Modern Applied Science
www.ccsenet.org/journal.html
Vol. 3, No. 1
January 2009

A Study on the Effects of Higher Order Mode Wave on Mufflers Performance

Hongguang Liu, Senlin Lu & Falin Zeng
School of Automobile and Traffic Engineering
Jiangsu University
Zhenjiang, Jiangsu, China
Tel: 86-511-8878-0271 E-mail: lhg@ujs.edu.cn

Zhangping Lu
School of Mechanical Engineering;
Jiangsu University
Zhenjiang, Jiangsu, China

Jiangkun Zhou
School of Automobile and Traffic Engineering
Jiangsu University
Zhenjiang, Jiangsu, China

Abstract

Based on the existence of higher order mode waves (HOMW) in expansion mufflers is proved theoretically, a 3-D FEM is used to obtain the four-pole parameters of a muffler which are in turn used to calculate the transmission loss. With this method, the distribution of sound pressure in a muffler is calculated, the effects of HOMW on the muffler performance are investigated and a technique to improve the muffler performance is put forward. It is shown that, because of HOMW, the sound wave in expansion chamber is not in plane form; HOMW can descent capability of muffler, and the position of inlet and outlet is important to the performance of mufflers.

Keywords: Muffler, Higher order model wave, FEM

1. Introduction

Expansion mufflers have several advantages, such as back pressure is little and the noise elimination performance is steady, so they are used in lots of engines. At present, the formula using to analysis the expansion muffler performance was established by Davis in the early 1950's. This formula was built up on the basis of the plane wave model, assuming that the sound in the expansion chamber propagates in plane form in a stationary medium. Their analysis revealed that the transmission loss is related with the area expansion ratio *m* and the non-dimensional frequency parameter *kl* and is a periodic function. Namely

$$TL = 10\log_{10}[1+\frac{1}{4}(m-\frac{1}{m})^2 \sin^2 kl] \tag{1}$$

Where *k* is the wave number and *l* is the length of the expansion chamber.

However, due to the existent of higher order model wave(HOMW), when the frequency increases in certain extent, the sound waves in the expansion chamber is not linear anymore, and the muffler performance do not accord with the Eq.(1). With the development of the computer, the non-linear problems of sound field in mufflers are paid to more and more attention, including the effects of HOMW on muffler performance or the 3-D effects of the muffler and the effects of airflow on muffler performance and so on, and numerical calculation method is one of the important research methods. Narayana predicts muffler performance and the effects of HOMW with numerical method; Sahasrabudhe investigates the effects of HOMW by means of 3-D FEM and the four-pole network. Lu Senlin et al uses the 3-D FEM of the ANSYS process combining with four-pole network to research HOMW, and it acquires favorable effect. The method will be used to discuss HOMW and its effects on muffler performance in this paper.

2. HOMW in a tube

Most expansion mufflers consist of various tubes, so the HOMW merely in a rotundity tube is discussed simply now. Assuming that there is a tube, its diameter is 2a , r is the radius of polar coordinate ,θ is angle of polar coordinate and the Z coordinate is along the tube length (see Fig 1), the sound wave equation in the tube can be obtained by:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 p}{\partial\theta^2}+\frac{\partial^2 p}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \tag{2}$$

Let the general solution for this equation be

$$p = R(r)\Theta(\theta)Z(z)e^{j\omega t} \tag{3}$$

And Substituting Eq. (3) to Eq. (2), we obtain

$$\left.\begin{aligned} &\frac{d^2Z}{dz^2}+k_z{}^2 Z=0\\ &\frac{d^2\Theta}{d\theta^2}+m^2\Theta=0\\ &\frac{d^2R}{dr^2}+\frac{1}{r}\frac{dR}{dr}+\left(k_r^2-\frac{m^2}{r^2}\right)R=0 \end{aligned}\right\} \tag{4}$$

Where k_r and k_z are the wave number in the direction of r and z respectively. At the same time, both k_r and k_z satisfy the equation

$$k_r^2+k_z^2=k^2=\frac{\omega^2}{c^2} \tag{5}$$

Where c is sound speed.

For the equation with variable z in formula (4), the solution can be given by

$$Z(z)=A_z e^{-jk_z z} \tag{6}$$

For the equation with variable θ , we can obtain the solution of

$$\Theta(\theta)=A_\theta\cos(m\theta+\varphi_m) \tag{7}$$

Because the relation of $\Theta(\theta)=\Theta(m\theta+2\pi)$ should be satisfied, the m in Eq. (7) must be positive integer.

