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Accurate Solution and Characteristics for Electromagnetic Wave Propagation in Time-varying Media

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Abstract

Characteristics of wave propagation in some time-varying media have been studied. Using the separation of variables and hybrid method, accurate solutions of electromagnetic waves are obtained when $\varepsilon(t)$ and $\sigma(t)$ are monotone and periodic functions with respect to time. Furthermore, the classes of $\varepsilon(t)$ and $\sigma(t)$ on which wave equation has analytical solution are studied and listed. In time-varying media conditions, some time-varying parameters such as frequency shift, change of phase velocity, wave impedance are also discussed. The examples and rationale for the results are given.

Keywords: Wave Propagation, Separation of Variables and Hybrid Methods, Time-varying Media

1. Introduction

The media whose parameters such as permittivity, conductivity, permeability vary with time are called the time-varying media. Time-varying media actually exist in nature, which can be found in ferroelectric and magnetoelastic materials. When an electric system operates in an environment around a nuclear expansion, it will receive irradiation of gamma-ray and results in an Internal Electromagnetic Pulse (IEMP) in an enclosed metal structure(Qiao, 2003). In an ionosphere, the electron density influenced by solar flares or strong laser pulse is also a function of time. Wave propagation in time-varying media is of considerable interest for applications involving modulation of microwave power, ionized plasmas, space aviation, etc.

Characteristics of electromagnetic wave propagation in time-varying media have been studied in some papers. Morgenthaler has considered the propagation of waves within a time-varying dielectric media(Morenthaler,1958). Felsen and Whitman have studied the propagation within a dispersive media (Felsen and. Whitman, 1970). Nerukh, Sewell and Benson have studied the electromagnetic fields in longitudinal uniform dielectric waveguides with media whose permittivity changes abruptly with time in their core(Nerukh and Sewell,2004). The methods used include transform technique(Ruiz and Kozaki, 1978), Volterra integral technique(Nerukh and Sewell,2004), numerical computation technique(Taylor, 1969), analytic technique(Zhang, 2005,Chen, 2006), and method of geometric optics approximation(.Fante, 1971), etc. As for the waves propagation in multi-time-varying parameters or various types of time-varying media, there is no literature availability in this area. In this article, using the separation of variables and hybrid technique, we study the solution of wave equation when relative permittivity $\varepsilon(t)$ and conductivity $\sigma(t)$ change gradually with respect to time, including monotone and periodic variations. We find that analytical solution can be obtained only for a few wave equations including $\varepsilon(t)$ and $\sigma(t)$, most of them can not be solved analytically. Therefore, the combined method due to separation of variables, and numerical computation technique (for the equation in time domain) is adopted in our study. According to our studies, the field distribution, frequency and phase velocity of electromagnetic waves are all functions of time when $\varepsilon(t)$ and $\sigma(t)$ both vary with respect to time. Because of the reason that the plane waves are the basic building blocks for all wave problems and are also good approximations to many actual problems, the emphasis is put on the plane waves in time-varying media, furthermore, some time-varying parameters of the plane waves are also discussed in this paper.

2. Problem and Formulation

Let us consider a homogeneous and uniform medium having dielectric constant $\varepsilon_0\varepsilon(t)$, magnetic permeability μ_0, and conductivity $\sigma(t)$. The functions of relative permittivity and conductivity are time varying i.e. $\varepsilon(t)=\exp(a_1 t)$ and $\sigma(t)=a_2\exp(a_1 t)$ (where a_1 is an arbitrary constant, a_2 is non-negative constant), respectively.

It is assumed that $\vec{E}$, $\vec{H}$ and $\vec{D}$ are electric field intensity, magnetic field intensity and electric flux density, respectively. There usually exists a relationship $\vec{D}(x,y,z,t)=\varepsilon_0\varepsilon(t)\vec{E}(x,y,z,t)$ between $\vec{D}$ and $\vec{E}$. Thus, in a source free region, Maxwell's curl equations in terms of the electric and magnetic field intensities $\vec{E}$ and $\vec{H}$ are:

$$\nabla\times\vec{E}=-\mu_0\frac{\partial}{\partial t}\vec{H}, \tag{1}$$

$$\nabla\times\vec{H}=\sigma(t)\vec{E}+\varepsilon_0\frac{\partial}{\partial t}\varepsilon(t)\vec{E}. \tag{2}$$

