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Measuring Planck's Constant With Compton Scattering

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Abstract
Measured values of the electron mass and Compton wavelength yield a value of Planck's constant with a relative standard uncertainty of 3×10^{-10}. This is only slightly larger than the 1.3×10^{-10} relative standard uncertainty in measurements performed using the Kibble balance. Compton scattering presents an alternative pathway for improving the value of Planck's constant.
Natural units of length, mass, and time offer viable solutions for improving the values of physical constants. While extensive values of the Planck units lie beyond the reach of present-day instrumentation, certain product and quotient pairs of Planck units such as the speed of light can be measured with relatively high precision. Better measurements of certain unit pairs will improve the value of the gravitational constant.

Keywords: Compton scattering, Compton wavelength, electron mass, Kibble balance, metrology, natural units, Planck constant

1. Introduction

The System of International Units sets an exact value for Planck's constant based on measurements undertaken by the National Institute of Standards and Technology between 2015 and 2017 using the Kibble balance (Haddad et al., 2017). These measurements reduced uncertainty by more than twofold over previous measurements, achieving a relative standard uncertainty of 1.3×10^{-10}.

Although the Kibble balance is the preferred method for measuring Planck's constant, it is not the only experimental means for obtaining a high precision measurement. Planck's constant can also be determined from measurements of the electron's Compton wavelength and rest mass, each with a relative standard uncertainty of 3.0×10^{-10} (NIS, 2018). The relationship between these measurements and the reduced Planck constant is

$$\hbar = \lambda\!\!\!^{-}_C m_0 c = 1.054\ 571\ 8176... \times 10^{-34}\ kgm^2/s \tag{1}$$

2. Derivation of Results

Equation 1 is obtained by representing Planck's constant with natural units in the constant's unit dimensions—an implicit assumption of Planck's derivation (Humpherys, 2022)

$$\hbar = \frac{l_P^2 m_P}{t_P} = l_P m_P c. \tag{2}$$

Furthermore, it has been shown that an electron's Compton wavelength and rest mass are inversely proportional (Haug, 2022; Humpherys, 2021; Kepner, 2018), making the product of wavelength and mass invariant and equal to the product of Planck length and Planck mass

$$\lambda\!\!\!^{-}_C m_0 = l_P m_P. \tag{3}$$

Substituting 3 into 2 yields Equation 1.

Table 1 summarizes the CODATA values of the Compton wavelength and rest mass which produce Planck's constant according to equation 1. Measurements of the muon and tau Compton wavelengths and rest masses also produce the constant, but with less certainty: 2.2×10^{-8} and 6.8×10^{-5} relative standard uncertainties respectively (NIS, 2018).

Table 1. Lepton properties which determine the value of Planck's constant

Particle	Compton wavelength λ_C	Rest mass m_0	Reduced Planck constant $\lambda_C m_0 c$
Electron	$3.861\ 592\ 6796 \times 10^{-13}$	$9.109\ 383\ 7015 \times 10^{-31}$	$1.054\ 571\ 8176... \times 10^{-34}$
Muon	$1.867\ 594\ 306 \times 10^{-15}$	$1.883\ 531\ 627 \times 10^{-28}$	$1.054\ 571\ 817... \times 10^{-34}$
Tau	$1.110\ 538 \times 10^{-16}$	$3.167\ 540 \times 10^{-27}$	$1.054\ 57... \times 10^{-34}$

3. Planck Scale Metrology

It may be reasonably argued that extensive quantities of Planck length, mass, and time lie beyond the reach of experimental measurements (Adler, 2010). However, certain product and quotient *relationships* between pairs of Planck units are demonstrably within the reach of modern instrumentation, such as the ratio of Planck length to Planck time. The speed of light has been measured with a relative standard uncertainty of $1.6 \times 10^{-10}\ m/s$ (Jennings et al., 1987), which is an important consideration in the decision to define c in the System of International Units

$$\frac{l_P}{t_P} = c = 299,792,458\ m/s. \tag{4}$$

An accurate measurement of the speed of light is possible because the intensive ratio between distance and time can be measured on scales that are much larger than the Planck scale.

The product of Planck length and Planck mass also has a defined value as the ratio of two defined constants: Planck's constant and the speed of light

$$l_P m_P = \frac{\hbar}{c} = \frac{l_P m_P \cancel{c}}{\cancel{c}} = 3.517\ 672\ 9417... \times 10^{-43}\ kgm. \tag{5}$$

The inversely proportional relationship between wavelength and mass shown in Equation 3 is responsible for the invariance of $l_P m_P$ and Planck's constant.

The two universal constants give a third defined value in the product of Planck mass and Planck time

$$m_P t_P = \frac{\hbar}{c^2} = \frac{\frac{m_P \cancel{l_P^2}}{\cancel{t_P}}}{\frac{\cancel{l_P^2}}{t_P \cancel{t_P}}} = 1.173\ 369\ 3920... \times 10^{-51}\ kgs. \tag{6}$$

Table 2 summarizes the three defined pairs of Planck units and three pairs of units with uncertainties that depend on the precision of the gravitational constant.

