Decision Making in Ignorance and Consequent Market Outcomes: Equilibrium Analysis

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Abstract

In recent years a series of important work has stressed the importance of iterative planning with market experimentation and learning in markets characterised by uncertain and dynamic environments. The possibility of herding, anti-herding and erroneous social learning, collectively called informational cascades, can add further instability to unstable markets in which actors suffer from significant knowledge gaps, or from plain ignorance. The main innovation of this paper is to make rational decision-maker aware of the adverse consequences of informational, or information, cascades in markets characterised by ignorance due to their uncertain and dynamic environments. This paper offers for the first time, to the best of our understanding, a simple modelling of a market with uncertain and dynamic environments and explains the existence of an equilibrium that is immune from the vagaries of whims, fads and informational cascades. The implication cannot be overstressed: issues of stock-out and markdown losses, market crashes and failures can be pushed to the background if the smart decision-makers can home in on this cascade-free equilibrium. In future work, it is imperative to design a mechanism that can act as perpetual succour to the cascade-free, or what we call “blessed”, equilibrium in contradistinction to the famed “cursed equilibrium”.

Keywords: Ignorance, Informational cascades, Fixed-point, Cascade-free-equilibrium

1. Introduction

Ignorance about market conditions in uncertain and dynamic environments is a serious concern in many modern businesses characterised by relative instability especially in the fashion goods industry and financial markets as highlighted in the early work of Fisher and Raman (1996) and Hammond (1990, 1992). In such markets, a typical supplier must predict the evolving and unknown demand correctly and introduce new products just in time and will also need to pull out before the market gets oversaturated and prices and profits start plummeting. For being financially viable a supplier will also need to predict its rivals’ marketing strategies with reasonable degrees of precision. In many such markets the demand history and the supply responses of rivals are non-existent for a typical supplier. Ignorance about the market is the rule rather than an exception and it is not folly, by any measure, to be wise in such markets. Yet knowledge is sketchy and the relevant history gradually builds up and it also changes its course quite abruptly and unpredictably in many markets with financial markets being a classic example. Long-term decision making is not only fraught with missing information and information gaps, but actual trading often keeps releasing coarser information. In recent empirical work on the role of information in financial markets, Barras, Scailet and Wermers (2010) and Duan, Hu, and Mclean (2009) marshal crucial evidence that institutional trading has become significantly less informative over time. This finding is in consonance with Yan and Zhang (2009), who find that long-term institutions are significantly less informed than short-term institutions, which is due to the difficulty associated with collecting, processing and interpreting reliable information with durable values. Immediate consequences of ignorance are three-fold: first, suppliers heavily invest in Quick Response Strategies (QRS) in order to reduce manufacturing and distribution lead times (Note 1). Secondly, despite Quick Response, the mismatch between revealed demand and planned supplies constituted roughly about 25% of annual retail sales (Note 2). Thirdly, the unknowability of the market condition has led to the escalation of fire-sales in such markets – as an example, Pashigian (1988) estimated that markdowns as a percentage of sales since World War II have increased from 3% to 16% for many apparel segments in the US.
In the existing literature, exogenous demand shocks, or demand uncertainty, are postulated as the main source of ignorance for decision-makers. In the case of pure demand uncertainty, huge losses are reported in automobiles industry (Jordan and Graves, 1991 and White, 1991) and computer industry (Stewart, 1992). MacCormack and Verganti (2003) highlight the importance of market uncertainty, driven by unknown and evolving requirement of customers for new products, in the context software development. The fluidity of environments encourages decision-makers to experiment with their markets. Market experimentation in the context of demand uncertainty is a double-edged sword: first, it can trigger fads and herding as highlighted by Scharfstein and Stein (1990), Banerjee (1992), Bikhchandani et al. (1992), Welch (1992) among many. Secondly, in an important recent work Callander and Horner (2009) consider informational externalities in a market where customers are either fully informed or uninformed, with no waiting costs. They illustrate how rational inference by decision-makers in this setup can motivate them to “go against the flow”, what is known as anti-herding, by following the footsteps of a small minority of decision-makers. Note that both herding and anti-herding instincts represent informational cascades in which agents ignore their private signals and rationally, or otherwise, mimic each other.

