Applied Physics Research; Vol. 15, No. 2; 2023
ISSN 1916-9639 E-ISSN 1916-9647
Published by Canadian Center of Science and Education

Revolutionizing Foam Physics: A Cutting-Edge Drainage Equation Model for Wet Foam

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Received: June 10, 2023 Accepted: July 4, 2023 Online Published: July 26, 2023

doi:10.5539/apr.v15n2p18 URL: https://doi.org/10.5539/apr.v15n2p18

Abstract

Foam physics is a field of study that scientists and researchers are interested in due to the vast range of uses, e.g. foam-foamed materials, oil extraction, and food processing. This study proposes a new equation for the drainage of wet foam that could add to the science of foam. To improve our comprehension of the intricate behaviour of wet foam, this model expands on a theoretical derivation. The usage of a bubble size formula that was proposed using the experimental data is one of the model's distinguishing characteristics. The size of foam bubbles can be predicted using this formula more precisely. A thorough derivation of the theoretical model is provided in the paper. Finally, this work presents a novel wet foam drainage model that has the potential to enhance the field of foam physics. The results of this work have important implications for industries. Therefore, more study is needed for developing a two dimensional drainage equation.

Keywords: drainage, equation, wet, foam, bubble, permeability

1. Introduction

Foam drainage has a growing literature as a result of its significant technological applications in a variety of fields, including food and beverage, cosmetics, and foam-formed materials (Weaire and Hutzler, 1999; Cantat et. al., 2013; Exerowa and Kruglyakov, 1998; Rosen, 2004). Research has primarily focused on so-called "dry" foams, and much less is known about "wet" foams. Understanding foam behavior is critical for optimizing these products as well as generating novel materials and technologies (Mira et. al., 2014; Isarin et. al., 1985; Darton and Sun, 1999).

Foam drainage, the process by which the liquid in the foam drains out of the bubbles over time, is one of the major phenomena that affects foam behavior (Weaire and Hutzler, 1999). The foam drainage equation is a mathematical model that represents this process by taking into account variables such as bubble size and shape, liquid viscosity, and surface tension between the liquid and gas phases. The foam drainage equation is highly important because it gives a quantitative framework for understanding and predicting the behavior of foam under various conditions. This information is required for developing novel materials with specialized qualities and constructing more efficient and effective foam-based processes. As a result, the foam drainage equation is important in materials science, chemical engineering, and a variety of other research and technological sectors.

Over the years, a large amount of study has been devoted to understanding the foam drainage process and developing mathematical models to represent it. Verbist and his colleagues created a foam drainage model for dry foam (Verbist et. al., 1996). They proposed a nonlinear second order differential equation and it has been widely utilized in many applications, despite certain limitations and the fact that it does not account for many of the complicated aspects that might impact foam drainage such as the case of wet foam.

The theory of Verbist and Weaire has been successfully applied to fit data obtained from conductivity measurements used to determine the liquid fraction in a foam column subjected to free and forced drainage conditions (Weaire et. al., 1995).

Recently, (Hutzler et. al., 2005) investigates the process of liquid drainage in two-dimensional foam structures. The authors use a combination of experimental and computational methods to study the dynamics of foam drainage and to develop a mathematical model that can predict the rate of drainage. The results show that the theoretical and the computational models are in agreement.

2. Theoretical Model for Wet Foam

The amount of air in the foam affects whether it is "dry" or "wet." When the air content is greater than 80%, dry foam is created, indicating that the majority of the system is air and the liquid component is minimal. In this instance, the bubbles' form is polyhedral rather than spherical. Wet foam, on the other hand, is created when the system's air concentration is between 60 and 80 percent, and the bubbles have spherical shapes with rather thick layers of water between them.

The conservation of mass in a wet foam system is described by the continuity equation, a fundamental equation in fluid mechanics. According to the equation, the net rate of mass flow into or out of a system equals the rate of change of mass inside the system. The equation is expressed mathematically as:

$$\frac{\partial \varphi_l}{\partial t} + \vec{\nabla} \cdot (\vec{u}_m) = 0 \tag{1}$$

where φ_l is the liquid fraction of the foam, and u_m is the flow rate over sample area, and it is equal to $\langle \bar{u} \rangle \, \varphi_l$ where $\langle \bar{u} \rangle$ is the average over all pores of the average velocity in each pore.

The flow of fluids through porous material is described by Darcy's law, a fundamental principle of fluid mechanics. According to this, the permeability and pressure gradient both affect how quickly fluids move through porous media. Darcy's law can be used to explain how liquid drains through a foam structure in the case of wet foam.

$$\vec{u}_m = \frac{k}{\eta}\left(\rho_l \vec{\mathrm{g}} - \vec{\nabla}\, \mathrm{p}\right) \tag{2}$$

where k is the permeability of the foam, $\vec{\nabla}\,\mathrm{p}$ is the fluid pressure gradient, and η is the viscosity of the fluid in the foam.

