

An Optimum Filter for Mobile Digital Communication Systems

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Abstract

The conservation of bandwidth in multi channel mobile, digital communication systems for both civil and military services introduces stringent filtering requirements. To obtain security advantages or to aid the integration of a digital network with existing equipment, the multiplexing of digital channels is conventionally a hybrid system of time division multiplexing and frequency division multiplexing. With the allocation of channels on a frequency basis there is an inherent energy spillover between adjacent channels resulting in cross-talk effects. This degenerative effect is further accentuated in mobile systems due to the unpredictable nature of the physical position of particular transmitters and receivers in the communication complex. There is a likelihood of a situation such that in a given location a weak signal is being received and at the same time, from a short distance away, transmissions are taking place on an adjacent channel. This paper is concerned with the design of filters to effect adequate filtering of the signal before transmission whilst incurring a minimum degradation in system performance. An optimum filter is derived to satisfy typical performance criteria.

Keywords: Digital communication, Multiplexing, Security, Filters

1. Introduction

The transmission of information about an input waveform by digital code messages is generally referred to as pulse code modulation. The three essential stages in the modulation process are sampling, quantization and encoding. The sampling frequency is governed by the nature of the input message and determines the basic pulse width, t , to be transmitted (Schiller, J. H. 2000) (Salmasi, A., and Gilhousen, K.S. 1991). The width of quantization defines the possible accuracy of reconstruction of the message of the receiving terminal and the encoding process determines the form of the spectral density function of the signal to be transmitted. The choice of transmitted code to represent the quantized sample amplitude depends on a number of factors but in this Paper the final form of the digital signal to be transmitted is taken as a balanced non-return-to-zero pulse code modulated signal (Paulraj, A. J. and Papadias, C. B. 1997) (Balaban, P., and Salz, J. 1991). This configuration, shown in figure 1 avoids the need to transmit d.c. and affords the advantage that a detection decision depends only on the present level being received and is independent of past history. By assuming that the amplitude of the signal at the (switch-on) instant is E or $-E$ with equal probabilities, then the signals form a stationary process which will be assumed ergodic. If at each time quantum a (1 or 0) is considered equally likely then the autocorrelation of a message realization is given by:

$$\Phi_x(r) = \frac{E^2(t-|r|)}{t}, \quad 0 \leq |r| \leq t \quad (1)$$

$$= 0, \quad t < |r| \leq \infty$$

Giving a power spectral density function of the form:

$$I_x(\omega) = E^2 \cdot t \left[\frac{\sin(\omega t / 2)}{\omega t / 2} \right]^2 \quad (2)$$

Equation 2 indicates that the bandwidth required for the transmission of a P.C.M. signal cannot be defined in a simple manner. Clearly, to minimize the effects of extra channel energy it is desirable to limit the bandwidth of the transmitted signal. However, restricting the bandwidth reduces the rate of build up of pulses, thus reducing the discrimination between the (1) level and the (0) level at the receiving terminal. The optimum shaping of the signal spectrum for the allocated channel bandwidth must, therefore, be a compromise. Accepting the necessity to pass the digital signal through a filter before transmission, the requirements of the filter in terms of energy limiting capabilities and maximum waveform degradation criteria must be laid down (Kim, Y. and Shamsunder, S. 1998). In view of the severe restriction that a receiver and transmitter utilizing adjacent channels, may be in close proximity, it has been estimated- that less than 0.1% of the total signal energy should appear outside the allocated channel bandwidth. Only then could the resultant extra-band energy effects be reduced to

negligible proportions. The rise-time requirements of the filter must ultimately be determined by the level of noise present in the (R-F link) but provision for pulses to reach at least 90% of their pre-filtered amplitude will be sufficient in most cases. From the viewpoint of detection requirements the over swings should be as small as possible, say at least below 1% of the difference between the two levels being transmitted.

2. Performance of the Gaussian Filter

The Gaussian filter is generally recommended for use with rapidly varying signals on account of its ideal transient response, in its theoretical form; its frequency response function obeys the Gaussian error function law:

$$H(j\omega) = e^{-\left(\log_e \sqrt{2}\right) \left(\frac{\omega}{\omega_3}\right)^2} \quad (3)$$

