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Effects of Surface Tension and General Rotation on the Rayleigh-Taylor Instability of Three-Layer Flow

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Abstract

The stability of two interfaces separating three fluids, where the fluids are assumed to be incompressible, inviscid, and of constant density, has been investigated for a system that is acted upon by a general rotation. The effect of surface tension at the two interfaces is taken into account. A general dispersion relation for the system is obtained analytically by formulating the problem in terms of complex variables. Numerical calculations were performed for a hexane-NaCl-CCl_4 system to investigate stable case, and special cases that isolate the effect of various parameters on the growth rate of the Rayleigh-Taylor instability are discussed. It is found that the two cutoff wave numbers for the system with surface tension are unchanged by the addition of a general rotation, and that for the system considered, all growth rates are reduced in the presence of a general rotation.

Keywords: Rayleigh-Taylor instability, surface tension, inertial-confinement fusion

1. Introduction

The Rayleigh-Taylor instability (RTI) (Rayleigh, 1883; Taylor, 1950) refers to the instability of the plane interface between two fluids of different density which were superimposed one over the other and were subjected to gravity. When the upper fluid has a density greater than the lower fluid, the interface can be unstable to small perturbations. The amplification which is well described by linear theory with dependence on the density ratio and gravity, where the disturbance depends on the gravitational acceleration g and the Atwood number $A=\dfrac{\rho^{(2)}-\rho^{(1)}}{\rho^{(2)}+\rho^{(1)}}$ and grows exponentially in time like $\exp(nt)$ ($n=[gkA]^{1/2}$), where $\rho^{(1)}$, $\rho^{(2)}$ are the densities of the lighter and heavier fluids and k is the wave number. As the amplification continues and the problem enters the nonlinear regime, the fluid mixing can become chaotic, characterized by bubbles of lighter fluid rising up into the heavier fluid, and spikes of heavier fluid falling down into the lighter fluid, with regions of high vorticity near the spike/bubble interface. The most striking feature of RT turbulence is the formation of a turbulent mixing zone with width L (or amplitude) that grows quadratically with time, i.e. $L=\alpha Agt^2$, where α is constant (Sharp, 1984).

Over the past years growing interest in the Rayleigh-Taylor instability, because Such a phenomenon is present in a wide variety of situations such as in physics (e.g., inertial confinement fusion, for example, in inertial-confinement fusion (ICF) (Lindl, 1995) a directed high energy density provided by a set of laser beams, is used to strongly compress a small pellet filled with deuterium-tritium in order to initiate nuclear burn. The perturbations which are generated in various locations in the pellet may grow with time through RT-type instabilities. On the other hand, the interest of RT instability goes beyond the above applications) or in astrophysics (e.g., supernovae, Fryxell et al., 1991; supernova remnants Chevalier et al., 1995; Jun et al., 1995) SNRs, HII regions (Williams et al., 2001). Indeed, this is a classical problem in fluid dynamics. The appearance of such instabilities in previous topics has inspired us to study it and this is the main motivation of this work.

The role that plays the surface tension on Rayleigh-Taylor instability problem on linear stage has been studied by several authors (Bellman, 1954; Chandrasekhar, 1961; Daly, 1976; Mikaelian, 1983). Recently, the effect of surface tension and rotation (about the *z-Axis*) on Rayleigh-Taylor instability of two superposed fluids with suspended particles studied by (Sharma et al, 2010). All results show that, the presence of surface tension will bring about stability on the growth rate of unstable configuration, also the surface tension has critical strength to suppress the instability completely.

The effect of rotation with and without other factors in the *z-Axis* or *x-Axis* and in *x-Axis* and *z-Axis* on interface between two superposed fluids studied by several authors (Hide, 1954; Chandrasekhar, 1961; Chakraborty et al., 1974; Davalos, 1993). Also, the results show that, the presence of rotation will bring about stability on the growth rate of unstable configuration, while the rotation has no critical strength to suppress the instability completely.

The effect of a general rotation with and without surface tension or horizontal magnetic field on linear stage for two fluids was considered by (Davalos et al., 1989; Davalos, 1996) and in the presence of surface tension through porous media was studied by (Hoshoudy, 2011).

The Rayleigh-Taylor instability problem in multi-layer fluids was considered by (Chakaraborty et al., 1976; Mikaelian, 1982; Khater et al., 1984; El-Shehawey et al., 1985; Mohamed et al., 1986; Mikaelian, 1990; Obied-Allah, 1991; Parhi et al., 1991; Mohamed et al., 1994; Cherfils et al., 2000; Defne et al., 2000; El-Ansary et al., 2002; Hoshoudy, 2007). The effect of general rotation on the Rayleigh-Taylor instability of three fluids through porous media under the influence of uniform magnetic field was investigated by Hoshoudy (2011). The Rayleigth-Taylor instability of a system three fluids separated by one unstable and one stable interface is investigated experimentally by (Jacobs et al., 2005).

In this paper, we consider the linear Rayleigh-Taylor instability for three incompressible fluids in the presence of both the surface tension and the general rotation $\Omega = (\Omega_x, \Omega_y, \Omega_z)$. The system consists of three fluids of constant densities. The goal is to obtain the dispersion relation that determines the growth rate n as a function of the physical parameters of the system considered. It also determines the role of vertical and horizontal components of rotation on instability of the system considered.

