Modern Applied Science
www.ccsenet.org/journal.html
Vol. 3, No. 2
February 2009

Motion Control of Underwater Supercavitating Projectiles in Vertical Plane

Daijin Li, Yuwen Zhang, Kai Luo & Jianjun Dang
Collage of Marine Engineering
Northwestern Polytechnical University
Xi'an 710072, China

Abstract

Supercavitating projectiles are characterized by substantially reduced hydrodynamic drag from the fully wetted underwater projectiles. Drag is localized at the nose of the projectile, where a cavitator generates a cavity that completely envelops the body (supercavity). While a supercavitating projectile moving in cruise phase, its motion modes is different from that of a usual wetted projectiles. Based on the motion characteristics, an indirect fixed-depth control via controlling the pitch angle feedback schemes adopting the margin steering control method is put forward. The control method is robust and easy to implement. The research may be useful for designing the control system of supercavitating projectiles.

Keywords: Supercavitating projectiles, Vertical plane, Motion control

CLC number: TP13; TJ630.1 Document code: A

1. Introduction

When underwater projectiles such as torpedoes and submarines move through water at sufficient speed, the fluid pressure drops locally below the level that sustains the liquid phase, and a low-density gaseous cavity forms. Flows exhibiting cavities entirely enveloping the moving body are called "supercavitating". (S. S. Ahn and M. Ruzzene, 2006, p.967-976).

In supercavitating flows, the liquid phase does not contact the moving body over most of its length, thus making the skin drag almost negligible. (John Dzielski and Anderew Kurdila, 2003, p.791-804). Several new and projected supercavitating underwater projectiles exploit supercavitation as a means to achieve extremely high submerged speeds and low drag. (Miller D, 1995, p.61-63). The size of existing or notional supercavitating high-speed bodies ranges from that of projectiles to heavyweight full-scale torpedoes. (Ashley S, 2001, p.52-53). Although extensive efforts have been devoted in the past to the analysis of the fluid dynamic characteristics of supercavitating projectiles (Harkins TK, 2001, p.7-11), very little research has been dedicated so far to the evaluation of the fixed-depth control of slender elastic bodies traveling underwater at high speed in supercavitating regimes. Much of the previous and current studies on guidance, control and stability have considered supercavitating projectiles as rigid bodies (Kirschner IN, Fine NE and Uhlman JS, 2001; Kirschner IN and Imas LG, 2002; Vasin AD, 2001). And scarcely any have addressed the hydrodynamic characteristics of the water/cavity system. (Semenenko VN, 2001). The small wetted surface area results in a significant reduction of drag. The projectile may be controlled through a combination of the deflection of the cavitator and control surface, and planning. The control of these bodies presents a number of significant challenges. Depending on the shape of the cavitator and the size and immersion of the control surfaces, the body may be inherently stable or unstable. If the body is unstable, active control of the cavitator or control surface in the aft of the projectile is required to achieve stability. Even if the body is designed to be stable, it is not guaranteed that the projectile will be stable when in contact with the cavity. Since the cavity wall causes a very large restoring force and the time that the projectile is in contact with the cavity wall is very short, suppressing these contacts requires very high performance actuators.

The goal of this paper is to investigate the dynamic characteristics of supercavitating bodies, then present a fixed-depth control method. The paper is organized in six sections. In the first section a brief introduction is given. Section 2 outlines the formulation for the equations of motion for the longitudinal or pitch-plane dynamic model. Section 3 presents a control-algorithm of indirect depth control via controlling the pitch angle feedback. Section 4 describes margin steering control method to put the control-algorithm into effect. Section 5 gives numerical simulation results, while Section 6 summarizes the main results of the work and gives recommendations for future research.

2. A longitudinal kinematics model for a supercavitating projectile

To derive the equations of motion, the following assumptions are made:

- The motion of the projectile is confined to a plane.
- The effect of rolling angular is negligible.
- The effect of added mass on the dynamics of the projectile is negligible.
- The motion of the projectile is not influenced by the presence of gas, water vapour or water drops in the cavity.
- The velocity and cavitation number are confined fixed values.