From (1), (2), the scalar equation of $\vec{H}$ is:

$$\nabla^2 H=\mu_0\sigma(t)\frac{\partial H}{\partial t}+\frac{\varepsilon'(t)}{c^2}\frac{\partial H}{\partial t}+\frac{\varepsilon(t)}{c^2}\frac{\partial^2 H}{\partial t^2}, \tag{3}$$

where $c=1/\sqrt{\mu_0\varepsilon_0}$, $\varepsilon'(t)$ is the derivative of $\varepsilon(t)$ with respect to t. For later part, the expression for other function has the same meaning. Here, attention will be paid to the time-varying media. No generality is lost, let $\partial H/\partial x=0$, $\partial H/\partial y=0$. The problem is reduced to one dimensional case (plane wave propagating in free space). It assumed that the initial conditions of H are:

$$H\big|_{t=0}=\varphi(z), \tag{3a}$$

$$H'\big|_{t=0}=\psi(z). \tag{3b}$$

Using separation of variables method and the initial conditions i.e. (3a) and (3b), (3) can be solved. Assuming $H(z,t)=H_1(z)H_2(t)$, substituting it into (3) and separating the variables, we obtain:

$$\frac{1}{H_1}\frac{d^2H_1}{dz^2}=\frac{1}{H_2}[\mu_0\sigma(t)\frac{dH_2}{dt}+\frac{\varepsilon'(t)}{c^2}\frac{dH_2}{dt}+\frac{\varepsilon(t)}{c^2}\frac{d^2H_2}{dt^2}]$$

The left side of this equation is a function of z only, and the right side is a function of t only. Since the variables z and t are independent of each other, the condition that holds the equation is to make the equation's both sides equal to a constant ($-\lambda^2$). We have:

$$\frac{d^2H_1}{dz^2}+\lambda^2H_1=0, \tag{4a}$$

$$\frac{d^2H_2}{dt^2}+(\mu_0c^2\frac{\sigma(t)}{\varepsilon(t)}+\frac{\varepsilon'(t)}{\varepsilon(t)})\frac{dH_2}{dt}+\frac{\lambda^2c^2}{\varepsilon(t)}H_2=0 \quad (4b)$$

where λ is an arbitrary constant which is independent of t and z. (4b) is the second order ordinary differential equation in time domain. Generating speaking, analytical solutions can be found only for few types of $\varepsilon(t)$ and $\sigma(t)$. Of course, numerical solutions can be obtained for most types of $\varepsilon(t)$ and $\sigma(t)$.(see below)

In order to obtain the analytical solution, taking $\varepsilon(t)=\exp(a_1t)$, $\sigma(t)=a_2\exp(a_1t)$, substituting them into Eq.(4b) and assuming: $b_1=\mu_0c^2a_2+a_1$, the solutions of Eq.(4a) and (4b) are respectively as follows:

$H_1(z)=C_1\exp(i\lambda z)$,

$H_2(t)=\exp(-b_1t/2)[AH^{(1)}(\nu,\frac{2|\lambda|c}{a_1}\exp(-a_1t/2))+BH^{(2)}(\nu,\frac{2|\lambda|c}{a_1}\exp(-a_1t/2))]$,

Where λ may be positive or negative, A and B are unknown coefficients, $H^{(1)}(\nu,x)$ and $H^{(2)}(\nu,x)$ are the first- and second-kind Hankel functions of order ν respectively, $\nu=b_1/a_1$ is usually non-integer.

Thus, the solution of (3) is:

$$H(z,t)=\exp(-\frac{b_1t}{2})\int_{-\infty}^{\infty}[AH^{(1)}(\nu,\frac{2|\lambda|c}{a_1}\exp(-a_1t/2))+BH^{(2)}(\nu,\frac{2|\lambda|c}{a_1}\exp(-a_1t/2))]\exp(i\lambda z)dz \cdot \quad (5)$$

Let $F_J=H^{(1)}(\nu,2|\lambda|c/a_1)$, $F_N=H^{(2)}(\nu,2|\lambda|c/a_1)$, $F_J^{'}=H^{(1)}{}'(\nu,2|\lambda|c/a_1)$, $F_N^{'}=H^{(2)}{}'(\nu,2|\lambda|c/a_1)$. Based on initial conditions, we obtain:

$$\int_{-\infty}^{\infty}[AF_J+BF_N]\exp(i\lambda z)d\lambda=\varphi(z), \quad (6a)$$

$$\int_{-\infty}^{\infty}[A(b_1F_J/2+|\lambda|cF_J^{'})+B(b_1F_N/2+|\lambda|cF_N^{'}]\exp(i\lambda z)d\lambda=-\psi(z) \cdot \quad (6b)$$

