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Cosmological Constant as a Variable Parameter: Spring Theory

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Abstract

A spring term is added into Newton's law of gravitation. The universe accelerates to luminal or superluminal speed at the outer rim of the Hubble sphere where no matters can be observed. Such a big bang is explained three dimensionally from which we obtain the Hubble constant $\mathbf{10^{-17.5}\,/s}$ which is the square root of the cosmological constant $\mathbf{10^{-35}\,/s^2}$. The missing mass of galaxies in the rotation curves can be clarified via the virial theorm in which galaxies mass $\mathbf{10^{41}\ kg}$ and their spring constant $\mathbf{10^{-31}\,/s^2}$ match with other authors. Under certain conditions in Sect.5, the Schroedinger equation can be reduced to 1st order for long range interaction and 2nd order(both time and spatial part) for micro interaction; whereas the latter has the same form as the Klein-Gordan equation. Upon a simple modification of the classical field theory, we derive the equation V(r) = a *ln*(1 + b/r) + C which is compatible with the well known Cornell potential in quark confinement. Such a modified field theory can furtherly apply to planetary motions by adding a spring term in the Binet equation to estimate different spring constants of the sun for the inner planets;they are $\mathbf{10^{-14}\ /s^2 (Mercury), 10^{-15}/ s^2\ (Venus)\ amd\ 10^{-16}\ \ /s^2\ \ (Earth)}$. Since spring is a fluid which cannot break except its value k decreases at farther distance. A comparison with the Fischbach's fifth force is also dicussed in the conclusion.

Keywords: Schwarzschild/de Sitter solution, expanding universe, missing mass of galaxies, short range and long range spring theory

1. Introduction

Notions such as pure space, dark matter, dark energy, aether, cosmological constant, spring and all these are of the same dark fluid. They are different manifestations of the same entity (Zhao 2007), just like the compression and extension of a spring. The existence of aether can explain the null result of the Michelson-Morley experiment; that is, the earth carries the aether while revolving around the sun. A simple form of the Schwarzschild/de Sitter solution is written as

$$\frac{\mathrm{GM}}{r^2} \pm kr = a \tag{1}$$

where the spring constant k is a positive value. The signs denote either compression or extension of the spring which connects the two interacting bodies. For instance, the spring is being extended once the object leaves the source causing a downward restoring force. Conversely, it is under compression in case of a free falling object, inducing an upward restoring force against gravity. Different sources have their own spring constant. In the case of expanding universe, Λ refers to the spring constant of the universe, also known as the cosmological constant.

In large scale measurements, Perlmutter (2003) suggested that 3-D space is sufficient to depict the physics of the universe instead of general relativity (GR).

2. Theoretical and Experimental Evidence

The concept of pure space in the form of Ricci tensors was proposed by Yang (1974):

$$R_{ab;c} \ = \ R_{ac;b} \tag{2}$$

By operating g^{aa} on (2) and set a = c, we get

$R^{a}{}_{b;a} \ = \ R_{;b}$, same as the Bianchi

contraction below for $R_{;b} = 0$

In fact, (2) can also be verified by contracting the Bianchi Identity

$$R^{n}{}_{ikl;m} + R^{n}{}_{ilm;k} + R^{n}{}_{imk;l} = 0$$

to

$$2R^{k}{}_{m;k} = R_{;m}$$

Pavelle (1974) pointed out that (2) are non-physical unless the cosmological constant remains in the field equations. An ideal experiment to verify the existence of spring is the Pound-Rebka experiments that are mentioned in most of the university textbooks. The Jefferson Physical Laboratory at Harvard used a ^{57}Fe source being placed at a height of H= 22.6 m above the detector within a designed vacuum environment.

Data were collected when the gamma rays dropped onto the detector:

E = $h\nu_0$ = 14.4 keV source energy

ΔE = h$\Delta\nu$= 3.5 x 10^{-11} eV energy change after drop

acceleration a = $\frac{c^2 \Delta\nu}{\nu_0 \ H}$= 9.67 m/$s^2$

Pound and Snider refined the apparatus to obtain the energy shifts of the upward and downward path

$$\frac{\Delta E}{E} \text{ (down) - } \frac{\Delta E}{E} \text{ (up)} = 4.905 \times 10^{-15}$$

The acceleration and deceleration equations are expressed as

$$\frac{GM}{r^2} - \omega^2 \ R \ cos^2\theta \ \mp \ kR = a \text{ (- for acceleration,+ for deceleration)} \tag{1a}$$

where the latitude of Massachusetts

$$\theta = 42^0$$

The spring constant on the earth surface can be calculated as

$$k = 1.2 \times 10^{-8}/s^2$$

3. The Expanding Universe

Hitherto, there is yet no orthodox conclusion to explain why all matters inside our universe were initially compressed into a dense lump and released, causing the big bang. A simple way to describe the acceleration is the restoring force of the compressed spring;

$$\Lambda r - (\frac{4}{3})\pi r^3 \rho G/r^2 = \text{acceleration } a = v\frac{\mathrm{dv}}{\mathrm{dr}} \tag{3}$$

(Note: Rigorously speaking, the 2nd term on LHS is not suitable for all matters inside a sphere.)