In this paper we examine market ignorance as a situation in which agents find it difficult to predict their individual demand due to unknown supply responses of their rivals, which can lead to high stock-out and markdown losses. The main innovation of this paper is to introduce the adverse consequences of informational cascades, or herding and anti-herding instincts, on the real returns of decision-makers. In the context of financial crisis, as a concrete example from the real world, we assume that an investor is also wary about the possible economic ruins that financial crisis and market collapse can beget from erroneous herding, or anti-herding. Any rational agent must take into account this potential hazard of going with, or against, the herd. If decision-makers are wary of crisis, what is expected then is that the rational agent can take adequate measures to avoid the pitfall of crowd behaviour. Herding and anti-herding, or cascades, will not be then a product of rational decision-making. The consequent equilibrium will be free of informational cascades and we call it cascade-free, or “blessed equilibrium”. It is important to realise that informational cascades can unleash devastating consequences for market participants. Cascades can and do cause market crashes and consequent rolling of heads. Rational agents should take this possible adverse effect into account while making their decisions. Can rational actions taken by rational agents prevent informational cascades (Note 3)? We provide an affirmative answer to this question. In this paper we also characterise the cascade-free, blessed equilibrium for the very first time in the literature. The layout of the paper is as follows: Section 2 reviews the literature and outlines the model and Section 3 offers an extension and Section 4 concludes.

2. Foundation of Decision Making in Ignorance

2.1 Related Literature

2.1.1 Market Experimentation

An undeniably important development in such markets is market experimentation by suppliers in which suppliers usually observe a fraction, say 20%, of demand in the initial phase to forecast the overall demand and then commit to a production strategy for the rest of the planning horizon (see Fisher and Raman, 1996). In their work Fisher and Raman formulated the production planning problem as a two-stage stochastic decision-making in order to obtain feasible solutions for decision-makers for minimizing the stock-out and markdown costs. It is important to note that Miller (1986) and Lovejoy (1990) examined demand uncertainty in the context of forecast updating. Bradford and Sugrue (1990) and Eppen and Iyer (1992) introduced the decision to produce several products in this context of demand uncertainty. In a very early work Hausman and Peterson (1972) considered the decision-theoretic problem of (demand) forecast updating in the presence of capacity constraints. In their work MacCormack and Verganti (2003) noted that the success of new software products is critically predicated upon early technical and market feedbacks and subsequent updating of these products by software firms. Veeraraghavan and Debo (2009) demonstrate how informational externalities emerge - when decision-makers are ignorant due to imperfect signals - by analysing a queue choice model without any waiting costs in the absence of any other externalities, which can seriously complicate decision-making in ignorance. A critical element of market success for products for which there is insufficient knowledge, or what we call ignorance, about their demand and supply characteristics is that successful firms tend to adopt iterative strategies that are propelled by experimentation, learning and adaptation - see Iansiti and MacCormack (1997) and MacCormack, Verganti and Iansiti (2001). This paper provides a comprehensive methodology to understand the process of learning and adaptation in the social context, which is a context in which firms learn from each other’s market experiments over time. The heart of the problem for us is that firms can behave in a herd-like, anti-herd, fashion that can exacerbate the intrinsic uncertainty in markets already characterised by unstable, uncertain and dynamic environments (Note 4).
2.1.2 Ignorance and Informational Cascades

Herd behaviour is a well-received doctrine in modern markets. Scharfstein and Stein (1990) modelled sequential investment by agents/investors concerned about their reputation as good forecasters. If these agents have correlated signals conditionally on the state of the world, they will imitate, or copy, the behaviour of the first investor. This kind of modelling has come to be known as reputational herding. In models of statistical herding, agents maximise expected profits in a common value environment and have access to conditionally independent private signals of bounded precision, while still watching the behaviour of others. Eventually, the accumulated evidence from observing earlier decisions is sufficiently strong to undermine the private information of a single decision maker. Callander and Horner (2009) consider informational externalities in a market where customers are either fully informed or uninformed, with no waiting costs and establish how anti-herding instincts can develop in such markets.