The third equation in the formula (4) is a standard Bessel equation, in which the variable is $(k_r r)$, and the general solution is given by:

$$R(k_r r)=A_r J_m(k_r r)+B_r N_m(k_r r) \tag{8}$$

Where $J_m(k_r r)\ and\ N_m(k_r r)$ are Bessel function and Norman function respectively? According to Norman-function emanative at the zero point, B=0 should be admitted, then simplified the Formula (7) to

$$R(k_r r)=A_r J_m(k_r r) \tag{9}$$

So the sound pressure in the tube can be obtained by

$$p_m=A_m J_m(k_r r)\cos(m\theta-\varphi_m)e^{j(\omega t-k_z z)} \tag{10}$$

Then the corresponding radial velocity is

$$v_{rm}=A_m\frac{jk_r}{\rho_0\omega}\left[\frac{dJ_m(k_r r)}{d(k_r r)}\right]\cos(m\theta-\varphi_m)e^{j(\omega t-k_z z)} \tag{11}$$

Assuming the muffler wall is rigid, that is when $r=a$, $v_r=0$, thus gained

$$\left[\frac{dJ_m(k_r r)}{d(k_r r)}\right]_{(r=a)}=0 \tag{12}$$

According to Bessel function is a recursive function, the following equations can be obtained:

$$J_{m-1}(k_r a)=J_{m+1}(k_r a)\quad (m>0) \tag{13}$$

$$J_1(k_r a) = 0 \qquad (m = 0) \tag{14}$$

To solute above equations, a series roots can be obtained. Using two integer *m* and *n* to represent the serial numbers of these roots, and recording corresponding k_r as k_{mn}, then the part of roots of the equation can be obtained which are shown in Table 1.

Consequently, the sound pressure can be written in following form

$$p_{mn} = A_{mn} \cos(m\theta - \varphi_m) J_m(k_{mn} r) e^{j(\omega t - k_z z)} \tag{15}$$

Where $k_z = \sqrt{k^2 - k_{mn}^2}$

Eq.(15) express the (m, n)order simple waves may be generated in the tube. Among them the (0,0) order is the main wave, while the rest is the high order waves.

3. Numerical analysis of the muffler performance.

Because of HOMW in mufflers, the sound wave doesn't keep in plane wave, so the transmission loss doesn't accord with Eq.(1) anymore. In this paper, the transmission loss is calculated by means of 3-D FEM and four-pole network to investigate the effects of HOMW on muffler performance .

Transmission loss of a muffler can be denoted by four-pole parameters, namely

$$T = 20\lg\left[\frac{1}{2}\left(A + D + \frac{\rho c}{S_0}C + \frac{S_0}{\rho c}B\right)\right] \tag{16}$$

Where *A.B.C.D* is four-pole parameters of the muffler; S_0 is cross-sectional area of the Inlet/outlet tubes of mufflers; ρ is air density.

When the sound wave in expansion chamber is plane wave, the four-pole parameters are showed as follow respectively:

$$A = \cos kl, \quad B = j\frac{\rho c}{S}\sin kl, \quad C = j\frac{S}{\rho c}\sin kl, \quad D = \cos kl$$

Where *l* is the length of the tubes; S is cross-sectional area of the tubes.

Because of HOMW, the sound wave in expansion chamber is not plane wave, so the formula to calculate four-pole parameters by using the theory of plane wave is not suitable. Then 3-D FEM is employed to obtain the four-pole parameters of the muffler which are in turn used to calculate the transmission loss with Formula(16).

By the definition of four-pole parameters, we know that:

$$\begin{cases} A = \left.\dfrac{p_0}{p_r}\right|_{V_r = 0} \\ B = \left.\dfrac{p_0}{V_r}\right|_{p_r = 0} \\ C = \left.\dfrac{V_0}{p_r}\right|_{V_r = 0} \\ D = \left.\dfrac{V_0}{V_r}\right|_{p_r = 0} \end{cases} \tag{17}$$

Where p_0 and p_r are denoted the sound pressures at the inlet and outlet of the muffler respectively; V_0 and V_r are the volume speed of medium at the inlet and outlet of the muffler respectively.

Accordingly, FEM can be used to obtain the sound pressure and volume speed when the outlet sound pressure is zero(the short circuit state)and volume speed is zero(the open circuit state) respectively, and the Formula(17)can be employed to get the four-pole parameters.