The supervacitating projectiles longitudinal plane kinematics equation, make up the system model:

$$mv\dot{\Theta} = A^d k + A^\alpha \alpha + A^\omega \omega_z - G \tag{3}$$

$$J_z \dot{\omega}_z = M^d k + M^\alpha \alpha + M^\omega \omega_z \tag{4}$$

$$\dot{\theta} = \omega_z \tag{5}$$

$$\dot{y} = v\Theta \tag{6}$$

$$\Theta = \theta - \alpha \tag{7}$$

Where m is the mass of the projectile; Θ is the obliquity of the trajectory; θ is the pitch attitude; and J_z is the moment of inertia about z axis. The combined coefficients A^d , A^α , A^ω , M^d , M^α , M^ω are given by

$$A^d = 0.5C_D^d(\sigma, \alpha_N)\rho A_d v^2 \tag{8}$$

$$A^\alpha = 0.5C_D^\alpha(\sigma, \alpha_N)\rho A_b v^2 \tag{9}$$

$$A^\omega = 0.5C_D^\omega(\sigma, \alpha_N)\rho A_b v^2 \tag{10}$$

$$M^d = 0.5\rho v^2 A_d L m_d^d \tag{11}$$

$$M^\alpha = \begin{cases} 0.5\rho v^2 A_b L m_{bs}^\alpha, |\alpha| \le \alpha_S \\ 0.5\rho v^2 A_b L m_{bB}^\alpha, |\alpha| > \alpha_S \end{cases} \tag{12}$$

$$M^\omega = 0.5 m_b^\omega \rho A_b L^2 v \tag{13}$$

In Eq. (3) and (4), balanceable angle of attack and balanceable rudder angle can be gotten:

$$\alpha_0 = GM^d / (A^\alpha M^d - A^d M^\alpha) \tag{14}$$

$$d_0 = -GM^\alpha / (A^\alpha M^d - A^d M^\alpha) \tag{15}$$

Generally speaking, $|\alpha_0| \le \alpha_S$, so M^α should be fixed under the condition of $|\alpha| \le \alpha_S$. Because damping derivatives m_{bs}^α is negative, near by the α_0. The projectile is of static stable. While $|\alpha| > \alpha_S$ (For example, the control system of the projectile dose not work.), m_{bs}^α would become to positive number. The projectile loses static stability. (Jou-Yong Choi, 2006, p.1360-1370).

It is necessary to investigate the motion characteristics of the projectiles on a vertical plane in order to controlling the depth and the attitude. Under the fixed depth control mode, the condition of $|\alpha| > \alpha_S$ is not allowed. So the study should focus on motion characteristics of projectile in the case of $|\alpha| \le \alpha_S$. Also, the active control of the angle of nose rudder should be paid more attention for keeping from the condition of $|\alpha| > \alpha_S$ while moving in cruise phase. (Kirschner IN, Kring DC and Stokes AW, 2002, p.219-2).

The benchmark problem focuses exclusively on the vertical plane dynamics of the body which currently appear to present the most severe challenges. (John Dzielski and Anderew Kurdila, 2003, p.791-804).

Under the condition of $|\alpha| \le \alpha_S$, the transfer function of the system can be gotten striking a balance from the parameter of a_0 and d_0 :

$$\alpha(s) = G(s)(J_z s - M^{\omega}) / \bar{X}(s) + d(s)\cdot(-A^d J_z s + A^d M^{\omega} - A^{\omega} M^d + mvM^d) / \bar{X}(s) \tag{16}$$

$$\omega_z(s) = G(s) M^{\alpha} / \bar{X}(s) + d(s)\cdot(mvM^d s + A^{\alpha} M^d - A^d M^{\alpha}) / \bar{X}(s) \tag{17}$$

$$\theta(s) = \omega_z(s)/s \tag{18}$$

$$y(s) = -v\left[\theta(s) - \alpha(s)\right]/s \tag{19}$$

$$\bar{X}(s) = mvJ_z s^2 + (A^{\alpha} J_z - mvM^{\omega})s + A^{\omega} M^{\alpha} - A^{\alpha} M^{\omega} - mvM^{\alpha} \tag{20}$$

In the functions, not only d is the importer, which can be controlled, but G which is disturbance quantity. While a supercavitating projectiles moving in cruise phase, G is reducing with monotone curvature, according to the propellant reduction. So G can be treated as a steady negative slope signal, labeled:

$$G(s) = -k_G / s^2 \tag{21}$$

Where slope k_G represents $\dot{m}_f$ which is the propellant consumption per seconds.