In the context of anti-herding, agents herd against the majority by flowing with the minority of decision-makers. Herding and anti-herding arise since the observed behaviour of others affects the probability belief of an agent attached to different states of the world and also the payoff conditional on each state. Both herding and anti-herding indicate some sorts of informational cascade. An informational cascade signifies a situation in which subsequent agents, based on the observations of others, makes the same choice independent of their private signals. Information cascades are argued to engender erroneous mass behaviour and cause fragility in markets where there are information gaps: Prechter and Parker (2007) find that uncertainty about valuation may cause herding. Kultti and Miettinen (2006) set up a standard sequential decision model to argue that if the cost of the information about the predecessors’ actions is very expensive then agents will act according to their own signals. If observation is not costly then one is shown to choose the herding behaviour. This has important connotation: facing financial panic, investors may not have enough time to collect and analyse valuable information from many disorderly data and, as a result, investors may herd during financial panic. Though this model is logically consistent, Henker, Henker and Mitsios (2006) find no market wide herding in Australian market during panic attacks. Our goal in this context is to establish a cascade-free equilibrium that remains immune from the vagaries of social learning. We call this cascade-free equilibrium a “blessed equilibrium” that is the polar case of the “cursed equilibrium” associated with erroneous (anti)herding.

2.1.3 Strategic Communications

Our model also contributes to an important new line of research what is known as strategic communication (SC), see Sobel (2007), Krishna and Morgan (2008), and Ottaviani and Sorensen (2006) for surveys and references on strategic communication. The idea is simple: in complex organisations the flow of information is of critical importance. Yet the information is not centrally available. Information is needed to take multiple decisions, across physical distance and over time. Information does not reside with any single individual (or unit) within the organisation. It rather evolves through time and is fragmented across a large number of participants. This necessarily creates a structure in which agents act both as senders (of what they have gathered directly or have been given by others), and receivers of information. This literature studies the strategic side of information transmission in a large group of agents with imperfectly aligned interests. Information improves through time and accrues to different agents, each of whom has some information that is useful to all others. Recent work by Kartik (2009) incorporates lying costs into the SC model and thereby bridges the gap between costly signalling and cheap talk. Li (2007) considers sequential arrival of information in a sender-receiver set up. Her focus is on what the sequence of reports by the sender reveals about his ability (the quality of his signals). In our model, action reveals the information in a sequential manner, though there is no other direct communication, or flow of information. Agents can thus be strategic in choosing their action since action, in our model, is louder than words.

2.2 A New Model of Decision Making in Ignorance

Suppose there are n agents in a stylised, global financial market characterised by n assets, like n styles or fashions. Each agent can take position in one and only one asset at a point in time. Each agent acts as a seller of an asset given the exogenous demand for this asset (Note 5). We assume agent i takes a position in asset i, and he makes a return $\Pi_i$ from selling each unit of this asset. The story is that return $\Pi_i$ depends on an informational cascade: if too many agents herd on asset i everyone suffers a loss from selling asset i due to an increased competition and congestion. On the other hand, agent i collects a handsome return if he remains the sole agent of asset i. We also consider a ‘trigger rate’ $R^N$ that governs switching of agents from one asset to another. This simple situation can give rise to social mimicking, or anti-mimicking, since agents know that there is some unknown correlation between other agents’ decisions and their private information. In the existing literature the implicit assumption is that decision-makers ignore the correlation, which gives rise to the notion of “cursed equilibrium” as expounded by Eyster and Rabin (2005; 2009). They offer simple but convincing examples where a decision-maker will take an action seemingly inconsistent with both one’s own signal and the beliefs of all previous movers. Our model
posits that agents don’t neglect the above correlation, which is believed to create a cursed equilibrium. Our behavioural foundation is provided in assumptions 1, 2, 3 and 4. The time structure of the problem is given in Figure 1.

**ASSUMPTION 1:** The trigger rate $R_N$ is the rate such that if $\Pi_i > R_N$, then agent $i$ apprehends that other agents may herd on asset $i$ and thereby drive its return below $R_N$. Agent $i$ hence expects two possible returns from asset $i$. He expects the high return to be $\Pi_i=(R_N+\Delta) > R_N$ where $\Delta$ is the excess return over the trigger rate. This rate materializes if there are no future cascades. On the other hand, agent $i$ expects the rate to be $\Pi_i=(R_N-\Delta) < R_N$, if there is a cascade on asset $i$. Agent $i$ further expects that this high (ex ante) rate will materialize with a probability $(1-\lambda)$, or will fail to materialize with a probability $\lambda$.