It is assumed that the Fourier transforms of $\varphi(z)$ and $\psi(z)$ are $\overline{\varphi}(\lambda)$ and $\overline{\psi}(\lambda)$, given below respectively:

$$\overline{\varphi}(\lambda)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\varphi(z)\exp(-i\lambda z)dz, \quad (6c)$$

$$\overline{\psi}(\lambda)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\psi(z)\exp(-i\lambda z)dz \cdot \quad (6d)$$

From Eq.(6a), (6b), (6c) and (6d), the unknown coefficients A and B can be determined as

$$A=\frac{\overline{\varphi}(b_1F_N/2+|\lambda|cF_N^{'})+\overline{\psi}F_N}{|\lambda|c(F_N^{'}F_J-F_J^{'}F_N)}, \quad (7a)$$

$$B=\frac{\overline{\varphi}(b_1F_J/2+|\lambda|cF_J^{'})+\overline{\psi}F_J}{|\lambda|c(F_N^{'}F_J-F_J^{'}F_N)} \cdot \quad (7b)$$

When $t<0$, it is assumed that a monochromatic plane wave propagates in a time-invariant lossless media, that is, $\varepsilon(t)=1$, $\sigma(t)=0$. The magnetic field intensity is given by $\vec{H}_0(z,t)=\hat{y}\cos(\omega_0t-k_0z)/\eta_0$ (where $k_0=\omega_0/c$, $\eta_0=\sqrt{\mu_0/\varepsilon_0}$), which is a well known expression. At the moment of $t=0$, the whole media begin to vary in terms of: $\varepsilon(t)=\exp(a_1t)$ and $\sigma(t)=a_2\exp(a_1t)$. Thus, the initial conditions are:

$$\varphi(z)=\cos(k_0z)/\eta_0, \qquad \psi(z,t)=\omega_0\sin(k_0z)/\eta_0 \cdot$$

The Fourier transform of $\varphi(z)$ and $\psi(z)$ are:

$$\overline{\varphi}(\lambda)=\frac{1}{2\pi\eta_0}\int_{-\infty}^{\infty}\cos(k_0z)\exp(-i\lambda z)dz=\frac{1}{2\eta_0}[\delta(\lambda-k_0)+\delta(\lambda+k_0)], \quad (8a)$$

$$\overline{\psi}(\lambda)=\frac{\omega_0}{2\pi\eta_0}\int_{-\infty}^{\infty}\sin(k_0z)\exp(-i\lambda z)dz=\frac{\omega_0}{2i\eta_0}[\delta(\lambda-k_0)-\delta(\lambda+k_0)] \quad (8b)$$

Where $\delta(x)$ represents the Dirac delta function. Substituting Eq.(7a), (7b), (8a) and (8b) into Eq.(5), we obtain the analytical solution of the field as follows:

$$H(z,t)=\exp(-b_1t/2)\frac{H^{(1)}(\nu,2\omega_0\exp(-a_1t/2)/a_1)}{\eta_0\omega_0(F'_{N0}F_{J0}-F'_{J0}F_{N0})}[(b_1F_{N0}/2+\omega_0F'_{N0})\cos(k_0z)+\omega_0F_{N0}\sin(k_0z)]$$

$$-\exp(-b_1t/2)\frac{H^{(2)}(\nu,2\omega_0\exp(-a_1t/2)/a_1)}{\eta_0\omega_0(F'_{N0}F_{J0}-F'_{J0}F_{N0})}\times[(b_1F_{J0}/2+\omega_0F'_{J0})\cos(k_0z)+\omega_0F_{J0}\sin(k_0z)], \quad (9)$$

where $F_{J0}=H^{(1)}(\nu,2\omega_0/a_1)$, $F_{N0}=H^{(2)}(\nu,2\omega_0/a_1)$, $F'_{J0}=H^{(1)}{}'(\nu,2\omega_0/a_1)$, $F'_{N0}=H^{(2)}{}'(\nu,2\omega_0/a_1)$.

It is necessary to point out that the first part of (9) represents positive-traveling wave and the second part of (9) represents negative-traveling wave. As it is well known that function $H^{(1)}(\nu,x)$ is usually analogous to $\exp(ix)$ and $H^{(2)}(\nu,x)$ is usually analogous to $\exp(-ix)$ in the case of larger variable.

