Modern Applied Science
www.ccsenet.org/journal.html
Vol. 3, No. 3
March 2009

Kinematics and Dynamics of a Master Maipulator

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Abstract

I analyze the kinematics and Dynamics of Phantom Premium 1.5 made and sold by SenSable Technologies Inc., which is widely-used as the master manipulator in telerobotic systems. The forward kinematics is studied using Denavit-Hartenberg method, the manipulator Jacobian is also presented. The dynamic equations incorporating frictional effects of Premium 1.5 are derived.

Keywords: Master manipulator, Kinematics analysis, Dynamic model, Frictional effects

1. Introduction

The first telerobotic system was developed by Raymond C. Goertz in 1940s to let an operator handle radioactive materials behind a shielded wall, at National Argonne Laboratory in the US. In the past decades, teleoperation has found applications in many areas including space technologies, underwater explorations and assistance, surgery and rehabilitations, nuclear/toxic material handling and waste disposal, military/firefighting operation, mining. Recently, the applications of teleoperation systems have been extended to training, education, entertainment, and virtual reality areas as well.

As can be seen in figure 1, a teleoperation system generally has five components: operator, master, control system, slave, and environment. The main function of the system can be explained as follows: a control command is sent through the control system to the slave to make the remote manipulator perform a task as desired; to prevent damage, to reduce task completion time and to enhance performance, contact interaction from the remote site has to be transmitted to the operator.

As shown in figure 1, throughout an interaction, the master mechanism must perform the dual task of position measurement and force display. The kinematics especially the forward kinematics is of great importance for the control of the telerobotic system. While, when dealing with the dynamics of robotic manipulators, frictional effects is often neglected. This paper mainly discusses the forward kinematics using Denavit-Hartenberg method, and derives the dynamic equations incorporating frictional effects in the joints.

2. Kinematics analysis

Figure 2 is the photo and structural sketch of a typical master manipulator (PHANToM Premium 1.5), which can provide the operator with 3 DOF motion and three dimensional force feedbacks. As shown in figure 2(b), Premium 1.5 consists of three rotational joints, zero configuration of Premium 1.5 is shown in figure 2(b), spatial frame and tool frame are superposed in zero configuration. Figure 3 depicts the side and top views of the configuration of Premium 1.5. And, the reference frames are shown in figure 4, the D-H parameter is listed in table 1, where

a_i represents the distance from Z_i to Z_{i+1} measured along X_i,

α_i represents the angle from Z_i to Z_{i+1} measured about X_i,

d_i represents the distance from X_{i-1} to X_i along Z_i,

θ_i represents the angle from X_{i-1} to X_i measured about Z_i.

The general transformation matrix 1 comes directly from the D-H parameters (Craig, 2005 & Murray, 1998).

$$ {}^{i-1}_{i}T = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i\cos\alpha_{i-1} & \cos\theta_i\cos\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin\alpha_{i-1}d_i \\ \sin\theta_i\sin\alpha_{i-1} & \cos\theta_i\sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1} $$

And the final transformation matrix can be written in equation 2 with some mediated matrixes from ${}^{w}_{0}T$ to ${}^{3}_{t}T$ omitted.

$$ {}^{s}_{t}T = {}^{w}_{0}T\,{}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{t}T = \begin{bmatrix} \boldsymbol{R}(\theta) & \boldsymbol{p}(\theta) \\ \boldsymbol{0} & 1 \end{bmatrix} \tag{2} $$

Where

$$ \boldsymbol{R}(\boldsymbol{\theta}) = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1\sin\theta_3 & \sin\theta_1\cos\theta_3 \\ 0 & \cos\theta_3 & \sin\theta_3 \\ -\sin\theta_1 & -\cos\theta_1\sin\theta_3 & \cos\theta_1\cos\theta_3 \end{pmatrix} \tag{3} $$

$$ \boldsymbol{p}(\boldsymbol{\theta}) = \begin{pmatrix} \sin\theta_1(l_1\cos\theta_2 + l_2\sin\theta_3) \\ l_2 + l_1\sin\theta_2 - l_2\cos\theta_3 \\ -l_1 + \cos\theta_1(l_1\cos\theta_2 + l_2\sin\theta_3) \end{pmatrix} \tag{4} $$

Except the forward kinematics of the manipulator, the velocity relationship between the rotational joints and the end effector is often concerned. So, the spatial and body manipulator Jacobian is written here, moreover, they can be used to describe the relationship between the end point wrench and the joint torque.