**ASSUMPTION 2:** If $\Pi_i < R_N$, then agent $i$ expects a sluggish exit of other agents from asset $i$.

Informational cascades can have adverse effect on the return from asset $i$ and, hence, the return from asset $i$ will reach a lower rate $(R_N-\Delta)$ with a probability $\lambda$. The main intuition here is that a typical agent may face an economic disaster if other agents withdraw simultaneously from other assets to take position into this concerned asset. Such a decline in return can lead to bankruptcy, or at least to an economic disaster (Note 6).

**OBSERVATION 1:** Agent $i$ thus confronts the following gamble: either he receives $(R_N+\Delta)$ with a probability $(1-\lambda)$, or receives $(R_N-\Delta)$ with a probability $\lambda$.

**CLAIM 1:** The expected value of the gamble is $E(.)$:

$$E(.)=R_N+\Delta(1-2\lambda) \quad (1a)$$

Proof: $E(.)$ by definition is the following:

$$E(.)=(1-\lambda)(R_N+\Delta)+\lambda(R_N-\Delta) \quad (1b)$$

Simplification of (1b) yields (1a). Q.E.D.

**ASSUMPTION 3:** We assume that the probability of informational cascades ($\lambda$) has a positive relation with the ex ante return and we express this as:

$$\lambda=(\eta/2)\Pi_i=\eta/2(R_N+\Delta) \quad \text{where} \quad \eta > 0 \quad (1c)$$

Note that (1c) implies that the larger is the value of expected/ex ante return, $\Delta$, the larger will be the probability of informational cascade ($\lambda$) and the smaller will be the probability $(1-\lambda)$. The idea is simple as agents expect whether other investors are alert enough to detect good and bad returns. What we propose is that agents’ ability and incentives to detect will depend on the returns at stake, which will thereby drive informational cascades.

**CLAIM 2:** The expected value of the gamble is reduced to the following:

$$E(.)=R_N+\Delta(1-\eta R_N)-\eta \Delta^2 \quad (1d)$$

Proof: Substituting (1c) in (1b) yields (1d). Q.E.D.

**OBSERVATION 2:** The expected value of the gamble is a quadratic function of $\Delta$ and it attains its maximum for $\Pi_i^*=\Pi_i^*$:

$$\Pi_i^*=R_N+\Delta^* \quad (2a)$$

where

$$\Delta^*={1-\eta R_N \over 2\eta} \quad (2b)$$

Hence

$$\Pi_i^*=(R_N/2)+((1/2)\eta) \quad (2c)$$

(2b) is obtained from the first order condition of maximizing (1d) with respect to $\Delta$. From Assumption 1 we know $\Pi_i^*=R_N+\Delta^*$ and hence (2a) is obtained. Substituting (2b) into (2a) yields (2c).

**ASSUMPTION 4:** We postulate that the trigger rate $R_N$ at date $t$ is a weighted average of ex ante returns at date $t-1$:

$$R_N^t=\sum_{i=1}^n w_i \Pi_i^{t-1} \quad (2d)$$
\[ \sum_{i=1}^{n} w_i = 1 \]  \hspace{1cm} (2f)

Where \( R_{i}^{t} \) is the trigger rate at date \( t \) and \( \Pi_{i}^{t-1} \) is the ex ante return from asset \( i \) at date \( t-1 \) and \( w_i \) is the weight attached to the asset \( i \) in the determination of the trigger rate.

**PROPOSITION 1:** The dynamics of the ex ante return on asset \( j \) is given by:

\[ \Pi_{j}^{*} = \left( \frac{1}{2} \right) \sum_{i=1}^{n} w_i \Pi_{i}^{t-1} + \left( \frac{1}{2\eta} \right) \]  \hspace{1cm} (3a)

Proof: Substituting (2d) into (2c), after appropriate dating, yields (3a). QED.

**THEOREM 1:** The unique and stable fixed point of the dynamic process (3a) is given by the following:

\[ R_{N} = \Pi_{1}^{*} = \Pi_{2}^{*} = \ldots = \Pi_{n}^{*} = \left( \frac{1}{\eta} \right) \]  \hspace{1cm} (3b)

This fixed point of the dynamic process involving ex ante returns and the trigger rate is the cascades-free equilibrium since once this state is reached agents have no incentive to imitate, or follow, others. The fixed point, hence, is cascades-free.