2. Formulation of the Problem

Consider the motion of an incompressible, inviscous fluid in the presence of a general rotation $\Omega = (\Omega_x, \Omega_y, \Omega_z)$. Let $\vec{U}_1 = (u_{x1}, u_{y1}, u_{z1})$, $p1$, $\rho1$ denote the perturbations in the velocity, pressure p, and density ρ, respectively. The relevant linearized perturbations equations of a fluid flowing are (Davalos et al., 1989; Davalos 1996; Hoshoudy, 2011)

$$\rho_0 \frac{\partial \vec{U}_1}{\partial t} = -\vec{\nabla} p_1 + \rho_1 \vec{g} + \left(\vec{U}_1 \, x \, \vec{\Omega}\right), \tag{1}$$

$$\vec{\nabla} \cdot \vec{U}_1 = 0, \tag{2}$$

$$\frac{\partial \rho_1}{\partial t} + \vec{\nabla} \cdot \left(\rho_0 \vec{U}_1\right) = 0 \tag{3}$$

where g is the acceleration due gravity directed anti-parallel to *z-axis*.

If our system is arranged in horizontal strata, then we suppose that the density is a function of the vertical coordinate **z** only. Then the system of Equations (1)-(3) can be put as

$$\rho_0 \frac{\partial^2 u_{z1}}{\partial y \partial t} - \frac{\partial}{\partial z}\left(\rho_0 \frac{\partial u_{y1}}{\partial t}\right) = 2\frac{\partial}{\partial y}\Big(\rho_0 \left(\Omega_y u_{x1} - \Omega_x u_{y1}\right)\Big) - 2\frac{\partial}{\partial z}\Big(\rho_0 \left(\Omega_x u_{z1} - \Omega_z u_{x1}\right)\Big) - g\frac{\partial \rho_1}{\partial y}, \tag{4}$$

$$\rho_0 \frac{\partial^2 u_{z1}}{\partial x \partial t} - \frac{\partial}{\partial z}\left(\rho_0 \frac{\partial u_{x1}}{\partial t}\right) = 2\frac{\partial}{\partial x}\Big(\rho_0 \left(\Omega_y u_{x1} - \Omega_x u_{y1}\right)\Big) - 2\frac{\partial}{\partial z}\Big(\rho_0 \left(\Omega_z u_{y1} - \Omega_y u_{z1}\right)\Big) - g\frac{\partial \rho_1}{\partial x}, \tag{5}$$

$$\rho_0 \frac{\partial}{\partial t}\left\{\frac{\partial u_{y1}}{\partial x} - \frac{\partial u_{x1}}{\partial y}\right\} = 2\rho_0 \left\{\left(\Omega_x \frac{\partial}{\partial x} + \Omega_y \frac{\partial}{\partial y}\right) u_{z1} - \Omega_z \left(\frac{\partial u_{x1}}{\partial x} + \frac{\partial u_{y1}}{\partial y}\right)\right\}, \tag{6}$$

$$\frac{\partial u_{x1}}{\partial x} + \frac{\partial u_{y1}}{\partial y} + \frac{\partial u_{z1}}{\partial z} = 0 \tag{7}$$

$$\frac{\partial \rho_1}{\partial t} + u_{z1} \frac{d\rho_0(z)}{dz} = 0, \tag{8}$$

Considering the perturbation in the physical quantity takes the form

$$\left(u_{x1}(x,y,z,t), u_{y1}(x,y,z,t), u_{z1}(x,y,z,t) \right) = \left(u_{x1}(z), u_{y1}(z), u_{z1}(z) \right) \exp\left(ik_x\, x + ik_y\, y + n\, t \right),$$
$$p_1(x,y,z,t) = p_1(z) \exp\left(ik_x\, x + ik_y\, y + n\, t \right) \tag{9}$$

where k_x, k_y $(k^2 = k_x^2 + k_y^2)$ are horizontal wave numbers and n denotes the rate at which the system departs from the equilibrium.

Making of the expression (9) in Equations (4)-(8) and eliminating some variables, we get the following differential equation in u_{z1}

$$n^2 \left\{ \frac{d}{dz}\left(\rho_0 \frac{d}{dz} \right) - k^2 \rho_0 \right\} u_{z1} + \left\{ g\, k^2 + 2in\Omega^- \right\} u_{z1} \left(\frac{d\rho_0}{dz} \right) +$$
$$4i\rho_0\, \Omega^+ \left\{ \Omega_z \frac{d}{dz} + i\Omega^+ \right\} u_{z1} + 4\Omega_z \frac{d}{dz}\left\{ \rho_0 \left(\Omega_z \frac{d}{dz} + i\Omega^+ \right) u_{z1} \right\} = 0 , \tag{10}$$

$\Omega^+ = k_x\Omega_x + k_y\Omega_y$ and $\Omega^- = k_y\Omega_x - k_x\Omega_y$.