$$ \boldsymbol{J}^s(\boldsymbol{\theta}) = \begin{pmatrix} l_1 & -l_1\sin\theta_1\sin\theta_2 & \sin\theta_1(l_2 + l_1\sin\theta_2) \\ 0 & l_1\cos\theta_2 & l_1(\cos\theta_1 - \cos\theta_2) \\ 0 & -l_1\cos\theta_1\sin\theta_2 & \cos\theta_1(l_2 + l_1\sin\theta_2) \\ 0 & 0 & -\cos\theta_1 \\ 1 & 0 & 0 \\ 0 & 0 & \sin\theta_1 \end{pmatrix} \tag{5} $$

$$ \boldsymbol{J}^b(\boldsymbol{\theta}) = \begin{pmatrix} l_1\cos\theta_2 + l_2\sin\theta_3 & 0 & 0 \\ 0 & l_1\cos(\theta_2 - \theta_3) & 0 \\ 0 & -l_1\sin(\theta_2 - \theta_3) & l_2 \\ 0 & 0 & -1 \\ \cos\theta_3 & 0 & 0 \\ \sin\theta_3 & 0 & 0 \end{pmatrix} \tag{6} $$

3. Dynamic Model of the Master Manipulator

M. C. Cavusoglu (Cavusoglu, 2001) identified the mechanical structure of Premium 1.5 into seven segments A through G shown on figure 5. Note that the spatial frame used in dynamic calculations is centered at the intersection point of three axises of rotational joints.

And he calculated the kinetic and potential energy of each segment in the spatial frame, then wrote the dynamic equation of the system using Lagrange method.

$$ M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + N(\boldsymbol{\theta}) = \boldsymbol{\tau} \tag{7} $$

In equation 7, $M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}}$ is inertial force, $C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}}$ is Coriolis and centrifugal force, $N(\boldsymbol{\theta})$ is gravitational force, $\boldsymbol{\tau}$ is the vector of joint torque, each element of the matrix of ***M***, ***C***, and ***N*** is listed in Cavusoglu's paper (Cavusoglu, 2001). Note that the inertial force, Coriolis and centrifugal force and gravitational force are highly non-linear, making it difficult for dynamic parameter identification and controller algorithm design (McJunkin, 2007).

So, equation 7 could be linearly parameterized as follows:

$$ \boldsymbol{\tau}_d = \boldsymbol{Y}_d\left(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \ddot{\boldsymbol{\theta}}\right)\boldsymbol{\pi}_d \tag{8} $$

where ***Y*** is the regressor matrix and $\boldsymbol{\pi}$ is the vector 12 dynamic parameters defined as:

$$
\begin{aligned}
y_{12} &= (1+\cos 2\theta_3)\ddot{\theta}_1 - 2\sin 2\theta_3\dot{\theta}_1\dot{\theta}_3 \\
y_{13} &= (1-\cos 2\theta_3)\ddot{\theta}_1 + 2\sin 2\theta_3\dot{\theta}_1\dot{\theta}_3 \\
y_{15} &= (1+\cos 2\theta_2)\ddot{\theta}_1 - 2\sin 2\theta_2\dot{\theta}_1\dot{\theta}_2 \\
y_{16} &= (1-\cos 2\theta_2)\ddot{\theta}_1 + 2\sin 2\theta_2\dot{\theta}_1\dot{\theta}_2 \\
y_{17} &= \ddot{\theta}_1 \\
y_{18} &= (1+\cos 2\theta_2)\ddot{\theta}_1 - 2\sin 2\theta_2\dot{\theta}_1\dot{\theta}_2 \\
y_{19} &= (1-\cos 2\theta_3)\ddot{\theta}_1 + 2\sin 2\theta_3\dot{\theta}_1\dot{\theta}_3 \\
y_{10} &= 2\cos\theta_2\sin\theta_3\ddot{\theta}_1 - 2\sin\theta_2\sin\theta_3\dot{\theta}_1\dot{\theta}_2 \\
&\quad + 2\cos\theta_2\cos\theta_3\dot{\theta}_1\dot{\theta}_3
\end{aligned} \tag{9a}
$$

$$
\begin{aligned}
y_{24} &= \ddot{\theta}_2 \\
y_{25} &= \sin 2\theta_2\dot{\theta}_1^2 \\
y_{26} &= -\sin 2\theta_2\dot{\theta}_1^2 \\
y_{28} &= 2\ddot{\theta}_2 + \sin 2\theta_2\dot{\theta}_1^2 \\
y_{210} &= -\sin(\theta_2-\theta_3)\ddot{\theta}_3 + \sin\theta_2\sin\theta_3\dot{\theta}_1^2 \\
&\quad + \cos(\theta_2-\theta_3)\dot{\theta}_3^2 \\
y_{211} &= \cos\theta_2
\end{aligned} \tag{9b}
$$