3. The control-algorithm design

As obliquity of the trajectory Θ and angle of attack α are not easy to be measured in fact, (May A, 1974, p.75-2), the direct depth feedback control is impossible to achieve. So the other indirect depth control via controlling the pitch angle feedback should be discussed.

Transfer function of the control system (containing electric-actuator) can be expressed as follows:

$$G_c^y(s) = k(s)/\left(y_c(s) - y(s)\right) \tag{22}$$

Where, y_c is expectant depth. And transfer function of the closed-loop is given as:

$$y(s) = \frac{1}{\bar{M}_4 s^4 + \bar{M}_3 s^3 + \bar{M}_2 s^2 + 1} y_c(s) + \frac{\dfrac{M^{\alpha}}{(M^k A^{\alpha} - M^{\alpha} A^k) G_c^y(s)}}{\bar{M}_4 s^4 + \bar{M}_3 s^3 + \bar{M}_2 s^2 + 1} G(s) \tag{23}$$

Where, $\bar{M}$ is combinatorial coefficient. y is superposition output response for both y_c and G. For the output response of y_c, y achieves no-steady-state error tracking whatever $G_c^y(s)$ forms, because there are two integral actions in the transfer function forward channel. While, for the output response of G, which inputs in form of ramp, its steady-state error is given as:

$$\lim_{t\to\infty} y(s) = \lim_{s\to 0} \frac{\dfrac{M^{\alpha} - \left(J_z s - M^{\omega}\right)s}{\left(M^k A^{\alpha} - M^{\alpha} A^k\right) G_c^y(s)}}{\bar{M}_4 s^4 + \bar{M}_3 s^3 + \bar{M}_2 s^2 + 1} \frac{k_G s}{s^2} = \lim_{s\to 0} \frac{M^{\alpha} k_G}{\left(M^k A^{\alpha} - M^{\alpha} A^k\right) G_c^y(s) s} \tag{24}$$

Eq. (24) will tend to infinity, if there is no integral action in $G_c^y(s)$. Therefore, there are two choices to control $G_c^y(s)$, PI or PID.

Controlling $G_c^y(s)$ with PI control method, transfer function would add a negative real zero and a real zero-pole in the forward channel, else root locus curve would always has two branches in the right half plane, which can not guarantee system security. Root locus curve adjusted is shown as Figure 1.

Controlling $G_c^y(s)$ with PID control method, the control-algorithm is given as:

$$G_c^y(s) = k_P + T_D s + 1/T_I s \tag{25}$$

Where, k_P is Proportional gain; T_D is differential time and T_I is integral time. Transfer function would add a negative real zero and a pair of real zero-pole in the forward channel. Root locus curve adjusted is shown as Figure 2. Amplitude frequency curve cross the $0dB$ by $-20dB/dec$. Amplitude gain and phase margin are both abundant. System would keep stable controlling the gain not too large.

4. Margin steering control

To simplify the system and get better robustness, implement the fixed-depth control with limit rudder model could be considered, which is easy to achieve. (Liu Yun-Gang, 2007). From Eq.(25), the control-algorithm of fixed-depth can be gotten:

$$k_c = k_0 + \Delta k \cdot sign\left(k_P \Delta\theta + T_D \omega_z + \frac{1}{T_I}\int_0^t \Delta\theta dt \right) \tag{26}$$

Where, k_c represents command angle of the head rudder; k_0 is balanced angle of the head rudder; Δk is change angle of the head rudder; and $\Delta\theta$ is represents depth deviation. In order to reduce the depth deviation, the expectant pitch attitude angle can be set to the estimated value of the balanced attack angle.

