

# The Constant of the Universe Expansion Acceleration $\Gamma_H$ , the Einstein Variation of $c$ and Parameters $V_H$ , $\Omega$ , $\Lambda$ and $\gamma$ .

J. G. Lartigue<sup>1</sup>

<sup>1</sup> National University of Mexico, Mexico

Correspondence: J. G. Lartigue, National University of Mexico, Mexico. E-mail: jmlg5@hotmail.com

Received: January 31 2020

Accepted: March 25, 2020

Online Published: June 30, 2020

doi:10.5539/apr.v12n3p28

URL: <http://dx.doi.org/10.5539/apr.v12n3p28>

## Abstract

The Hubble-Lemaitre equation  $\mathbf{v} = H \cdot \mathbf{r}$  ( $\text{cm}\cdot\text{s}^{-1}$ ) represented a linear function of the radial Space expansion velocity, if  $H$  would be a constant. Sometimes it has been assumed as  $H = 1/t$ , which sends back to the classical  $\mathbf{v} = \mathbf{r}/t$ . However, the later discovered acceleration required the additional condition for  $H$  to be, also, a function of time; or, opposed, the existence of a not yet defined dark energy. In a previous paper [1] it had been proposed a provisional expression for a constant Universe expansion acceleration as function of distance:  $\Gamma = H^2(\text{cm}\cdot\text{s}^{-2})$ . Now, the substitution of  $\mathbf{r}$  as a function of time, takes to five new equations of  $H$ , the Hubble velocity  $\mathbf{v}_H$ , the Hubble acceleration  $\Gamma_H$  and the positive Hubble potential  $V_H$  of the Space. So the proposed Hubble functions for the Space:  $H$ ,  $\mathbf{r}_H$ ,  $\mathbf{v}_H$ ,  $\Gamma_H$  and  $V_H$  result higher than those in a gravitational field. All of these Hubble functions act in the total Space expansion though, into the Physical Universe,  $\Gamma_H$  is not perceived as it does, continuously, the opposed gravitational acceleration  $\mathbf{g}$ . Otherwise, a revision is made of the Einstein equation for the  $c$  value as function of the gravitational potential  $\phi$ . Additional proposals are made about the horizons definitions and parameters  $\Omega$ ,  $\Lambda$  and  $\gamma$ .

**Keywords:** Hubble-Lemaitre Equation, Critical Time, Proper Time.

## 1. Conventional Definitions

Hubble Flow: The outward displacement of external galaxies.

(H-L): The Hubble-Lemaitre equation.

Universe: the expanding Space that contains the Physical Universe.

Big Bang: an assumed singularity as the most probable origin of the Space, time, matter and physical laws.

## 2. The Hubble Functions: $H$ , Velocity, Acceleration and Radius

From de (H-L) equation:

$$\mathbf{v} = H \cdot \mathbf{r} \quad (1)$$

it is possible to obtain the Hubble parameter  $H$  if  $\mathbf{r}$  is expressed as a function of time and a function of the intensity of the expansive field  $\Gamma$ , at a given radius, in equation (1):

$$\mathbf{v} = H \cdot \mathbf{r} = H \cdot \Gamma \cdot t^2/2 = \Gamma \cdot t \left(\frac{\text{cm}}{\text{s}}\right) ; \text{ so: } H = 2/t \quad (2)$$

Then, substitution of (2) in (1) gives:

$$\mathbf{v}_H = \frac{2\mathbf{r}}{t} \quad (3)$$

so, the Hubble velocity in the accelerated expansion of the space could be expressed ed as

$$\mathbf{v}_H = 2\mathbf{v} \quad (4)$$

since the Space expansion velocity does not depend on gravitation. Therefore, the (H-L) equation (1) could be written as

$$v_H = Hr = \frac{2r}{t} = 2v \tag{4a}$$

In that concerning the (H-L) acceleration  $\Gamma_H$ , it may be obtained by substitution of equation (4) from a assumed Hubble acceleration equation:

$$\Gamma_H = v_H/t \tag{5};$$

substitution of equation (3) in (5) lets obtain the (H-L) acceleration of the Space expansion:

$$\Gamma_H = \frac{2r}{t^2} \tag{6}$$

$$\Gamma_H = H^2 r/2 \tag{7}$$

Even the Hubble radius of Space may be assumed from equation (4):

$$v_H = 2v; \text{ so, for a given time } t, \quad r_H = 2r \tag{8}$$

For example, in the accelerated Hubble field, if the present time is  $t_0 \sim 4.5 \times 10^{17}$  (s), the present (H-L) parameter would be  $H_0 \sim 4.4 \times 10^{-18}$  (s<sup>-1</sup>); if the Universe scale factor had been assumed to be about  $r_0 = 2 \times 10^{28}$  (cm), its constant (Lartigue, 2016) acceleration must be, from equation (6),  $\Gamma_H \sim 2 \times 10^{-7}$  (cm·s<sup>-2</sup>); it generates the Hubble expansion velocity of Space by the equation