The vector form of the field is:

$$\vec{H}(z,t)=\hat{y}H(z,t). \quad (9a)$$

Similarly, the time domain expression for $\vec{E}$ in time-varying media is

$$E(z,t)=\exp(-(a_1+b_1)t/2)\frac{H^{(1)}(\nu-1,2\omega_0\exp(-a_1t/2)/a_1)}{\omega_0(F'_{N1}F_{J1}-F'_{J1}F_{N1})}[((a_1+b_1)F_{N1}/2$$

$$+\omega_0F'_{N1})\cos(k_0z)+\omega_0F_{N1}\sin(k_0z)]-\exp(-(a_1+b_1)t/2)\times\frac{H^{(2)}(\nu-1,2\omega_0\exp(-a_1t/2)/a_1)}{\omega_0(F'_{N1}F_{J1}-F'_{J1}F_{N1})}$$

$$\times[((a_1+b_1)F_{J1}/2+\omega_0F'_{J1})\cos(k_0z)+\omega_0F_{J1}\sin(k_0z)], \quad (10)$$

where $F_{J1}=H^{(1)}(\nu-1,2\omega_0/a_1)$, $F_{N1}=H^{(2)}(\nu-1,2\omega_0/a_1)$, $F'_{J1}=H^{(1)}{}'(\nu-1,2\omega_0/a_1)$, $F'_{N1}=H^{(2)}{}'(\nu-1,2\omega_0/a_1)$.

$$\vec{E}(z,t)=\hat{x}E(z,t). \quad (10a)$$

3. Rationality of the Solution

The rationality of (9) and (10) can be verified indirectly by degenerating time-varying media into time invariant media. If a_1 and a_2 approach to zero, the time-varying media approach time-invariant media. When the parameter a_1 is very small, then $1/a_1$ will be very large accordingly. When x is very large i.e. in the range of $x\geq 50$, it has the following approximation formula

$$H^{(1)}(\nu,x)\approx\sqrt{\frac{2}{\pi x}}\exp(i(x-\nu\pi/2-\pi/4)),\ H^{(2)}(\nu,x)\approx\sqrt{\frac{2}{\pi x}}\exp(-i(x-\nu\pi/2-\pi/4)).$$

Substituting the above formula into (9) and after simplification it yields:

$$H(z,t)\approx\frac{G(t)}{2\eta_0}i\left[((\frac{b_1}{2\omega_0}+\frac{a_1}{2\omega_0^2}\nu-i)\cos(k_0z)+\sin(k_0z))\exp[i\frac{2\omega_0}{a_1}(\exp(-\frac{a_1}{2}t)-1)]\right.$$
$$\left.-((\frac{b_1}{2\omega_0}+\frac{a_1}{2\omega_0^2}\nu+i)\cos(k_0z)+\sin(k_0z))\exp[i\frac{2\omega_0}{a_1}(1-\exp(-\frac{a_1}{2}t))]\right] \quad (11a)$$

where $G(t)=\exp(-b_1t/2+a_1t/4)$. Substituting it into Eq.(9) and assuming that a_1 and a_2 approaches zero, yielding:

$$\lim_{a_1\to 0,a_2\to 0}\vec{H}(z,t)=\hat{y}\cos(\omega_0t-k_0z)/\eta_0.$$

The above result is the wave's equation in time-invariant media. (10) can also be verified by the same method.

Further, the validity of (9) can also be verified by the result from finite-difference time-domain (FDTD) simulation. The comparison between them is depicted in Fig.1, from the figure, we can affirm (9) is valid.

Besides $\varepsilon(t)$ and $\sigma(t)$ stated above, the other classes of $\varepsilon(t)$ and $\sigma(t)$ on which the analytical solutions can be obtained and the corresponding analytical solution functions are partly listed in Table 1.

For the sake of brevity we have not provided the detailed solutions of them one by one.

Wave equations for most of time-varying $\varepsilon(t)$ and $\sigma(t)$ can not be solved analytically. For example, when $\varepsilon(t)=\exp(a_1t)$ and $\sigma(t)=a_2\exp(a_3t)$ (where a_1, a_3 are arbitrary constants, $a_2\geq 0$ and $a_1\neq a_3$), the analytical solution of Eq.(3) could not be obtained. Here, the assistance of hybrid technique is taken i.e. separation of variables combined with numerical method. As mentioned above, the solution of (4b) can be obtained from Runge-Kutta's iteration method. Combining it with the solution of differential equation (4a), we can finally obtain the complete solution of the fields. The calculation show that the numerical solution has better accuracy. The comparison between analytical solution and numerical solution for an identical equation is shown in Fig.1. If the time step length is kept much less than that of the total time range considered, for example, the ratio being 1:15000, calculated results show that the relative error between them is less than 1%. The hybrid method can be used to solve wave equation with various time-varying media cases, including both analytical solution case and non-analytical solution case.