Obviously, the spring term of (3) overcomes the cosmos gravity hence (3) cam be reduced into the form

$$v = \Lambda^{1/2} r \tag{4}$$

which is the Hubble law having the Hubble constant as the square root of Λ. The maximium receding distance r= R ~10^{26}m is the present observed radius of our universe. Eq(4) yields the speed of light at this R. Either luminal or superluminal speed at the outer rim is acceptable by some cosmologists and there is no substantial proof of matter being observed beyond this Hubble sphere of radius $R = 10^{26}$ m (Kiang 2003, Davies & Linweaver 2004, Harrison 2003, Lewis & Pim van Oirschot 2012). We are observing galaxies receding away from us, as likewise, an observer in another galaxy also experiences his surrounding galaxies receding away from him, implying a 3-D lattice expansion governed by the same equation (4).

The radial Hubble law(4) can also be valid in the lateral case. Consider the isoceless Δ OAB, OA = OB = r > $chord\ AB$(~arc AB = θr)

where O is at the centre of the sphere. The lateral Hubble law rθ= vθH^{-1} ~H^{-1}v[sin(θ/2)]x2, along AB and BA direction.

Cosmology is a big topic. Our present purpose is to introduce the concept of a spring term which plays an important role in the big bang.

4. The Missing Mass in the Rotation Curve of Galaxies

We have examined 17 rotation curves, namely, 4 from Bless (1996), our galaxy from Binney & Tremaine (1987), 6 from Gessner (1992) and 6 from Bachhan group(2010). In large scale structure, the virial theorem including a spring term is a good solution:

$$< v^2 > = < \frac{\mathrm{GM(r)}}{r} > + < kr^2/2 > \tag{5}$$

where the spring term compensates the missing mass of the galaxy. Mass of each of these galaxies and their spring constants are obtained from the virial theorem (5);

$$\mathrm{M} = \frac{v^2}{\mathrm{G}} R \sim 10^{41} \text{ kg}$$

$$k \sim 10^{-31}/s^2$$

The results are reliable comparing with those authors. Both Gessner and the Bachhhan group used GR. The former author had input various spring constants to obtain the best fit values $k \sim 10^{-31}/s^2$ and M~ 10^{41} kg; while the latter research group obtained the range

$$k \sim 10^{-30}/s^2 \text{ to } 10^{-31}/s^2 \text{ and M} = 10^{37} kg \text{ to } 10^{40} \text{ kg.}$$

Due to discrepancies from direct measurement on these curves, we are satisfied with our results. Furthermore, the rotation curve of our galaxy is somehow flat. Further distance shows that the mass is still decreasing according to v^2- $kr^2/2$. Yet, good approximation can be obtained even though *v* remains almost constant across a few kpc's.

5. The 1st and 2nd Order Schroedinger Equation

The spring connecting two celestial bodies can be treated as a harmonic oscillator governed by the Schroedinger equation

$$\Psi'' - u^4 x^2 \Psi = \frac{-2m}{\hbar^2} E\Psi \tag{6}$$

where $u^2 = m\omega/\hbar$. The solution is

$$\Psi_n = \frac{1}{\sqrt{(n!2^n)}} (m\omega/\pi\hbar)^{1/4} H_n(ux) exp(\frac{-u^2x^2}{2}) \tag{7}$$

Setting $f = A_n H(ux)$, (7) becomes

$$\Psi_n = f \; exp(-u^2x^2/2) \tag{8}$$

Eq(6) can be re-written as

$$f'' - 2f' u^2 x - fu^2 = - \frac{2m}{\hbar^2} fE \tag{9}$$

From the Hermite polynomials: $H_0 = 1$, $H_1 = 2ux$, $H_2 = 4u^2x^2 - 2$, $H_3 = 8u^3x^3 - 12ux$……, the $(f'x)$ term dominates the (f'') term for large *x*. Eq(9) reduces to