$$
\begin{aligned}
y_{31} &= \ddot{\theta}_3 \\
y_{32} &= \sin 2\theta_3\dot{\theta}_1^2 \\
y_{33} &= -\sin 2\theta_3\dot{\theta}_1^2 \\
y_{39} &= 2\ddot{\theta}_3 - \sin 2\theta_3\dot{\theta}_1^2 \\
y_{310} &= -\sin(\theta_2-\theta_3)\ddot{\theta}_2 - \cos\theta_2\cos\theta_3\dot{\theta}_1^2 \\
&\quad + \cos(\theta_2-\theta_3)\dot{\theta}_2^2 \\
y_{312} &= \sin\theta_3
\end{aligned} \tag{9c}
$$

and

$$\pi_1 = I_{axx} + I_{efxx} \tag{10a}$$

$$\pi_2 = \frac{1}{2}\left(I_{ayy} + I_{efyy}\right) \tag{10b}$$

$$\pi_3 = \frac{1}{2}\left(I_{azz} + I_{efzz}\right) \tag{10c}$$

$$\pi_4 = I_{bxx} + I_{cdxx} \tag{10d}$$

$$\pi_5 = \frac{1}{2}\left(I_{byy} + I_{cdyy}\right) \tag{10e}$$

$$\pi_6 = \frac{1}{2}\left(I_{bzz} + I_{cdzz}\right) \tag{10f}$$

$$\pi_7 = I_{gyy} \tag{10g}$$

$$\pi_8 = \frac{1}{2}m_a l_1^2 + \frac{1}{8}m_b l_1^2 \tag{10h}$$

$$\pi_9 = \frac{1}{8}m_a l_2^2 + \frac{1}{2}m_b l_3^2 \tag{10i}$$

$$\pi_{10} = \frac{1}{2}m_a l_1 l_2 + \frac{1}{2}m_b l_1 l_3 \tag{10j}$$

$$\pi_{11} = m_a g l_1 + \frac{1}{2} m_b g l_1 - m_{cd} g l_4 \tag{10k}$$

$$\pi_{12} = \frac{1}{2} m_a g l_2 + m_b g l_3 - m_{ef} g l_5 \tag{10l}$$

The physical and geometric parameters I_{axx}, I_{ayy}, I_{azz}, I_{bxx}, I_{byy}, I_{bzz}, I_{cdxx}, I_{cdyy}, I_{cdzz}, I_{efxx}, I_{efyy}, I_{efzz}, I_{gyy}, m_a, m_b, m_{cd}, m_{ef}, l_1, l_2, l_3, l_4, l_5 can be found in the literature (Cavusoglu, 2001), while g is the acceleration gravity.

4. Inclusion of Frictional Effects

Friction effect in the joints of Premium 1.5 is neglected in Equation 7 and 8, as a matter of fact, the friction effect should not be ignored. Here I use Coulomb friction plus viscous damping (Armstrong, 1994 & Olsson, 1998) to include friction effect in the dynamic model of the device (Tahmasebi, 2005).

$$\boldsymbol{\tau}_f = \boldsymbol{\pi}_{fc}\,\mathrm{sgn}(\dot{\boldsymbol{\theta}}) + \boldsymbol{\pi}_{fv}\dot{\boldsymbol{\theta}} \tag{11}$$

or

$$\boldsymbol{\tau}_f = \begin{bmatrix} \tau_{f1} \\ \tau_{f2} \\ \tau_{f3} \end{bmatrix} = \begin{bmatrix} \tau_{fc1}\,\mathrm{sgn}(\dot{\theta}_1) + \tau_{fv1}\dot{\theta}_1 \\ \tau_{fc2}\,\mathrm{sgn}(\dot{\theta}_2) + \tau_{fv2}\dot{\theta}_2 \\ \tau_{fc3}\,\mathrm{sgn}(\dot{\theta}_3) + \tau_{fv3}\dot{\theta}_3 \end{bmatrix} = \boldsymbol{Y}_f \boldsymbol{\pi}_f \tag{12}$$

where

$$\boldsymbol{Y}_f = \begin{pmatrix} \mathrm{sgn}(\dot{\theta}_1) & 0 & 0 & \dot{\theta}_1 & 0 & 0 \\ 0 & \mathrm{sgn}(\dot{\theta}_2) & 0 & 0 & \dot{\theta}_2 & 0 \\ 0 & 0 & \mathrm{sgn}(\dot{\theta}_3) & 0 & 0 & \dot{\theta}_3 \end{pmatrix} \tag{13}$$

$$\boldsymbol{\pi}_f = \begin{bmatrix} \pi_{fc1} & \pi_{fc2} & \pi_{fc3} & \pi_{fv1} & \pi_{fv2} & \pi_{fv3} \end{bmatrix}^T \tag{14}$$

$\boldsymbol{Y}_f$ is the regressor matrix, $\boldsymbol{\pi}_f$ is 6 dimensional parameter vector.