Using analytical and numerical technique, the field distribution with time can be obtained. From them, time-varying parameters, such as the frequency, phase velocity and the impedance of the field can be deduced. These parameters are partly listed in table 2 as follows:

4. Example

As an example, take the initial frequency $f_0 = 2.5\times10^9$ Hz, $z_0 = 1.5$ m, $a_1 = 2.0\times10^8$, $a_2 = 0.003$. The results for magnetic field intensities from analytical method and numerical method(Runge-Kutta's iteration method) are shown in Fig.2 when $\varepsilon(t) = \exp(a_1 t)$, $\sigma(t) = a_2\exp(a_1 t)$. Comparing the results from analytical method with that of numerical method, our results are justified.

The result for another example in which magnetic field intensity is from only numerical method i.e. no analytical solution case as shown in Fig.3, when $\varepsilon(t) = \exp(a_1 t)$, $\sigma(t) = a_2\exp(a_3 t)$, $a_1 = 2.0\times10^8$, a_2 =0.003, and $a_3 = 0.6$.

Figures 2 and 3 show that plane wave in time-varying media is not a single oscillatory function with constant amplitude and periodicity, its amplitude and frequency change with time, when the relative permittivity and conductivity vary with time as exponential function. When they increase with time, the frequency, wave impedance, phase velocity and amplitude based on definitions stated above decrease with time as shown in Fig.4. When they decrease with time, we can also obtain available results by the same method.

Next, let us analyze periodic time-varying media. We consider $\varepsilon(t) = b_0 + b_1\cos(b_2 t)$, $\sigma(t) = b_3 + b_4\cos(b_5 t)$ ($b_0 = 2$, $b_1 = 1$, $b_2 = 2.5\times10^8$, $b_3 = 0.004$, $b_4 = 0.0034$, $b_5 = 2.0\times10^8$) and the corresponding results are shown in Fig.5.

Figure.5 (a) depicts the change of magnetic field intensity with respect to time. Because of the reason that the media change with respect to time periodically, the change of magnetic field intensity with time is much different from that in Fig.1 i.e. its waveform is not 'monotone', but 'periodic'. Similarly, from Fig.5 (b),(c),(d), the changes of frequency, phase velocity, and wave impedance with respect to time have 'periodical' characteristics that are much different from that in Fig. 4

Finally, let us simulate (9) by the following expression:

$$H_y(z,t) = |A(t)|_{loc\max}\cos(\theta(t) - \theta_0) \tag{12}$$

Where $|A|_{loc\max}$ is time-varying amplitude, $\theta(t)$ is time-varying total phase, θ_0 is the initial phase . The two results are shown in Fig.6.

Figure 6 shows the result from (9) is in good agreement with that from (12). Therefore, we can conclude that the solution of the field and the definitions of time-varying parameters are reasonable.

5. Conclusion

Using separation of variables, the accurate solutions of a plane electromagnetic wave have been obtained when the time-varying parameters $\varepsilon(t)$ and $\sigma(t)$ vary with time as monotone functions. Furthermore, the types of time-varying media in which the wave equation can be analytically solved are studied as well as listed. Next, the hybrid method of separation of variables and numerical method has also been introduced, which can be utilized to solve wave equation with various kinds of time-varying media. Then, some time-varying parameters, such as frequency, phase velocity, amplitude, wave impedance, etc, are defined and for some kinds of time-varying media in which the wave equation has analytical solution, the approximate analytical expressions of time-varying parameters are also listed. Moreover, the rationality of solution for wave equation in time-varying media and the time-varying parameters defined are verified based on appropriately derivative process and results from references.