The author used the ANSYS to obtain the parameters p_0, p_r, V_0 and V_r under this two states. In calculation process, the p_0 is given by the form of mechanical boundary condition of the inlet. In order to calculate simply, let $p_0 = 1$, and the p_r is obtained by using the FEM directly. For getting the V_0 and the V_r, we can calculate sound pressure at three equidistant point along the axis of Inlet/Outlet tube of the muffler, and then calculate the volume speeds, the V_0 and the

V_r of the muffler by Formula (18) and (19)

$$V = -j\frac{S_0}{\rho\omega}\frac{\partial p}{\partial n} \tag{18}$$

$$\frac{\partial p}{\partial n} \approx \frac{-p_3 + 4p_2 - 3p_1}{2h} \tag{19}$$

Where p_1, p_2 and p_3 are the sound pressures at three equidistant points following the direction of the axes and near the end of inlet/outlet tubes, *h* is the distance between two points, V is volume speed at the end of inlet/outlet tubes.

Because the diameter of the inlet /outlet tubes of muffler is much small, and then the sound wave transmits in plane form in the tubes, the volume speed have enough precision which obtained by using the above formulas.

The calculation process of finite element consists of building model, generating mesh, applying load and boundary conditions and solutions. The work should be divided into two parts; First, to calculate the sound pressure of the muffler under the open circuit state ; Second, to obtain the sound pressure of the outlet tube of muffler under the short circuit state with the mechanical boundary condition $p_r = 0$; Lastly, we get the data by using the Matlab interface program and second post-processing procedures to calculate the four-pole parameters and the transmission loss according to the above Formulas.

4. The effects of HOMW on muffler performance

From the table 1,we know that the cut-off frequency of the minimum HOMW in circular tube corresponds to the non-dimensional parameter of $ka = k_{1,0}\ a = 1.84$. If the value is less than 1.84, the sound wave is in plane form, or else is not in plane form anymore.

Fig2.shows the acoustic pressure contours in a coaxial expansion muffler obtained by making use of the FEM while $ka = 1.5$ and $ka = 4$ respectively. Obviously, when *ka* equals 1.5, the sound wave is in plane form in the whole muffler except the outlet and inlet of the expansion chamber. However, when *ka* is equals to 4, due to the effect of HOMW, the sound wave in the expansion chamber is not in plane form anymore, so 1-D sound wave theory is not suitable, but the sound wave is also in plane form in the Inlet/Outlet tubes.

From formula (1), we know that transmission loss of the simple expansion muffler is periodic function of $kl = \pi$ which is shown by the broken lines in Fig3. The real lines in Fig(3) show the transmission loss by making use of FEM and four-pole parameters. From the Fig(3), we conclude that when the frequency of the sound is less then the cut-off frequency (corresponding the *ka*=1.84), the two lines are tallied. When *ka* is larger than 1.84, HOMW begins to come forth in muffler, but its effect is small at beginning, and the difference of the result from FEM and plane wave analysis is little. When the (2,0)mode existent, the value of academic is different from that of FEM, but they are not obvious. However, when *ka* is larger than the value corresponding the (0,1)mode the disparity of them is clear, so the muffler comes invalidated..

From the table(1), we conclude that the number of cut-off frequency *ka* of the (0,1) mode is 3.83. when *a*=0.12m and *c*=400m/s, the cut-off frequency is about 2000Hz, and that is to say HOMW affects performance above the middle-high frequency about 2000Hz mainly. Although the basic frequency of the exhausting noise is in low-frequency, the airflow noise of muffler has more energy in the middle-high frequency. Moreover, considering the fact that the human ear is less sensitive to low frequency than middle-high frequency of the audible range , and A-weighting filter network provides less respond to low-frequency range, the noise of the middle-high frequency takes more advantages.

5. The improvement of the muffler performance.

Commonly, HOMW of the muffler is inevitable. However, the transmission of the sound wave in conduit depends not only on the geometrical dimension of the conduit, but also on the location of the source and the acoustic boundary conditions . The above-mentioned effect of HOMW on the muffler performance aims at the coaxial mufflers, namely the sound source is located on the center line of the chamber. If we change the location of sound source, the transmission of the sound wave in the expansion chamber is altered at the same time. In order to make the frequency of the excited HOMW is higher than certain frequency of the exhaust noise, we can change the configurations of mufflers to suppress the excitation of some HOMW. For example, when inlet and/or outlet tubes are coaxial with the expansion chamber, (1, 0) and (2, 0) modes are not excited effectively. However, when the Inlet tube offsets the axes of an expansion chamber by a distance of 0.63 times the radius, the (0,1)mode can be suppressed effectively. The above conclusion is also suitable for the outlet tube. Accordingly, if we put the one of outlet tube or inlet tube on the axes of the expansion chamber and another at the location offsetting the axes a distance of 0.63 times the radial of chamber, the disabled frequency of the muffler can be increased and the muffler performance is improved. The broken lines in Fig(4) show the transmission lose corresponding the outlet tube is coaxial with the expansion chamber, and the inlet tube offsets a certain distance. From the Figure (4) we conclude that when the outlet tube is coaxial with expansion chamber and the inlet tube offsets a certain distance, the muffler performance is improved obviously, and its noise reduction capacity keeps large until *ka* equals to 6.5, above which the energy of the exhaust noise is very small.