Proof: To derive the fixed point of (3a) we set \( \Pi_{1}^{*} = \Pi_{2}^{*} = \ldots = \Pi_{n}^{*} = \Pi \) for both dates \( t \) and \( t-1 \) and substitute them into (3a) to yield (3b). The stability of the fixed point is ensured since the slope of (3a) at the fixed point is \( (1/2) \). QED.

**ASSUMPTION 4:** We assume that there are two agents and two assets in our stylised model and the trigger rate \( R_{N}^{t} \) at date \( t \) is a non-linear function of ex ante rates \( \Pi_{i}^{*} \)s of date \( t-1 \):

\[ R_{N}^{t} = 2a(\Pi_{1}^{*})^2 + 2b(\Pi_{2}^{*})^2 \]  \hspace{1cm} (4a)

**THEOREM 2:** The dynamic path of ex ante returns on assets under Assumption 4 is given by

\[ \Pi_{i}^{*} = a(\Pi_{i}^{t-1})^2 + b(\Pi_{j}^{t-1})^2 + \left( \frac{1}{2\eta} \right) \]  \hspace{1cm} (4b)

The fixed points of the above path are given by:

\[ \Pi_{1}^{*} = \Pi_{2}^{*} = 1 \pm \sqrt{[1 - 2(a + b)/\eta]/[2(a + b)]} \]  \hspace{1cm} (4c)

The stable fixed point is given by

\[ \Pi_{1}^{*} = \Pi_{2}^{*} = 1 - \sqrt{[1 - 2(a + b)/\eta]/[2(a + b)]} \]  \hspace{1cm} (4d)

The larger value of the two roots given by (4c) is an unstable fixed point.

Proof: Substituting \( \Pi_{1}^{*} = \Pi_{2}^{*} = \Pi_{1}^{t-1} = \Pi_{2}^{t-1} = \Pi \) into (4b) yields

\[ (a + b)\Pi^2 - \Pi + \left( \frac{1}{2\eta} \right) = 0 \]  \hspace{1cm} (4e)

The roots of (4e) are the fixed points as outlined in (4d). For fixed point stability, we differentiate (4b) w.r.t \( \Pi_{i}^{t-1} \) and find that this derivative must be less than \( 1/[2(a + b)] \) which is true only for the smaller value of these roots. For the higher value, the derivative is greater than \( 1/[2(a + b)] \). Thus, the larger root is an unstable fixed point. We depict the fixed points and their stability property in Figure 2. QED.

Figure 2 is a one-agent representation of the model. In Figure 2 we depict ex ante rate along the horizontal axis and the trigger rate along the vertical axis. OA is the 45° Line and \( E_1 \) and \( E_2 \) are the two fixed points. \( E_1 \) is stable and \( E_2 \) is unstable. It is instructive to note whether the dynamic path of ex ante returns reaches the stable fixed point \( E_1 \) in Figure 2 depends on history and expectations. \( E_2 \) is the unstable fixed point. So long as the initial ex ante return for each asset is less than \( (1 + \sqrt{[1 - 2(a + b)/\eta]/[2(a + b)]}) \) and actual and ex ante returns are always contained within this bound, the dynamic path reaches the critical point \( E_1 \) that is a cascades-free equilibrium. The possibility of cascades remains open if ex ante, or ex post, returns from any of these assets is greater than \( (1 + \sqrt{[1 - 2(a + b)/\eta]/[2(a + b)]}) \). If the history, or expectations, somehow take the system beyond the unstable
fixed point $E_2$, rational agents will fail to reach the cascades-free equilibrium $E_2$ and may fail to prevent informational cascades ravaging returns from their assets.

3. An Extension: An Endogenous “Average Opinion”

At date $t$ agents may have different expectations about $R^N$ and, therefore, ex ante returns will fail to equalize across assets. The fixed point is defined to be the end-state that collectively confirms all ex ante returns so that the trigger rate equals the common ex ante return. We will now characterize the fixed point by utilizing the following definitions:

**DEFINITION 1:** We define the ‘average opinion’ $C$ as the following:

$$C = \frac{\sum_{i=1}^{n} \Pi_i^*}{n}$$

(5a)

**DEFINITION 2:** We define the ‘notional loss’ of agent $i$, $L_i$, from the average opinion as (Note7):

$$L_i = C - \frac{\sum_{j=1}^{n} (\Pi_j^* - \Pi_i^*)}{n}$$

(5b)