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Table 1. Main expressions for analytical solution and its parameters

Parameter expressions	Main components of field solution
$\varepsilon(t) = a_0 \exp(a_1 t)$, $\sigma(t) = 0$	$J(1,t)$, $N(1,t)$, $\exp(i\lambda z)$
$\varepsilon(t) = a_0$, $\sigma(t) = b_0 \exp(b_1 t)$	$I(\nu_1,t)$, $K(\nu_1,t)$, $\exp(i\lambda z)$
$\varepsilon(t) = a_0 + a_1 t$, $\sigma(t) = 0$	$J(\nu_2,t)$, $N(\nu_2,t)$, $\exp(i\lambda z)$
$\varepsilon(t) = a_0 + a_1 t$, $\sigma(t) = b_0 + b_1 t$	$KU(\lambda,\nu_3,t)$, $KM(\lambda,\nu_3,t)$, $\exp(i\lambda z)$
$\varepsilon(t) = a_0$, $\sigma(t) = b_0 / t$	$J(\nu_4,t)$, $N(\nu_4,t)$, $\exp(i\lambda z)$

Note: 1. a_0 , a_1 , b_0 , b_1 are constants.

2. $J(1,t)$, $N(1,t)$ are the first- and second-kind Bessel functions of first order respectively.

3. $I(\nu_1,t)$ and $K(\nu_1,t)$ are the first- and second-kind modified Bessel functions of ν_1 order , respectively.

4. $J(\nu_i,t)$, $N(\nu_i,t)$, i = 2 , 4 , are the first- and second-kind Bessel functions of ν_i order respectively, ν_i is non-integral number.

5. KM and KU are the first- and second- Kummer's functions, ν_3 is non-integral number.

Table 2. Time-varying parameters and its approximate conditions (from analytical methods)

Parameter expressions		Approximate conditions	Time-varying parameters		
$\varepsilon(t)$	$\sigma(t)$		$f(t)$	$V_p(t)$	$Z(t)$
$\exp(a_1 t)$	$a_2\exp(a_1 t)$	$\omega_0/a_1 > 25$	$f_0\exp(-a_1 t/2)$	$\omega_0\exp(-a_1 t/2)/k_0$	$\eta_0\exp(-a_1 t/2)$
$a_0\exp(a_1 t)$	0	$a_0=1, \omega_0/a_1 > 25$	$f_0\exp(-a_1 t/2)$	$\omega_0\exp(-a_1 t/2)/k_0$	$\eta_0\exp(-a_1 t/2)$
$a_0+a_1 t$	0	$a_0=1, \omega_0/a_1 > 25$	$f_0/\sqrt{a_0+a_1 t}$	$\omega_0/k_0\sqrt{a_0+a_1 t}$	$\eta_0/\sqrt{a_0+a_1 t}$
a_0	b_0/t	$a_0=1$	f_0	ω_0/k_0	η_0
$a_0+a_1 t$	b_0	$a_0=1, \omega_0/a_1 > 25$	$f_0/\sqrt{a_0+a_1 t}$	$\omega_0/k_0\sqrt{a_0+a_1 t}$	$\eta_0/\sqrt{a_0+a_1 t}$

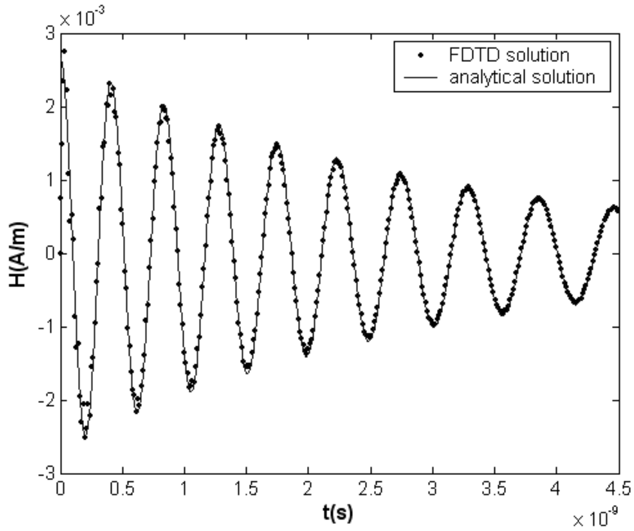


Figure 1. The comparison of magnetic field by analytical method and FDTD method
when $\varepsilon(t)=\exp(a_1 t), \sigma(t)=a_2\exp(a_1 t)$ ($a_1 = 2.0\times 10^8$, $a_2=0.003$)