3. The Instability for Three Layers

In this section we consider the effect of a general rotation and surface tension on RTI for three-layers, where the 3rd top and 1st bottom layers are bounded by a two horizontal parallel panels (two interfaces), where the first panel at $z = -h$ and the other one at $z = h$ (see Figure 1). In other words the two interfaces are horizontal on the direction of gravity. Moreover, the density is constant in each region. i. e., we specialize to the case of three constant-densities, which could be written in the form:

$$\begin{aligned} \rho_0 &= \rho^{(1)}, & z &\le -h, \\ \rho_0 &= \rho^{(2)}, & -h \le z &\le h,. \\ \rho_0 &= \rho^{(3)}, & z &\ge h. \end{aligned} \tag{11}$$

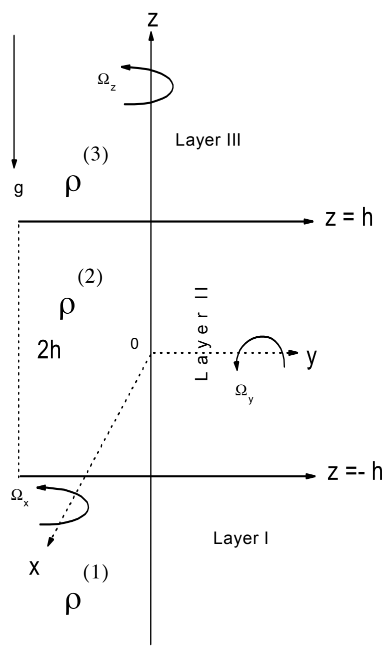


Figure 1. The geometry of the problem

For the case of constant-density equation (10) becomes

$$\left\{n^2+4\Omega_z^2\right\}\frac{d^2u_{z1}}{dz^2}+8i\Omega^+\Omega_z\frac{du_{z1}}{dz}-\left\{k^2n^2+4(\Omega^+)^2\right\}u_{z1}=0\ , \tag{12}$$

The general solution of Equation (12) is $u_{z1}=A\exp(q_1z)+B\exp(q_2z)$. Using the boundary conditions that the velocity is zero at large distances above and below the interface. Then in three region the vertical velocity takes the from

$$\begin{aligned} &(u_{z1})_1=A_1\exp(q_1z), && z\le -h,\\ &(u_{z1})_2=B_1\exp(q_1z)+B_2\exp(q_2z), && -h\le z\le h,\\ &(u_{z1})_3=C_1\exp(q_2z), && z\ge h, \end{aligned} \tag{13}$$

where A_1, B_1, B_2 and C_1 are constants

$$\begin{aligned} q_1&=\frac{1}{n^2+4\Omega_z^2}\left\{-4i\Omega^+\Omega_z+\left[n^2k^2(n^2+4\Omega_z^2)+4n^2(\Omega^+)^2\right]^{\frac{1}{2}}\right\}\\ q_2&=\frac{-1}{n^2+4\Omega_z^2}\left\{4i\Omega^+\Omega_z+\left[n^2k^2(n^2+4\Omega_z^2)+4n^2(\Omega^+)^2\right]^{\frac{1}{2}}\right\} \end{aligned} \tag{14}$$

The boundary conditions which are to be satisfied at the two interfaces $z=\pm h$ are

(i) At the interfaces between two various fluids we consider that u_{z1} is continuous at $z=\pm h$.

(ii) In the presence of surface tension, the jump conditions at the two interfaces $z=-h$ and $z=h$ are, respectively

$$\begin{aligned} &\left\{n^2+4\Omega_z^2\right\}\left\{\rho^{(2)}\frac{d(u_{z1})_2}{dz}-\rho^{(1)}\frac{d(u_{z1})_1}{dz}\right\}+4i\Omega^+\Omega_z\left\{\rho^{(2)}(u_{z1})_2-\rho^{(1)}(u_{z1})_1\right\}+\\ &\qquad\left\{\rho^{(2)}-\rho^{(1)}\right\}\left\{gk^2+2in\Omega^-\right\}(u_{z1})_{z=-h}-k^4T_{12}(u_{z1})_{z=-h}=0\\ &\hspace{20em},\\ &\left\{n^2+4\Omega_z^2\right\}\left\{\rho^{(3)}\frac{d(u_{z1})_3}{dz}-\rho^{(2)}\frac{d(u_{z1})_2}{dz}\right\}+4i\Omega^+\Omega_z\left\{\rho^{(3)}(u_{z1})_3-\rho^{(2)}(u_{z1})_2\right\}+\\ &\qquad\left\{\rho^{(3)}-\rho^{(2)}\right\}\left\{gk^2+2in\Omega^-\right\}(u_{z1})_{z=h}-k^4T_{23}(u_{z1})_{z=h}=0 \end{aligned} \tag{15}$$

where, T_{12} and T_{23} are the surface tension at the lower and upper interfaces, respectively.