6. Conclusion

From above analysis, we conclude that:

(1) Due to the existence of HOMW, the sound wave in mufflers is not in plane form in the middle-high range of frequency, so the 1-D acoustics formula is not suitable anymore

(2) HOMW does effects the noise reduction performance of expansion mufflers in the middle-high frequency range.

(3) Choosing the appropriate configuration of muffler can decrease the effects of HOMW and improve the performance of mufflers.

References

Du, Gonghan, Zhu Zemin & Gong, Xiufeng. *the Foundation of Acoustic*s. Nanjing: the Publishing Company of Nanjing University.

Lu, Senlin & Liu, Hongguang. Numerical analysis of engine muffler with 3-D FEM. [J]. *Transaction of CSICE.* 2003, 21(5): 346-351. (in Chinese).

Narayana TSS, Munjala ML. prediction and measurement of the four-pole parameters of a muffler including higher order mode effects [J]. *Noise Control Engineering Journal.* 2005. 53(6): 240-246.

Sahasrabudhe A D, Anantha Ramus, Munjal ML. Matrix condensation and transfer matrix techniques in 3-D analysis of expansion chamber muffler[J]. *Journal of Sound and Vibration.* 1991, 143(3): 371-394.

Sahasrabudhe AD, Munjal ML & Anatha Ramu S. Design of expansion chamber mufflers incorporating 3-D effects. [J]. *Noise Control Engineering Journal*, 1992, 38(1): 27-38.

Table1. Values of $k_{mn}a$

n	m		
	0	1	2
0	0	1.841	3.054
1	3.884	5.322	6.705
2	7.051	8.536	9.965

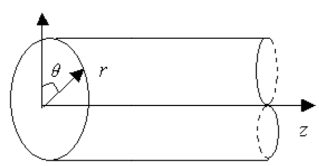

Figure 1. a sound pipe

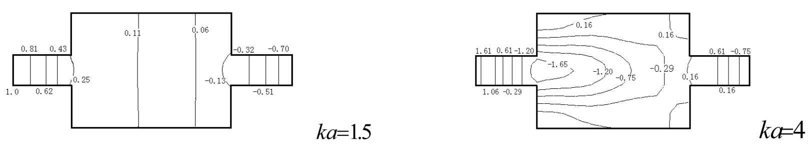

Figure 2. Acoustic pressure contours in a muffler

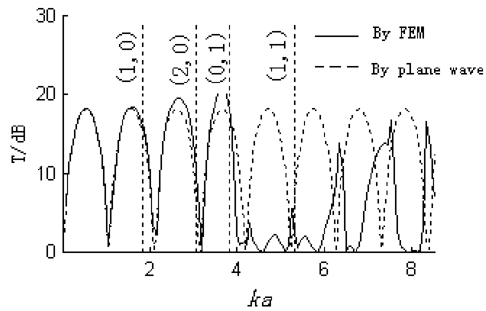

Figure 3. TL results expected from plane wave analysis and FEM

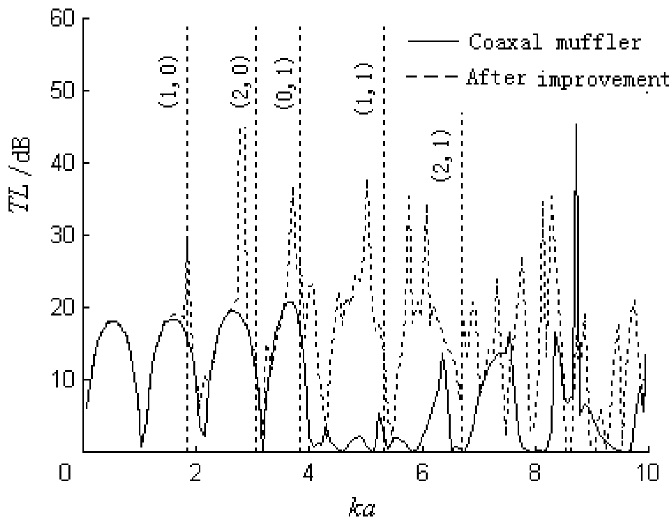

Figure 4. TL comparison of inlet-offset and coaxial expansion muffler