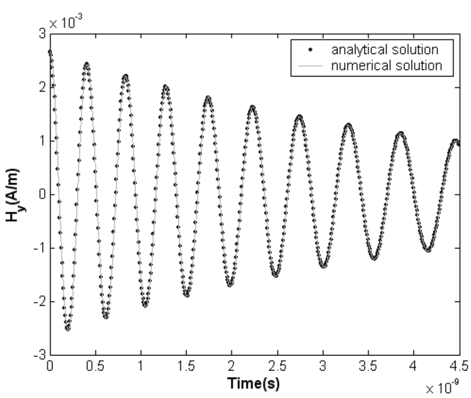


Figure 2. The magnetic field intensity with time, by analytical method (dotted curve)
and numerical method (solid curve) when $\varepsilon(t)=\exp(a_1 t)$, $\sigma(t)=a_2\exp(a_1 t)$ ($a_1 = 2.0\times 10^8$, $a_2=0.003$)

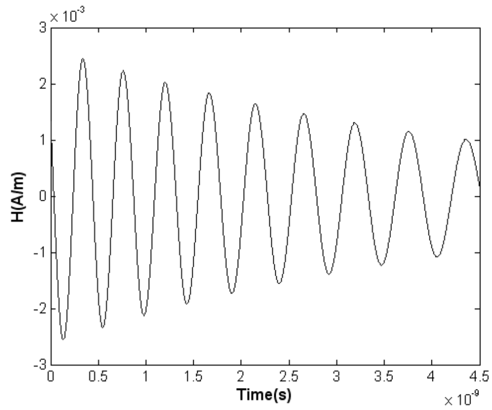
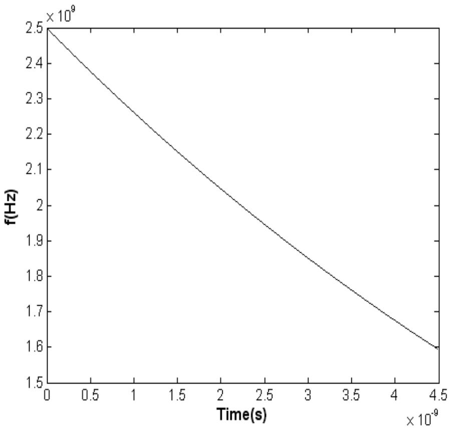
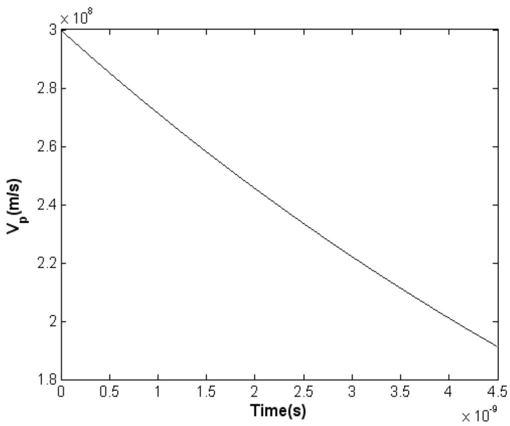


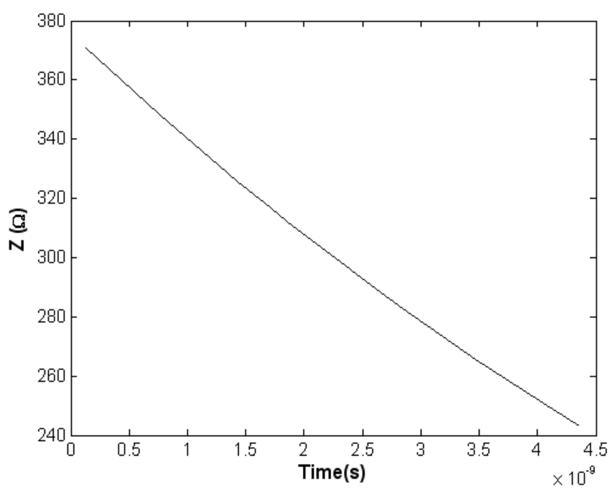
Figure 3. The magnetic field intensity with time, by numerical method when $\varepsilon(t) = \exp(a_1 t)$, $\sigma(t) = a_2 \exp(a_3 t)$ ($a_1 = 2.0 \times 10^8$, $a_2 = 0.003$, $a_3 = 0.6$).