In addition, considering $|\alpha| > \alpha_S$ is not allowed in the control process, overshoot of step-response system φ, and response gain $k_{k\alpha}$ from α to k are defined. Then restriction of Δk is as follows:

$$\Delta k \le \left(\alpha_s - \alpha_0\right) / \left[k_{k\alpha}\left(1+2\varphi\right)\right] \tag{27}$$

5. Numerical simulation

The following values have been used in the numerical simulation:

- Mass of the projectile $m = 100kg$.
- Depth of firing beneath the water surface $= 3m$.
- Sailing time $= 4s$.
- Shooting speed $= 44m/s(t=0)$.
- When $t = 0.23s$, the projectile achieves invariable speed $= 203Kn$.
- Thrust of solid rocket motor $= 25KN/0.5s$, specific impulse $= 220s$.
- Thrust misalignment angle of solid rocket motor $= 0.4^{\circ}$ (in the horizontal plane).
- Initial transverse-roll angle $= 20^{\circ}$.
- With indirect fixed-depth control (pitch attitude control mode).
- Conversion time of limit rudder angle of the head rudder $= 20\text{ms}$.
- Sampling period of pitch attitude control $= 240\text{ms}$.

Based on control-algorithm Eq.(26), simulation of projectile motion is shown in Figures 3 and 4. Figure 3 shows the characteristics of the head rudder and depth with PI control. Figures 4 shows the characteristics with PID control.

From the result of system simulations, it can be seen that both of the indirect depth control via controlling the pitch angle feedback are efficacious. Generally speaking, both the PI and PID control methods are well implemented with little range of projectile. While the range of the projectile is large, the system counteracts the variety of gravity vigorously with the PID control method. (Zhu ji-hua, Su Yu-min, Li Ye and Tian Yu, 2007).

With the indirect depth control via controlling the pitch angle feedback, the system running steady intensifies consumedly. (Yu Jian-cheng and Zhang Ai-qun, 2007). The system security can be ensured. The superiority of the control method is proved by experimentation. The system constitution and controlling algorithm is simple and well implemented.

6. Conclusion

The conclusion can be obtained from the analysis and example above: (1) The simple control algorithm should be designed to enlarge the sampling frequency for the stability of supercavitating projectiles motion characteristics on vertical plane; (2) Under the condition of lack of damp, attention should be paid to that attack angle must not exceed the limit restrictions in control process. And the control force can not be too violent; (3) The nicer control quality is not easy acquired by fixed-depth control directly; (4) The lag of system that is brought by the too slow respond speed of the head rudder drive machine will enlarge the regulation overshoot and dithering.

The control algorithm described in this paper is simple, robust and the capability and realization are good. (Yu Jian-cheng and Zhang Ai-qun, 2007). The results presented herein are intended to provide design guidelines which will help estimating the motion stability limits for the considered class of projectiles. Further investigations on how to construct the basis function vectors and somewhat simply the structure of the stabilizing functions, typically in high-order systems, may be needed in the future practical applications.

Acknowledgements

This work was supported by Foundation of China Shipbuilding Industry Corporation.

References

Ashley S. Warp-drive underwater. *Scientific American,* May 2001.

Harkins TK. Hydroballistics: development, theory and some test results. *19th international symposium on ballistics. Nterlaken.* Switzerland, May 2001, 7–11.

John Dzielski, Anderew Kurdila. A benchmark control for supercavitating projectiles and an initial investigation of solutions. *Journal of Vibration and control.* 2003, 9: 791-804.

Jou-Yong Choi, Massimo Ruzzene. Stability analysis of supercavitating underwater projectiles with adaptive cavitator. *International Journal of Mechanical Sciences.* 48 (2006). 1360-1370.

Kirschner IN, Fine NE, Uhlman JS, Kring DC. Numerical modeling of supercavitating flows. *RTO AVT lecture series on supercavitating flows.* Brussels, Belgium: Von Karman Institute; February 2001.

Kirschner IN, Imas LG. Plume-supercavity interactions. *The international summer scientific school "high-speed hydrodynamic".* Cheboksary, Russia, June 2002.

Kirschner IN, Kring DC, Stokes AW, Fine NE, Uhlman JS. Control strategies for supercavitating projectiles. *Journal of Vibration and Control* 2002; 8: 219–42.

Liu Yun-Gang. Output-feedback adaptive control for a class of nonlinear system with unknown control directions. *Acta Automatica Sinca*. Vol. 33, No.12, Dec, 2007.

May A. Water entry and the cavity-running behavior of missiles. *Naval Sea Systems Command. Arlington*, VA, SEAHAC TR 75-2; 1974.

Miller D. Going to war in a bubble. *International Defense Review.* 1995; 61–63.