$$v_H = \Gamma_H \cdot t_0 \tag{9}$$

that gives  $v_H = 3c$ ; so,  $r_H = 4 \times 10^{28}$  (cm). The space expansion velocity reached the  $c$  value at a time  $t_c = t_0/3$ , when the Physical Universe had not completed its formation. Therefore, the present concepts of horizons should be re-defined: the particles horizon, usually assumed as the radius of the observable Universe, would mainly depend on the range and precision of the instrumentation. The events horizon has been defined as the distance where the fare objects could be detected in the future; however, it is assumed that, in an accelerated Universe, the light emitted from the Physical Universe limit will never reach the Earth, because the expansion velocity must be higher than  $c$ , a situation anyway occurred in the Physical Universe since the time  $t_c = 1/3$  of the present time  $t_0$ . Therefore, the events horizon definition needs to be revised. Firstly, the electromagnetic radiation does not come from the Big Bang (except the CMB) but from astronomical objects, i. e. it is not generated in the external Space (a vacuum). Therefore, it is always emitted and eventually detected into the Physical Universe. Besides, it is evident that the physical fields do not alter the  $c$  velocity, i.e. it is a Physical Universe constant. So, in spite of the expansion velocity of such Universe, it always exists the possibility to detect the light from a fare object. To date, the farer astronomical object has been detected with a factor  $z \sim 12$  (Keck MOSFIRE Spectroscopy, 2013), i. e. about  $1.4 \times 10^{19}$ (cm). It means that the Observable Universe only covers the last  $4 \times 10^5$  years of the expansion.

The today best definitions of horizons are those given by reference (Sartory, 1996):

If

$$\int_{t_0} dt/R(t) \tag{10a}$$

is finite or  $\infty$ , it is an event horizon.

If

$$\int^0 dt/R(t) \tag{10b}$$

is finite, it is a particle horizon.

So, the present particle horizon could correspond to the last mentioned figure of  $4 \times 10^5$  (y). The respective not mentioned limits could be ( $t > t_0$ ) or  $\infty$  in the first case; and the Big Bang time ( $t_B$ ) in the second case. Then, equation (10a) covers future events and equation (10b) the past ones. Therefore, both horizons presents an always growing value to be determined.

\* In reference [1] it was proposed the constancy of  $\Gamma_H$ .

### 3. The The Hubble Potential $V_H$ and Parameters $\Omega$ , $\Lambda$ , $c$ and $\gamma$

The space expansion acceleration  $\Gamma_H$  (including the contained Physical Universe) would necessarily be the intensity of a (H-L) field  $V_H$ , that must be positive since it is opposed to the gravitational field of the Physical Universe:

$$V_H = \int r \Gamma_H \cdot dr \tag{11}$$

So, from (6)

$$V_H = H^2 r^2 / 4 \text{ (cm/s)}^2 . \tag{12}$$

The  $\Omega_t$  function has been defined as the ratio of the Universe density at a time t and the critical density of the Physical Universe (Lidsey, 2014) as:

$$\Omega_t = \rho_t / \rho_{cr} \tag{13}$$

This equation could be applied to the present time as:

$$\Omega_o = \rho_o / \rho_{cr} \tag{14}$$

If  $\rho_o > \rho_{cr}$ ,  $\Omega_o > 1$  and  $k > 0$ , that implies a future collapse of the Universe. If  $\rho_o < \rho_{cr}$ ,  $\Omega_o < 1$  and  $k < 0$ , it produces a forever expansion. If  $\rho_t = \rho_{cr}$ ,  $\Omega = 1$  and  $k = 0$ , that means a plane geometry.

The equation to determine  $\rho_{cr}$  has been deduced from a Friedmann equation where the parameter k is assumed 0, in order to get a plane geometry:

$$\rho_{cr} = 3H^2 / 8\pi G \tag{15}$$

Since H is a function of time (equation 2), the critical density could exist at a critical time as:

$$\rho_{cr} = 3 / 2\pi G (t_{cr}^2) \tag{16}$$

Then, the only way to obtain an actual  $t_{cr}$  value it should be to assume that  $\Omega_o = 1$ ; so, it would be necessary to substitute  $\rho_{cr}$  by  $\rho_o$  in equation (16). The  $\rho_o$  value has been estimated in  $1 \times 10^{-27} \text{ (g}\cdot\text{cm}^{-3}\text{)}$  if the Physical Universe would have a total mass of  $3 \times 10^{58} \text{ (g)}$  and a radius of  $2 \times 10^{28} \text{ (cm)}$ ; so, the critical time must be  $t_{cr} \sim t_o / 5$ . Condensing the above constants it gives another expression of equation (16):