Using Equation (13) in the above boundary conditions and eliminating the constants A_1, B_1, B_2 and C_1. The dispersion relation is given by equation

$$\left\{\left[\rho^{(2)}-\rho^{(1)}\right]\left[g k^{2}+2in\Omega^{-}\right]-\left[\rho^{(2)}+\rho^{(1)}\right]\left[4n^{2}(\Omega^{+})^{2}+n^{2}k^{2}(n^{2}+4\Omega_{z}^{2})\right]^{\frac{1}{2}}-k^{4}T_{12}\right\}$$
$$\left\{\left[\rho^{(3)}-\rho^{(2)}\right]\left[g k^{2}+2in\Omega^{-}\right]-\left[\rho^{(3)}+\rho^{(2)}\right]\left[4n^{2}(\Omega^{+})^{2}+n^{2}k^{2}(n^{2}+4\Omega_{z}^{2})\right]^{\frac{1}{2}}-k^{4}T_{23}\right\}-$$
$$\left\{\left[\rho^{(2)}-\rho^{(1)}\right]\left\{\left[g k^{2}+2in\Omega^{-}\right]+\left[4n^{2}(\Omega^{+})^{2}+n^{2}k^{2}(n^{2}+4\Omega_{z}^{2})\right]^{\frac{1}{2}}\right\}-k^{4}T_{12}\right\} \quad . \tag{16}$$
$$\left\{\left[\rho^{(3)}-\rho^{(2)}\right]\left\{\begin{matrix}\left[g k^{2}+2in\Omega^{-}\right]- \\ \left[4n^{2}(\Omega^{+})^{2}+n^{2}k^{2}(n^{2}+4\Omega_{z}^{2})\right]^{\frac{1}{2}}\end{matrix}\right\}-k^{4}T_{23}\right\}\exp\left\{2(q_{2}-q_{1})h\right\}=0$$

Setting $\Omega_x=\Omega_y=0$, Equation (16) reduces to Equation (24) of (El-Ansary et al., 2002). If we also set $\Omega_z=0$ we recover Equation (12) of (Mikaelian, 1990). If, in addition, we set $T_{12}=T_{23}=0$, we recover Equation (7) of (Mikaelian,1982).

Inversion symmetry, meaning the dispersion relation is invariant under $\rho^{(2)}\rightarrow\rho^{(1)}\rho^{(3)}/\rho^{(2)}$, is valid in the absence of surface tension (Mikaelian,1982, 1990). Setting $T_{12}=T_{23}=0$ in Equation (16) and dividing it by $\rho^{(2)}$ we see that in the resulting equation $\rho^{(2)}$ occurs only in the combination $\rho^{(2)}+\rho^{(1)}\rho^{(3)}/\rho^{(2)}$ and we conclude that inversion symmetry is valid in presence of general rotation.

Looking for the cut-off wave number k_c, i. e., the wave number where n vanishes, we note that Equation (16), with the surface tension terms included, is independent of $\Omega^{\pm}$ or Ω_z when we set n=0, and therefore the discussion given (29) concerning k_c is unchanged by the addition of rotation. In particular, $k_c=\sqrt{g(\rho^{(2)}-\rho^{(1)})/T_{12}}$ and/ or $k_c=\sqrt{g(\rho^{(3)}-\rho^{(2)})/T_{23}}$ whenever one or both growth rates vanish.

Now we consider the next definitions

$$\Omega^{+}=k\,\Omega\cos(\theta-\alpha), \tag{17}$$

$$\Omega^{-}=k\,\Omega\sin(\theta-\alpha), \tag{18}$$

where θ is the angle of the propagation of perturbation with respect to the *x-axis* and α is the angle of the horizontal component of rotation with respect to the *x-axis* also. Thus in the non-dimensional form, we can rewrite equation (16) as

$$\left\{\left[1+2iN\sin(\theta-\alpha)\right]\left[\rho^{(2)}-\rho^{(1)}\right]-\left[N^{4}+4\begin{pmatrix}F_{z}^{2}+\\ F^{2}\cos^{2}(\theta-\alpha)\end{pmatrix}N^{2}\right]^{\frac{1}{2}}\left[\rho^{(2)}+\rho^{(1)}\right]-F_{T_{12}}\right\}$$
$$\left\{\left[1+2iN\sin(\theta-\alpha)\right]\left[\rho^{(3)}-\rho^{(2)}\right]-\left[N^{4}+4\begin{pmatrix}F_{z}^{2}+\\ F^{2}\cos^{2}(\theta-\alpha)\end{pmatrix}N^{2}\right]^{\frac{1}{2}}\left[\rho^{(3)}+\rho^{(2)}\right]-F_{T_{23}}\right\}- \tag{19}$$
$$\left\{\left[\left[1+2iN\sin(\theta-\alpha)\right]+\left[N^{4}+4\left(F_{z}^{2}+F^{2}\cos^{2}(\theta-\alpha)\right)N^{2}\right]^{\frac{1}{2}}\right]\left[\rho^{(2)}-\rho^{(1)}\right]-F_{T_{12}}\right\}$$
$$\left\{\left[\begin{matrix}\left[1+2iN\sin(\theta-\alpha)\right]- \\ \left[N^{4}+4\left(F_{z}^{2}+F^{2}\cos^{2}(\theta-\alpha)\right)N^{2}\right]^{\frac{1}{2}}\end{matrix}\right]\left[\rho^{(3)}-\rho^{(2)}\right]-F_{T_{23}}\right\}\exp(-4QK)=0$$

Where

$$Q=\frac{1}{\left(N^2+4F_z^2\right)}\left\{N^4+4\left(F_z^2+F^2\cos^2(\theta-\alpha)\right)N^2\right\}^{\frac{1}{2}}, \tag{20}$$

$$N=\frac{n}{(kg)^{\frac{1}{2}}},\quad F=\frac{\Omega}{(kg)^{\frac{1}{2}}},\quad F_z=\frac{\Omega_z}{(kg)^{\frac{1}{2}}},\quad F_{T_{12}}=\frac{k^2}{g}T_{12},\, F_{T_{23}}=\frac{k^2}{g}T_{23}\ \text{ and }\ kh=K \tag{21}$$