where $(\omega_3 = 2\pi f_3)$, is the 3 db point and is arbitrarily used to normalize the frequency scale, A linear phase characteristic is obtained in theory, in general, in this investigation of possible filter configurations a linear phase characteristic is assumed because phase distortion is not a problem as it can be cancelled to any required degree by means of phase correcting networks. The most severe digital signal from pulse rise-time considerations is the signal of format ...0101010... And the response of the Gaussian filter to this waveform will indicate the pulse height degradation in the filtered signal. By modifying the Fourier series coefficients of the input signal by equation 3, the maximum height of the output is readily obtained (Liang, J., W., Chen, J. T., and Paulraj, A. J. 1998)(Thomas, T. A. and Zoltowski, M. D. 1997). This is plotted in figure 2 as a percentage of the unfiltered amplitude for a range of pulse widths. Evidently to satisfy the 90 amplitude criterion the maximum permissible bit-rate for propagation through a Gaussian filter is 2.3 f₃. by combining equations 2 and 3, it is possible to evaluate the performance of a Gaussian filter as an energy limiting device for a P.C.M. signal. Figure 3 shows the extent of the extra-band energy as a Percentage of total signal energy for channel bandwidths normalized to the frequency f₃ Curves are shown for a range of bit-rates. The necessary integrations for the production of this data were performed numerically on an Atlas computer. The 'flats' in the curves (a), (b) and (c) correspond to the zeros in the signal spectral density function of equation 2. Curve (c), representing a normalized bit-rate of 2.5 f₃, is clearly the threshold curve for efficient band limiting properties of the Gaussian filter down to the 0.1% energy level. For bit-rates less than this minimum value, one or more inflexions occur in the critical region of the energy distribution diagram. The Gaussian filter is evidently sub-optimal for the specified performance criteria as two conflicting bounds to the signaling rate are obtained, Used inefficiently at the 2.3 f₃ bit-rate its figure of merit, F, defined as maximum bit-rate divided by channel bandwidth, is obtained as 0,85. The theoretical maximum value of this figure is, of course(Slock, D. T. M. 1996).

3. Filter Optimization

The design criteria specified for the performance of a typical mobile P.C.M. communication system is insufficient for an optimum filter to be derived analytically in closed form. The analysis of the Gaussian filter, however gives insight into possible numerical methods for the location of an optimum filter attenuation profile. An approach to the problem which has yielded satisfactory results will now be described. Basically the method was to describe numerically on a digital computer a class of channel attenuation profiles. The profiles were representative of all possible profiles and each was subjected to tests which yielded values of pertinent performance parameters from which the best curve could be selected. A quantization grid, the fineness of which was ultimately governed by the computing time available on the Atlas computer, was super-imposed. On the attenuation-frequency plane of a single communication channel Linear segment approximations of all possible attenuation curves passing through points on the grid were then generated. Certain trivial possibilities were, however, excluded and in addition each curve was constrained to be monotonic for ease of approximation and realization. A linear phase characteristic was assumed on the grounds of the concept of phase equalization(Molnar, K. J. and Bottomley, G. E. 1996)(Raleigh, G. G. and Paulraj, A. J. 1995). Curves were then rejected which did not permit 90% pulse amplitude buildup for a range of practical signaling rates, Precluded from further consideration were attenuation characteristics which exhibited excessive overshoots to a step change in signal level. Of the remaining curves, the one maximizing the figure of merit, F, was chosen as optimum. Because of the quantization of the channel attenuation-frequency plane, the description of the optimum attenuation profile is limited in accuracy. To improve this Accuracy, a finer quantization net was described in the restricted region of the neighborhood of the optimum and the optimization procedure repeated. After three iterations of the method an optimum was pinpointed to within the accuracy detectable in the response of the filter to a P.C.M. signal. The extra-channel attenuation characteristics were then adjusted to satisfy the 0.1% energy limitation requirement. The optimum filter is shown in Figure 4. It has no points of inflection in the energy distribution diagram: which resembles curve d of Figure 3. The value of maximum bit-rate divided by channel hand width is found to be 1.3.

4. Filter Realization

The production of a network having the amplitude frequency response of the optimum filter raises the dual problems of approximation and realization. The process of finding a function to approximate the required characteristic is simplified by the fact that, with the specified tolerances, a cosine squared function fits the curve of Figure 4. It remains to realize a network having a cosine squared amplitude response. Denoting the real frequency transfer function of the optimum filter by $Z_T(j\omega)$ we have

$$|Z_T(j\omega)| = \cos^2\left(\frac{\pi}{2} \cdot \frac{\omega}{\omega_o}\right) \tag{4}$$

Where ω_o is the frequency at which the right hand side of equation 4 first equals Zero. For this approximation the relation between ω_o and the angular bandwidth of the channel ω_c is

$$\omega_o = 1.29 \omega_c \tag{5}$$

Darlington has indicated the reconstruction of a transfer function from knowledge of its magnitude. It merely requires to note that the poles and, zeros of $|Z_T(s)|^2 = Z_T(-s)Z_T(s)$, occur in positive and negative pairs. By associating the poles and zeros in the left-half plane with $Z_T(s)$ and their mirror images in the right half-plane with $Z_T(-s)$, the transfer function can be obtained. A special form of $Z_T(s)$ exists when all its zeros are at infinity, its only critical frequencies being poles in the left half plane. Such a configuration can be synthesized as a simple LC ladder network whose characteristics can be described by a continued fraction expansion. Because of this simplification equation (4) is expressed as

$$|Z_T(j\omega)|^2 = \frac{1}{\sec^4\left(\frac{\pi}{2} \cdot \frac{\omega}{\omega_o}\right)} \tag{6}$$

