambda}}{\partial x^{\bar{\nu}}} x_{\lambda}^{\sigma}. \tag{79}$$

If all coordinate systems satisfy Binary Law, I get

$$x^{\bar{\mu}}_{\bar{\nu}} = \frac{\partial x^{\bar{\mu}}}{\partial x^{\nu}}\frac{\partial x^{\nu}}{\partial x^{\bar{\nu}}}x^{\nu}_{\nu} \tag{80}$$

from (79). As I compare x^{ν}_{ν} with $x^{\bar{\mu}}_{\bar{\nu}}$ here and become same about the second index each, it is problem. I think that I change x^{ν}_{ν} to x^{ν}_{μ} in order to avoid this. Therefore I rewrite dummy index ν in μ for (80) and get

$$x^{\mu}_{\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}x^{\nu}_{\mu}. \tag{81}$$

I get the conclusion that (81) does satisfy Binary Law from Definision6. I get the conclusion that (81) is an equation of the tensor satisfying Binary Law because (81) does satisfy Binary Law.

Proposition12. When all coordinate systems satisfy Binary Law, $x_{\mu\nu\nu} = \frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}x_{\nu\mu\mu} = \frac{\partial x^{\mu}}{\partial x^{\nu}}x_{\nu\mu\mu}$ is established for $x_{\mu\nu\nu}$ covariant components of a tensor satisfying Binary law of the third rank.

Proof. I mark free index of Definision16 by the same reason which I expressed in Proposition1 and get

$$x_{\bar{\mu}\bar{\nu}\bar{\sigma}} = \frac{\partial x^{\lambda}}{\partial x^{\bar{\mu}}}\frac{\partial x^{\iota}}{\partial x^{\bar{\nu}}}\frac{\partial x^{\epsilon}}{\partial x^{\bar{\sigma}}}x_{\lambda\iota\epsilon}. \tag{82}$$

If all coordinate systems satisfy Binary Law, I get

$$x_{\bar{\mu}\bar{\nu}\bar{\nu}} = \frac{\partial x^{\nu}}{\partial x^{\bar{\mu}}}\frac{\partial x^{\nu}}{\partial x^{\bar{\nu}}}\frac{\partial x^{\nu}}{\partial x^{\bar{\nu}}}x_{\nu\nu\nu} \tag{83}$$

from (82). As I compare $x_{\nu\nu\nu}$ with $x_{\bar{\mu}\bar{\nu}\bar{\nu}}$ here and become same about the second, the third index each, it is problem. I think that I change $x_{\nu\nu\nu}$ to $x_{\nu\mu\mu}$ in order to avoid this. Therefore I rewrite dummy index ν in μ for (83) and get

$$x_{\mu\nu\nu} = \frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}x_{\nu\mu\mu} = \frac{\partial x^{\mu}}{\partial x^{\nu}}x_{\nu\mu\mu}. \tag{84}$$

I get the conclusion that (84) does satisfy Binary Law from Definision6. I get the conclusion that (84) is an equation of the tensor satisfying Binary Law because (84) does satisfy Binary Law.

I rewrite (84) using Definision2,Definision3 and get

$$x^{\mu}_{\mu\nu} = \frac{\partial x_{\nu}}{\partial x_{\mu}}x^{\nu}_{\nu\mu}. \tag{85}$$

I get the conclusion that $x_{\mu\nu\nu}$ and $x_{\nu\mu\mu}$ are equivalent in a component of the tensor satisfying Binary Law of rank 1 than (84),(85).

Proposition13. When all coordinate systems satisfy Binary Law, $x^{\mu}_{\nu\nu} = \frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}x^{\nu}_{\mu\mu}$ is established for $x^{\mu}_{\nu\nu}$ components of a mixed tensor satisfying Binary law of the third rank.

Proof. I mark free index of Definision17 by the same reason which I expressed in Proposition1 and get

$$x^{\bar{\mu}}_{\bar{\nu}\bar{\sigma}} = \frac{\partial x^{\bar{\mu}}}{\partial x^{\lambda}}\frac{\partial x^{\iota}}{\partial x^{\bar{\nu}}}\frac{\partial x^{\epsilon}}{\partial x^{\bar{\sigma}}}x^{\lambda}_{\iota\epsilon}. \tag{86}$$