We now consider some special cases

(1) In the case of short wave-length ($\lambda << h$), where $k=\frac{2\pi}{\lambda}$ and $F_{T_{12}}=F_{T_{23}}=0$, then the dispersion relation (19) takes two modes

$$\left\{\rho^{(2)}+\rho^{(1)}\right\}\left\{N^4+4\left(F_z^2+F^2\cos^2(\theta-\alpha)\right)N^2\right\}^{\frac{1}{2}}=\left\{\rho^{(2)}-\rho^{(1)}\right\}\left\{1+2i\,NF\sin(\theta-\alpha)\right\} \tag{22}$$

$$\left\{\rho^{(3)}+\rho^{(2)}\right\}\left\{N^4+4\left(F_z^2+F^2\cos^2(\theta-\alpha)\right)N^2\right\}^{\frac{1}{2}}=\left\{\rho^{(3)}-\rho^{(2)}\right\}\left\{1+2i\,NF\sin(\theta-\alpha)\right\} \tag{23}$$

Equation (22) is the similar to Equation (3) of the work (Davalos, 1996a).

(2) For the case of long wavelength ($\lambda >> h$) and F_{T12}=F_{T23}=0, Equation (19) becomes

$$\left\{\rho^{(3)}+\rho^{(1)}\right\}\left\{N^4+4\left(F_z^2+F^2\cos^2(\theta-\alpha)\right)N^2\right\}^{\frac{1}{2}}=\left\{\rho^{(3)}-\rho^{(1)}\right\}\left\{1+2i\,NF\sin(\theta-\alpha)\right\} \tag{24}$$

where the growth rate independent on $\rho^{(2)}$.

(3) From equation (19) N is purely imaginary (i. e. $N = iI$), then the dispersion relation takes the form

$$\begin{aligned}&\left\{\left[1-2IF\sin(\theta-\alpha)\right]\left[\rho^{(2)}-\rho^{(1)}\right]-\left[I^4-4\left(F_z^2+F^2\cos^2(\theta-\alpha)\right)I^2\right]^{\frac{1}{2}}\left[\rho^{(2)}+\rho^{(1)}\right]-F_{T_{12}}\right\}\\&\left\{\left[1-2IF\sin(\theta-\alpha)\right]\left[\rho^{(3)}-\rho^{(2)}\right]-\left[I^4-4\left(F_z^2+F^2\cos^2(\theta-\alpha)\right)I^2\right]^{\frac{1}{2}}\left[\rho^{(3)}+\rho^{(2)}\right]-F_{T_{23}}\right\}-\\&\left\{\left[\left[1-2IF\sin(\theta-\alpha)\right]+\left[I^4-4\left(F_z^2+F^2\cos^2(\theta-\alpha)\right)I^2\right]^{\frac{1}{2}}\right]\left[\rho^{(2)}-\rho^{(1)}\right]-F_{T_{12}}\right\}\\&\left\{\left[\begin{matrix}\left[1-2IF\sin(\theta-\alpha)\right]-\\\left[I^4-4\left(F_z^2+F^2\cos^2(\theta-\alpha)\right)I^2\right]^{\frac{1}{2}}\end{matrix}\right]\left[\rho^{(3)}-\rho^{(2)}\right]-F_{T_{23}}\right\}\exp(-4Q'K)=0\end{aligned} \tag{25}$$

$$Q' = \frac{1}{\left(4F_z^2 - I^2\right)}\left\{ I^4 - 4\left(F_z^2 + F^2\cos^2(\theta-\alpha)\right)I^2 \right\}^{\frac{1}{2}}, \tag{26}$$

Then the stability condition is

$$I^4 \geq 4\left\{ F_z^2 + F^2\cos^2(\theta-\alpha) \right\}I^2 \quad \text{or} \quad |N|^2 \geq 4\left\{ F_z^2 + F^2\cos^2(\theta-\alpha) \right\}. \tag{27}$$