By expanding the denominator of equation 6 by a Taylor series expansion about the frequency $\omega = 0$,

a number approximating polynomials to $\sec^4\left(\frac{\pi}{2} \cdot \frac{\omega}{\omega_o}\right)$ can be obtained. Since $|Z_T(j\omega)|$ is an even function of (ω) only terms in even powers of ω are present. Denoting

$$\sec^4\left(\frac{\pi}{2} \cdot \frac{\omega}{\omega_o}\right) = \sum_{n=0}^{\infty} a_{2n} \left(\frac{\omega}{\omega_o}\right)^{2n} + \dots$$

Approximations up to order 7 have been calculated with the results:

$a_0 = 1.00$	$a_2 = 4.935$	$a_4 = 14.21$	$a_6 = 31.38$
$a_8 = 4, 59.07$	$a_{10} = 99.90$	$a_{12} = 156.5$	$a_{14} = 231.5$

The location of the zeros of this polynomial for values of n in the range $1 \leq n \leq 7$ are given in Table 1. The realization procedure is completed by combining together the roots with negative real parts in Table 1 and performing a partial fraction expansion to determine the element values of the LC lossless ladder network. The component values will of course be normalized with respect to channel bandwidth and terminating resistance. The low frequency filter model is transformed to an actual bandpass filter for multichannel use by using well known techniques. If desired, predistortion of the roots of table I may be affected to produce elements which have an associated loss component. The optimum filter is found to have an almost linear phase characteristic and in most applications no phase correction will be necessary. A seven section filter is the minimum order of approximation to satisfy the stringent 0.1% extra-band energy requirement. With less strict band limiting criteria, lower order approximations will be sufficient because accuracy of approximation near $\omega = 0$ is maintained and desirable transient performance is still obtained.

Conclusion

There is a likelihood of a situation such that in a given location a weak signal is being received and at the same time, from a short distance away, transmissions are taking place on an adjacent channel. This paper is concerned with the design of filters to effect adequate filtering of the signal before transmission whilst incurring a minimum degradation in system performance. An optimum filter is derived to satisfy typical performance criteria.

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Table 1. The Location of the Polynomial Values

Order	n	Numerical Value of Zeros	
1		- 0.4013	
2		- 0.4685 ±	j 0.2142
3		- 0.5382 - 0.6576 1	j 0.3537
4		- 0.5558 1 - 0.4544 +	j 0.1510 j 0.4312
5		- 0.5977 - 0.5521 - 4, - 0.5301 +	j 0.2665 j 0.4056
6		- 0,5383 - - 0.6122 + - 0.5892 +	j 0.3576 j 0.1203 j 0.3803
7		- 0.6416 4 - 0.6116 - - 0.6363 4- - 0.5201 IT	j 0.2186 j 0.3563 j 0.4312

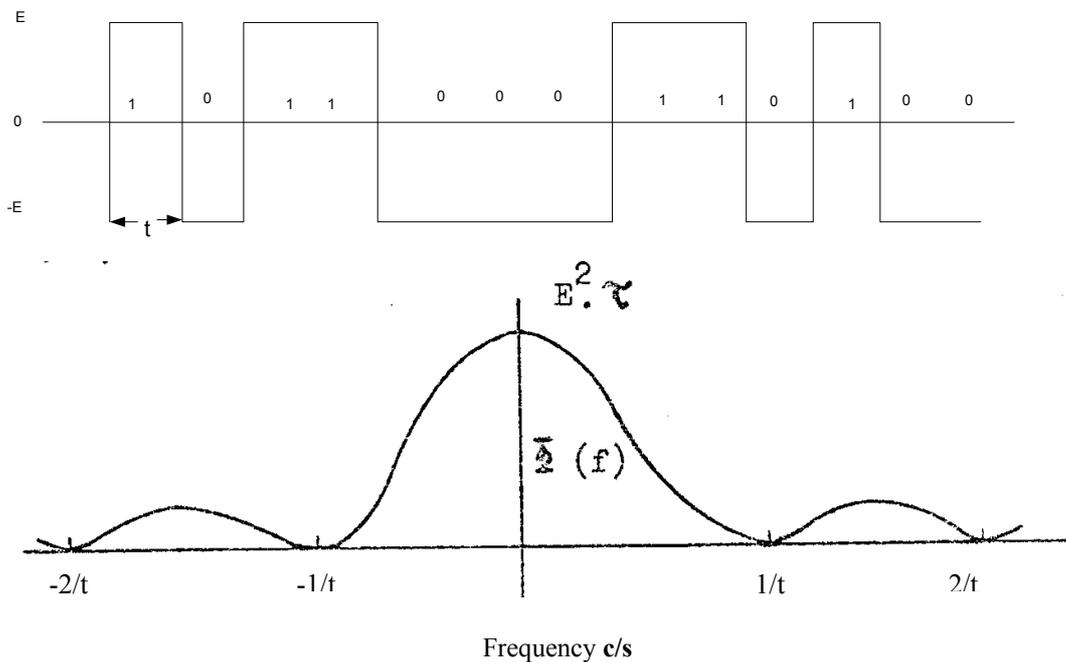
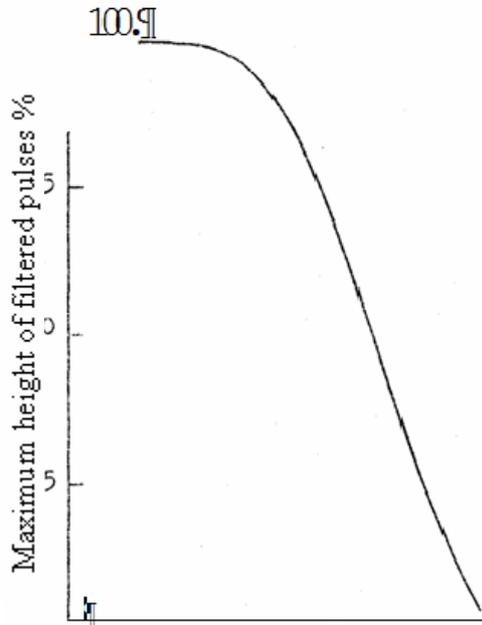