$$\rho_{cr} = 7.1 \times 10^6 / (t_{cr}^2) \text{ (g/cm}^3\text{)}. \tag{16a}$$

From the assigned values  $\Omega_o = 1$  and the known value of  $\rho_o$ , substitution of  $\rho_{cr}$  by  $\rho_o$  in equation (16a), it gives a critical time  $t_{cr} = 8.4 \times 10^{16} \text{ (s)}$ ; so  $t_{cr} \sim t_o / 5$ . Therefore, while the critical times would remain lower than the successive present times, the accelerated expansion will maintain a plain geometry.

Otherwise, since most of the Space is empty, the density concept of the Physical Universe has not a significant meaning for the total Space, which will probably continue its Hubble accelerated expansion forever, even if it happens any Physical Universe collapse.

The **c** variation. Contrary to his postulate of Special Relativity, Einstein proposed an equation to determine the variation of the present  $c_o$  value as a function of the gravitational potential  $\phi$  (Einstein, 1923):

$$c_\phi = c_o (1 + \phi / c_\phi^2) \tag{17}$$

$$\phi = - GM / r_c \tag{18}$$

They may occur 3 possible states, accordingly to the  $\phi$  value:

- a) If  $\phi = 0$  (when  $r_c \rightarrow \infty$ ),  $c_\phi = c_o$ .
- b) Since  $\phi < 0$  (in a gravitational field)  $c_\phi < c_o$ .
- c) If  $\phi / c_\phi^2 = - 1$ ,  $c_\phi = 0$ .

Even worse, a problem may appear if the value of the light velocity would depend, rather, on the moduli difference of gravitational and Hubble fields intensity, as:

$$c = (g - \Gamma_H) \cdot t \tag{19}$$

So, at a time  $t \sim 10^{18} \text{ (s)}$  and a Universe radius  $r \sim 10^{29} \text{ (cm)}$ , both intensities will mutually cancel, so generating a light velocity  $c = 0$ . Then it seems preferable to assume, independently, the expansion velocity of the Space as  $v_H = \Gamma_H t$  and the Physical Universe to be pulled as a whole into the Space at the same velocity, while the light speed **c** remains constant into the Physical Universe. Otherwise, the Observable Universe radius has a  $z \sim 12$  (Keck MOSFIRE Spectroscopy, 2013). i. e.  $r_{ob} \sim 1.4 \times 10^{19} \text{ (cm)}$  or  $10^{-9}$  times lower than the foreseeable Universe radius; so, its gravitational potential has not varied enough, in the last century, to detect an internal variation of **c**, if equation (17) would be valid.

However, the deviation of light rays in the nearness of astronomical bodies shows the mass property of photons; the corresponding equation (Tagliaferro, 2015) for gravitational lens shows the angle deviation as functions of the potential and the impact parameter, without any change in the light velocity; furthermore, if the impact parameter is 0, it appears the Einstein ring. Otherwise, as an electromagnetic wave, it happens a deviation in the plane of the photons direction in the Faraday’s experiment, without any change in their velocity. That is the reality in the Physical Universe:  $c = \text{constant}$  in a vacuum, though it could be different in the external Space.

The  $\gamma$  factor was discussed in reference (Hawkins, 1988) as a Special Relativity concept:

$$\gamma = 1/(1 - v^2/c^2)^{0.5} \tag{20}$$

where  $v$  is the velocity of a mass, measured at the reference frame time  $t_s$  ; this time, multiplied by the  $\gamma$  value, gives the proper time  $\tau$  by the equation:

$$\tau = \gamma t_s \text{ (s)} \tag{21}$$

Otherwise, if  $v = c$ , the proper time becomes  $\infty$ , which could be assumed as a frozen time. If  $v > c$ , it would give an imaginary proper time, mentioned by reference (Lartigue, 2018) as a coordinate in the Euclidian space. To avoid an imaginary time, it would be necessary to modify the equation (20) for the external Space as:

$$\gamma_s = (1 - \frac{(nc)^2}{(\Gamma_H \cdot t)^2})^{-.5} \tag{22}$$

where  $v = nc$  and  $(n \geq 1)$  would be the increasing factor of  $c$ , at a time  $t > t_0$  . The fraction should always be smaller than 1 since matter velocity ( $nc$ ) at time  $t_s$  cannot be higher than the space expansion velocity ( $\Gamma_H \cdot t_s$ ), at least till  $n > 600$ . So, the proper time in the external space (outside the gravitational field) would anyway remain as a real and positive number. This today impossibility in the Physical Universe, could be feasible for a future matter eventually traveling in the external space.