(a) $f(t)$

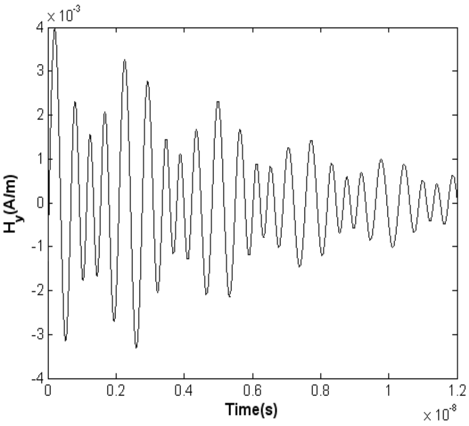


(b) $V_p(t)$

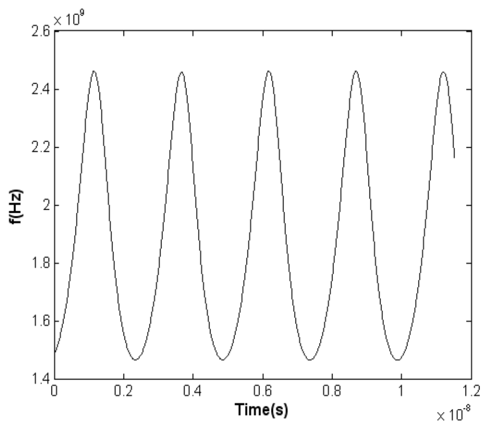


(c) $Z(t)$

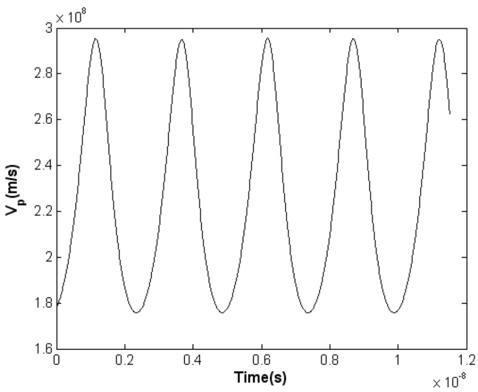
Figure 4. Changes of frequency, phase velocity and impedance with time when $\varepsilon(t) = \exp(a_1 t)$, $\sigma(t) = a_2 \exp(a_1 t)$ ($a_1 = 2.0 \times 10^8$, $a_2 = 0.003$)



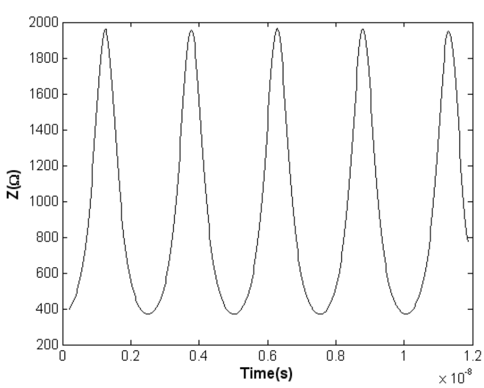
(a) H_y



(b) $f(t)$



(c) $V_p(t)$



(d) $Z(t)$

Figure 5. Changes of magnetic field intensity and its frequency, phase velocity, amplitude and wave impedance, respectively when $\varepsilon(t) = b_0 + b_1 \cos(b_2 t)$, $\sigma(t) = b_3 + b_4 \cos(b_5 t)$ ($b_0 = 2$, $b_1 = 1$, $b_2 = 2.5 \times 10^8$, $b_3 = 0.004$, $b_4 = 0.0034$, $b_5 = 2.0 \times 10^8$).

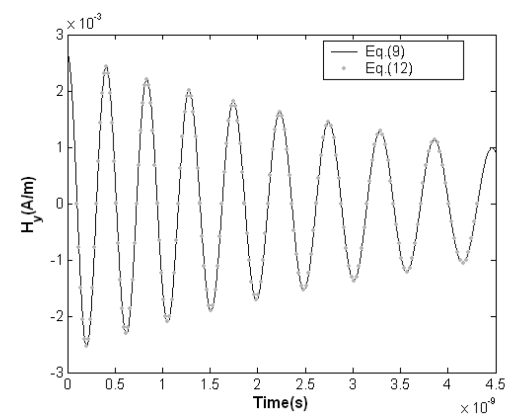


Figure 6. Comparison between analytical solution of magnetic field (dotted curve) and its simple expression $H_y(z,t) = |A(t)|_{loc\max} \cos(\theta(t) - \theta_0)$ (solid curve) when $\varepsilon(t) = \exp(a_1 t)$, $\sigma(t) = a_2 \exp(a_1 t)$ ($a_1 = 2.0$, $a_2 = 0.003$)