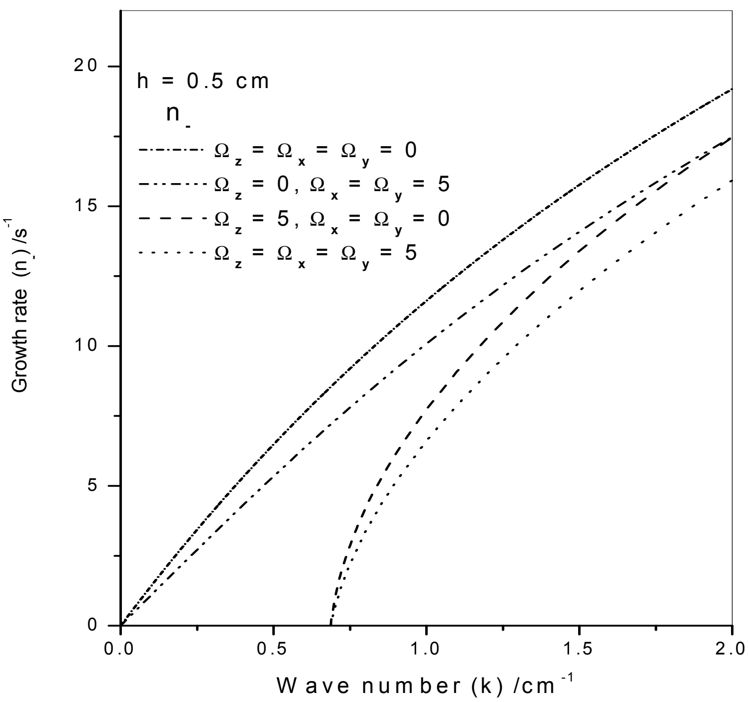
4. Discussion

Equation (16) is the dispersion relation of three fluids under the effect of general rotation, where the roots of this equation are complex numbers, which mean physically, that we have unstable perturbation.

We consider the case Ω^-=0 for which Equation (16) yields two real solutions denoted by $n_\pm^2$. Only for very short wavelengths, $\lambda << h$, do these solutions separately describe the growth of perturbations at each interface, independently of the other. For intermediate and long wavelengths each interface evolves under the influence of both growth rates and the interfaces are said to be "coupled" (Mikaelian, 1982, 1990). We shall plot the growth rates obtained numerically from Equation (16) for different values of thickness, surface tension, and rotation.

In order to investigate the effect of both the surface tension with the general rotation on the system considered, the dispersion relation (16) in the case of Ω^-=0 is to be numerically solved. At Ω^-=0 we find that $k_y\Omega_x = k_x\Omega_y$. If we put Ω_x=Ω_y, we have $k_x = k_y$. Thus, in equation (16) we put $\Omega^+ = \sqrt{2}k\Omega_x$ or $\sqrt{2}k\Omega_y$. Numerical calculations are presented in Figures 2–4. Our numerical example is the Hexane-NaCl solution-carbon tetrachloride system which has densities ($\rho^{(1)}$,$\rho^{(2)}$,$\rho^{(3)}$) = (0.66,1.027,1.593) gcm^{-3} and the surface tensions are (T_{12}, T_{23}) = (26, 22) $dyn{\cdot}cm^{-1}$. Also we will consider the acceleration of gravity is g = 980 cm^{-2}, Ω_z=Ω_x=Ω_y 5 or 10 $rads^{-1}$, $k_{critical} = k_c$ is the critical value for instability (at the point k_c the growth rate goes to zero) and k_{coup} is wave number at which the growth rate changes its behavior.

Figures 2(a) and 2(b) present the numerical results (positive roots (n_-, n_+)) of equation (16) in the absence of surface tension and presence of the general rotation at $h = 0.5\,cm$. One can see that both vertical and horizontal components of rotation have a stabilizing influence on the growth rate unstable configuration for inviscous fluids. For n (Figure 2(a)) and for the same values the effect of the vertical component of rotation is greater than the effect of horizontal components for short wave number values, while for n_+ (Figure 2b) the effect of horizontal components of rotation is greater than the effect of vertical component for all wave number values considered. The effect of Ω_z suppresses the perturbations for n_- only for $k < 0.6\,cm^{-1}$ which are unstable otherwise.



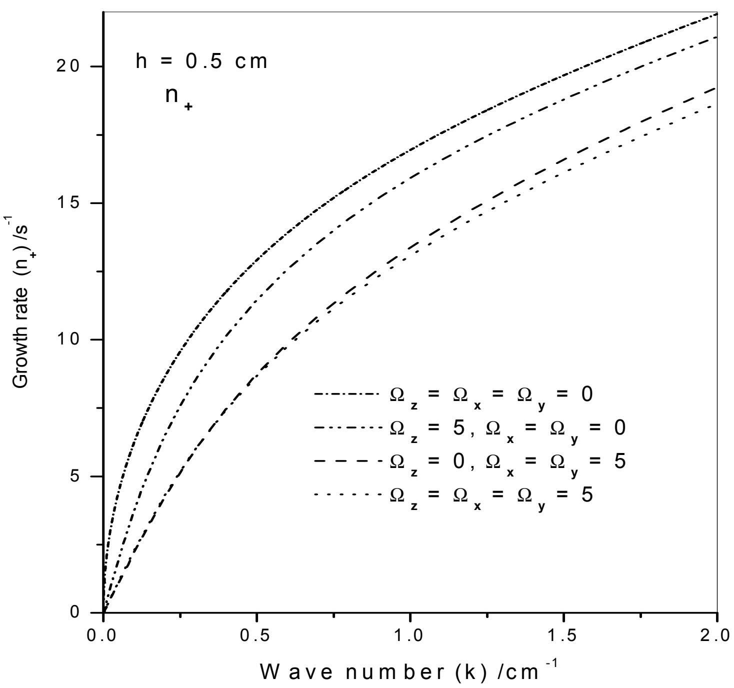


Figure 2. The effect both vertical and horizontal components of rotation on the $(\rho^{(1)},\rho^{(2)},\rho^{(3)}) = (0.66,1.027,1.593)$ gcm^{-3} system in the absence of surface tension ($T_{12} = T_{23} = 0$) at $h = 0.5cm$, $0 \le k \le 2$, we consider $\Omega_z = 5$ and $\Omega_x = \Omega_y = 0$, $\Omega_z = 0$ and $\Omega_x = \Omega_y = 5$ and $\Omega_z = \Omega_x = \Omega_y = 5$ (a) for n_- (b) for n_+