If all coordinate systems satisfy Binary Law, I get

$$x^{\bar{\mu}}_{\bar{\nu}\bar{\nu}} = \frac{\partial x^{\bar{\mu}}}{\partial x^{\nu}}\frac{\partial x^{\nu}}{\partial x^{\bar{\nu}}}\frac{\partial x^{\nu}}{\partial x^{\bar{\nu}}}x^{\nu}_{\nu\nu} \tag{87}$$

from (86). As I compare $x^{\nu}_{\nu\nu}$ with $x^{\bar{\mu}}_{\bar{\nu}\bar{\nu}}$ here and become same about the second, the third index each, it is problem. I think that I change $x^{\nu}_{\nu\nu}$ to $x^{\nu}_{\mu\mu}$ in order to avoid this. Therefore I rewrite dummy index ν in μ for (87) and get

$$x^{\mu}_{\nu\nu}=\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}x^{\nu}_{\mu\mu}. \tag{88}$$

I get the conclusion that (88) does satisfy Binary Law from Definision6.

I rewrite (88) using Definision4,Definision5 and get

$$x^{\mu}_{\mu\mu}=\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial(-x^{\nu})}{\partial(-x^{\mu})}\frac{\partial(-x^{\nu})}{\partial(-x^{\mu})}x^{\nu}_{\nu\nu}$$

$$=\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{\partial x^{\nu}}{\partial x^{\mu}}x^{\nu}_{\nu\nu}=\frac{\partial x^{\nu}}{\partial x^{\mu}}x^{\nu}_{\nu\nu}. \tag{89}$$

I get the conclusion that $x^{\mu}_{\nu\nu}$ and $x^{\nu}_{\mu\mu}$ are equivalent in a component of the tensor satisfying Binary Law of rank 1 than (88),(89).

Proposition14. When all coordinate systems satisfy Binary Law, $x^{\mu}_{\nu\nu\nu}=\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}x^{\nu}_{\mu\mu\mu}$ is established for $x^{\mu}_{\nu\nu\nu}$ components of a mixed tensor satisfying Binary law of the fourth rank.

Proof. I mark free index of Definision18 by the same reason which I expressed in Proposition1 and get

$$x^{\bar{\mu}}_{\bar{\nu}\bar{\sigma}\bar{\lambda}}=\frac{\partial x^{\bar{\mu}}}{\partial x^{\iota}}\frac{\partial x^{\epsilon}}{\partial x^{\bar{\nu}}}\frac{\partial x^{\alpha}}{\partial x^{\bar{\sigma}}}\frac{\partial x^{\beta}}{\partial x^{\bar{\lambda}}}x^{\iota}_{\epsilon\alpha\beta}. \tag{90}$$

If all coordinate systems satisfy Binary Law, I get

$$x^{\bar{\mu}}_{\bar{\nu}\bar{\nu}\bar{\nu}}=\frac{\partial x^{\bar{\mu}}}{\partial x^{\nu}}\frac{\partial x^{\nu}}{\partial x^{\bar{\nu}}}\frac{\partial x^{\nu}}{\partial x^{\bar{\nu}}}\frac{\partial x^{\nu}}{\partial x^{\bar{\nu}}}x^{\nu}_{\nu\nu\nu} \tag{91}$$

from (90). As I compare $x^{\nu}_{\nu\nu\nu}$ with $x^{\bar{\mu}}_{\bar{\nu}\bar{\nu}\bar{\nu}}$ here and become same about the second, the third, the fourth index each, it is problem. I think that I change $x^{\nu}_{\nu\nu\nu}$ to $x^{\nu}_{\mu\mu\mu}$ in order to avoid this. Therefore I rewrite dummy index ν in μ for (91) and get

$$x^{\mu}_{\nu\nu\nu}=\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}x^{\nu}_{\mu\mu\mu}. \tag{92}$$

I get the conclusion that (92) does satisfy Binary Law from Definision6. I get the conclusion that (92) is an equation of the tensor satisfying Binary Law because (92) does satisfy Binary Law.

I rewrite (92) using Definision4,Definision5 and get

$$x^{\mu}_{\mu\mu\nu}=\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial(-x^{\nu})}{\partial(-x^{\mu})}\frac{\partial(-x^{\nu})}{\partial(-x^{\mu})}\frac{\partial x^{\mu}}{\partial x^{\nu}}x^{\nu}_{\nu\nu\mu}$$

$$=\frac{\partial x^{\mu}}{\partial x^{\nu}}\frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{\partial x^{\nu}}{\partial x^{\mu}}\frac{\partial x^{\mu}}{\partial x^{\nu}}x^{\nu}_{\nu\nu\mu}=x^{\nu}_{\nu\nu\mu}. \tag{93}$$

I rewrite (93) using Definision2,Definision3 and get

$$x^{\mu\nu}_{\mu\nu}=x^{\nu\mu}_{\nu\mu}. \tag{94}$$

I get the conclusion that $x^{\mu}_{\nu\nu\nu}$ and $x^{\nu}_{\mu\mu\mu}$ are equivalent in a component of the tensor satisfying Binary Law of rank 0 than (92),(93),(94).