Table 2. Of the six product and quotient relationships between the Planck units, three have exact values based on the exact values of Planck's constant and the speed of light. The other three relationships have uncertainties comparable to the uncertainty in the gravitational constant.

Planck Unit Pair	Formula	Value	Rel. std. uncertainty
$\frac{l_P}{t_P}$	c	$299,792,458\ m/s$	defined
$l_P m_P$	$\frac{\hbar}{c}$	$3.517\ 672\ 9417 \times 10^{-43}\ kgm$	defined
$m_P t_P$	$\frac{\hbar}{c^2}$	$1.173\ 369\ 3920 \times 10^{-51}\ kgs$	defined

$\frac{l_P}{m_P}$	$\frac{G}{c^2}$	$7.426\,160 \times 10^{-28}\ m/kg$	2.2×10^{-5}
$\frac{m_P}{t_P}$	$\frac{c^3}{G}$	$4.036\,978 \times 10^{35}\ kg/s$	2.2×10^{-5}
$l_P t_P$	$\frac{\hbar G}{c^4}$	$8.713\,629 \times 10^{-79}\ ms$	2.2×10^{-5}

The significance of these three defined values is much more than academic; the Planck units offer an overlooked pathway for obtaining more accurate values of the gravitational constant and other constants that depend on G. Like Planck's constant, the gravitational constant can be represented in natural units of length, mass, and time

$$G = \frac{l_P}{m_P} c^2. \tag{7}$$

The formula indicates that the uncertainty in G lies in the ratio of Planck length to Planck mass, given an exact value of c. Consequently, the gravitational constant has a relative standard uncertainty of 2.2×10^{-5} which is comparable to the 1.1×10^{-5} relative standard uncertainty in the CODATA values of Planck length and Planck mass.

Improving the value of G requires a more accurate measurement of at least one of the three undefined values listed in Table 2. This is because the three defined values only provide enough information to constrain the proportions among the Planck units and do not reveal the extensive values themselves.

To see why this is the case, consider the three pairs of Planck units with defined values in Table 2. Note that the set contains *either* a product relationship *or* a quotient relationship between a given pair of units, but *not both*. For example, the ratio l_P/t_P is defined but $l_P t_P$ has a large uncertainty by comparison.

If we had precision measurements for both the product and quotient relationships between a pair of units, we could determine a value for the two units with the same level of precision. This is easy to observe in the following manner. Let a and b represent high-precision values of the product and quotient relationships between Planck length and Planck time

$$\frac{l_P}{t_P} = a \tag{8}$$

and

$$l_P t_P = b. \tag{9}$$

From this information we can solve for l_P and t_P. Restating Eq. 8

$$l_P = t_P a \tag{10}$$

and plugging into 9 yields the solution

$$t_P^2 = \frac{b}{a}. \tag{11}$$

Our misfortune lies in having defined values for three Planck unit pairs without a single set of product and quotient relationships.

The result is that we have greater precision in the proportions among Planck units than in the unit values themselves. This is apparent in the CODATA values of Planck length and Planck time which give a ratio of $299,792,423$ for the speed of light. Although we know this ratio is inaccurate, we can only improve it with better measurements.

Figure 1 illustrates the degree to which the CODATA values of Planck length, mass, and time are proportionally inaccurate. In the figure, each node of the triangle represents the current value of a Planck unit and the equilateral triangle formed by these points represents a proportionally accurate relationship between them. Three triangles overlaying the equilateral triangle indicate the degree to which two of the units are out of proportion given the value of a first unit. For example, the blue triangle with a node on the Planck length indicates that, given the current value of Planck length, the value of Planck mass is too small and the value of Planck time is too large.