Figure 1. Digital signal format and its associated spectral density function



Bit rate F3

Figure 2. Guessing filtering of 0101010

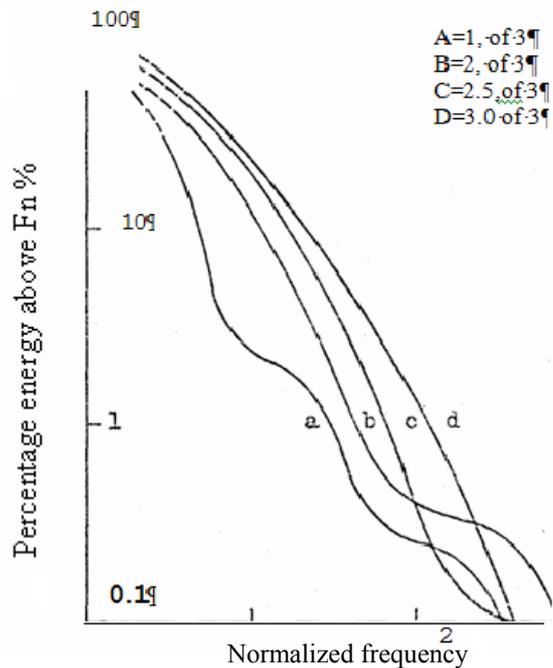


Figure 3. energy distribution diagram after

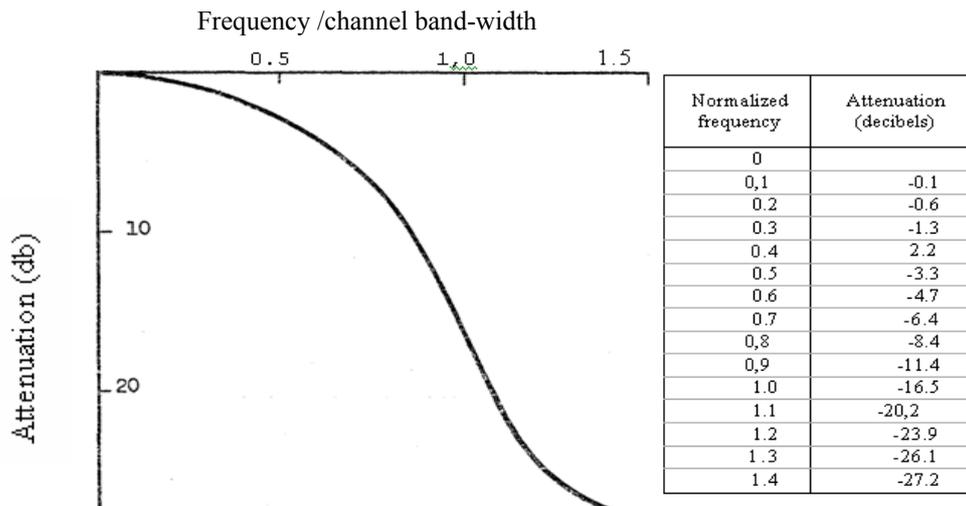


Figure 4. Optimum filter attenuation characteristic