$$dln \; f = n \; dln \; x \tag{10}$$

where $E = (n+1/2)\hbar\omega$. The result

$$f = x^n \tag{11}$$

gives the ground state wavefunction

$$\Psi_0 = (m\omega/\pi\hbar)^{\frac{1}{4}} exp(\frac{-u^2x^2}{2}) \tag{12}$$

For long range Schroedinger equation, once the f'' term being ignored, the wave equation can then be reduced into a first order derivative wave equation

$$\Psi' = (\frac{n}{x} - u^2x^2)\Psi \tag{13}$$

For n = 0, the probability equation

$$\int_0^R \| \Psi \|^2 \, dx = A^2 \int_0^R exp(-u^2x^2) dx = 1 \tag{14}$$

can easily be solved for large R to get

$$\mathrm{A} = \left(\frac{4m\omega}{\pi\hbar}\right)^{1/4} \tag{15}$$

which is the amplitude of the oscillator.

The spring wave belongs to intermediate range type different from the gravitational waves of GR. Besides the 1st order differential equation(13) for long range, under certain conditions in the case of micro-interaction, the Schroedinger equation can become a 2nd order differential equation(both time and spatial part) by operating $\partial/\partial t$ on

$$-\frac{\hbar^2}{2m} \nabla^2 + \mathrm{V} = i\hbar\frac{\partial}{\partial t} \tag{16}$$

to get

$$i\omega\frac{\hbar^2}{2m}\nabla^2 - i\omega\mathrm{V} = i\hbar\frac{\partial^2}{\partial t^2} \tag{17}$$

where $\psi \sim \exp(-i\omega t)$

is the wavefunction of the time part.

Re-arrange (17) we get

$$\frac{1}{C^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{\omega}{\hbar c^2}\mathrm{V} = 0 \tag{18}$$

having the same form as the Klein-Gordan equation for the case of a ground state harmonic oscillator of energy

$$\hbar\omega = 2mc^2$$

In fact, nature provides all sorts of microscopic springs (Susskind & Friedman 2014). Many molecules consist of two atoms – for example, a heavy atom and a light one. There are forces holding the molecules in equilibrium with the atoms separated by a certain distance. When the light atom is displaced, it will be attracted back to the equilibrium location. The molecule is a miniature version of the weight-and-spring system.

6. Modification of Classical Field Theory

In micro-electromagnetism, Coulomb's law fails, particularly energy blows up to infinity at r = 0 (see Feynman Lectures on Physics Vol.2 Chapter 5.8-1).

Renormalization is a tedious way due to its mathematical complication. To solve such a problem classically, we start from the relation between the electric field intensity E and the electric field energy density w induced by a charge q in the form of

$$\mathrm{w} = \boldsymbol{\beta}E^2 \tag{19}$$

which is also proportional to the charge density ρ_e of the source in the form of

$$\nabla \cdot E = 4\pi\rho_e = \alpha E^2 \tag{20}$$

Eqs (19) and (20) are applicable to both electrostatics and mechanics depending on ρ,α and β.Upon integration, (20) becomes

$$E = \frac{1}{br^2}\left(1 + \frac{\alpha}{br}\right)^{-1} \tag{21}$$

where b is a constant of integration. Upon integrating (19) over the whole space, the total field energy

$$\mathrm{W} = \beta\int_0^\infty \frac{4\pi}{b^2r^2}\left(1 + \frac{\alpha}{br}\right)^{-2}\mathrm{d}r \tag{22}$$

For r =0, W≠∞, renormalization is not necessary.

(i) Electrostatics:

Eq(22) equates to δmc^2 which is the energy of the electromagnetic mass of the charge. Such an electromagnetic mass always goes along with the charge, that is: q = 0,δm = 0. The nature and structure of δm is beyond the scope of this paper. However, its value can be estimated by applying (25) into Bohr model of hydrogen atom to get δm $\sim -10^{-37}\ kg$ which seems to be the mass of an anti-electron neutrino. With the help of (21), we get

$$\beta = \frac{\varepsilon_0}{2},\ \alpha = \frac{q}{2\delta mc^2} \ \text{ and } \ \frac{1}{b} = q \tag{23}$$

For convenience, $4\pi\varepsilon_0$ is not always appear throughout this paper. With the help of (21), ie, $-\nabla\emptyset = E$, the potential becomes

$$\varphi = \frac{2\delta mc^2}{q}\ln(1+ \frac{q^2}{2\delta mc^2 r}) \tag{24}$$

$$\text{or } = \frac{q}{r} - \frac{q^3}{2\delta mc^2 r^2} + \ldots \text{ (for long range)} \tag{25}$$

(ii) Newton mechanics:

Eq(22) equates to Mc^2 which is the energy of the source M. We get

$$\beta = \frac{1}{4\pi G},\ \alpha = \frac{1}{c^2} \text{ and } \frac{1}{b} = GM \tag{26}$$

The potential becomes

$$\varphi = c^2 ln(1 + \frac{GM}{rc^2}) \tag{27}$$

$$\text{or } = \frac{GM}{r} - \frac{G^2M^2}{2r^2c^2} + \cdots.(for\ long\ range) \tag{28}$$

7. Short Range Interaction

In micro-interaction, the well established Cornell potential is claimed to be the standard formula for quark confinement (Martin & Shaw 2008)

$$V(r) = \frac{a}{r} + br \text{ (Cornell)} \tag{29}$$

also known as the unified potential for quarkonia, mesons and baryons.

The form (24) and (27) plus a spring term can be generalized as

$$V(r) = aln\ (1 + \frac{b}{r}) + C \text{ where } C = \frac{1}{2}mkr^2. \tag{30}$$

Notice that the particle is not isolated but immersed inside the aether sea, same concept as the earth carries the aether while revolving around the sun.

The parameters *a*, *b* and *C* are obtained experimentally. Apparently, *C* is the energy of the particle- spring system in which the oscillating range equals to the wavelength λ of the oscillator, same as the Klein-Gordan equation;

$$\frac{1}{2}\ mkr^2 = mc^2$$

or

$$k = \frac{2c^2}{\lambda^2} \sim 10^{46}/s^2$$

which is so strong to imprison the quarks inside the confinement. Once the range diminishes to 10^{-15}m, gravitation will become a strong-spring type of interaction (Treder 1975). From Figure 1, we choose some suitable points(dots on the curve) to fit into (30): for charmonium, C = 3.3GeV, a = -1.15GeV, b = 4.36 fm and for bottomonium C = 9.3 GeV, a = -2.8 GeV, b = 10.9 fm. The constant C refers to the energy of charmonium or bottomonium. Obviously, (30) is equivalent to the Cornell (29) and the Natural Log formula for *r* ranging 0.1 to 1.4 fm while V(r) ranging from -1 to 1.3 GeV as shown in the figure.

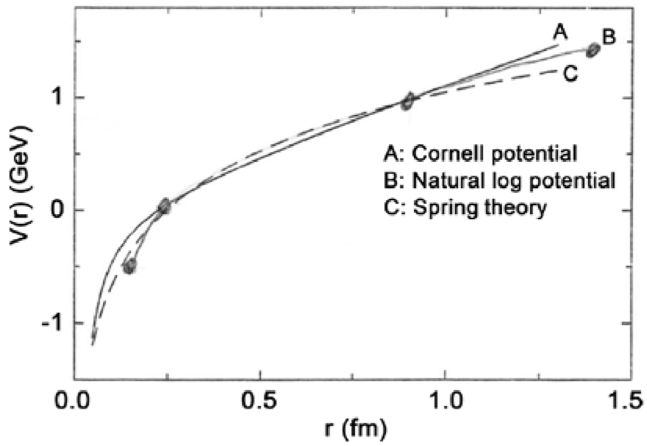

Figure 1. Quarkonium potential from fitting the energy levels (Martin and Shaw 2008)

8. Spring Theory in the Solar System

Recent technology had acquired accurate observational data for the perihelion advance of the first three inner planets, say, Mercury, Venus and Earth. The Binet equation can be written as, selecting the first two terms from (28) plus a spring term:

$$\frac{d^2u}{d\varphi^2} + u = \frac{GM}{h^2} - \frac{G^2M^2u}{c^2h^2} - \frac{k}{h^2u^3} \quad (31)$$

Following the procedures from Adler et al(1975) to solve the differential equation(31);

$$u = \frac{GM}{h^2} + kD \cos[(1-\frac{G^2M^2}{h^2c^2})^{1/2}(1-\frac{3kh^6}{2G^4M^4})]\phi - \frac{kh^4}{G^3M^3} \quad (32)$$

where

$$kD = \frac{e}{a(1-e^2)}$$

There are 2 ways to look for the values of the solar *k:*

Mehod (i)

The most reasonable way is to set kD be zero for eccentricity e = 0 especially the orbits of the first three inner planets are nearly circula. Through a lengthy algebraic manipulation, (32) yields the values of the solar spring constants:

$$10^{-14}/s^2 \text{ (Mercury)}, 10^{-15}/s^2\ (Venus)\ and\ 10^{-16}/s^2 \text{ (Earth)}$$

Differently, other authors like Sereno & Jetzer (2006), Iorio (2006) together with Adkins & Mcdonnell (2007) used different approach based on GR to calculate the solar spring k via the three inner planetary orbits but the first planet Mercury has the value $k\sim 10^{-24}\ /s^2$, which is much smaller than our result. Cardona & Tejeiro (1998) even obtained the same value as the cosmological constant via the Mercury orbit. Other theories pointed out that the sun-planet distance, influence of other planets as well as the non-conservation of the orbital angular momentum should be taken into account (Nyambuya 2010).