The capillary pressure, or the differential in pressure between the liquid in the foam and the surrounding gas or liquid, is what determines the pressure gradient in Darcy's law for wet foam. The geometry of the foam structure, such as the size and shape of the pore, and the interfacial tension between the liquid and gas or liquid phases are the two factors that cause capillary pressure. It can be calculated as:

$$\vec{\nabla}\,\mathrm{p} = -\vec{\nabla}\left(\frac{\gamma}{r}\right) \tag{3}$$

where γ is the is the surface tension and r is the radius of the bubble.

The average bubble size in mixing flows is measured by the Sauter mean radius rather than the arithmetic average, which is an important factor in the research on foam stability and drainage. Al-Qararah et al. (2013) proposed an equation that accounts for the effects of surface tension, rotation speed of the mixing, and the air content on the foam structure to determine the Sauter radius of bubbles in wet foam.

$$r_{32} = \frac{\mathrm{C}\,\gamma}{\mathrm{N}(\frac{1}{\phi} - 1)}. \tag{4}$$

where C is a geometrical factor that depends on the vessel and the initial suspension volume, γ is the surface tension, N is the rotation speed, and ϕ is the air content. The correlation between the liquid fraction and the air content is: $\varphi_l = 1 - \phi$. Thus, Eq. (5) can be written as:

$$r_{32} = \frac{\mathrm{C}\,\gamma(1-\varphi_l)}{\mathrm{N}\,\varphi_l} \tag{5}$$

A porous medium's permeability is a measurement of how easily fluids can pass through it. The Carman-Kozeny equation, a popular empirical formula that links the permeability of a porous media to its porosity and specific surface area, can be used to calculate the permeability in the situation of sphere packing. The permeability is determined as:

$$k = \frac{{\varphi_l}^3 d^2}{180\,(1-\varphi_l)^2} \tag{6}$$

where φ_l is the liquid fraction of the foam, and d is the diameter of bubble size.

By Substituting the formula of the bubble size into Eq. (6) gives:

$$k = \frac{C^2\gamma^2}{45\,N^2}\varphi_l \tag{7}$$

Substituting Eqs. (6) and (7) into Eq. (2) to get:

$$\vec{u}_m = \frac{C^2\gamma^2\varphi_l}{45\,\eta\,N^2}\left[\rho_l\vec{g} + \frac{N}{C}\vec{\nabla}\left(\frac{\varphi_l}{1-\varphi_l}\right)\right] \tag{8}$$

The foam drainage equation for wet foam can be obtained by substituting Eq.(8) into Eq. (1). The final equation is:

$$\frac{\partial\varphi_l}{\partial t} + \frac{C^2\gamma^2\rho_l}{45\,\eta\,N^2}\vec{\nabla}\cdot(\varphi_l\vec{g}) + \frac{C\gamma^2}{45\eta N}\vec{\nabla}\cdot\left(\varphi_l\vec{\nabla}\left(\frac{\varphi_l}{1-\varphi_l}\right)\right) = 0 \tag{9}$$

It is challenging to understand and solve this partial differential equation since it has numerous variables. Reducing this equation to a one-dimensional form make it easier to understand. This requires making the assumption that the foam is homogeneous in one dimension, usually the vertical direction. The comparison of experimental results with theoretical models is made easier because to this simplification, which also enables a more straightforward examination of foam drainage behavior. In one dimension Eq.(9) becomes:

$$\frac{\partial\varphi_l}{\partial t} + A\frac{\partial\varphi_l}{\partial \mathrm{z}} + \mathrm{B}\frac{\partial}{\partial \mathrm{z}}\left(\frac{\varphi_l}{(1-\varphi_l)^2}\frac{\partial\varphi_l}{\partial \mathrm{z}}\right) = 0 \tag{10}$$

where $A = \frac{C^2\gamma^2\rho_l g}{45\,\eta\,N^2}$ and $B = \frac{C\gamma^2}{45\eta N}$.

This partial differential equation can be solved with numerous methods. This will be discussed in forthcoming paper along with the comparison of the results obtained with this model and the experimental data.

3. Conclusions

The discipline of foam science may advance as a result of the novel equation for the drainage of wet foam that is put out in this work. However, the absence of meaningful experimental data on wet foam drainage makes this study difficult. Wet foam drainage behavior may be better understood by experiments employing a wider variety of foam compositions, surfactants, and additives. Although the work offers helpful insights into the foam drainage equation, more testing of the model in real-world applications may be necessary. The applicability and dependability of this model would be increased by testing it with various industrial wet foams or under real world conditions.

More study may be conducted to a better understanding the parameters that determine foam stability, such as the effects of surfactant concentrations and types of the surfactant that used. This would help us to a better understanding of wet foam drainage behaviour. Investigating the influence of various additives, such as fibres, and nanoparticles on wet foam drainage might give insights into their potential for improving wet foam stability and managing drainage rates.

By solving these issues and looking into these areas of study, we may learn more about how wet foam drains, which will improve the stability and uses of wet foam.

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