**DEFINITION 3:** We define a steady state as a vector of ex ante returns $(\Pi_1^*, \Pi_2^*, \ldots, \Pi_n^*)$ such that:

$$C = \Pi_i^*$$

for all $i \forall 1...n$ (5c)

and hence $L_1 = L_2 = \ldots = L_i = \ldots = L_n = 0$ (5d)

It is important to note that investment by an agent is solely driven by his ex ante return ($\Pi_i^*$) and not by $C$, that is, his ‘average opinion’. We construct $C$ to characterize the steady state in terms of $L_i$: $L_i$ is similar to the notion of excess demand in the Walrasian theory of general equilibrium. In the Walrasian theory, the equilibrium is defined as a state in which excess demand for each good is zero. The endogenous dynamics in the Walrasian theory arises when there is non-zero excess demand for at least one good. Such a non-zero excess demand engenders the endogenous dynamics known as the Walrasian tâtonnement. The Walrasian equilibrium is a state characterized by the absence of the endogenous dynamics, or tâtonnement process. To recap: unless all the predictions coincide, $L_i$ is non-zero for some agents who expect to gain from moving off their initial positions. Such movements will impinge on the dynamic path involving ex ante returns and trigger rates. We intend to examine if the dynamic path has a fixed point that can act as a cascades-free equilibrium.

**ASSUMPTION 5:** We assume the trigger rate at date $t$, $R^N(t)$, to be a function of ex ante returns, $\Pi^*(t-1)$:

$$R^N(t) = W \cdot \Pi^* (t-1) + h$$

(6a)

where $\Pi^* = \Pi_1^*, \Pi_2^* \ldots \Pi_n^*$; and $h = h_1, h_2, \ldots, h_n$. We know $\Pi^*$ is the vector of ex ante returns, $h$ is a vector of exogenous variables, $W$ is a set of weights chosen by the market forces.

**DEFINITION 4:** The fixed point of the dynamic path is a trigger rate $R^N*$ such that $L_i(R^N*) = L_2(R^N*) = \ldots = L_n(R^N*) = 0$. At the fixed point, $R^*$, each agent has fulfilled expectations and, hence, each agent correctly predicts $R^*$ on the basis of his $\Pi_i^*$. By construction, this fixed point is the cascades-free equilibrium.

**DEFINITION 5:** The market chooses a weight $w_i$ for industry $i$, in equilibrium, such that $0 \leq w_i \leq 1$ for all $i \forall 1 \ldots n$. We define set $M = \{1,2,\ldots,n\}$ as a finite set of indices and define $\Omega$ as the $(n-1)$ dimensional simplex such that

$$\Omega = w \in R^n : \Sigma_i w_i = 1$$

(6a’)

The vertices of $\Omega$ are denoted by $V_i$ for $i \in M$ and for each non-empty set $z \subseteq M$, the convex hull formed out of the $V_j$, for $j \in z$, is described as $Z$.

**OBSERVATION 3:** The notional loss function of the $i^{th}$ agent is a non-linear function of the market weights $W$.

Hence we write, given the ex ante returns $\Pi^*$

$$L_i = L_i(W \Pi^* + h) = F_i(W)$$

(6b)
At the fixed point, if it exists, we express $l_i$ as the following:

$$l_i = \frac{f_i(w)}{\max \left| F_j \right|} \text{ for } j \in 1,2..n. \quad (6c)$$

Two properties of $l_i$ are in order:

**PROPERTY 1:** \( l_i = f_i(w) \leq 1. \) \quad (6d)

Secondly, from the nature of the notional loss function $L_i$ we know the following:

**PROPERTY 2:** \( \sum l_i = 0. \) \quad (6d)

**DEFINITION 6:** We define \( H = \{H_i: i \in M\} \) as the labelling of the simplex $\Omega$ (Shapley, 1981, pp. 368) such that:

$$H_i = \{w \in \Omega: \text{Minimum of } f_j(w) = f_i(w) \text{ for all } j \in M \} \quad (7a)$$

**LEMMA 1:** If \( w_j = 0 \) implies that \( l_j = f_j \geq 0, \) then the labelling $H$ is called a proper labelling.

Proof: If $w \in z$, then the above condition implies that $f_j(w) \geq 0$ for all $j \in M \mid z$. Since $\Sigma f_i(w) = 0$, for one element $k$ in $z$ $f_k(w) \leq 0$. Hence the following is true:

$$\min f_i(w) = \min f_i(w)$$

$w \in z$  $w \in M$

The condition $(7b)$ is the definition of the proper labelling (see Shapley, 1981, pp. 368). The implication of lemma 1 has important ramification. It states that if the market picks a weight $w_j = 0$, then the agent holding asset $j$ has return expectation above the ‘average opinion’. Q.E.D.