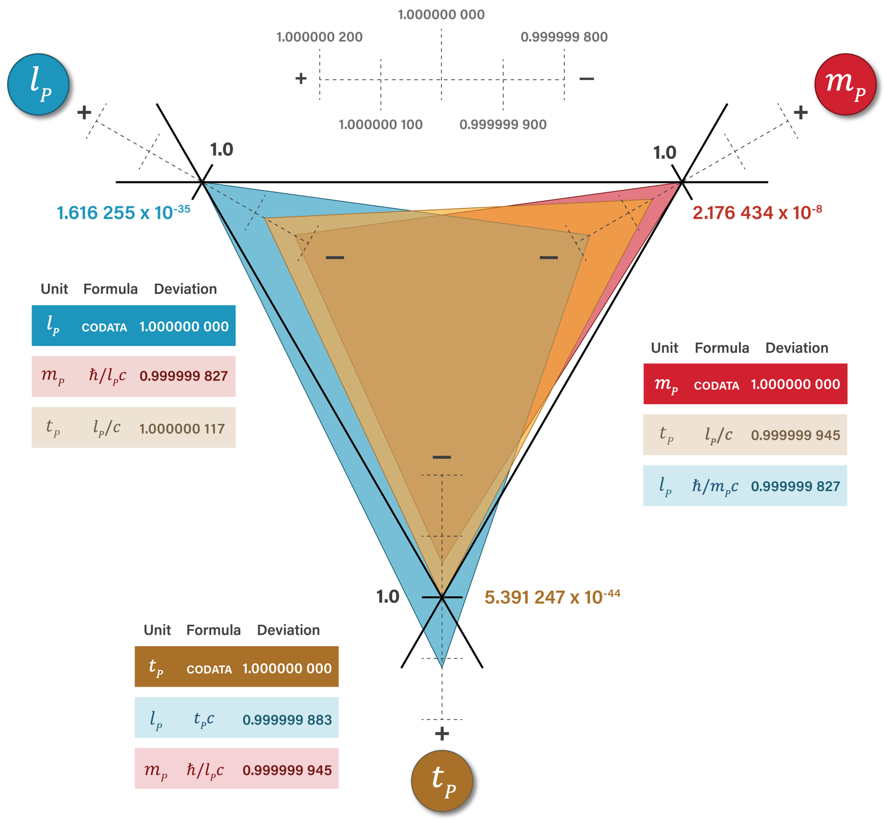

Figure 1. The speed of light and Planck's constant constrain the proportions of Planck length, mass, and time. The equilateral triangle represents a proportionally accurate relationship among the units.

Table 3 gives the formulas for calculating proportionally accurate values of the second and third Planck units when given the value of a first unit.

Table 3. Formulas for calculating the values of any two Planck units when given the value of a first unit. Defined values of Planck's constant and the speed of light provide the required constraints.

Given unit value	l_P	m_P	t_P
Planck length	-	$\frac{\hbar}{l_P c}$	$\frac{l_P}{c}$
Planck mass	$\frac{\hbar}{m_P c}$	-	$\frac{l_P}{c}$
Planck time	$t_P c$	$\frac{\hbar}{l_P c}$	-

3.1 New Measurement Approaches

The natural units present an opportunity for devising new measurement solutions to improve the accuracy of physical constants while shedding new light on the structure of natural phenomena. Measurements of the universal constants are also measurements of the relationships between natural units, and an improvement in one elevates the other.

One approach to improving the accuracy of the gravitational constant is to continue refining the instruments and methods for measuring l_P/m_P. However, such measurements depend on accurate measurements of the mass and radius of two bodies and it remains challenging to obtain more precise measurements of the gravitational field between bodies of measurable mass.

An alternate pathway to improve the value of G is to devise new measurement techniques aimed at determining more precise values of m_P/t_P or l_Pt_P. A more precise measurement of either quantity yields a commensurate increase in the precision of G. This is because a better measurement of any undefined pair in Table 2 improves the values of the Planck units using the formulas in Table 4. In particular, we need better values of l_P and m_P given the exact value of c^2. An improvement in these two values will improve the value of G according to Equation 7.

Table 4. Formulas for calculating values of the three Planck units with the same precision as a measurement of the Planck unit pairs in the first column. Better values of the Planck units improve the values of universal constants.

Planck unit pair	l_P	m_P	t_P
$\frac{l_P}{m_P}$	$\sqrt{\frac{l_P}{m_P}\frac{\hbar}{c}}$	$\sqrt{\frac{m_P}{l_P}\frac{\hbar}{c}}$	$\sqrt{\frac{l_P}{m_P}\frac{\hbar}{c^3}}$
$\frac{m_P}{t_P}$	$\sqrt{\frac{t_P}{m_P}\hbar}$	$\sqrt{\frac{m_P}{t_P}\frac{\hbar}{c^2}}$	$\sqrt{\frac{t_P}{m_P}\frac{\hbar}{c^2}}$
l_Pt_P	$\sqrt{l_Pt_Pc}$	$\sqrt{\frac{\hbar^2}{l_Pt_Pc^3}}$	$\sqrt{\frac{l_Pt_P}{c}}$

The ratio m_P/t_P is found in the unit dimensions of force and opens up the possibility of applying more precise measurements of the electromagnetic interaction to these two units. This is perhaps the most promising way to improve the value of G. A greater challenge, however, is measuring l_Pt_P which does not appear in the unit dimensions of common natural phenomena.

4. Conclusion

The present study is an examination of the structural relationship between universal constants and natural units, while also demonstrating the application of theory by calculating Planck's constant from the electron mass and Compton wavelength. New research may derive further insights by restating universal constants in natural units and examining the formulas in each unit dimension. This approach yields more granular information than setting the constants equal to 1. Such research might also produce a better understanding of physical constants and what they represent.

A better understanding of the natural units may also produce new measurement techniques for improving the values of Planck's constant, the gravitational constant, and other constants which embody natural units. High precision measurements obtained through Compton scattering offer a viable alternative to the Kibble balance for determining Planck's constant because the product of electron mass and Compton wavelength is conserved, and because the ratio of length to time in the speed of light is constant.

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