Including the friction effect of the device and combing equation 8 with 12, I could write

$$\boldsymbol{\tau} = \boldsymbol{Y}\left(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \ddot{\boldsymbol{\theta}}\right)\boldsymbol{\pi} \tag{15}$$

where

$$\boldsymbol{Y} = \left[\boldsymbol{Y}_d \,\vdots\, \boldsymbol{Y}_f\right] \tag{16}$$

$$\boldsymbol{\pi} = \left[\boldsymbol{\pi}_d \,\vdots\, \boldsymbol{\pi}_f\right]^T \tag{17}$$

Equation 15 is named the dynamic model of Premium 1.5 with joint friction. Each of the element of the parameter vector is identified as shown in table 2.

5. Conclusion

In telerobotic systems, the kinematics especially the forward kinematics is of great importance for the control of teleoperaton, frictional effects is often neglected in dynamic equations. This paper mainly discusses the forward kinematics using D-H method, the manipulator Jacobian is also presented. And I derive the dynamic equations incorporating frictional effects in the joints.

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Table 1. D-H parameter of Premium 1.5

Frame	α_{i-1}	a_{i-1}	θ_i	d_i
0	$-\pi/2$	0	$-\pi/2$	0
1	0	$-l_1$	θ_1	l_2
2	$\pi/2$	0	θ_2	0
3	0	l_1	$-\pi/2-\theta_2+\theta_3$	0
tool	$-\pi/2$	l_2	$\pi/2$	0

Table 2. Dynamic Parameter of Premium 1.5($\times 10^{-4}$)

π_1	π_2	π_3	π_4	π_5	π_6
7.5784	0.3154	3.3572	12.0490	5.5095	0.2981
π_7	Π_8	π_9	π_{10}	π_{11}	π_{12}
11.8700	6.1644	0.8572	4.5693	-160	-739
π_{13}	π_{14}	π_{15}	π_{16}	π_{17}	π_{18}
707	251	248	-57	-35	-5

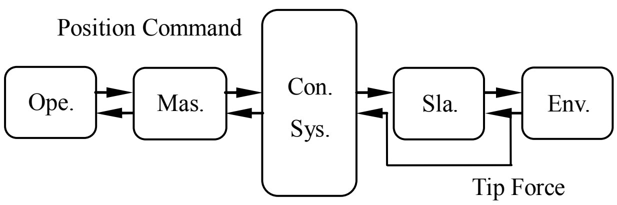

Figure 1. Diagram of the master-slave teleoperation system

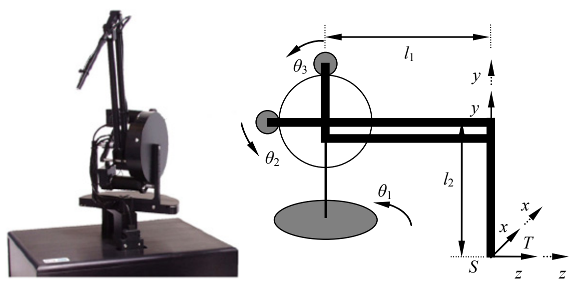

Figure 2. Photo and structural sketch of Premium1.5

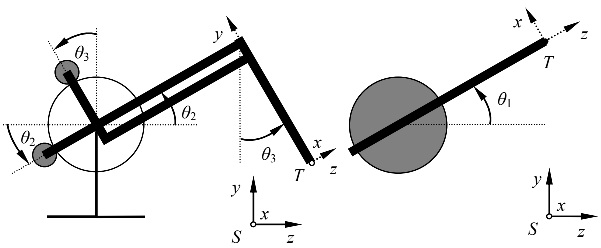

Figure 3. Configuration of Premium 1.5

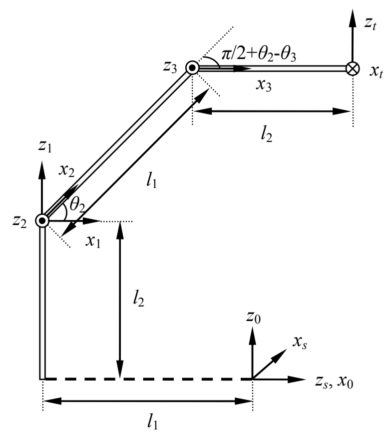

Figure 4. Reference frames of Premium1.5

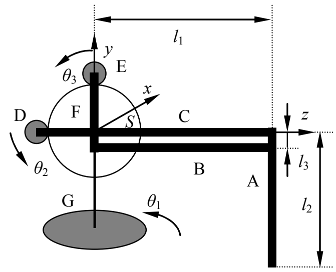

Figure 5. Segments of Premium 1.5