**THEOREM 2:** Since $H$ is a proper labelling of $\Omega$, there exists a set of weights $W^* \in \Omega$ such that $l_1(w^*) = l_2(w^*) = \ldots = l_n(w^*) = 0$. This set of weight $w^*$ determines the profit signals $R^*$ which induces identical ex ante profit rates $\Pi^*$.

Proof: Since $H$ is a proper labelling, from the K-K-M theorem (see Shapley, 1981; pp. 369) we know that $\cap H_i \neq \emptyset$. Therefore, there exists at least one $w^*$ which belongs to all $H_i$. Hence all $l_i(w^*)s$ are equal due to Definition 4. Since the sum of $l_i$s is always zero, the result follows. Q.E.D.

This section proves in sufficient generality the existence of a cascade-free equilibrium that characterizes the postulated global setting. In this equilibrium, agents do not suffer from adverse consequences of herding, or informational cascades. Forward-looking agents, driven by their fear of the adverse consequences of informational cascades, engage in trades with sufficient caution and thereby unintentionally create the cascade-free, or blessed, equilibrium.

**4. Discussion**

In the context of unstable markets characterised by uncertainty, unpredictability and uncharted dynamics, decision-makers are forced to take decisions in ignorance about both current and evolving demand and supply characteristics. In the context of fashion goods and financial products, decision-makers have adopted iterative decision-making strategies with relevant updating of their beliefs. A serious problem in such markets is social learning and collective adaptation as decision-makers willy-nilly share the time profiles of their information sets. The possibility of information sharing and social observations of individual decisions open the lid off the Pandora’s box as decision-makers can be driven by fads, herding and anti-herding instincts what are collectively called information, or informational, cascades. It has been widely recognised that herding and informational cascades can cause serious problems in unstable markets. The major innovation of our work is to allow a rational decision-maker to be a little wary of the adverse consequences of information cascades. The pertinent question then is whether this fear of the adverse consequences of cascades can motivate rational decision-makers to act in such a fashion such that the resultant market outcome becomes efficient and cascade-free? Our answer to this question is affirmative. We establish the existence and convergence properties of a cascade-free equilibrium in a setting in which decision-makers iteratively make decisions and are aware of the possible adverse consequences of informational cascades.

We also underscore two important findings in this context: first, the state space is divided by boundary, or *separatrix*. This separatrix divides the state space into stable and unstable zones for this cascade-free equilibrium. If pecuniary returns from an action are less than a critical value, then the system converges to the cascade-free
equilibrium, and, hence, painful issues concerning fads, fashion, herding and anti-herding fade in the thin air, which we may like to call the “blessed equilibrium” – in contradistinction to the “cursed equilibrium” of Eyster and Rabin (2005). On the other hand, the system fails to reach this blessed equilibrium if returns from an action exceed this critical value. Thus the initial condition, or history, will decide whether the blessed, or cascade-free, equilibrium is reachable or not. If the blessed equilibrium is not reached, then informational cascades cannot be prevented by smart decision-makers, which will gradually take the system to the cursed equilibrium. Secondly, we also note that large changes in expected returns from an action can easily make the system unstable since the blessed equilibrium becomes unattainable in this case. These two findings are in consonance with earlier results in the context of learning. Our work thus provides an analytical foundation to the important observations by MacCormack and Verganti (2003), Iansiti and MacCormack (1997) and MacCormack, Verganti and Iansiti (2001) among others that the success of new products in unstable markets is critically predicated upon early technical and market feedbacks and subsequent updating of forecasts by relevant decision-makers who apply iterative planning strategies with market experimentation and learning by doing. Our theoretical model also shows the limitation of iterative learning mechanism as the success of iterative learning is propelled by the initial condition as well as changes in individual (financial) returns in unstable markets. In future work, it is imperative to design a *mechanism* that can be succour to this cascade-free equilibrium in unstable markets so that firms can reduce, even stamp out, the pecuniary costs of ignorance like stock-out and markdown losses. Our model also contributes to an important new line of research what is known as *strategic communication* (SC): see Sobel (2007), Krishna and Morgan (2008), and Ottaviani and Sorensen (2006) for surveys and references on strategic communication. Recent work by Kartik (2009) incorporates lying costs into the SC model thereby bridging the gap between costly signalling and cheap talk. Li (2007) considers sequential arrival of information in a sender-receiver set up. Her focus is on what the sequence of reports by the sender reveals about his ability (the quality of his signals). On the contrary, our paper develops a general model that can explain the equilibrium behaviour in strategic communication that can stabilise markets, which are otherwise unstable since decision-makers are ignorant of some critical market characteristics.