The  $\Lambda$  concept has varied from the Einstein’s one (a constant necessary for a static Physical Universe), to be excluded by Friedmann in his dynamical equations; though, after the Hubble discovery of the Universe expansion,  $\Lambda$  has been proposed again to represent a mysterious dark energy or quintessence, which nature and mathematical expressions, as functions of time or distance, have not yet been specified (Peebles, 2014). Instead, the  $\Gamma$  acceleration constant was proposed (Lartigue, 2016) as function of the Hubble parameter  $H$  and distance  $r$ ; and, in this article,  $\Gamma_H$  as function of  $r$  and  $t$  in equations (6) and (7).

**4. Conclusions**

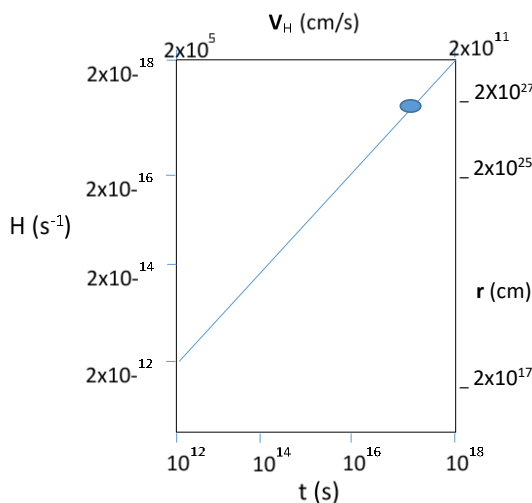


Figure 1. The scale factor  $r$  (cm) of Space and expansion velocity  $v_H$  (cm/s) as functions of  $H$  and time. The blue point corresponds to the present time  $t_0$ . The quotient of both abscissas gives the acceleration constant  $\Gamma_H = 2 \times 10^{-7} \text{ (cm/s}^2\text{)}$ . The product of both abscissas gives the scale factor  $r$  of the Space

**4.1-**The relationship between the Hubble parameter and time has been given by equation (2) as  $H = 2/t \text{ (s}^{-1}\text{)}$ . The distances, in the Hubble field, results to be  $r_H = 2r$ . The Hubble velocity is  $v_H = 2v$ . The space expanding

acceleration has been expressed as a constant by equation (7):  $\Gamma_H = H^2 r/2$  (cm/s<sup>2</sup>). The positive Hubble potential of the Space expansion was deduced in equation (10) as:  $V_H = H^2 r^2/4$  (cm/s)<sup>2</sup>. The present value of the Space expansion velocity is  $3c$ . The previous horizons concepts have been questioned for an expanding Universe.

**4.2** Regarding the  $\Omega$  parameter, it has been proposed an equation (16) for the critical density as a function of the critical time; it was shown that the critical time has occurred at  $1/5$  of the present time, which guarantees a future expansion in a plane geometry.

**4.3** It was proposed an equation (22) for the  $\gamma$  parameter in the spatial case (where it could be  $v > c$ ) in order to obtain, always, a positive proper time  $\tau$ .

**4.4** Equation (2) has several implications:  $H$  diminishes as time increases; Hubble radius ( $r_H = c \cdot t$ ) would become, if  $v_H > c$ , ( $r_H = v_H \cdot t$ ). Otherwise, the light cone will continuously expand to become a circular surface that would cancel the “elsewhere” zones in the Loedel diagram.

**4.5** Figure 1 shows the values of  $R$ ,  $H$ ,  $v_H$  and  $\Gamma_H$  as time functions.

### Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

### References

- Einstein, A. (1923). *The Principles of Relativity* (p.106). Dover Pub. U.S.A.
- Hawkins, S. W. (1988). *Historia del tiempo* (p. 179) Ed. Crifica, México.
- Keck MOSFIRE Spectroscopy*. (2013). UDFj-29546284, Wikipedia.
- Lartigue, J. G. (2016). The Hubble Field versus Dark Energy. *Journal of Modern Physics*, 7, 1610.
- Lartigue, J. G. (2018). A photonic model of the Big Bang. *Journal of Modern Physics*, 9.
- Lidsey, J. E. (2014). *Cosmology Course ASTM*, 108.
- Peebles, P. J. E. (2014). The Natural Science of Cosmology. *Journal of Physics, Conference Series*, 484.
- Sartory, L. (1996). *Understanding Relativity* (p. 315). UCLA, Los Angeles.
- Tagliaferro, T. A. (2015). *Lentes gravitacionales fuertes*. Tesis, Cap. 1.2, Universidad Complutense, Madrid.

### Copyrights

Copyright for this article is retained by the author(s), with first publication rights granted to the journal.

This is an open-access article distributed under the terms and conditions of the Creative Commons Attribution license (<http://creativecommons.org/licenses/by/4.0/>).