S. S. Ahn, M. Ruzzene. Optimal design of cylindrical shells for enhanced buckling stability: Application to supercavitating underwater projectiles. *Finite Elements in Analysis and Design.* 42(2006) 967-976.

Semenenko VN. Dynamic processes of supercavitation and computer simulation. *RTO AVT lecture series on supercavitating flows.* Brussels, Belgium: Von Karman Institute; February 2001.

Vasin AD. Some problems of supersonic cavitation flows. In: *Proceedings of the fourth international symposium on cavitation. Pasadena CA.* June 20–23, 2001.

Yu Jian-cheng, Zhang Ai-qun. Direct adaptive control of underwater projectiles based on fuzzy neural networks. *Acta Automatica Sinca.* Vol. 33, No.8, Aug, 2007(In Chinese).

Zhu ji-hua, Su Yu-min, Li Ye, Tian Yu. Integral variable structure control and simulation for near-surface moment of AUV. *Journal of System simulation.* Vol. 19, No.22, Nov, 2007(In Chinese).

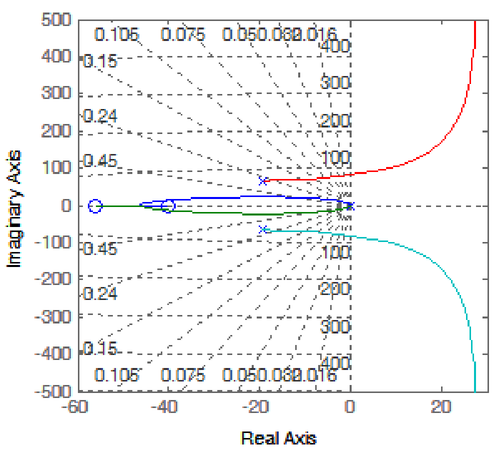

Figure 1. Root locus of depth by PI control

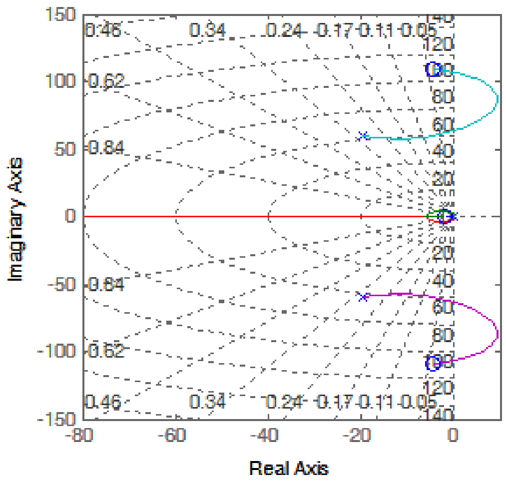

Figure 2. Root locus of depth by PID control

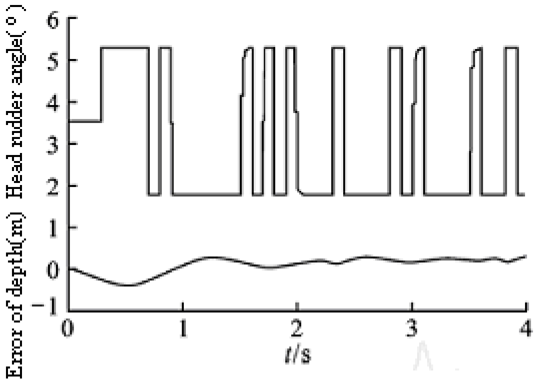

Figure 3. Characteristics of the head rudder and depth with PI control

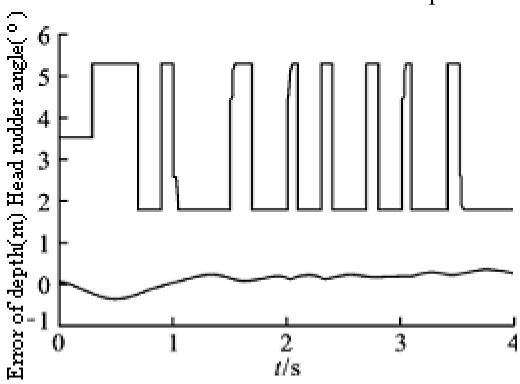

Figure 4. Characteristics of the head rudder and depth with PID control