6. Discussion

About Voluntary Contraction

The index which can appear in consideration of Definision6 becomes only two kinds of μ,ν in the tensor satisfying Binary Law. When I consider the contraction as $\mu=\nu$ in tensor satisfying Binary Law $M^{\mu}_{\nu\nu}$. As Binary Law is not established for $\mu=\nu$, the contraction in the tensor satisfying Binary Law is not established. However, I rewrite $M^{\mu}_{\nu\nu}$ in the tensor satisfying Binary Law by Definision4 and get $M^{\mu}_{\nu\nu}=M^{\mu}_{\mu\mu}=M_{\mu}$. Thus, I do it without establishment of $\mu=\nu$ and can carry out a contraction of $M^{\mu}_{\nu\nu}$. Furthermore, I rewrite $M_{\mu\nu}$ in the tensor satisfying Binary Law by Definision2 and can get $M_{\mu\nu}=M^{\mu}_{\mu}=M$. Thus, I do it without establishment of

$\mu = \nu$ and can carry out a contraction of $M_{\mu\nu}$. The contraction which I introduced here is different from the contraction in Definision21. Therefore, I call it a "Voluntary Contraction" to distinguish it from a contraction in Definision21. In the voluntary contraction, the contraction for the tensor of the type except the mixed tensor is possible by $M_{\mu\nu} = M^{\mu}_{\mu} = M$ mentioned above.

7. Conclusions

I consider the contraction as $\mu = \nu$ for A^{μ}_{ν} which is tensor of rank 2. I illustrate flow of the contraction operation and get

$$A^{\mu}_{\nu} \rightarrow A^{\mu}_{\nu}\colon (\mu = \nu) = A^{\mu}_{\mu}. \tag{95}$$

As A^{μ}_{ν} is unchangeable even if I consider Binary Law in A^{μ}_{ν}, A^{μ}_{ν} is tensor satisfying Binary Law. When A^{μ}_{ν} is tensor satisfying Binary Law, $A^{\mu}_{\nu}\colon (\mu = \nu)$ cannot be established in (95). If $\mu = \nu$ is established, Binary Law cannot be established. In other words, the technique of the contraction by the establishment of $\mu = \nu$ in the tensor analysis is impracticable in the tensor satisfying Binary Law. I consider the contraction as $x^{\nu} = x_{\mu}$ for $A^{\mu\nu}$ which is tensor satisfying Binary Law of rank 2. I illustrate flow of the contraction operation and get

$$A^{\mu\nu} = A^{\mu\nu}\colon \left(x^{\nu} = x_{\mu}\right) = A^{\mu}_{\mu}. \tag{96}$$

The number of free index is reduced to 0 from 2 together in (95) and (96). I regard the operation in (96) as a contraction like (95). As $\mu = \nu$ is not mathematics law and is information, it is expressed like $A^{\mu}_{\nu} \rightarrow A^{\mu}_{\mu}$ in (95). As $x^{\nu} = x_{\mu}$ is mathematics law, it is expressed like $A^{\mu\nu} = A^{\mu}_{\mu}$ in (96). I show a contraction for other tensor satisfying Binary Law in

$$A^{\mu}_{\nu} = A^{\mu}_{\nu}\colon (x_{\nu} = x^{\mu}) = A^{\mu\mu}, \tag{97}$$

$$A^{\mu}_{\nu} = A^{\mu}_{\nu}\colon \left(x_{\nu} = -x_{\mu}\right) = -A^{\mu}_{\mu}, \tag{98}$$

$$A^{\mu}_{\nu\nu} = A^{\mu}_{\nu\nu}\colon \left(x_{\nu} = -x_{\mu}\right) = A^{\mu}_{\mu\mu}, \tag{99}$$

$$A^{\mu}_{\nu\nu\nu} = A^{\mu}_{\nu\nu\nu}\colon \left(x_{\nu} = -x_{\mu}\right) = A^{\mu}_{\mu\mu\nu}\colon \left(x_{\mu} = x^{\nu}\right) = A^{\mu\nu}_{\mu\nu}. \tag{100}$$

The contraction of A^{μ}_{ν} is not possible in the tensor satisfying Binary Law from (97),(98). The number of free index is reduced to 1 from 3 in (99). The number of free index is reduced to 0 from 4 by two times of contractions in (100).

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