Figures 3(a) and 3(b) show our numerical results of the growth rate (n_-, n_+) with respect to the wave number at different thickness (*h*=0.1 and *h*=0.5 cm) for hexane-NaCl-CCl_4 systems under the influence of T_{12} and T_{23} with and without the vertical and horizontal components of rotation. It clearly shows that behavior of n_- (Figure 3a) in the presence of or absence of general rotation n_- increases with increasing *h*. Also $k_c = 3.72\,cm^{-1}$ for all cases. In Figure 3(b) we note that the magnitude of n_+ decreases with increasing h below $k_{coup} = 3.9\,cm^{-1}$ in the presence of surface tension and in the absence of general rotation, while above $k_{coup} = 3.9\,cm^{-1}$ the magnitude of n_+ increases with increasing h. The same phenomenon holds in the presence of general rotation but at $k_{coup} = 3.5\,cm^{-1}$ and $k_c = 5.02\,cm^{-1}$ for all cases.

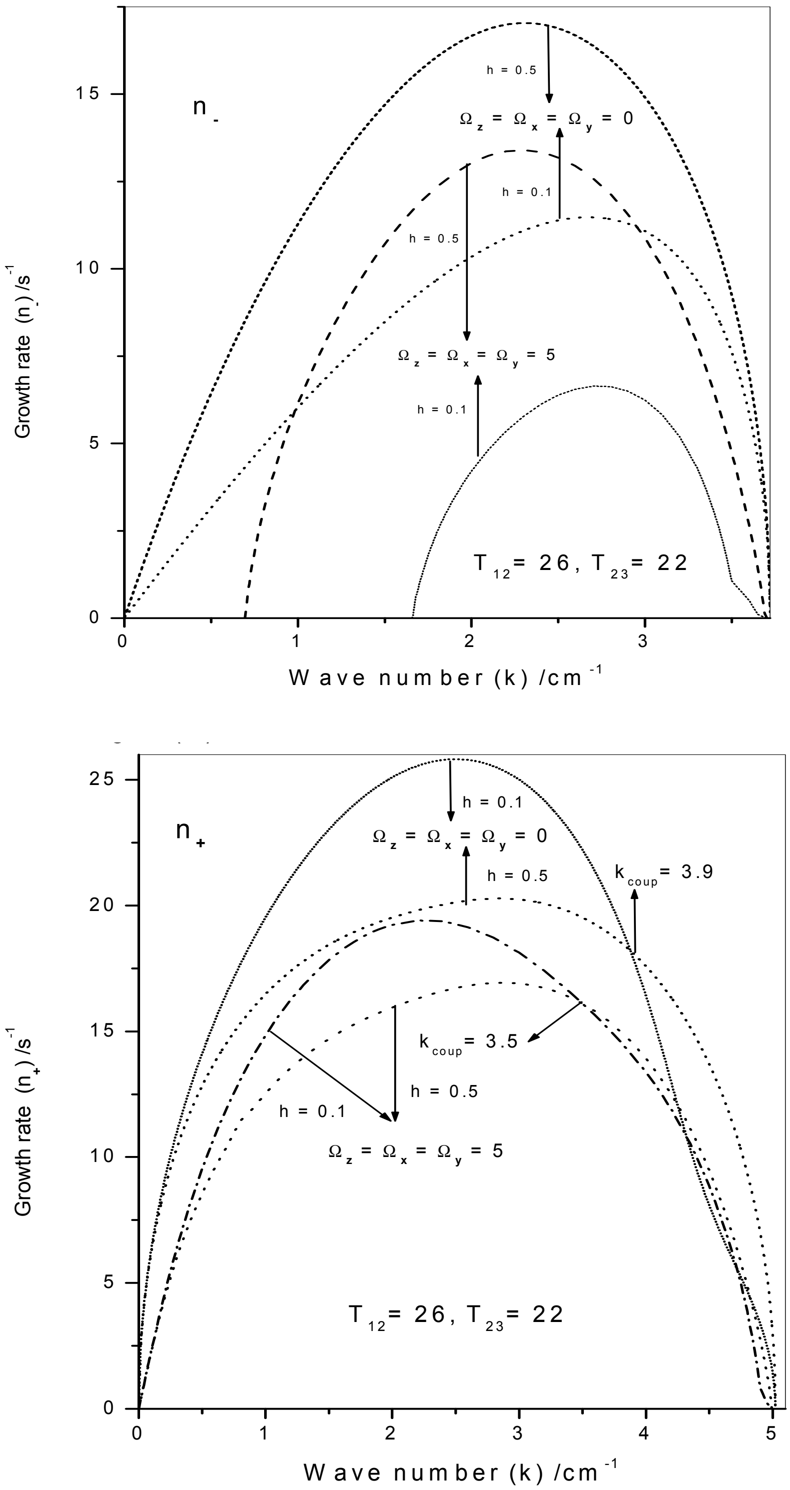


Figure 3. The effect both vertical and horizontal components of rotation on the $(\rho^{(1)},\rho^{(2)},\rho^{(3)}) = (0.66, 1.027, 1.593)$ gcm^{-3} system in the presence of surface tension ($(T_{12},T_{23}) = (26,22)\,dyn\,cm^{-1}$) at $h = 0.1\,cm$, $h = 0.5\,cm$ and $\Omega_z = \Omega_x = \Omega_y = 5$ (a) for n_-, $0 \le k \le 3.71$ (b) n_+, $0 \le k \le 5.02$