Fedi (2019) linked up the viscous force of the aether fluid acting on an orbiting spherical planet with the Stoke's law

$$F(aether) = -6\pi r v \eta$$

where the coefficient of dynamic viscosity η is replaced by the Lorentz dilation factor.

Method (ii)

In case the cosine part of (32) is the main contribution of the perihelion advance, then

$$\Delta\varphi = 2\pi\left(\frac{G^2M^2}{2h^2c^2} + \frac{3kh^6}{2G^4M^4}\ldots.\right) \tag{33}$$

in which the 1st term resembles the form of GR:

$3G^2 M^2/h^2c^2$. Perhaps suitable choice of α= n/c^2 in(26)can get this GR term but in this paper,we select n=1. From the following observed data,

	Mercury	Venus	Earth
Observed shift	43"11	8"4	5" per century

the solar spring values within the first three inner planets range from

$$k\sim\ 10^{-22}/s^2\ \text{ to }\ 10^{-23}/s^2$$

respectively from Mercury,Venus to Earth.These values seem to match with the above-mentioned authors, except Cardonna. Let's consider a simple solution of a circular motion having the form

$$\omega^2 \pm \mathrm{k} = \frac{GM}{R^3}$$

Substituting the physical data of the first three planets into the above equation, we get the results of the solar k:

Mercury ~$10^{-14}/s^2$,

Venus~ $10^{-15}/s^2$

and Earth ~$10^{-16}/s^2$,

same as method(i).

Another simple way to find k = k(r) of a free falling object is to use the change of kinetic energy at different r

$$\mathrm{d}(v^2/2) = \mathrm{d}(GM/r)$$

to give

$$v\,dv/dr = -GM/r^2$$

and then substitute the above into (1) to obtain

$$\mathrm{k}\,(\mathrm{r}) = 2GM/r^3$$

which is a good approximation. Through Mercury, the solar k~$10^{-15}/s^2$ which is acceptable as we did in method(i); on the solar surface k∼ $10^{-7}/s^2$, which is slightly different from the bemding of light as below, ie ∼$10^{-8}/s^2$ amd on the earth surface k~$10^{-6}/s^2$, which is different from $10^{-8}/s^2$ of Pound-Rebka in Sect 2.

We are puzzled by the dilemma of method(i) and (ii) eventhough the former seems to be logical. Concerning the bending of light while grazing the sun, the observed total deflection is

$$2\delta = 1''9 = \frac{4G^2M^2}{Rc^2} + \frac{2\mathrm{kR}^2}{c^2} \tag{34}$$

in which the 1st term is selected from GR . The spring on the surface of the sun is found to be k∼$10^{-8}/s^2$.

9. Concclusion

Since the cosmological constant is connected with dark energy (Peebles & Bharat 2003); or treating it as a variable parameter to fit the observational data ;or theories relating it to the vacuum energy (Carroll 2001) or viscous aether(Kuang & Lin 2009, Fedi 2019), our proposed spring theory should not be considered as hypothesis. The spring force, also known as the fifth force, is of intermediate range which differs from the Fischbach's Yukawa type whose potential is written as (Fischbach et al 1986, Fischbach & Talmadge 1992)

$$\psi = \frac{GM}{r}\ [1 + \alpha\ \exp(-r/b)] \tag{35}$$

where α, b are constants.Besides the lacking of experimental support, their theory cannot define r once the acceleration a = 0 whereas our (1) seems logical to show that k ∝ r^{-3}. The lunar surface can provide a frictionless free fall test to verify the existence of the fifth force. The time taken through a height H

$$T = \left(\frac{2H}{g-kH}\right)^{1/2} \tag{36}$$

will take longer time than the conventional one without a spring constant. Moreover, the same equation can be used to study the muon decay while darting through the atmosphere onto the earth. Detail discussion of (36) can be found in the work Tsang (2012).

Finally, we have no intention to decline GR other than replacing the cosmological constant by a variable parameter *k* in the Schwarzschild-de Sitter solution;

$$g_{00} = \left(1 - \frac{2GM}{rc^2} - \frac{kr^2}{2c^2}\right),$$

$$g_{11} = -(g_{00})^{-1},$$

$$k = \frac{2\Lambda}{3} \sim \Lambda.$$

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