References


Endnotes


Note 2. In an early study Frazier (1986) estimated that the annual cost of inventory carrying, excess supply and shortages at market prices stood at 425b in the US apparel industry. These mammoth costs associated with market unpredictability seem to have outweighed their manufacturing costs in the apparel industry.


Note 4. Let us take an example at this stage: suppose you and many other people are on the way to find a cosy restaurant for the Valentine’s Day dinner. All of you have some private information about the quality of a restaurant and also some public information about where people are off to. If all of you turn up at the same spot, or a few spots, each suffers from congestion, queuing and poor quality of services. Rational agents - experience
congestions will like to switch to ferret out a nicer place. There is nothing sacrosanct that one will end up with a nice dinner due to possible cascades that can goad them to cluster around a few spots again. Cascades can thus cause terrible personal losses. The question that prompts this paper is as follows: can all of you intelligently make choices such that no one has incentives to switch from the restaurant after the first round of choices?

Note 5. An alternative way of modelling this problem is to assume that each agent takes position on an asset as a buyer. In this scenario, a cascade on an asset will create excess returns on it. Similarly, an asset will earn low returns if agents cascade away from this asset.

Note 6. Blatt (1983) highlighted such uncertainty as the utility of being hanged on the gallows. His fable is compelling: consider an ‘illegal venture, say importing heroin in Singapore, which is highly profitable if undetected, but leads to the death penalty upon discovery’ (pp. 253). Thus, the criminal either makes a handsome profit or a sheer disaster. Such an economic disaster has been highlighted by Adam Smith (1776) who masterfully drew the analogy between bankruptcy and gallows (pp. 325).

Note 7. Li is a ‘notional loss’ and not actual loss since an agent uses his ex ante return ($\pi^*_i$), and not $C$, for investing. The ‘notional loss’ would have been actual loss if an agent uses $C$ for taking position on an asset. In case agents choose investment on the basis of $C$, then the model would have reduced to the Keynesian model of beauty contest (see Moulin, 1986). If agents invest on the basis of $C$, then their decisions are not predicated on the “fundamentals” but on what they think what everyone else thinks what $C$ is (Keynes, 1936, Chapter 12). Our results of multiple equilibria is well-established when agents use $C$ for framing their decisions (see Nagel, 1995). However, we will establish the possibility of multiple and unstable equilibria even when agents use $\pi^*_i$.

Figure 1. Time-structure of Decisions and Events

Observations: Figure 1 describes the arrow of time, at any Date $t$, past important variables are represented by history, current decisions are taken at Date $t$ on the basis of the expectations about future values of the relevant variables expected to materialise at Date $t+1$. 

- Ex ante Return ($\pi_t^{e,i}$) at date ($t-1$)
- Agent $i$ takes position on asset $i$ for all $i=1,2,...,n$
- Formation of $\pi_t^i$ at date $t$ and trigger rate $R^N$ at date $t$
- Possible Cascades and new moves by others as agents take new positions on assets
- Formation of actual returns
- Formation of $\pi_{t+1}^i$ at date $t+1$
- Possible cascades
- Formation of actual returns at date $t+1$
- The system evolves to $t+2$...
Figure 2. Stability of Fixed Points

Description: Figure 2 is a one-agent representation of the model. In Figure 2 we depict *ex ante* rate along the horizontal axis and the *ex post* rate along the vertical axis. OA is the 45° Line and E₁ and E₂ are the two fixed points.