Figure 4. shows n_-, n_+ against the wave number at $h=0.5cm$ for various values of the vertical and horizontal components of rotation ($\Omega_z = \Omega_x = \Omega_y = 5$ and $\Omega_z = \Omega_x = \Omega_y = 10$). It is clear that both n_-, n_+ decrease with $\Omega_z, \Omega_x, \Omega_y$ increase.

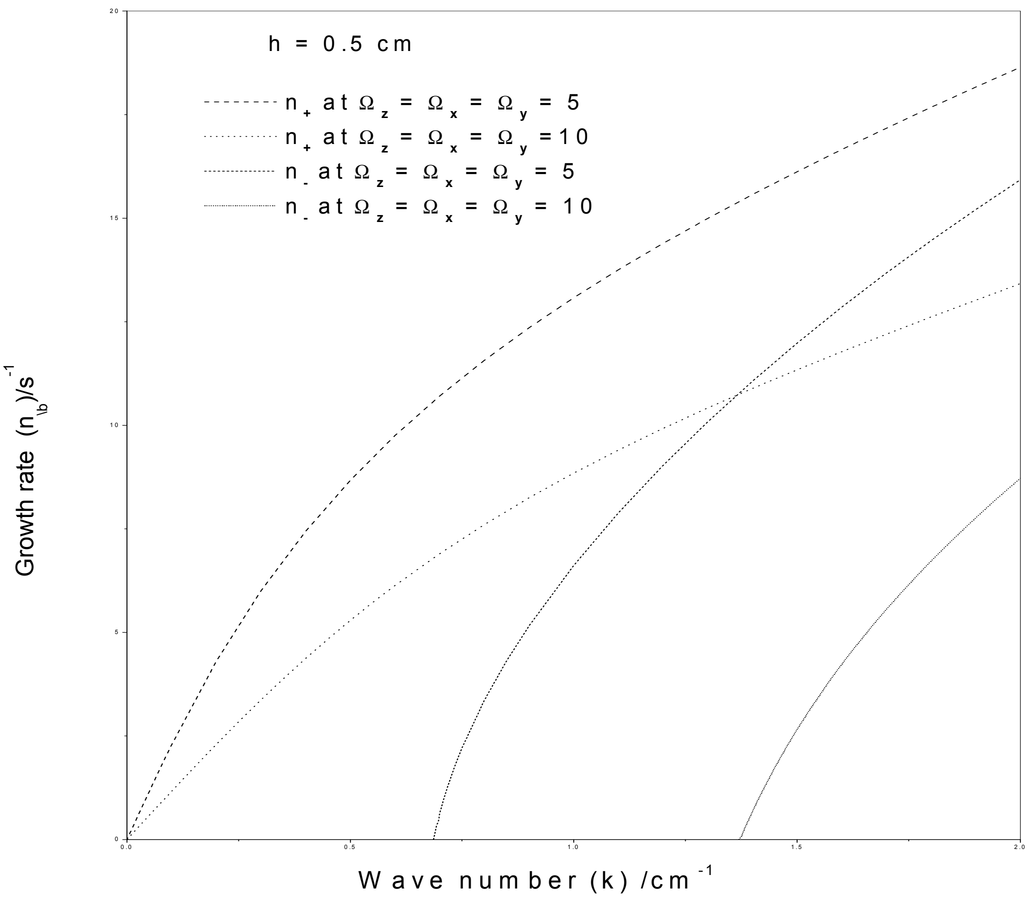


Figure 4. The values of n_-, n_+ at different values of the vertical and horizontal components of rotation ($\Omega_z = \Omega_x = \Omega_y = 5$ and $\Omega_z = \Omega_x = \Omega_y = 10$) for $(\rho^{(1)}, \rho^{(2)}, \rho^{(3)}) = (0.66, 1.027, 1.593)\, gcm^{-3}$ system in the absence of surface tension at $h=0.5cm$ and $0 \le k \le 2$

5. Conclusion

We have presented the analytical results of the Rayleigh-Taylor instability with general rotation for a system consisting of three non-viscous fluids in the presence of surface tension. Numerical calculations for the Hexane-NaCl solution-carbon tetrachloride system have been analyzed, where the growth rate is plotted against the wave number. Whereas the surface tension has a critical strength to suppress the instability completely, while the general rotation has no such strength.

Finally, the general rotation has no essential influence on the general behavior of the growth rate, but the magnitudes of the growth rate in the presence of a general rotation are slightly less than their magnitudes in absence of the general rotation. In the presence of surface tension, the value of the critical point still that as in the presence and absence of general rotation, while the coup point in the presence of general rotation is less than its value with no general rotation. Also, we have noticed that, the influence of general rotation is felt more for small wave numbers. The inversion symmetry property is conserved in the presence of general rotation and in the absence of surface tension, which is given by (Mikaelian, 1982, 1990).

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